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Homework 4 - Q9 to Q11

Question 9:

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a) Exercise 4.1.3, sections b, c

b. $f(x) = 1/\left(x^2 - 4\right)$
This f(x) is not a funcion. It is becuase when x is equal to 2 or -2, the function is not defined.

c. $f(x) = \sqrt{x^2}$
This f(x) is a function, f:R $\rightarrow$ R^+. The range is [0, ∞].

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b) Exercise 4.1.5, sections b, d, h, i, l

b. {4, 9, 16, 25}

d. {0, 1, 2, 3, 4, 5}

h. {(2, 1), (1, 1), (3, 1), (1, 2), (2, 2), (3, 2), (1, 3), (3, 3), (2, 3)}

i. {(2, 2), (1, 2), (3, 2), (1, 3), (2, 3), (3, 3), (1,4), (3, 4), (2, 4)}

l. {∅, {2}, {3}, {2,3} }

Question 10:

I. a) Exercise 4.2.2, sections c, g, k

c. h: Z ⟶ Z. h(x) = x^3

This function is one-to-one, not onto.

Because for each value of x in Z, there is a corresponding y value that is x^3, but not all y has a correspongding x value, for instance, if y = 2, there is no such corresponding x such that x^3 = 2.

g. f: Z × Z ⟶ Z × Z, f(x, y) = (x+1, 2y)

This function is one to one, not onto.

Because for each value pair of (x, y) there is a only corresponding (x+1, 2y) value. but not all (x+1, 2y) has a corresponding (x, y) value, for instance, if (x+1, 2y) is equal to (0, 3) there is not (x, y) pair in Z.

k. f: Z^+× Z^+ ⟶ Z^+, f(x, y) = 2^x + y.

This function is one-to-one, not onto.

Because for each value of (x, y) in the range Z+, there is a only corresponding 2^x+y value, but not all 2^x+y value has a corresponding (x,y) value. for example, if 2^x+y is equal to 1, there is no such (x, y) value in the range of Z^+.

I. b) Exercise 4.2.4, sections b, c, d, g

b.

This function is not onto, and also not one-to-one.

Since two different x values will have the same y value, for example, both f(010) and f(110) will point to 110, this function is not one-to-one. At the same time, there will be some y value that will not be pointed within the range, such as 010, so this function is not onto.

c.

This function is one-to-one and onto.

Because each x value in the range has a only different corresponding y value within the range.

d.

This function is one-to-one, not onto.

Because there are some y value that has no corresponding x valye, for instance, if y = 0111, there is no such x in the range that points to this y value.

g.
This function is not onto and not one-to-one.
Because some different x value will point to the same y value, for example, if x1={2,3}, x2={1,2,3}, they both have the same y value {2, 3}. But there are also some y value that will not be pointed within the range, for example if y = {1, 2, 3}, no such x will point to this y value.

II.
a) one-to-one, but not onto. f: $Z \rightarrow Z^{+}$
$f(x) = 2 * |x| + 1$

b) onto, but not one-to-one. f: $Z \rightarrow Z^{+}$
$f(x) = \sqrt{x^2}$

c) one-to-one and onto. f: $Z \rightarrow Z^{+}$
$f(x) = 2 * |x| \ \ if\ x\ is\ negative$
$\quad\quad 2x + 1 \ \ if\ x\ is\ non-negative$

d) neither one-to-one nor onto. f: $Z \rightarrow Z^{+}$
$f(x) = x^4 + 1$

Question 11:

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a) Exercise 4.3.2, sections c, d, g, i

c. f: R → R. f(x) = 2x + 3
This function have an inverse in this range. $f^{-1}: f(x) = (x-3)/2$

d.
This function does not have an inverse in this range. because the function is not one-to-one and not onto within the range. so, f^{-1} is not defined.

g.
This function have an inverse in this range since it is bijective. $f^{-1}: f(x) = \{1, 0\}^3$

i. f: Z × Z → Z × Z, f(x, y) = (x+5, y-2)
This function have an inverse in this range since it is bijective. $f^{-1}: f(x, y) = (x-5, y+2)$

