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Homework 3 - Q7 to Q11

Question 7:

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a) Exercise 3.1.1, sections a-g

1.a $27 \in A$: True

1.b $27 \in B$: Flase

1.c $100 \in B$: True

1.d $since\ E \subseteq C\ is\ false,\ C \subseteq E\ is\ false,\ E \subseteq C\ or\ C \subseteq E\ is\ False.$

1.e $E \subseteq A$: True

1.f $A \subset E$: False

1.g $E \in A$: False

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b) Exercise 3.1.2, sections a-e

2.a $15 \subset A$ False

2.b $\{15\} \subset A$ True

2.c $\varnothing \subset A$ True

2.d $A \subseteq A$ True

2.e $\varnothing \in B$ False

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c) Exercise 3.1.5, sections b, d

5.b $let\ A = \{3, 6, 9, 12... \}$
$A = \{x \in Z:\ x\ is\ an\ integer\ multiply\ of\ 3\ and\ x \geq 3\}$

the set is infinite.

5.d $let\ B = \{0,\ 10,\ 20,\ ...,\ 1000\}$
$B = \{x \in Z:\ x\ is\ an\ integer\ multiply\ of\ 10\ and\ 0 \leq x \leq 1000\}$
this set is finite, $|B| = 101$

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d) Exercise 3.2.1, sections a-k

1.a $2 \in X$ True

1.b $\{2\} \subseteq X$ True

1.c $\{2\} \in X$ False

1.d $3 \in X$ False

1.e $\{1, 2\} \in X$ True

1.f $\{1, 2\} \subseteq X$ True

1.g $\{2, 4\} \subseteq X$ True

1.h $\{2, 4\} \in X$ False

1.i $\{2, 3\} \subseteq X$ False

1.j $\{2, 3\} \in X$ False

1.k |X| = 7 False

Question 8:

Exercise 3.2.4, section b

4.b since A={1,2,3} , $P(A) = \{\emptyset, \{1\},\ \{2\},\ \{3\},\ \{1,2\},\ \{1,3\},\ \{2,3\},\ \{1,2,3\}\}$
$let\ B = \{X \in P(A):\ 2 \in X\}$
$B = \{\{2\},\ \{1,2\},\ \{2,3\},\ \{1,2,3\}\}$

Question 9:

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a) Exercise 3.3.1, sections c-e

1.c $A \cap C = \{-3,\ 1,\ 17\}$

1.d $A \cup (B \cap C) = A \cup (\{-5,\ 1\}) = \{-3,\ -5,\ 0,\ 1,\ 4,\ 17\}$

1.e $A \cap B \cap C = \{1,\ 4\} \cap C = \{1\}$

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b) Exercise 3.3.3, sections a, b, e, f

3.a $\cap_{i=2}^{5} A_i = A_2 \cap A_3 \cap A_4 \cap A_5$

$A_2 = \{1,\ 2,\ 4\}$
$A_3 = \{1,\ 3,\ 9\}$
$A_4 = \{1,\ 4,\ 16\}$
$A_5 = \{1,\ 5,\ 25\}$
$\therefore\ \cap_{i=2}^{5} A_i = A_2 \cap A_3 \cap A_4 \cap A_5 = \{1\}$

3.b $\cup_{i=2}^{5} A_i = A_2 \cup A_3 \cup A_4 \cup A_5$

$A_2 = \{1,\ 2,\ 4\}$
$A_3 = \{1,\ 3,\ 9\}$
$A_4 = \{1,\ 4,\ 16\}$
$A_5 = \{1,\ 5,\ 25\}$
$\therefore\ \cup_{i=2}^{5} A_i = A_2 \cup A_3 \cup A_4 \cup A_5 = \{1, 2, 3, 4, 5, 9, 16, 25\}$

