REVIEW MATERI

1] Integral Riemann

Tentukan luas daerah di bawah $y = x + 5$ pada interval $[1, 5]$ menggunakan jumlah Riemann di mana:

a) $n = 4$ partisi $\longrightarrow$ $A(R_4) = \sum_{i=0}^{4-1} f(x_i) . \Delta x$

b) $n \to \infty$

$$\Delta x = \frac{5-1}{4} = \frac{4}{4} = 1$$

$$x_i = 1 + \frac{4}{4} i = 1 + i$$

$$= \sum_{i=0}^{3} f(1+i) . 1$$

$$f(x) = x + 5$$

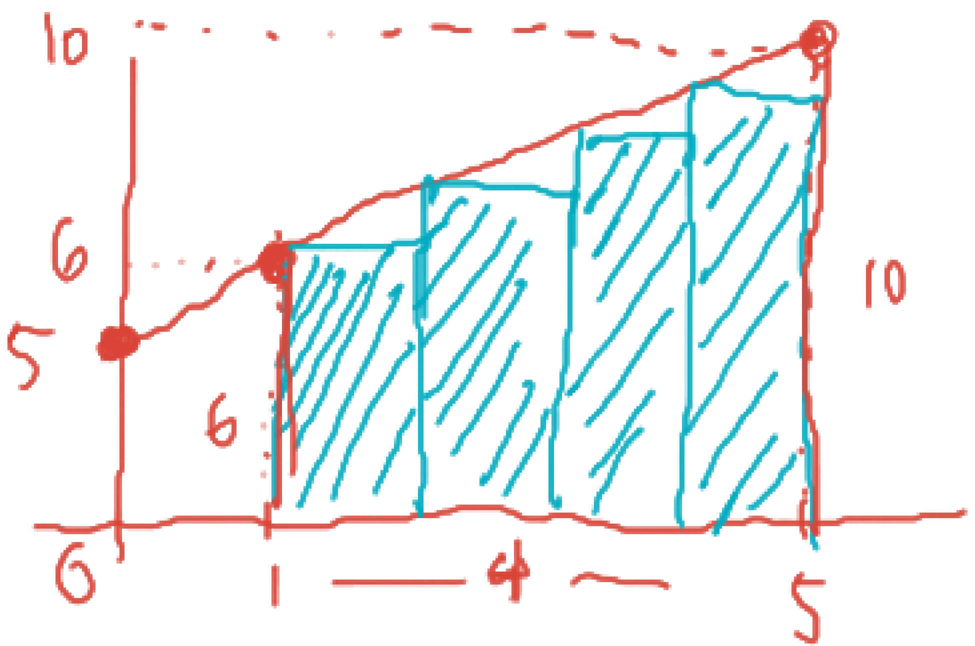
$$f(1+i) = (1+i) + 5$$
$$= i + 6$$

$$= \sum_{i=0}^{3} i + 6$$

$$= 6 + 7 + 8 + 9$$

$$= 30$$

gambar



b) $n \to \infty$, kita buat terlebih dahulu n partisi

$$\Delta x = \frac{5-1}{n} = \frac{4}{n} \qquad x_i = 1 + \frac{4}{n} i$$

$$y = x + 5$$

$$f\left(1 + \frac{4}{n} i\right) = \left(1 + \frac{4i}{n}\right) + 5 = \frac{4}{n} i + 6$$

$$A(R_t) = \sum_{i=0}^{n-1} f\left(1 + \frac{4}{n} i\right) \cdot \frac{4}{n}$$

$$= \sum_{i=0}^{n-1} \left(\frac{4i}{n} + 6\right) \cdot \frac{4}{n}$$

$$= \sum_{i=0}^{n-1} \left(\frac{16i}{n^2} + \frac{24}{n}\right) = \sum_{i=0}^{n-1} \frac{16}{n^2} i + \sum_{i=0}^{n-1} \frac{24}{n}$$

