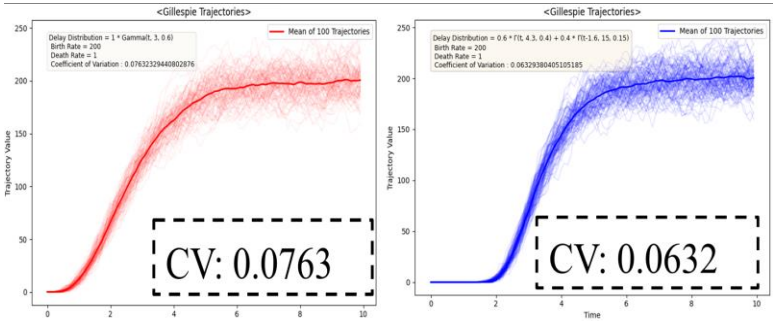
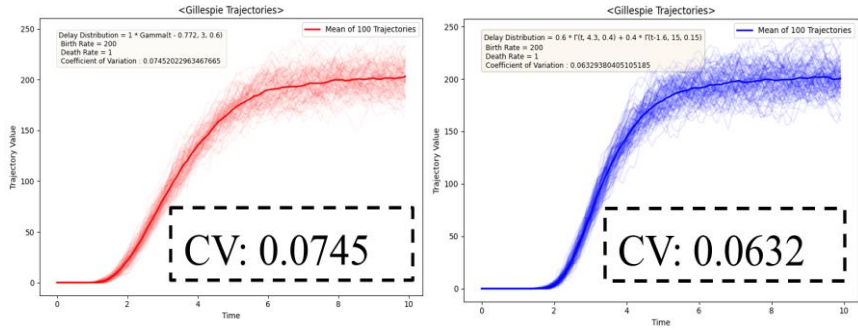


# Review: No significant difference between unimodal CV vs bimodal CV at single timepoint.

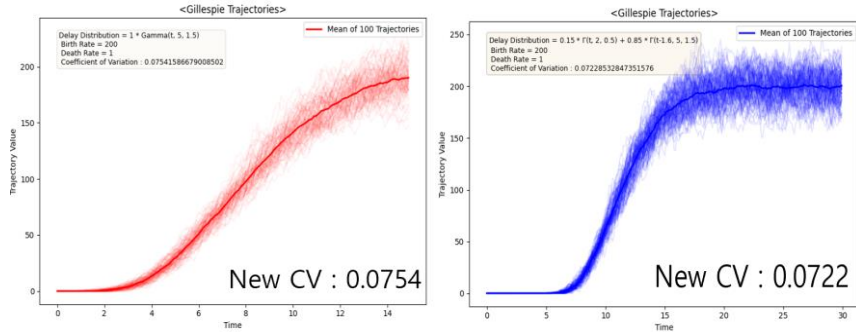
Original [Paper]