3.e $\cap_{i=1}^{100} C_i = C_1 \cap C_2 \cap C_3 .... \cap C_{100}$

$C_1 = \{X \in R:\ -1/1 \leq x \leq 1/1\} = \{X \in R: -1 \leq x \leq 1\}$
$C_2 = \{X \in R:\ -1/2 \leq x \leq 1/2\}$
$C_3 = \{X \in R:\ -1/3 \leq x \leq 1/3\}$
.
.
.
$C_{100} = \{X \in R:\ -1/100 \leq x \leq 1/100\}$
$\cap_{i=1}^{100} C_i = C_1 \cap C_2 \cap C_3 .... \cap C_{100} = \{x \in R:\ -1/100 \leq x \leq 1/100\}$
$\because\ x\ in\ the\ range\ -1/100 \leq x \leq 1/100\ is\ part\ of\ all\ C_1 \cap C_2 \cap C_3 .... \cap C_{100}$

3.f $\cup_{i=1}^{100} C_i = C_1 \cup C_2 \cup C_3 .... \cup C_{100}$

$C_1 = \{X \in R: \ -1/1 \leq x \leq 1/1\} = \{X \in R: -1 \leq x \leq 1\}$

$C_2 = \{X \in R: \ -1/2 \leq x \leq 1/2\}$

$C_3 = \{X \in R: \ -1/3 \leq x \leq 1/3\}$

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$C_{100} = \{X \in R: \ -1/100 \leq x \leq 1/100\}$

$\cup_{i=1}^{100} C_i = C_1 \cup C_2 \cup C_3 .... \cup C_{100} = \{x \in R: \ -1 \leq x \leq 1\}$

$\because$ *x in the range* $-1 \leq x \leq 1$ *covers all elements in* $C_1 \cup C_2 \cup C_3 .... \cup C_{100}$

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c) Exercise 3.3.4, sections b, d

4.b $P(A \cup B)$

$A \cup B = \{a, b, c\}$

$\therefore P(A \cup B) = \{\varnothing, \{a\}, \ \{b\}, \ \{c\}, \ \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$

4.d $P(A) \cup P(B)$

$P(A) = \{\varnothing, \{a\}, \{b\}, \{a, b\}\}$

$P(B) = \{\varnothing, \{c\}, \{b\}, \{c, b\}\}$

$P(A) \cup P(B) = \{\varnothing, \{a\}, \{b\}, \{a, b\}, \{c\}, \{c, b\}\}$

Question 10:

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a) Exercise 3.5.1, sections b, c

1.b $B \times A \times C = (foam,\ tall,\ non-fat)$

1.c $B \times C = \{(b,c) \mid b \in B \wedge c \in C\}$

$= \{(foam,\ non-fat),\ (foam,\ whole), (no-foam, non-fat), (no-foam, whole)\}$

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b) Exercise 3.5.3, sections b, c, e

3.b $Z^2 \subseteq R^2 \quad True$

3.c $Z^2 \cap Z^3 = \varnothing \quad True$

3.e $prove\ that\ A \subseteq B \rightarrow A \times C \subseteq B \times C$

$A \times C = \{(a,c):\ a \in A \wedge c \in C\}$

$B \times C = \{(b,c):\ b \in B \wedge c \in C\}$

$\because A \subseteq B \therefore a \in A\ and\ a \in B$

$\therefore A \times C \subseteq B \times C\ is\ True$

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c) Exercise 3.5.6, sections d, e

6.d $x \in \{0\} \cup \{0\}^2$

$\{0\}^2 = \{0\} \times \{0\} = \{00\}$

$\{0\} \cup \{0\}^2 = \{0,\ 00\}$

$x \in \{0,\ 00\}$

$y \in \{1\} \cup \{1\}^2$

$\{1\}^2 = \{1\} \times \{1\} = \{11\}$

$\{1\} \cup \{1\}^2 = \{1,\ 11\}$

$y \in \{1,\ 11\}$

$let\ A = \left\{xy: where\ x \in \{0\} \cup \{0\}^2\ and\ y \in \{1\} \cup \{1\}^2\right\}$

