Fungsi Transenden

A. Eksponen Alami/Natural $\rightarrow e^x$, $e = 2{,}718\ldots$

B. Logaritma Natural $\rightarrow \ln x$, $\ln \rightarrow {}^{e}\log$

C. Eksponen Umum $\rightarrow a^x$, a bilangan real, Ex: $2^x, 3^x, \sqrt{2}^x$
$(\frac{1}{2})^x, (3{,}21)^x$

D. Logaritma umum $\rightarrow {}^{a}\log x$

E. Fungsi Invers dan Gradiennya $\rightarrow f^{-1}(x)$ dan $(f^{-1})'(x)$

F. Fungsi Trigonometri dan Inversnya
$\rightarrow \sin x$
$\rightarrow \cos x$
$\rightarrow \tan x$
$\rightarrow \csc x$
$\rightarrow \sec x$
$\rightarrow \cot x$

A. Fungsi Eksponen Natural

$$f(x) = e^x \quad \text{di mana } e = 2,718\ldots$$

1) Turunan : $f'(x) = e^x$

2) Integral : $\int e^x \, dx = e^x + C$

Contoh :

1. $f(x) = e^{2x} \longrightarrow f'(x) = e^{2x} \cdot 2 = 2e^{2x}$

2. $f(x) = e^{\sqrt{x}} \longrightarrow \dfrac{df(x)}{dx} = e^{\sqrt{x}} \cdot \frac{1}{2}x^{-\frac{1}{2}} = \dfrac{1}{2\sqrt{x}} e^{\sqrt{x}}$ $x^{1/2}$

3. $\int e^{5x} \, dx$

$= \frac{1}{5} e^{5x} + C$

4. $\int x e^{x^2} \, dx$

$u = x^2$

$du = 2x \, dx$

$\dfrac{du}{2x} = dx$

$= \int \cancel{x} e^u \dfrac{du}{2\cancel{x}}$

$= \frac{1}{2} \int e^u \, du$

$= \frac{1}{2} e^u + C$

$= \frac{1}{2} e^{x^2} + C$

B. Fungsi Logaritma Natural

$f(x) = \ln x \quad , \; x > 0$

1) Turunan : $f'(x) = \frac{1}{x}$

2) Integral : $\int \frac{1}{x} \, dx = \ln |x| + C$

atau

$\ln x + C \quad , \; x > 0$

$\cdot \int \frac{1}{x^2} \, dx = \frac{x^{-2+1}}{-2+1} + C$

x^{-2}

$= \frac{x^{-1}}{-1} + C$

$= -\frac{1}{x} + C$

$\cdot \int \frac{1}{x} \, dx = \cancel{\frac{x^{-1+1}}{-1+1} + C}$

$\cancel{= \frac{x^0}{0} + C}$

* Fakta : $\ln x$ saling invers dg e^x shg

1) $\cancel{e}^{\cancel{\ln} x} = x$

2) $\cancel{\ln (e}^x) = x$

Contoh :

Rumus

(1) $y = \ln 3x \rightarrow y' = \frac{1}{\cancel{3}x} \cdot \cancel{3} = \frac{1}{x}$ $\longrightarrow$ $y = \ln ax$, maka $y' = \frac{1}{x}$

(2) $y = \ln x^2 \rightarrow y' = \frac{1}{x^2}\, 2x = \frac{2}{x}$ $\longrightarrow$ $y = \ln x^n$, maka $y' = \frac{n}{x}$

Cara lain : $y = \ln x^2 = 2 \ln x = 2 \cdot \frac{1}{x} = \frac{2}{x}$

(3) $y = \ln(\sqrt{e^x}) = \frac{1}{\cancel{\sqrt{e^x}}}\, \cancel{e^{\frac{1}{2}x}}\, \frac{1}{2} = \frac{1}{2}$

$e^{\frac{1}{2}x}$

Cara lain : $y = \ln(\sqrt{e^x}) = \ln e^{\frac{1}{2}x} = \frac{1}{2}x$

$\frac{dy}{dx} = \frac{1}{2}$

④ $\int \cancel{e^{\ln}} x \, dx = \int x \, dx = \frac{x^2}{2} + C$

⑤ $\int \frac{1}{x+1} \, dx = \int \frac{1}{u} \frac{du}{1} = \frac{1}{1} \ln|u| + c = \ln|x+1| + C$