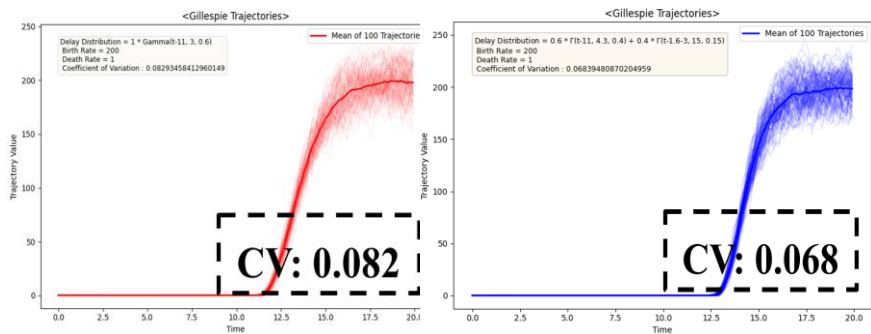
Same Mean



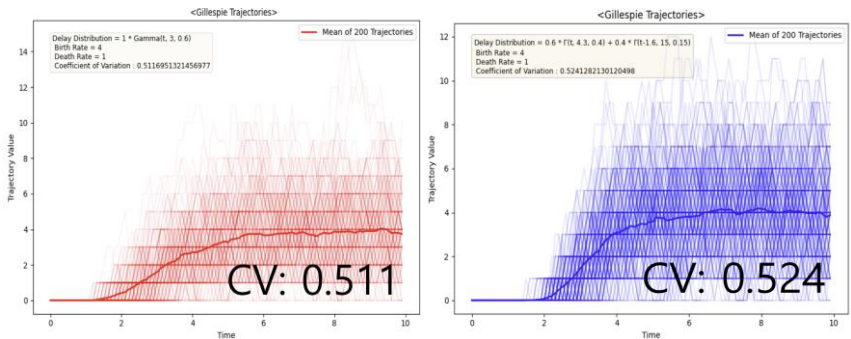
Increased Variance



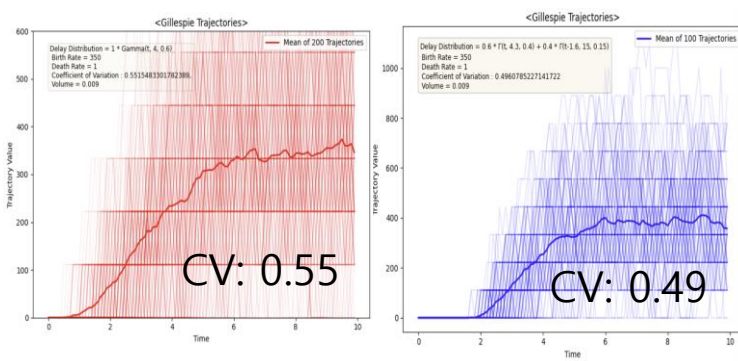
Horizontal Shift



Lower Birth/Death Ratio (4:1)

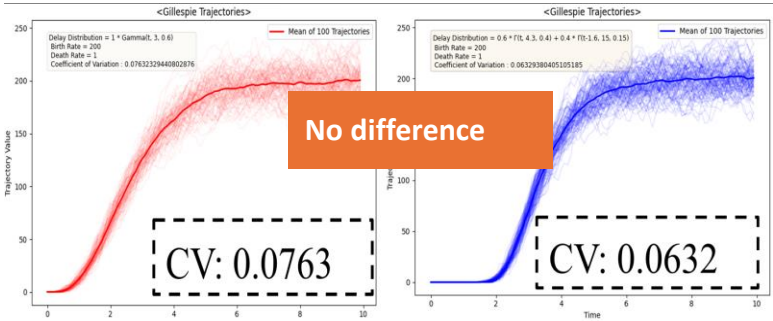


Lower Volume (1 → 0.009)

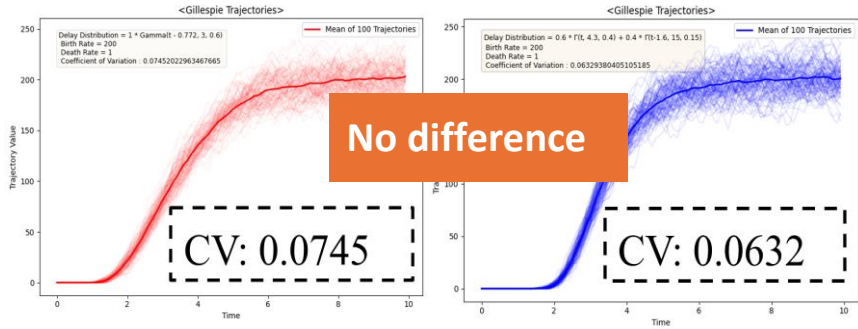


# Review: No significant difference between unimodal CV vs bimodal CV at single timepoint.

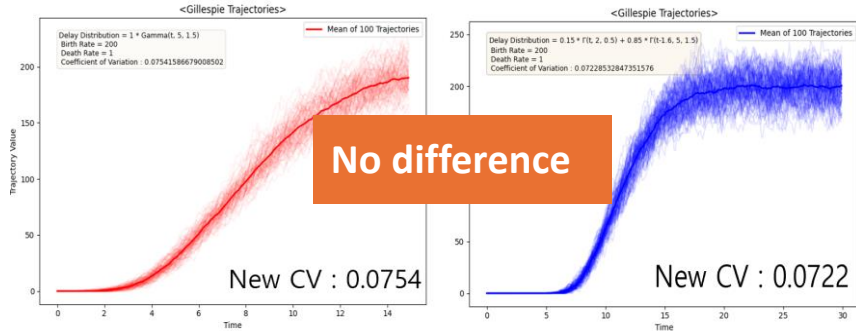
Original [Paper]



