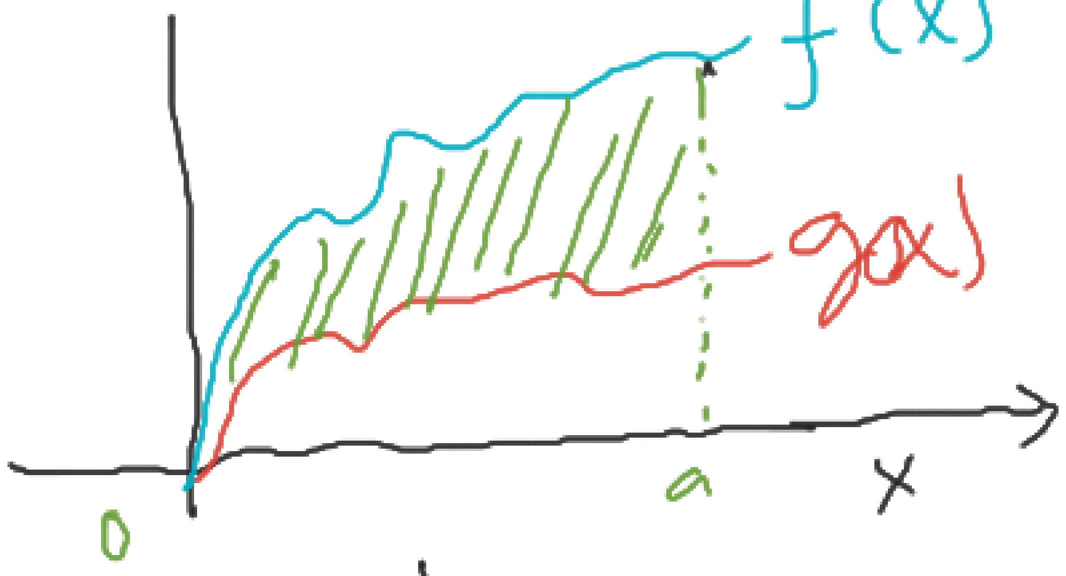
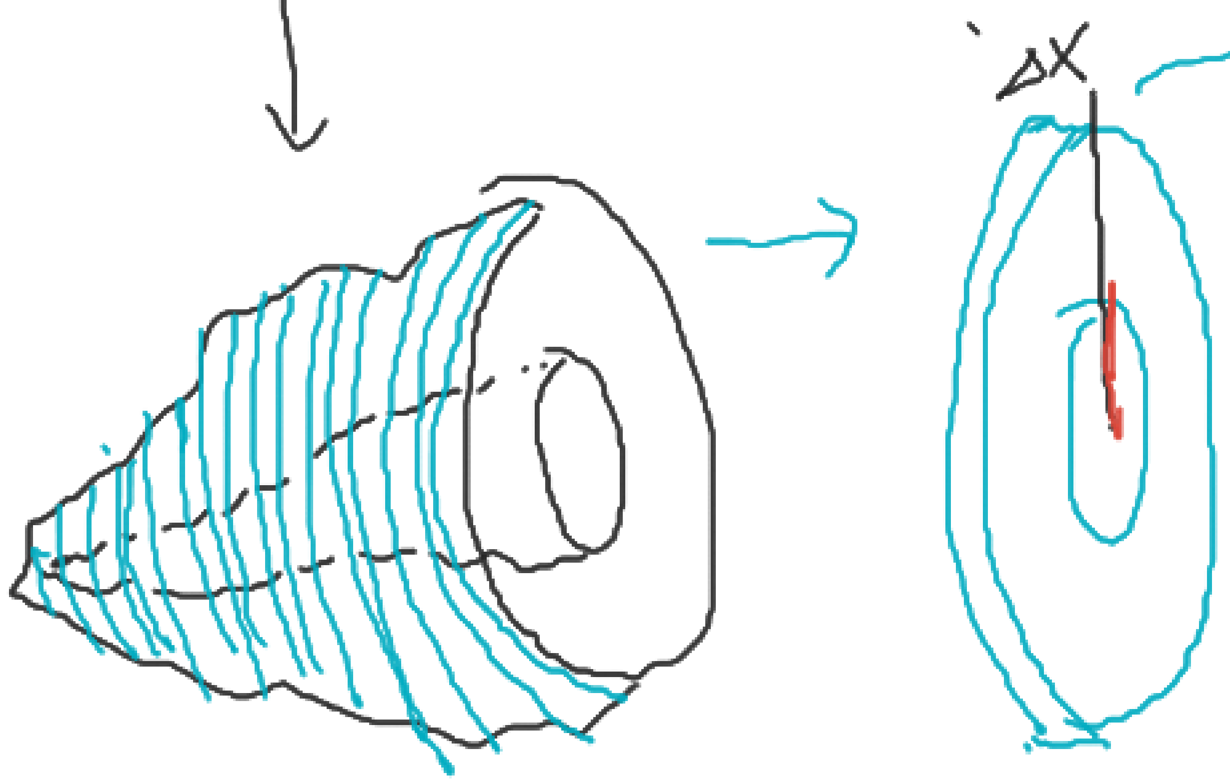
B. Volume Benda Putar