$$= \frac{16}{n^2} \sum_{i=0}^{n-1} i + \frac{24}{n} \sum_{i=0}^{n-1} 1$$

$$= \frac{16}{n^2} (0 + 1 + 2 + \dots + n-1) + \frac{24}{n} (\overbrace{1 + 1 + 1 + \dots + 1}^{n \text{ buah}})$$

$$= \frac{\cancel{16}^{8}}{n^2} \left(\frac{n}{\cancel{2}} (0 + n - 1)\right) + \frac{24}{\cancel{n}} \cdot \cancel{n}$$

$$A(R_n) = \frac{8}{n}(n-1) + 24 = 8 - \frac{8}{n} + 24 = 32 - \frac{8}{n}$$

a) $n=4 \longrightarrow A(R_4) = 32 - \frac{8}{4} = 32 - 2 = 30$

b) $n \to \infty \Longrightarrow A(R_n) = \lim_{n\to\infty} 32 - \frac{8}{n} = 32 - \frac{8}{\infty} = 32 - 0 = 32$

Integral $= \int_1^5 (x+5)\,dx$

$$= \frac{x^2}{2} + 5x\Big]_1^5 = \frac{25}{2} + 25 - \left(\frac{1}{2} + 5\right)$$

$$= \frac{24}{2} + 20 = 12 + 20 = 32$$

2] Teorema Dasar Kalkulus

a) $\int_1^2 \left[\frac{1}{2x^2} - \frac{3x}{4} + \frac{1}{\sqrt{2x}} - 1000\right] dx = \frac{1}{2}\frac{x^{-1}}{-1} - \frac{3}{4}\frac{x^2}{2} + \frac{1}{\sqrt{2}}\frac{x^{\frac{1}{2}}}{\frac{1}{2}} - 1000x\Big]_1^2$

$\frac{1}{2}x^{-2} - \frac{3}{4}x + \frac{1}{\sqrt{2}}x^{-\frac{1}{2}} - 1000$

$= -\frac{1}{2x} - \frac{3}{8}x^2 + \frac{2}{\sqrt{2}}\sqrt{x} - 1000x\Big]_1^2$

$= -\frac{1}{4} - \frac{3}{2} + 2 - 2000 -$

$\left(-\frac{1}{2} - \frac{3}{8} + \frac{2}{\sqrt{2}} - 1000\right)$

$= -1000,28$

b) $\frac{d}{du}\int_{\sqrt{e}}^{\sqrt{u}} \frac{1}{2}v \, dv$

$u^{\frac{1}{2}}$

$\frac{dy}{du} = \frac{1}{2}u^{-\frac{1}{2}}$

$= \frac{1}{2}\sqrt{u} \cdot \frac{1}{2\sqrt{u}}$

$= \frac{1}{4}$

c) $\frac{d}{du}\int_{\sqrt{u}}^{\sqrt{e}} \frac{1}{2}v \, dv = -\frac{d}{du}\int_{\sqrt{e}}^{\sqrt{u}} \frac{1}{2}v \, dv$

$= -\frac{1}{4}$

3] Metode substitusi dan Parsial

a) $\int x\sqrt{1-2x}\,dx$

$u = x$	$dv = (1-2x)^{1/2}\,dx$
$du = -1$	$v = \frac{1}{-2}\frac{(1-2x)^{3/2}}{3/2}$
0	$-\frac{1}{3}\left(\frac{1}{-2}\frac{(1-2x)^{5/2}}{5/2}\right)$