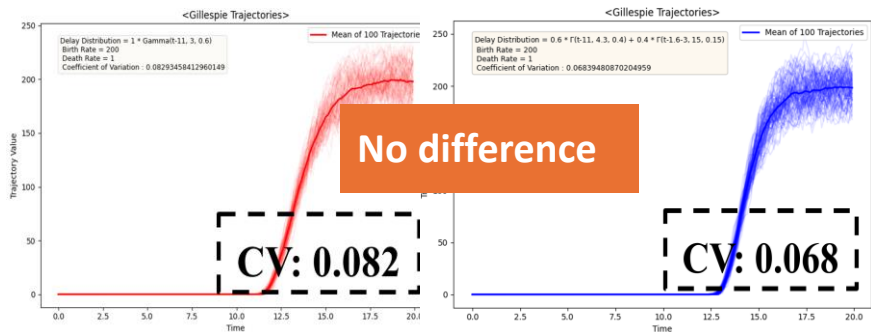
Same Mean



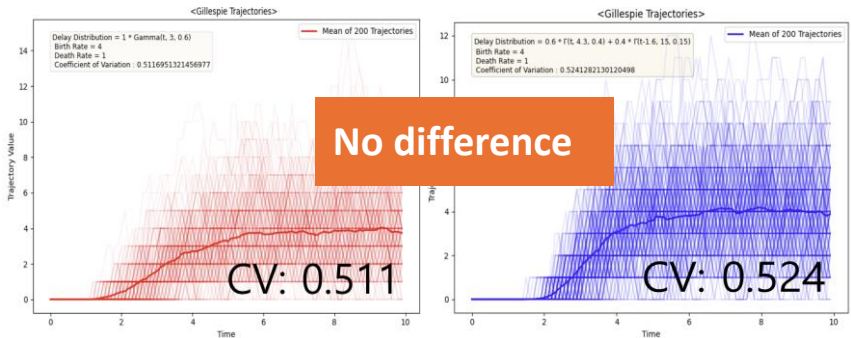
Increased Variance



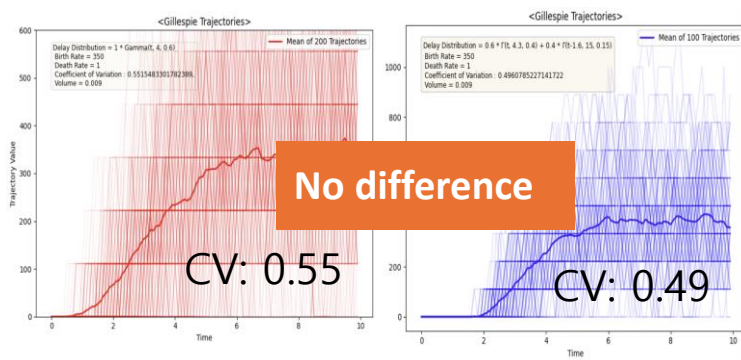
Horizontal Shift



Lower Birth/Death Ratio (4:1)

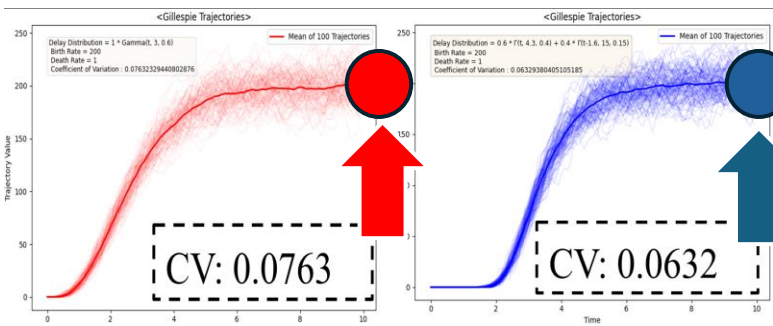


Lower Volume (1 → 0.009)