- → Metode Cakram/tabung/Cincin
- → Metode selimut/kulit tabung

1. Metode cakram

(i) Diputar thd sumbu-x





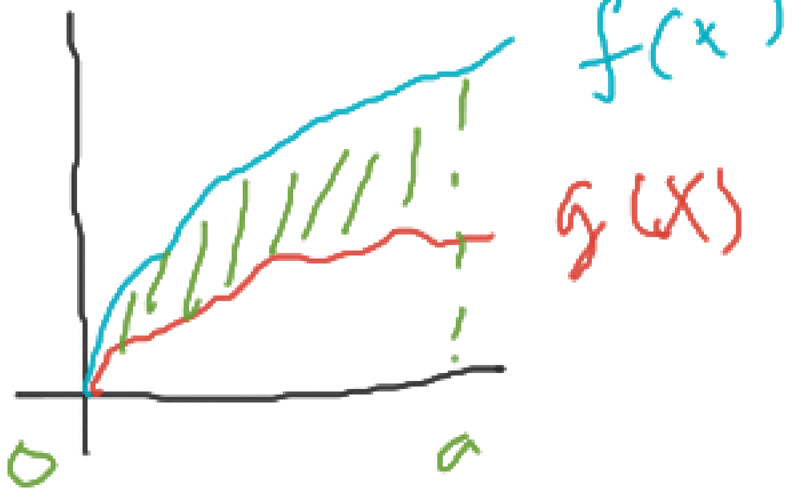
Hamp tabung tipis/cakram

$$V_p = \pi r_1^2 t - \pi r_2^2 t = \pi (r_1^2 - r_2^2) t$$

$$V_p = \pi (f(x)^2 - g(x)^2) \Delta x$$

$$V = \pi \int_0^a (f(x)^2 - g(x)^2)\, dx$$

Luas vs Volume



$$L = \int_0^a \left(f(x) - g(x)\right) dx$$

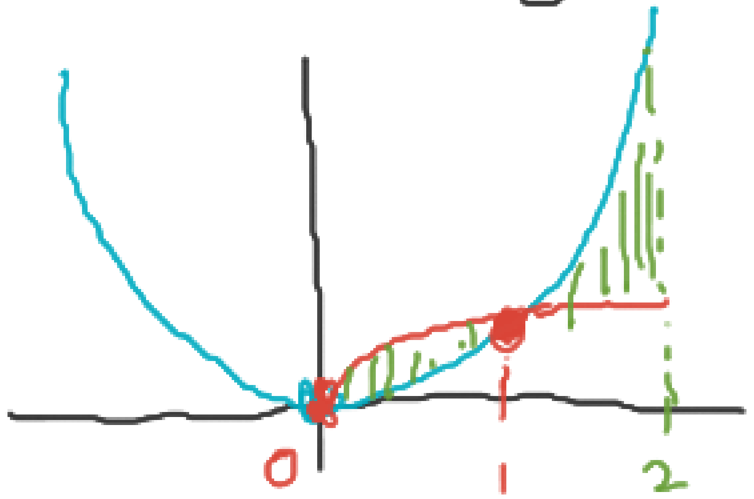
$$V = \pi \int_0^a \left(f(x)^2 - g(x)^2\right) dx$$