$$= -\frac{x}{3}(1-2x)^{3/2} - \frac{1}{15}(1-2x)^{5/2} + C$$

b) $\int x\sqrt{\underbrace{1-2x^2}_{u}}\,dx$

$u = 1-2x^2$

$du = -4x\,dx$

$-\frac{1}{4x}\,du = dx$

$$= \int \cancel{x}\,u^{1/2}\left(-\frac{1}{4\cancel{x}}\right)du$$

$$= -\frac{1}{4}\int u^{1/2}\,du$$

$$= -\frac{1}{4}\cdot\frac{2}{3}u^{3/2} + C$$

$$= -\frac{1}{6}(1-2x^2)^{3/2} + C$$

4) Luas daerah dan Volume diputar thd sumbu-x

Batas : $y = x^2+2x+1$, $y = 7-3x$, sumbu-y

• $y=0 \Rightarrow 0 = x^2+2x+1$

$0 = (x+1)(x+1)$

$\boxed{x=-1}$

• $x^2 \rightarrow \cup$

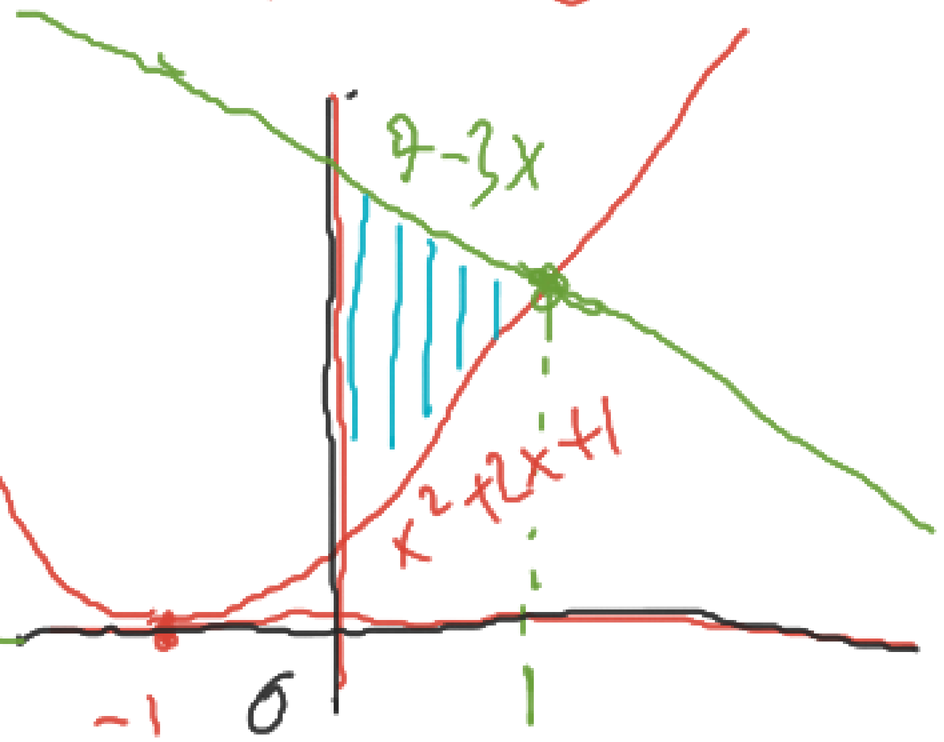
$x^2+2x+1 = 7-3x$

$x^2+5x-6=0$

$(x+6)(x-1)=0$

$x=-6 \quad x=1$

• $-3x \rightarrow \diagdown$



a) Luas daerah

$$L = \int_0^1 [(7-3x) - (x^2+2x+1)]\,dx$$

$$= 7x - \frac{3}{2}x^2 - \frac{x^3}{3} - \frac{2x^2}{2} - x \Big]_0^1$$

$$= 7 - \frac{3}{2} - \frac{1}{3} - 1 - 1 - 0$$

$$\approx 5{,}17$$

b) Volume diputar sb-x 360°

$$L = \pi \int_0^1 [(7-3x)^2 - (x^2+2x+1)^2]\,dx$$

$$= \pi \int_0^1 [49 - 42x + 9x^2 - x^4 - 4x^2 - 1 - 4x^3 - 2x^2 - 4x]\,dx$$

5] Volume diputar thd sumbu-y (metode selimut tabung)