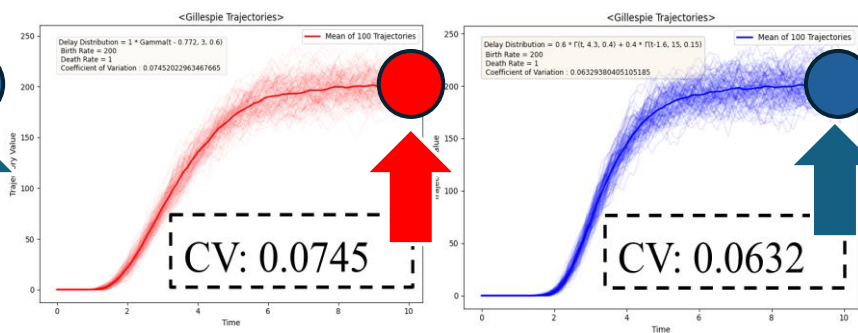


# Review: No significant difference between unimodal CV vs bimodal CV at single timepoint.

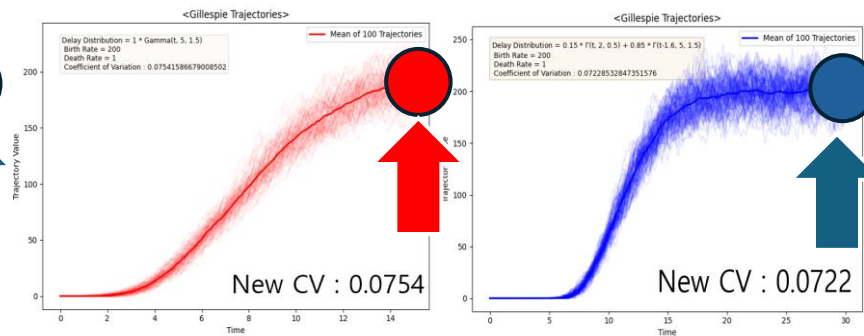
Original [Paper]