$u = x+1$
$du = 1 \cdot dx$
$\frac{du}{1} = dx$

Cara cepat: $\int \frac{1}{1x+1} \, dx = \frac{1}{1} \ln|x+1| + C$

⑥ $\int \frac{x}{x^2+e} \, dx = \int \frac{\cancel{x}}{u} \frac{du}{2\cancel{x}} = \frac{1}{2} \int \frac{1}{u} \, du = \frac{1}{2} \ln|u| + c$

$= \frac{1}{2} \ln|x^2+e| + C$

$u = x^2 + e$
$du = 2x \, dx$
$\frac{du}{2x} = dx$

C. Fungsi Eksponen Umum

$$f(x) = a^x \quad , \quad a \text{ bilangan real}$$

1) turunan : $f'(x) = a^x \ln a$

2) Integral : $\int a^x \, dx = \dfrac{a^x}{\ln a} + C$

Contoh.

① $f(x) = e^x \rightarrow f'(x) = e^x \cancel{\ln e} = e^x$

② $f(x) = \pi^x \rightarrow f'(x) = \pi^x \ln \pi$

③ $f(x) = 2^{3x} \rightarrow f'(x) = 2^{3x} \ln 2 \cdot 3$

$= 3 \cdot 2^{3x} \ln 2$

④ $\int 3^x \, dx = \dfrac{3^x}{\ln 3} + C$

⑤ $\int 3^{5x} \, dx = \dfrac{1}{5} \cdot \dfrac{3^{5x}}{\ln 3} + C$

$= \dfrac{3^{5x}}{\ln 3^5} + C$

$= \dfrac{3^{5x}}{\ln 243} + C$

⑥ $\int x \left(\frac{1}{2}\right)^{x^2} dx$

$u = x^2$

$du = 2x \, dx$

$\dfrac{du}{2x} = dx$

$= \int \cancel{x} \left(\frac{1}{2}\right)^u \dfrac{du}{2\cancel{x}}$

$= \dfrac{1}{2} \dfrac{\left(\frac{1}{2}\right)^{x^2}}{\ln \frac{1}{2}} + C$

D. Fungsi Logaritma Umum

$$f(x) = {}^{a}\log x$$

1) <u>turunan</u> : $f'(x) = \frac{1}{x \ln a}$

* Fakta :

1) logaritma dan eksponen umum saling invers

$$\cancel{a}^{{}^{a}\log x} = x \quad \text{dan} \quad \cancel{{}^{a}\log (a}^{x}) = x$$

2) $${}^{a}\log x = \frac{\log x}{\log a} = \frac{\ln x}{\ln a}$$

3) $$\ln x = \frac{\log x}{\log e}$$

Contoh :

1) $y = {}^{2}\log x$

$$\frac{dy}{dx} = \frac{1}{x \ln 2}$$

2) $y = {}^{e}\log x = \ln x$

$$y' = \frac{1}{x \cancel{\ln e}} = \frac{1}{x}$$

3) $y = {}^{3}\log x^2$

$$y' = \frac{1}{x^2 \ln 3} \cdot 2x$$

$$= \frac{2}{x \ln 3}$$

E. Fungsi Invers dan Gradiennya

Fungsi Invers

1) $f(x) = ax+b \longrightarrow f^{-1}(y) = \frac{y-b}{a}$

$f^{-1}(x) = \frac{x-b}{a}$

$y = ax+b$

$y-b = ax$

$\frac{y-b}{a} = x$

2) $f(x) = x^2 \longrightarrow f^{-1}(y) = \sqrt{y}$

$f^{-1}(x) = \sqrt{x}$

$y = x^2$

$\pm\sqrt{y} = x$

3) $f(x) = e^x \rightarrow f^{-1}(y) = \ln y$

$f^{-1}(x) = \ln x$

4) $f(x) = a^x \longrightarrow f^{-1}(y) = {}^a\log y$

$f^{-1}(x) = {}^a\log x$

dan masih banyak lagi

Gradien fungsi Invers

$f(x) \longrightarrow$ invers $f^{-1}(y)$

gradien : $(f^{-1})'(y)$

di mana $y = f(x)$

Teorema gradien fungsi Invers

$$(f^{-1})'(y) = \frac{1}{f'(x)}$$

Contoh :

