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Homework 6 - Q7 to Q10

Question 7:

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a) Exercise 6.1.5, sections b-d

b.

$$p(E) = \frac{|E|}{|S|}$$

$|E| = \binom{13}{1}*\binom{4}{3}*\binom{12}{2}*\binom{4}{1}*\binom{4}{1} = 54318$

$choose\ one\ rank\ from\ 13\ ranks * choose\ three\ same\ rank\ from\ 4\ different\ suits * other\ than\ the\ 4\ same\ rank\ card,\ we\ can\ choose\ left\ 1\ card\ from\ 48\ cards * since\ the\ two\ cards\ left\ cannot\ have\ the\ same\ rank\ as\ the\ three\ same\ rank\ cards\ and\ the\ other\ chosen\ cards,\ we\ only\ have\ 44\ cards\ left\ for\ choosing.$

$|S| = \binom{52}{5} = 2598960$

$$\therefore p(E) = \frac{|E|}{|S|} = \frac{13*4*48*44}{2598960} = 0.021$$

c.

$$p(E) = \frac{|E|}{|S|}$$

$for\ all\ five\ cards\ have\ the\ same\ suit,\ we\ know\ that\ in\ the\ deck,\ there\ are\ 13\ different\ cards\ of\ the\ same\ suit.$

$|E| = \binom{13}{5}*\binom{4}{1} = 5148$

$|S| = \binom{52}{5} = 2598960$

$$p(E) = \frac{|E|}{|S|} = \frac{5148}{2598960} = 0.00198$$

d.

$$p(E) = \frac{|E|}{|S|}$$

$|E| = \binom{13}{1} * \binom{4}{2} * \binom{12}{3} * \binom{4}{1} * \binom{4}{1} * \binom{4}{1}$

$|S| = \binom{52}{5} = 2598960$

$$p(E) = \frac{|E|}{|S|} = 0.423$$

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b) Exercise 6.2.4, sections a-d

a. The hand has at least one club.

$$P(E) = 1 - \frac{|\overline{E}|}{|S|}$$

$|\overline{E}| = \textit{the hand that there is no club} = \binom{39}{5} = 575757$

$|S| = \binom{52}{5} = 2598960$

$$P(E) = 1 - \frac{|\overline{E}|}{|S|} = 1 - 0.222 = 0.778$$

b.The hand has at least two cards with the same rank.

$$P(E) = 1 - \frac{|\overline{E}|}{|S|}$$

$|\overline{E}| = \textit{the hand that only one cards with the same rank} = \binom{13}{5} * 4^5 = 1317888$

$|S| = \binom{52}{5} = 2598960$

$$P(E) = 1 - \frac{|\overline{E}|}{|S|} = 1 - 0.507 = 0.493$$

c. The hand has exactly one club or exactly one spade.

$$P(\textit{exactly one club}) = \frac{|E|}{|S|} = \frac{\binom{13}{1} * \binom{39}{4}}{\binom{52}{5}} = \frac{1069263}{2598960}$$

$$P(\textit{exactly one space}) = \frac{|E|}{|S|} = \frac{\binom{13}{1} * \binom{39}{4}}{\binom{52}{5}} = \frac{1069263}{2598960}$$

since p(E1 U E2) = p(E1) + p(E2) − p(E1 ∩ E2)

$$P(\textit{exactly one club and exactly one space}) = \frac{|E|}{|S|} = \frac{\binom{13}{1} * \binom{13}{1} * \binom{26}{3}}{\binom{52}{5}} = \frac{439400}{2598960}$$

P(The hand has exactly one club or exactly one spade) = $\frac{1069263}{2598960}+\frac{1069263}{2598960}-\frac{439400}{2598960}$=0.654

d. The hand has at least one club or at least one spade.

P(at least one club or at least one spade) = 1 - P(there is no club and no spade)

$$P(\textit{there is no club and no spade}) = \frac{\binom{26}{5}}{\binom{52}{5}} = 0.023$$

$$P(\textit{at least one club or at least one spade}) = 1 - P(\textit{there is no club and no spade}) = 0.977$$

Question 8:

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a) Exercise 6.3.2, sections a-e

a.

$$P(A) = \frac{|A|}{|S|} = \frac{6!}{7!} = 0.143$$

$$P(B) = \frac{|B|}{|S|} = \frac{\left(\frac{7!}{2!}\right)}{7!} = 1/2 = 0.5$$

$$P(C) = \frac{|C|}{|S|} = \frac{5!}{7!} = 0.024$$

b.

$$P(A|C) = \frac{P(A \cap C)}{P(C)} = \frac{\frac{3!+3!}{7!}}{\frac{5!}{7!}} = 0.1$$

c.

$$P(B|C) = \frac{P(B \cap C)}{P(C)} = \frac{\frac{0.0024}{2!}}{\frac{5!}{7!}} = 0.5$$

d.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{5!*3}{7!}}{\frac{5!}{7!}} = \frac{0.0714}{0.5} = 0.143$$

e.