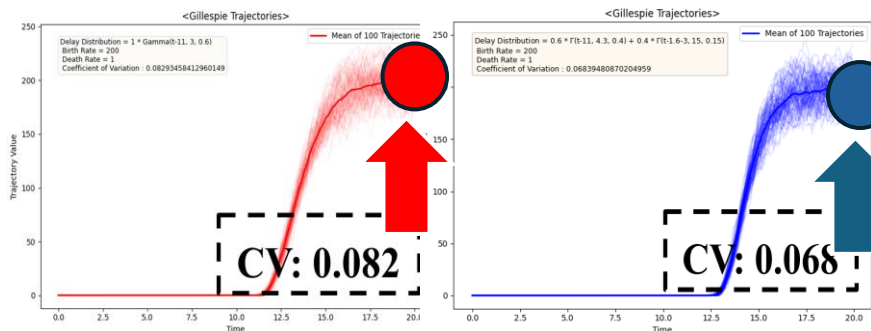
Same Mean



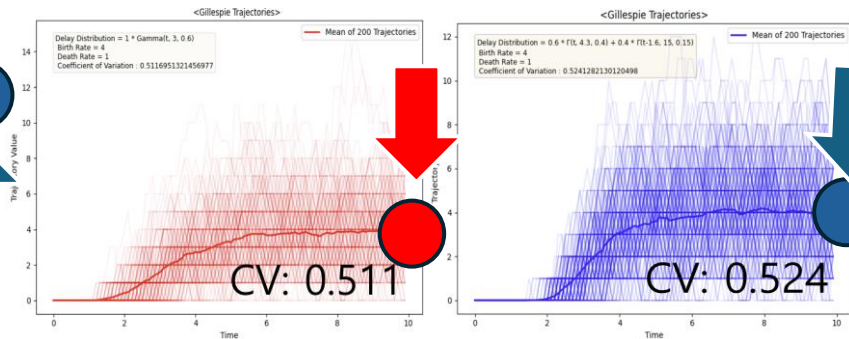
Increased Variance



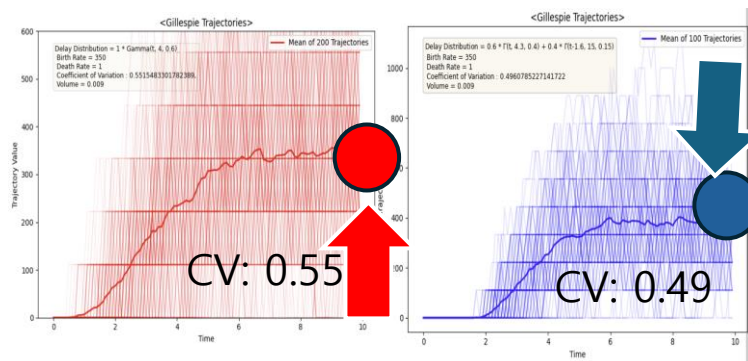
Horizontal Shift



Lower Birth/Death Ratio (4:1)



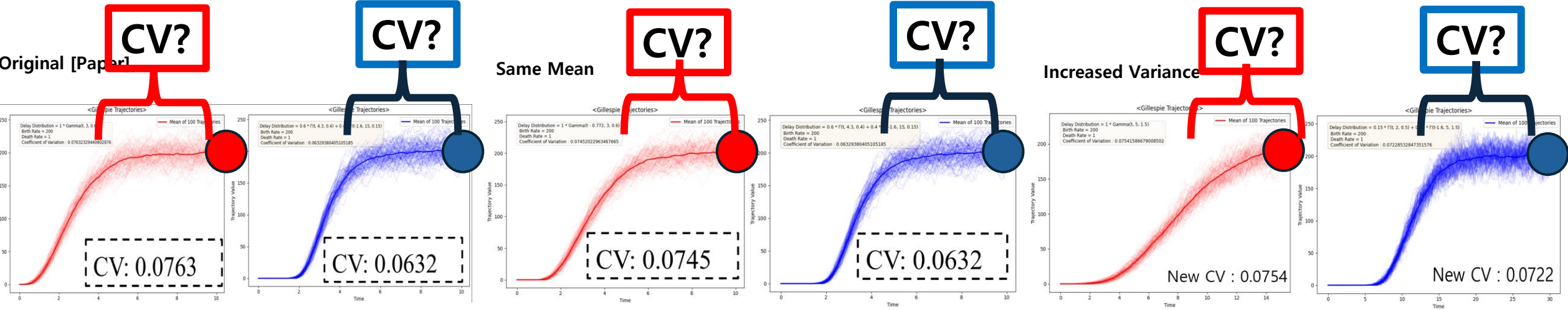
Lower Volume (1 → 0.009)





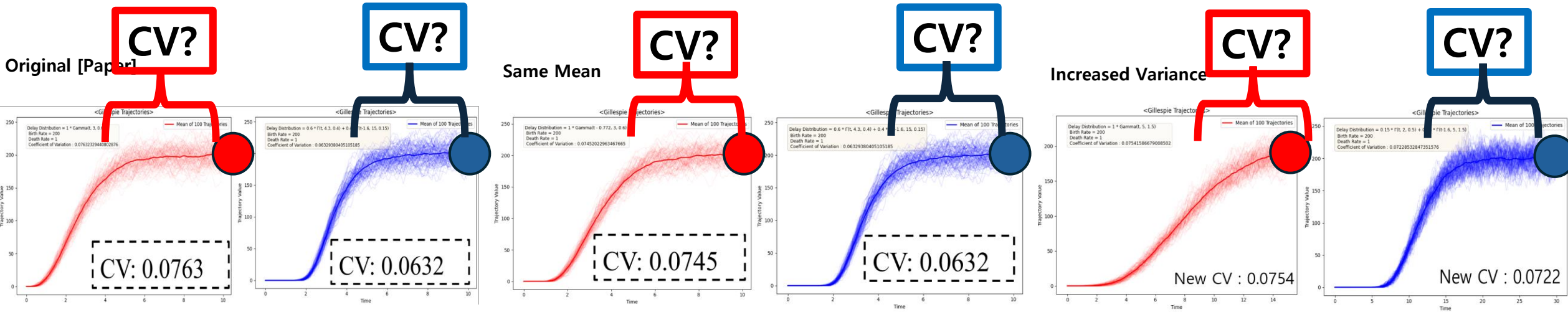
Review: No significant difference between unimodal CV vs bimodal CV at single timepoint.