(1) tentukan gradien dari invers fungsi $f(x) = 2x + 7$ saat $y = 0$

$\Rightarrow f^{-1}(y) = \frac{y-7}{2} = \frac{1}{2}y - \frac{7}{2}$

$\Rightarrow$ gradien $(f^{-1})'(y) = \frac{1}{2} \Longrightarrow (f^{-1})'(0) = 1/2$

Cara teorema :

$f(x) = 2x + 7 \rightarrow f'(x) = 2$

$\Rightarrow f^{-1}(y) = \frac{1}{f'(x)} = \frac{1}{2}$

$f^{-1}(0) = \frac{1}{2}$

(2) $f(x) = x^2$ tentukan gradien saat $y = 4$

$f^{-1}(y) = \sqrt{y} = y^{1/2}$

$(f^{-1})'(y) = \frac{1}{2} y^{-1/2} = \frac{1}{2\sqrt{y}}$

$(f^{-1})'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{2 \cdot 2} = \frac{1}{4}$

Cara teorema :

$f(x) = x^2 \rightarrow f'(x) = 2x$

$y = 4 = x^2$

$\pm 2 = x$

$\rightarrow (f^{-1})'(y) = \frac{1}{2x}$

$\rightarrow (f^{-1})'(4) = \frac{1}{2(2)} = \frac{1}{4}$

atau $= \frac{1}{2(-2)} = \frac{1}{-4}$

(3) $y = x^2 + 5x + 7 \rightarrow f'(x) = 2x + 5$

gradien saat $y = 13$

$13 = x^2 + 5x + 7$

$0 = x^2 + 5x - 6$

$(x+6)(x-1)$

$x = -6 \quad x = 1$

$(f^{-1})'(y) = \frac{1}{2x+5}$

$(f^{-1})'(13) = \frac{1}{2(-6)+5} = \frac{1}{-7}$

atau $= \frac{1}{2(1)+5} = \frac{1}{7}$

F) Fungsi Trigonometri dan Inversnya

Fungsi Trigonometri

	turunan	integral
1) $y = \sin x$	$y' = \cos x$	$\int \sin x \, dx = -\cos x + C$
2) $y = \cos x$	$y' = -\sin x$	$\int \cos x \, dx = \sin x + C$
3) $y = \tan x$	$y' = \sec^2 x$	$\int \sec^2 x \, dx = \tan x + C$
4) $y = \csc x$	$y' = -\csc x \cot x$	$\int \csc x \cot x \, dx = -\csc x + C$
5) $y = \sec x$	$y' = \sec x \tan x$	$\int \sec x \tan x \, dx = \sec x + C$
6) $y = \cot x$	$y' = -\csc^2 x$	$\int \csc^2 x \, dx = \cot x + C$

Contoh : ① $\int x \sec^2 x^2 \, dx = \int \cancel{x} \sec^2 u \frac{du}{2\cancel{x}}$

$u = x^2 \rightarrow du = 2x \, dx$

$\frac{du}{2x} = dx$

$= \frac{1}{2} \tan u + C$

$= \frac{1}{2} \tan x^2 + C$

② $y = \cot x$. gradien inversnya saat $y = 1$

$1 = \cot x = \frac{1}{\tan x}$

$f'(x) = -\csc^2 x$

$\tan x = 1 \Rightarrow x = 45°$

$\Rightarrow$ gradien invers :

$(f^{-1})'(y) = \frac{1}{-\csc^2 x} \Rightarrow (f^{-1})'(1) = \frac{1}{-\csc^2(45°)} = -\sin^2(45°)$

berkebalikan

$= -(\frac{1}{2}\sqrt{2})^2$

$= -\frac{1}{2}$

Invers Fungsi trigonometri :

$f(x) = \sin x \longrightarrow f^{-1}(x) = \sin^{-1}(x) = \arcsin(x)$

$f(90^\circ) = \sin 90^\circ$

$= 1$

$f^{-1}(1) = \sin^{-1}(1)$

$= 90^\circ$

$f(x) = \tan x \longrightarrow f^{-1}(x) = \tan^{-1}(x) = \arctan x$

$f^{-1}(1) = \tan^{-1}(1)$

$= 45^\circ$ atau 225°