Contoh 1 : $y = x^2$, $y = \sqrt{x}$, pada $[0, 2]$

• $y = 0 \rightarrow 0 = x^2$

$0 = x$

• $x^2 \rightarrow (+)\ \cup$



• $x^2 = \sqrt{x}$

$x^4 = x$

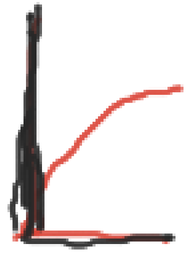
$x^4 - x = 0$

$x(x^3 - 1) = 0$

$x = 0$ $\quad x^3 = 1$

$x = 1$

• $\sqrt{x} \rightarrow$



$$L = \int_0^1 (\sqrt{x} - x^2)\, dx + \int_1^2 (x^2 - \sqrt{x})\, dx$$

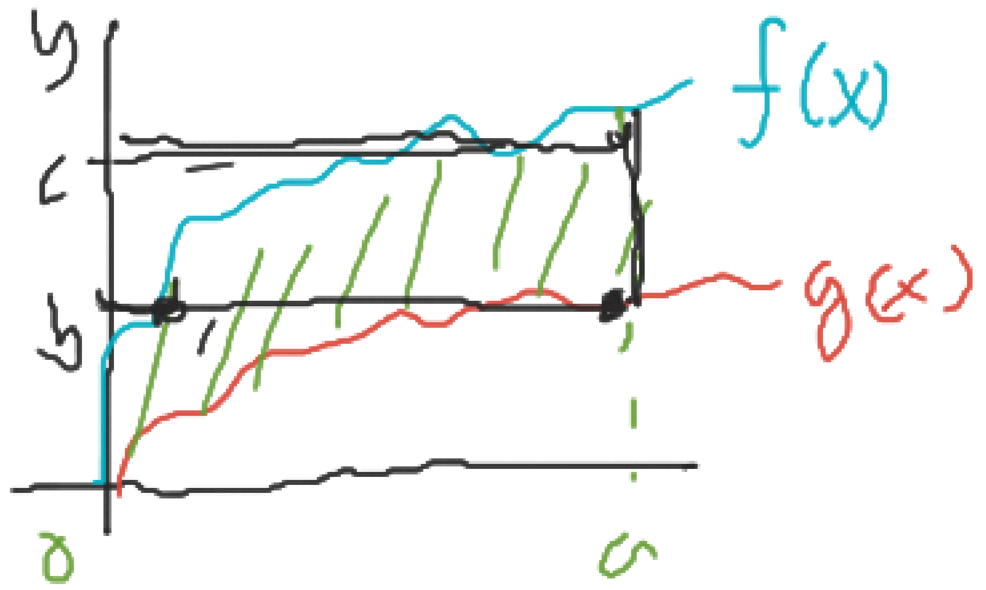
Volume jika diputar 360° thd sb-x

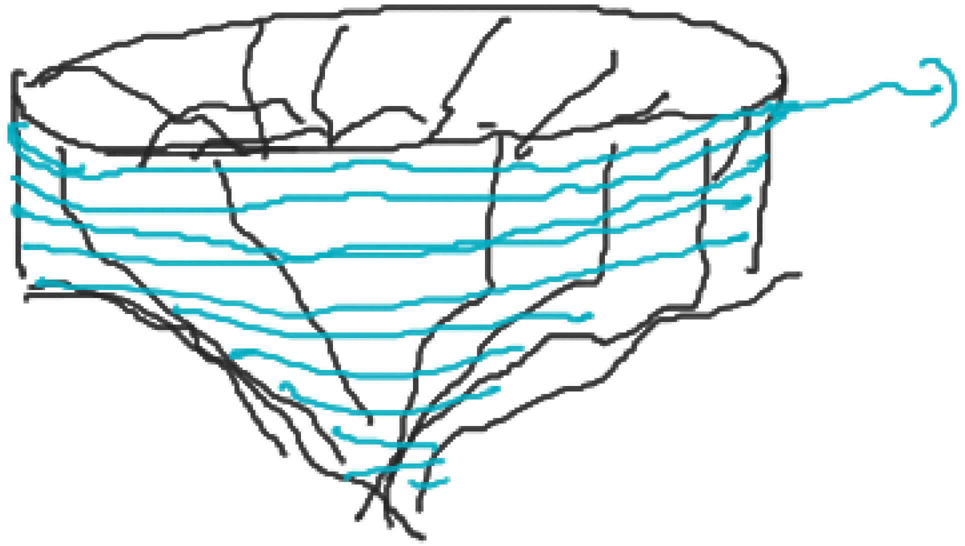
$$V = \pi \left[\int_0^1 (x - x^4)\, dx + \int_1^2 (x^4 - x)\, dx \right]$$

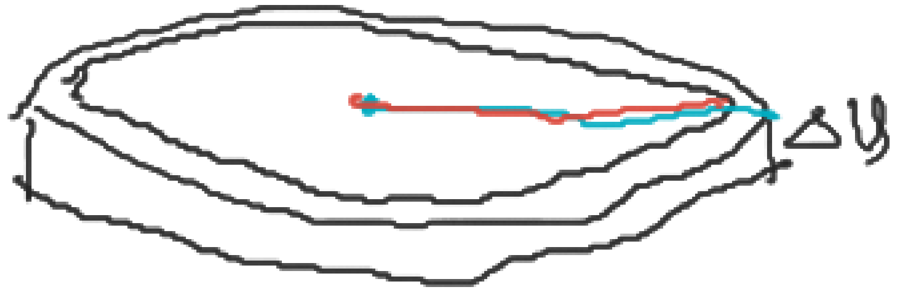
$$= \pi \left(\left[\frac{x^2}{2} - \frac{x^5}{5} \right]_0^1 + \left[\frac{x^5}{5} - \frac{x^2}{2} \right]_1^2 \right.$$

$$= \pi \left[\frac{1}{2} - \frac{1}{5} - 0 + \frac{32}{5} - 2 - \left(\frac{1}{5} - \frac{1}{2} \right. \right.$$

(ii) Dipotar thd sumbu y







$$V_{p_1} = \pi(r_1^2 - r_2^2)\, t$$
$$= \pi\left(a^2 - (f^{-1}(y))^2\right)\Delta y$$