Event A and B are independent event since P(A|B)=P(A)=0.143

Event B and C are independent event since P(B|C)=P(B)=0.5

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b) Exercise 6.3.6, sections b, c

b.

$$\left(\frac{1}{3}\right)^5 * \left(\frac{2}{3}\right)^5$$

c.

$$\left(\frac{1}{3}\right) * \left(\frac{2}{3}\right)^9$$

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c) Exercise 6.4.2, section a

a.

Assume event A represent that the die is a fair die, P(A)=$P(\overline{A}) = 0.5$
Assume event B represent that the number rolled after six times are 4, 3, 6, 6, 5, 5

we need to know what is the probability of the fair die given the result, ie. P(A|B)

Based on the Bayes' theorem:

$$P(A|B) = \frac{P(B|A)\ P(A)}{P(B|A)\ P(A) + P(B|\overline{A})\ P(\overline{A})}$$

$$P(B|A) = \left(\frac{1}{6}\right)^6$$

$$P(B|\overline{A}) = (0.15)^4 * (0.25)^2$$

$$P(A) = P(\overline{A}) = 0.5$$

$$\therefore P(A|B) = \frac{P(B|A)\ P(A)}{P(B|A)\ P(A) + P(B|\overline{A})\ P(\overline{A})}$$

$$= \frac{\left(\frac{1}{6}\right)^5 * 0.5}{\left(\frac{1}{6}\right)^5 * 0.5 + (0.15)^4 * (0.25)^2 * 0.5} \approx 0.404$$

Question 9:

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a) Exercise 6.5.2, sections a, b

a.

the range of A is {0, 1, 2, 3, 4}

b.

the distribution of the random variable A is:

$$\left(0, \frac{\binom{48}{5}}{\binom{52}{5}}\right), \left(1, \frac{\binom{4}{1}\binom{48}{4}}{\binom{52}{5}}\right), \left(2, \frac{\binom{4}{2}\binom{48}{3}}{\binom{52}{5}}\right), \left(3, \frac{\binom{4}{3}\binom{48}{2}}{\binom{52}{5}}\right), \left(4, \frac{\binom{4}{4}\binom{48}{1}}{\binom{52}{5}}\right)$$

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b) Exercise 6.6.1, section a

a.

E[G] = 1 * P(one girls chosen) + 2 * P(two girls chosen) + 0 * P(zero girls chose)

$$= 1 * \left(\frac{\binom{7}{1}\binom{3}{1}}{\binom{10}{2}}\right) + 2 * \left(\frac{\binom{7}{2}}{\binom{10}{2}}\right) = \frac{7}{5}$$

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c) Exercise 6.6.4, sections a, b

a.

$$E[X] = 1*\left(\frac{1}{6}\right) + 4*\left(\frac{1}{6}\right) + 9*\left(\frac{1}{6}\right) + 16*\left(\frac{1}{6}\right) + 25*\left(\frac{1}{6}\right) + 36*\left(\frac{1}{6}\right) = 15.167$$

b.

$$E[Y] = 0 * \left(\frac{1}{8}\right) + 1 * \left(\frac{3}{8}\right) + 4 * \left(\frac{3}{8}\right) + 9 * \left(\frac{1}{8}\right) = 3$$

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d) Exercise 6.7.4, section a

a.

Assume A be the random variable denoting the number of children who get his or her own coat, and

$A_i = 1$ *denoting that the* $i-th$ *child get his or her own coat*, $A_i = 0$ *denoting that the* $i-th$ *child does not get his or her own coat.*

$$E[A_i] = 1 * \frac{9!}{10!} = \frac{1}{10}$$

$$E[A] = E[A_1] + E[A_2] + \dots + E[A_{10}] = 10*\frac{1}{10} = 1$$

Question 10:

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a) Exercise 6.8.1, sections a-d

a.

based on Burnolli trial probabilities

$\binom{100}{2} * F^2 * S^{98} = \binom{100}{2} * (0.01)^2 * (0.99)^{98}$

b.

P(at least two defect) = 1 - P(only one defect) - P(no defect)

$= 1 - \binom{100}{1} * (0.001) * (0.99)^{99} - 0.99^{100}$

$= 1 - 0.99^{99} - 0.99^{100}$

c.

E[X] = 100 * 0.01 =1

d.

$1 - \binom{50}{0} * (0.01)^0 * (0.99)^{50} = 1 - (0.99)^{50}$

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b) Exercise 6.8.3, section b

b.

the incorrect conclusion is reached if there are more than three heads, the probability p is

$1 - \left((0.7)^{10} + 10 * 0.3 * 0.7^9 + \binom{10}{2} * (0.3)^2 * (0.7)^8 + \binom{10}{3} * (0.3)^3 * (0.7)^7\right)$