[ Other CV? (Steady State) ]

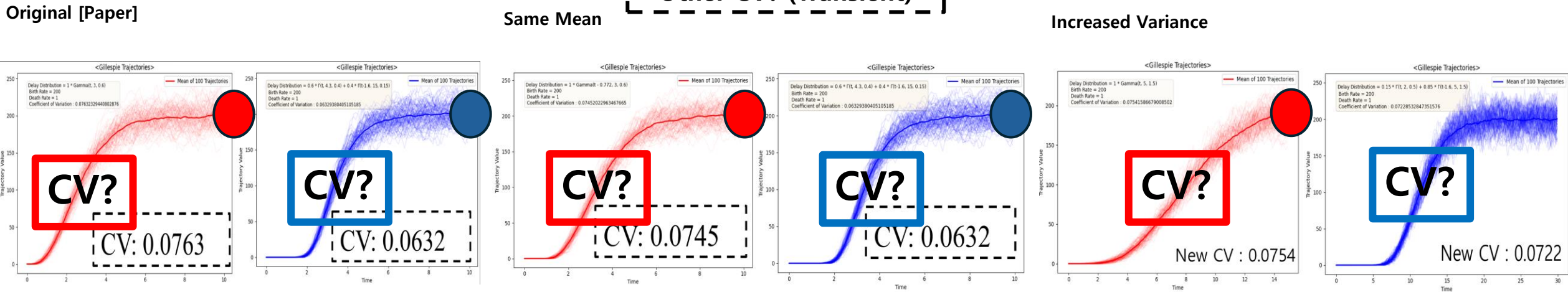


Review: No significant difference between unimodal CV vs bimodal CV at single timepoint.

Other CV? (Steady State)

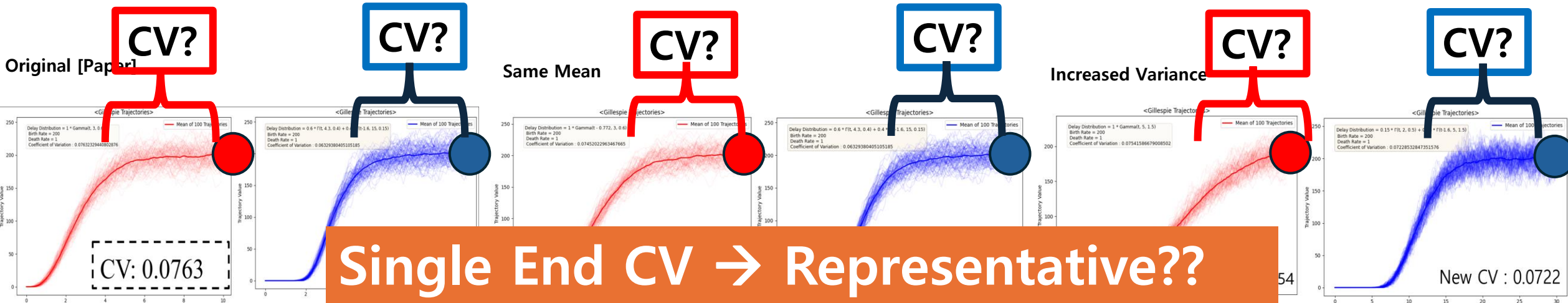


Other CV? (Transient)