$$V_{p_2} = \pi\left(g^{-1}(y)^2 - f^{-1}(y)^2\right)\Delta y$$

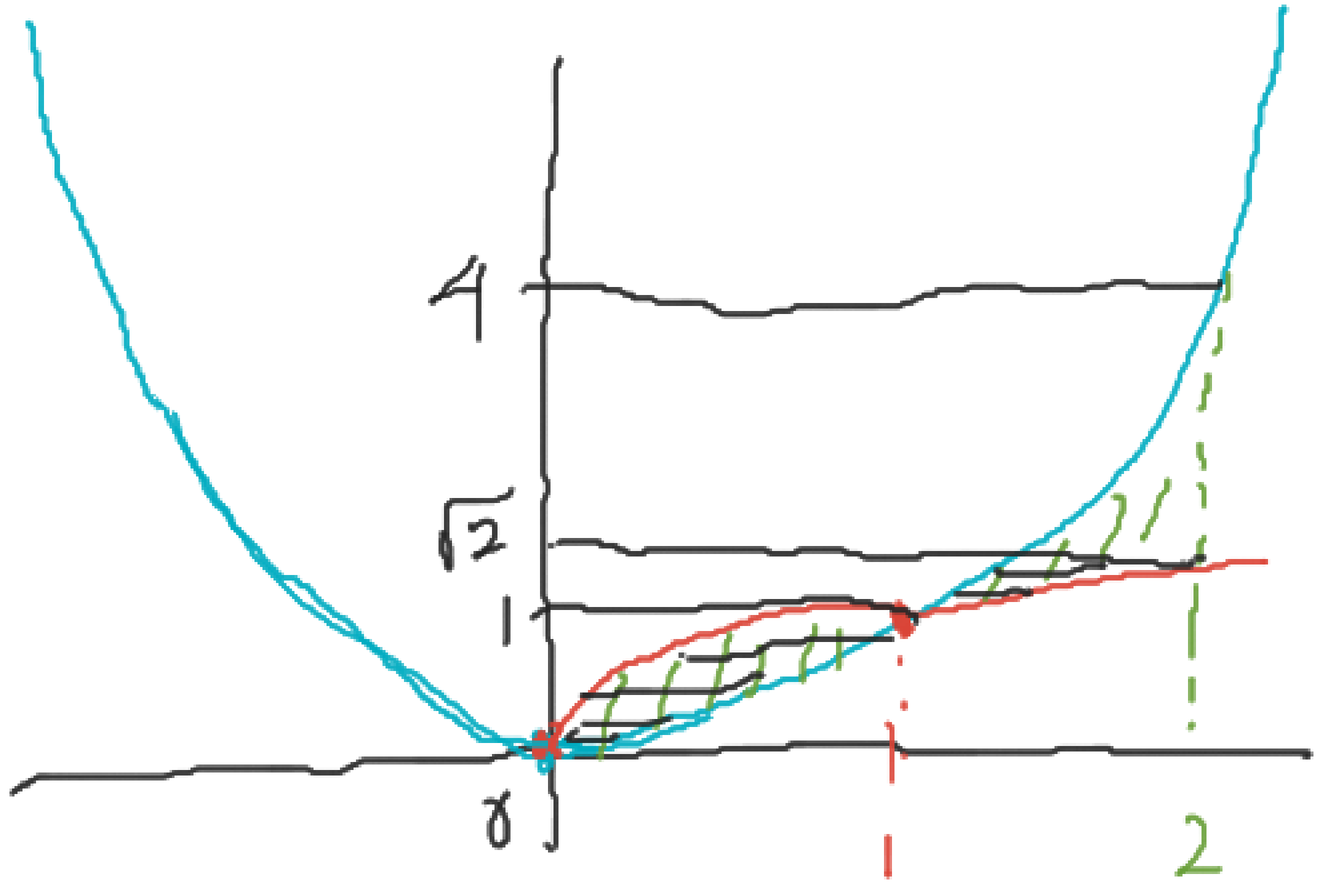
Luas vs Volume

$$L = \int_0^b \left(g^{-1}(y) - f^{-1}(y)\right) dy + \int_b^c \left(a - f^{-1}(y)\right) dy$$

$$\Rightarrow V = \pi\left[\int_0^b \left(g^{-1}(y)^2 - f^{-1}(y)^2\right) dy + \int_b^c \left(a^2 - f^{-1}(y)^2\right) dy\right]$$

Contoh : $y = x^2$, $y = \sqrt{x}$, pada $[0,2]$

$y = x^2 \to \sqrt{y} = x$ (biru)
$y = \sqrt{x} \to y^2 = x$ (merah)



Diputar thd sumbu-y 360°

$$\to L = \int_0^1 (\sqrt{y} - y^2)\,dy + \int_1^{\sqrt{2}} (y^2 - \sqrt{y}) + \int_{\sqrt{2}}^4 (2 - \sqrt{y})\,dy$$

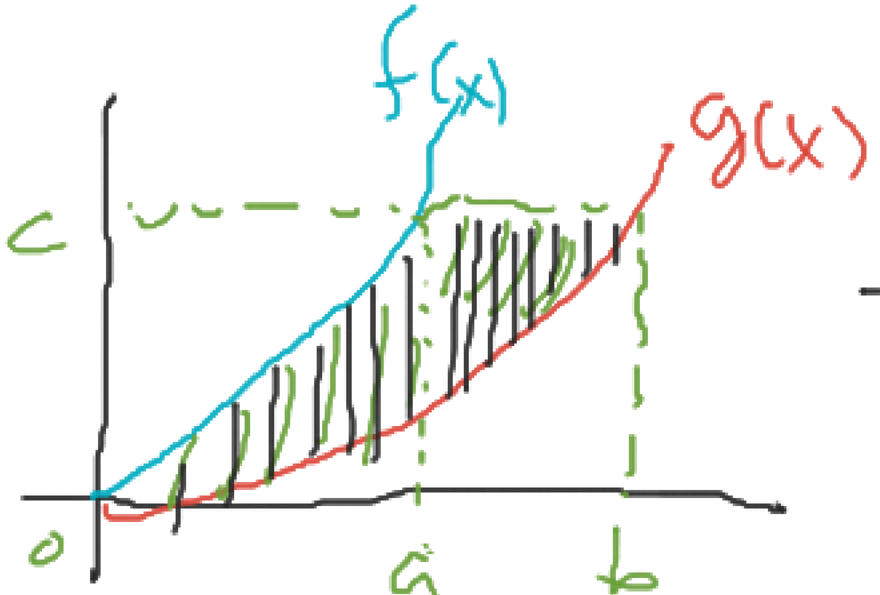
$$V = \pi\left[\int_0^1 (y - y^4)\,dy + \int_1^{\sqrt{2}} (y^4 - y)\,dy + \int_{\sqrt{2}}^4 (4 - y)\,dy\right]$$

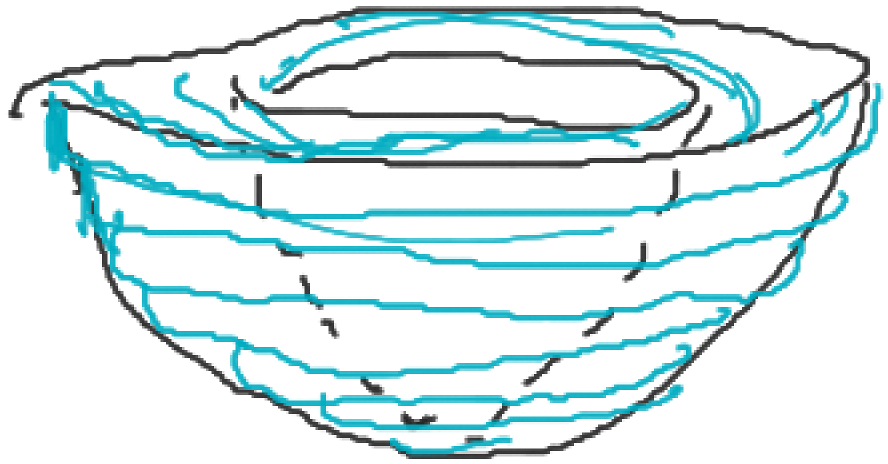
$$= \pi\left[\frac{y^2}{2} - \frac{y^5}{5}\Big)_0^1 + \frac{y^5}{5} - \frac{y^2}{2}\Big)_1^{\sqrt{2}} + 4y - \frac{y^2}{2}\Big)_{\sqrt{2}}^4\right]$$