Batas : $y = x^2+2x+1$, $y = 7 - x$. sumbu-x , sumbu-y , pada $[0, 2]$

Volume diputar thd sumbu-y 360°

metode selimut tabung : $V = 2\pi \int_a^b x \cdot f(x)\, dx$

$$V = 2\pi \left[\int_0^1 x \cdot (x^2+2x+1)\, dx + \int_1^2 x \cdot (7-x)\, dx \right]$$

$$= 2\pi \left[\int_0^1 (x^3 + 2x^2 + x)\, dx + \int_1^2 (7x - x^2)\, dx \right]$$

$$= 2\pi \left[\frac{x^4}{4} + 2\frac{x^3}{3} + \frac{x^2}{2} \right]_0^1 + 7\frac{x^2}{2} - \frac{x^3}{3} \Big]_1^2$$

$$= 2\pi \left[\frac{1}{4} + \frac{2}{3} + \frac{1}{2} - 0 + 14 - \frac{8}{3} - \frac{7}{2} + \frac{1}{3} \right]$$

$$= 2\pi\,(9{,}58) = 60{,}19$$

Luas :

$$L = \int_0^1 (x^2+2x+1)\, dx + \int_1^2 (7-x)\, dx$$

$= \ldots$

6] Fungsi Transendens

a) $\int 3^{\sqrt{e^x}}\, e^{x/2}\, dx = \int 3^u\, \cancel{e^{x/2}}\; 2\cdot\frac{1}{\cancel{e^{\frac{1}{2}x}}}\, du = 2\int 3^u\, du$

$u = \sqrt{e^x} = e^{\frac{1}{2}x}$

$du = e^{\frac{1}{2}x}\cdot 1\cdot\frac{1}{2}\, dx$

$2\cdot\frac{1}{e^{\frac{1}{2}x}}\, du = dx$

$= 2\cdot\frac{3^u}{\ln 3} + C$

$= \frac{2}{\ln 3}\, 3^{\sqrt{e^x}} + C$

b) y' dari $y = \log(\underbrace{\ln 10^{e^x}}_{e^x \ln 10})$

$y' = \frac{1}{\ln 10^{e^x}\, \cancel{\ln 10}}\cdot e^x\, \cancel{\ln 10}$

$= \frac{1}{\cancel{e^x}\,\ln 10}\cdot \cancel{e^x} = \frac{1}{\ln 10} = \log e$

c) Tentukan kemiringan fungsi invers dari $y = \frac{1}{2}\cos 2x$ saat $y = \frac{1}{2}$

$$y' = \frac{1}{\cancel{2}}(-\sin 2x \cdot \cancel{2})$$

$$\boxed{y' = -\sin 2x}$$

$$(f^{-1})'\left(\tfrac{1}{2}\right) = \frac{1}{f'(x)}$$

$$= \frac{1}{-\sin 2x}$$

$$= \frac{1}{-\sin 0^\circ} = \frac{1}{-1} = -1$$

$$\frac{1}{\cancel{2}} = \frac{1}{\cancel{2}}\cos 2x$$

$$1 = \cos 2x$$

$$2x = 0^\circ$$

7 Pertumbuhan dan Peluruhan eksponen

$y_0 = 1$ Milyar

$i = 5\%$ p.a. terkonversi setiap saat

Tentukan ;

a) y saat 10 tahun

$$y = 1 . e^{5\% . 10}$$

$$\approx 1{,}649 \text{ Miliar}$$

b) Kapan $y = 2$ Miliar

$$y = y_0 \, e^{it}$$

$$2 = 1 \, e^{5\% t}$$

$$2 = e^{5\% t}$$

$$\ln 2 = \cancel{\ln e}^{5\% t}$$

$$\ln 2 = 5\% t \rightarrow t = \frac{\ln 2}{5\%} = 13{,}86 \text{ tahun}$$

$\times 12$

13,86 tahun $\approx$ 13 tahun 10 bulan