$= \{01, 011, 001, 0011\}$

6.e

$y \in \{a\} \cup \{a\}^2$

$\{a\}^2 = \{a\} \times \{a\} = \{aa\}$
$\{a\} \cup \{a\}^2 = \{a,\ aa\}$
$y \in \{a,\ aa\}$

$$let\ A = \left\{xy:\ where\ x \in \{aa, ab\}\ and\ y \in \{a\} \cup \{a\}^2\right\}$$
$$= \{aaa, aaaa, aba, abaa\}$$

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d) Exercise 3.5.7, sections c, f, g

7.c $(A \times B) \cup (A \times C)$
$A \times B = \{ab, ac\}$
$A \times C = \{aa, ab, ad\}$
$(A \times B) \cup (A \times C) = \{aa, ab, ac, ad\}$

7.f $P(A \times B)$
$A \times B = \{ab, ac\}$
$P(A \times B) = \{\varnothing, \{ab\}, \{ac\}, \{ab, ac\}\}$

7.g $P(A) \times P(B)$
$P(A) = \{\varnothing, \{a\}\}$
$P(B) = \{\varnothing, \{b\}, \{c\}, \{b, c\}\}$
$P(A) \times P(B) = \{(\varnothing, \varnothing),\ (\varnothing, \{b\}), (\varnothing, \{c\}), (\varnothing, \{b, c\}),\ (\{a\}, \{b\}), (\{a\}, \{c\}), (\{a\}, \{b, c\}),$
$(\{a\}, \varnothing)\}$

Question 11:

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a) Exercise 3.6.2, sections b, c

2.b $(B \cup A) \cap (\overline{B} \cup A) = A$

$(B \cup A) \cap (\overline{B} \cup A)$	Hypothesis
$(A \cup B) \cap (A \cup \overline{B})$	Commutative laws
$A \cup (B \cap \overline{B})$	Distributive laws
$A \cup \varnothing$	Complement laws
A	Identity law

2.c $\overline{A \cap \overline{B}} = \overline{A} \cup B$

$\overline{A \cap \overline{B}}$	Hypothesis
$\overline{A} \cup \overline{\overline{B}}$	De Morgan's laws
$\overline{A} \cup B$	Double complement laws

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b) Exercise 3.6.3, sections b, d

3.b $A - (B \cap A) = A$

$let\ A = \{1, 2, 3, 4, 5\}$

$let\ B = \{3, 4\}$

$\therefore\ B \cap A = \{3, 4\}$

$\therefore\ A - (B \cap A) = \{1, 2, 5\}$ *which is not equal to* $A = \{1, 2, 3, 4, 5\}$

so, the set equation is not identical.

3.d $(B - A) \cup A = A$

$let\ A = \{3, 4, 6\}$

$let\ B = \{1, 2, 3, 4, 5\}$

$\therefore\ B - A = \{1, 2, 5\}$

$\therefore (B - A) \cup A = \{1, 2, 5, 3, 4, 6\}$ *which is not equal to* $A = \{3, 4, 6\}$

so, the set equation is not identical.

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c) Exercise 3.6.4, sections b, c

4.b $A \cap (B - A) = \varnothing$

$A \cap (B - A)$	Hypothesis
$A \cap (B \cap \overline{A})$	Subtraction laws
$A \cap (\overline{A} \cap B)$	Commutative laws
$(A \cap \overline{A}) \cap B$	Associative laws
$\varnothing \cap B$	Complement laws
$B \cap \varnothing$	Commutative laws
$\varnothing$	Domination laws

4.c $A \cup (B - A) = A \cup B$

$A \cup (B - A)$	Hypothesis
$A \cup (B \cap \overline{A})$	Subtraction laws
$(A \cup B) \cap (A \cup \overline{A})$	Distributive laws
$(A \cup B) \cap U$	Complement laws
$A \cup B$	Identity laws