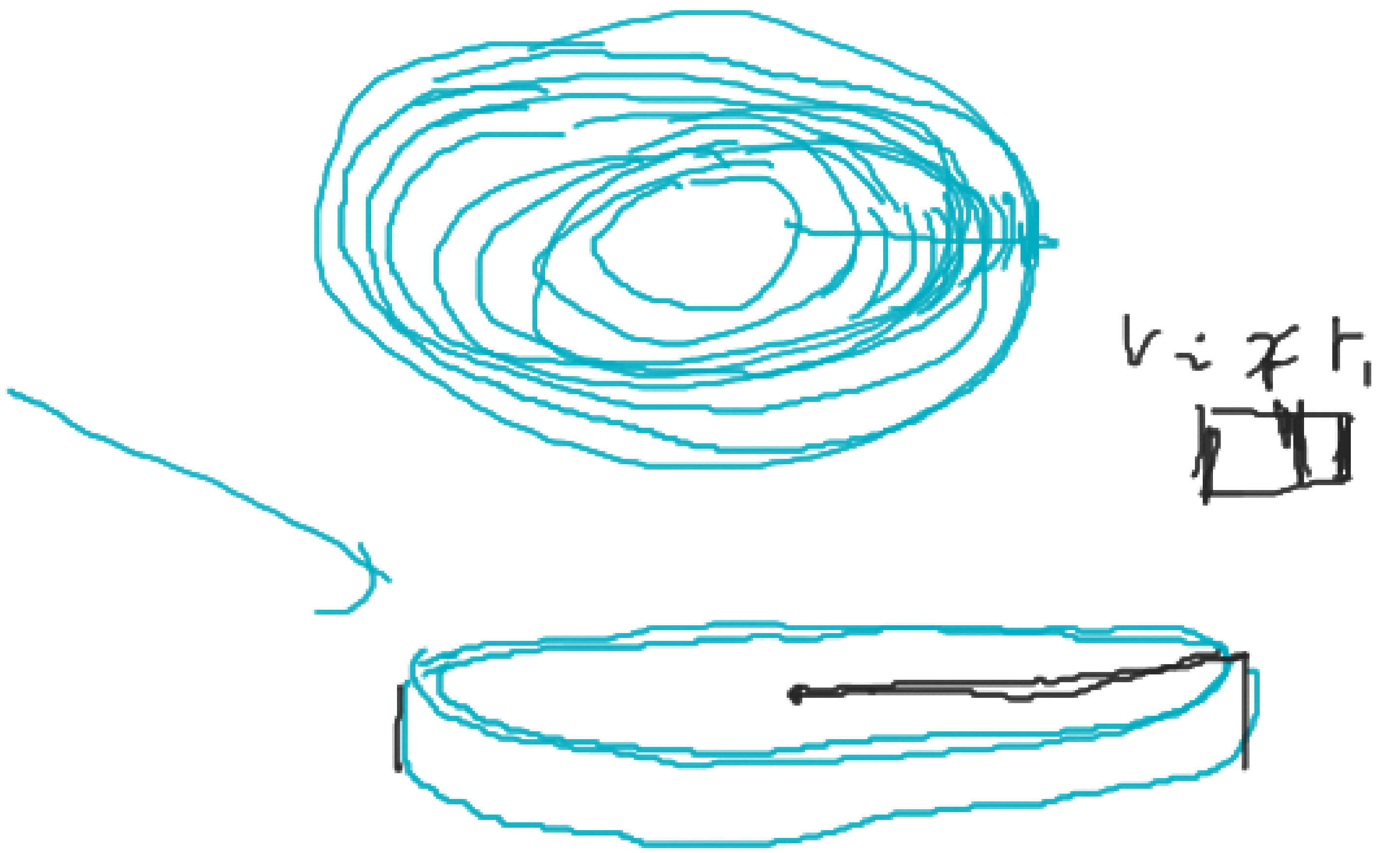
$$= \pi\left[\frac{1}{2} - \frac{1}{5} - 0 + \frac{\sqrt{2}^5}{5} - 1 - \left(\frac{1}{5} - \frac{1}{2}\right) + 16 - 8 - (4\sqrt{2} - 1)\right]$$