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b) Exercise 4.4.8, sections c, d
f(x) = 2x + 3
g(x) = 5x + 7
h(x) = x^2 + 1

c. f o h
(f o h) $Z \to Z: (f\ o\ h)\ (x)\ = 2\left(x^2+1\right)+3 = 2x^2+5$

d. h o f
(h o f) $Z \to Z: (h\ o\ f)\ (x)\ = (2x+3)^2+1 = 4x^2+12x+10$

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c) Exercise 4.4.2, sections b-d

b. Evaluate f o h(52)
(f o h) $Z \to Z: (f\ o\ h)\ (x)\ = \lceil x/5 \rceil^2$
when x = 52, (f o h) (52) = $11^2 = 121$

c. Evaluate g o h o f(4)
(g o f o h) $Z \to Z: (g\ o\ f\ o\ h)\ (x)\ = 2^{\lceil x^2/5 \rceil}$
when x = 4, $(g\ o\ f\ o\ h)\ (x)\ = 2^{\lceil x^2/5 \rceil} = 16$

d.Give a mathematical expression for h o f.
(h o f) $Z \to Z: (f\ o\ h)\ (x)\ = \lceil x^2/5 \rceil$

d) Exercise 4.4.6, sections c-e

c. What is h o f(010)?

(h o f) $\{0,1\}^3 \rightarrow \{0,1\}^3$: $(h\ o\ f)\ (010)\ = h(f(010) = h(110) = 111$

d. What is the range of h o f?

$\{0,1\}^3$	f	h o f	range
(000)	(100)	(101)	(101)
(001)	(101)	(101)	(101)
(010)	(110)	(101)	(101)
(011)	(111)	(111)	(111)
(100)	(100)	(101)	(101)
(101)	(101)	(101)	(101)
(110)	(110)	(111)	(111)
(111)	(111)	(111)	(111)

so the range of (h o f) is {111, 101}

e. What is the range of g o f?

$\{0,1\}^3$	f	g o f	range
(000)	(100)	(001)	(001)
(001)	(101)	(101)	(101)
(010)	(110)	(011)	(011)
(011)	(111)	(111)	(111)
(100)	(100)	(001)	(001)
(101)	(101)	(101)	(101)
(110)	(110)	(011)	(011)

(111)	(111)	(111)	(111)

so the range of (g o f) is {111, 011, 101, 001}

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e) Extra Credit: Exercise 4.4.4, sections c, d

c.
Is it possible that f is not one-to-one and g o f is one-to-one? Justify your answer. If the answer is "yes", give a specic example for f and g.

$f: X \to Y \ \& g: Y \to Z$

we assume that f is not one-to-one, such that
$\exists x1, x2 \in f: X \to Y \quad x1 \neq x2 \to f(x1) = f(x2)$, and g o f is one to one such that
$\forall x1,\ x2 \in g \ \ o\ f: Y \to X \quad x1 \neq x2 \to (gof)(x1) \neq (gof)(x2)$.

Since there are x1 and x2 that are different value but they have the same f(x) value, then (g o f)(x1) will have the same value of (g o f)(x2) , which contradict the assumption.

So, it is not possible that f is not one-to-one but g o f is one to one.

d.
Is it possible that g is not one-to-one and g o f is one-to-one? Justify your answer. If the answer is "yes", give a specic example for f and g.

We assume that g is not one-to-one, such that
$\exists x1, x2 \in g: Y \to X \quad x1 \neq x2 \to g(x1) = g(x2)$, and g o f is one to one such that
$\forall x1,\ x2 \in g \ \ o\ f: Y \to X \quad x1 \neq x2 \to (gof)(x1) \neq (gof)(x2)$.

There are x1 and x2 in g that produce the same g(x) value. if f(k1) and f(k2), in which k1 is different from k2, are same as the x1 and x2 value, (gof)(k1) and (gof)(k2) will have the same result. this example contradict the assumption.

So, it is not possible that g is noot one-to-one and g o f is one-to-one.