$$= \pi\,[8{,}6] \approx 27{,}02$$

2. Metode Selimut tabung



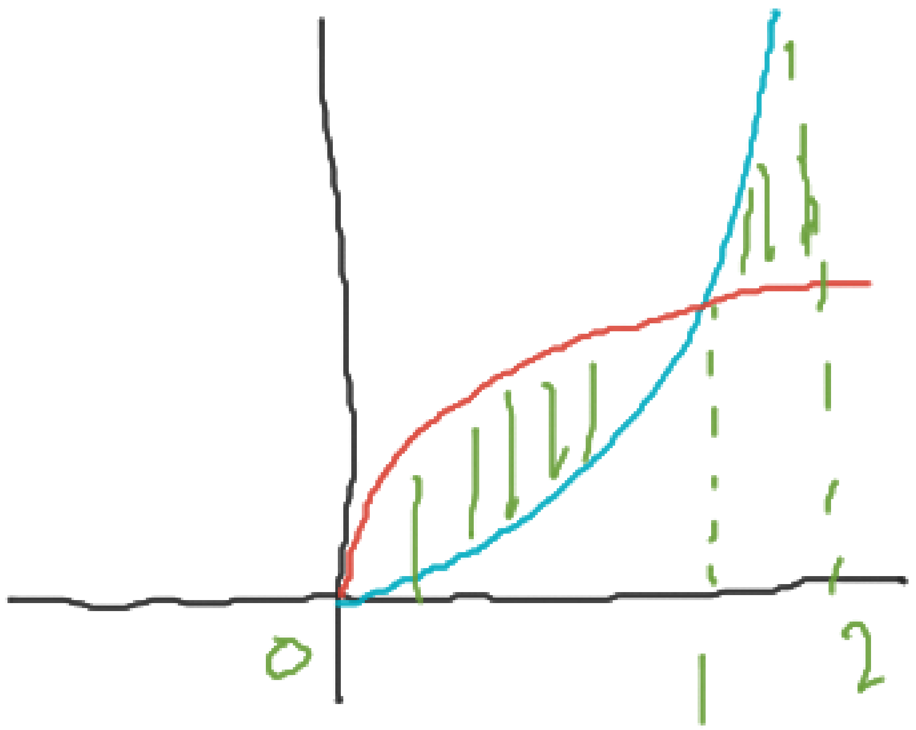




$$V = 2\pi \int_0^a x \cdot (f(x) - g(x))\, dx + 2\pi \int_a^b x \cdot (c - g(x))\, dx$$

$$
\begin{cases}
V_{p_1} = \pi r^2 \cdot t \\
\quad = \pi (r_1^2 - r_2^2)(c - g(x)) \\
\quad = \pi (r_1 + r_2)(r_1 - r_2)(c - g(x)) \\
\quad = 2\pi \dfrac{(r_1 + r_2)}{2} \Delta r \, (c - g(x)) \\
\quad = 2\pi \cdot x \, (c - g(x)) \, \Delta x \\
V_{p_2} = 2\pi \cdot x \, (f(x) - g(x)) \, \Delta x
\end{cases}
$$

Contoh ! $y = x^2$, $y = \sqrt{x}$, pada $[0, 2]$



$\longrightarrow$ Volume diputar thd sumbu-y 360°.

$$V = 2\pi \int_0^1 x \cdot (\sqrt{x} - x^2)\, dx \; +$$

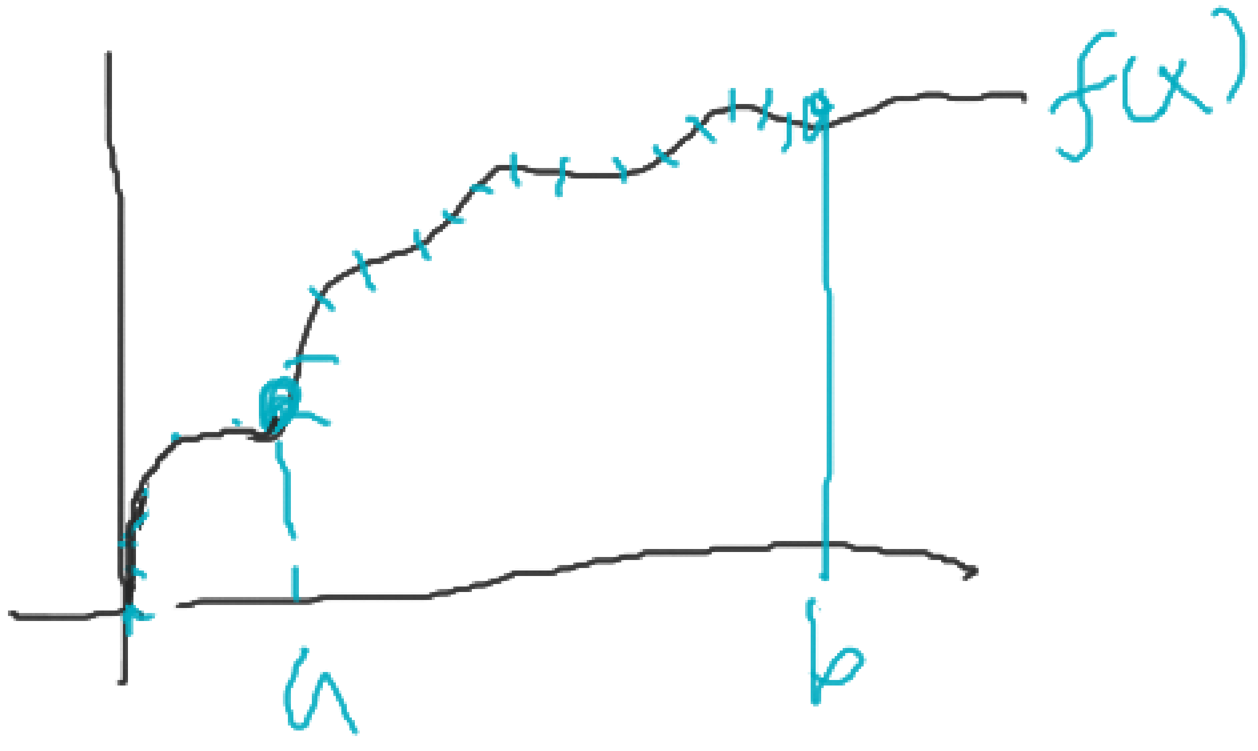
$$2\pi \int_1^2 x \cdot (x^2 - \sqrt{x})\, dx$$

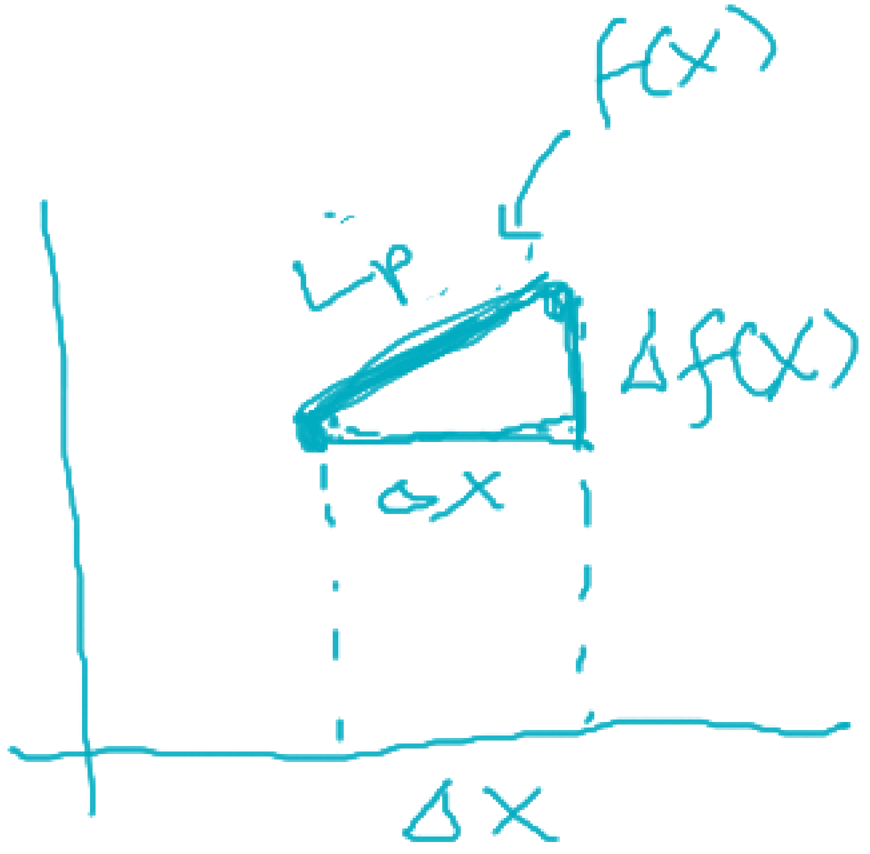
$$V = 2\pi \int_0^1 (x^{3/2} - x^3)\, dx + 2\pi \int_1^2 (x^3 - x^{3/2})\, dx$$

$$= 2\pi \left[\frac{x^{5/2}}{5/2} - \frac{x^4}{4} \right)_0^1 + \frac{x^4}{4} - \frac{x^{5/2}}{5/2} \Big]_1^2 = 2\pi \left(\frac{2}{5} \cdot 1 - \frac{1}{4} - 0 + 4 - \frac{2}{5}\sqrt{32} - \left(\frac{1}{4} - \frac{2}{5} \right) \right.$$

$$= 2\pi (12{,}8) = 80{,}42$$

Panjang Lintasan/Kurva





$$\rightarrow \; l_p = \sqrt{\Delta x^2 + (\Delta f(x))^2}$$

$$= \sqrt{\left(1+\left(\frac{\Delta f(x)}{\Delta x}\right)^2\right)\Delta x^2}$$

$$= \sqrt{1+\left(\frac{\Delta f(x)}{\Delta x}\right)^2}\,\Delta x$$

$\frac{\Delta f(x)}{\Delta x} \rightarrow \frac{df}{dx}$ turunan

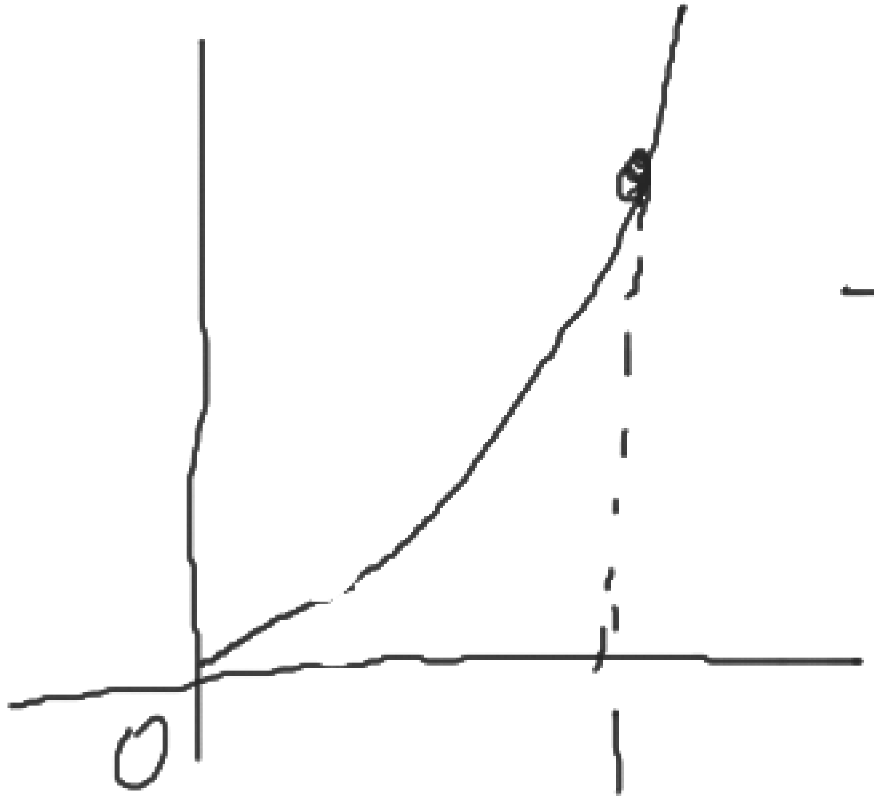
total $\boxed{l = \int_a^b \sqrt{1+f'(x)^2}\,dx}$

Contoh: Akan dicari panjang lintasan kurva $y = \frac{1}{2}x\sqrt{x}$ pada $[0, 1]$

$$= \frac{1}{2}x^{3/2}$$

$$y' = \frac{1}{2} \cdot \frac{3}{2} x^{1/2}$$

$$y' = \frac{3}{4} x^{1/2}$$



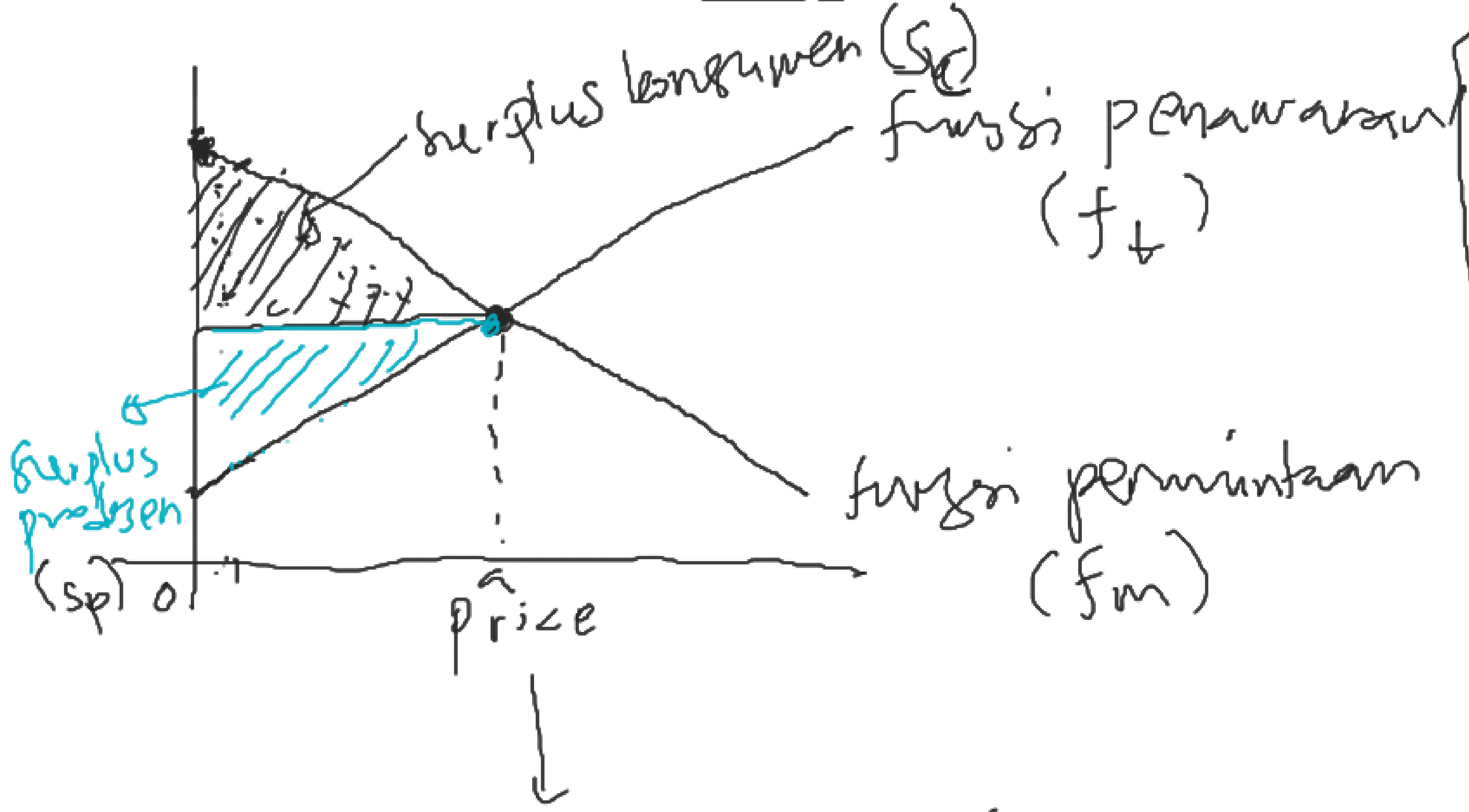
$$\rightarrow l = \int_0^1 \sqrt{1 + \left(\frac{3}{4}x^{\frac{1}{2}}\right)^2}\, dx$$

$$= \int_0^1 \left(1 + \frac{9}{16}x\right)^{1/2} dx$$

$$= \frac{1}{9/16} \frac{\left(1 + \frac{9}{16}x\right)^{3/2}}{3/2} \Bigg]_0^1 = \frac{16}{9} \cdot \frac{2}{3} \left(\left(1 + \frac{9}{16}\right)^{3/2} - (1+0)^{3/2} \right)$$

$$= \frac{32}{27}\left(\left(\frac{25}{16}\right)^{3/2} - 1\right) = \frac{32}{27}\left(\left(\frac{5}{4}\right)^3 - 1\right) = 1,13$$

Surplus produsen dan konsumen



titik a → $f_m = f_t$

Total surplus

$= S_p + S_c$

$= \int_0^a (f_m - f_t)\, dt$