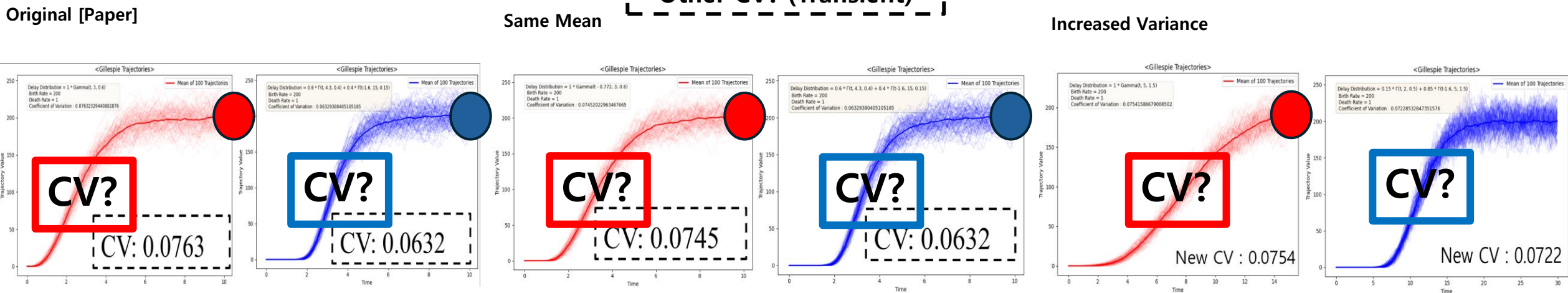


Review: No significant difference between unimodal CV vs bimodal CV at single timepoint.

Other CV? (Steady State)



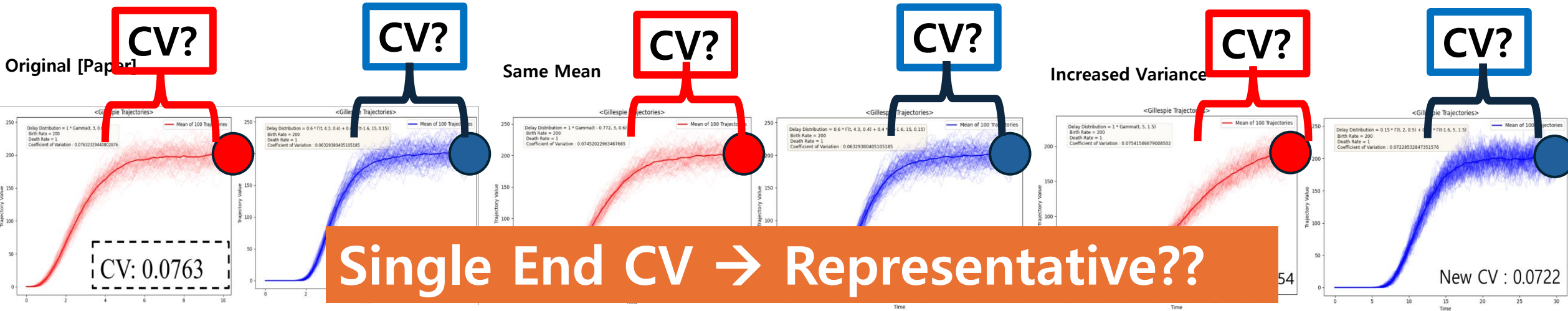
Other CV? (Transient)





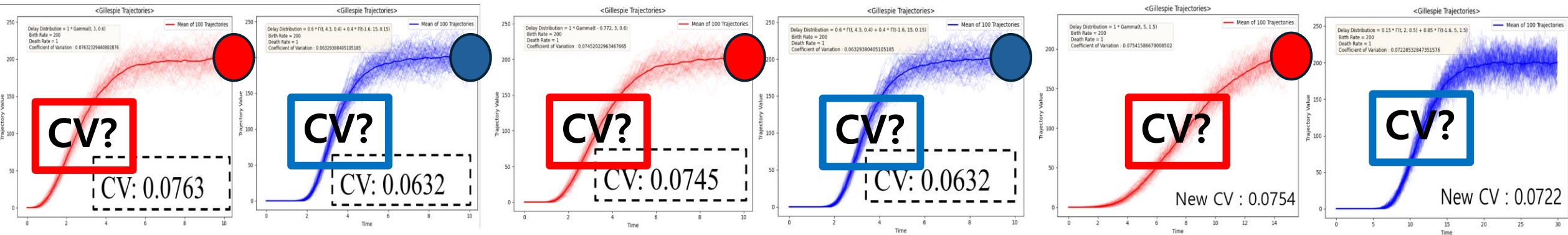
Review: No significant difference between unimodal CV vs bimodal CV at single timepoint.

[ Other CV? (Steady State) ]

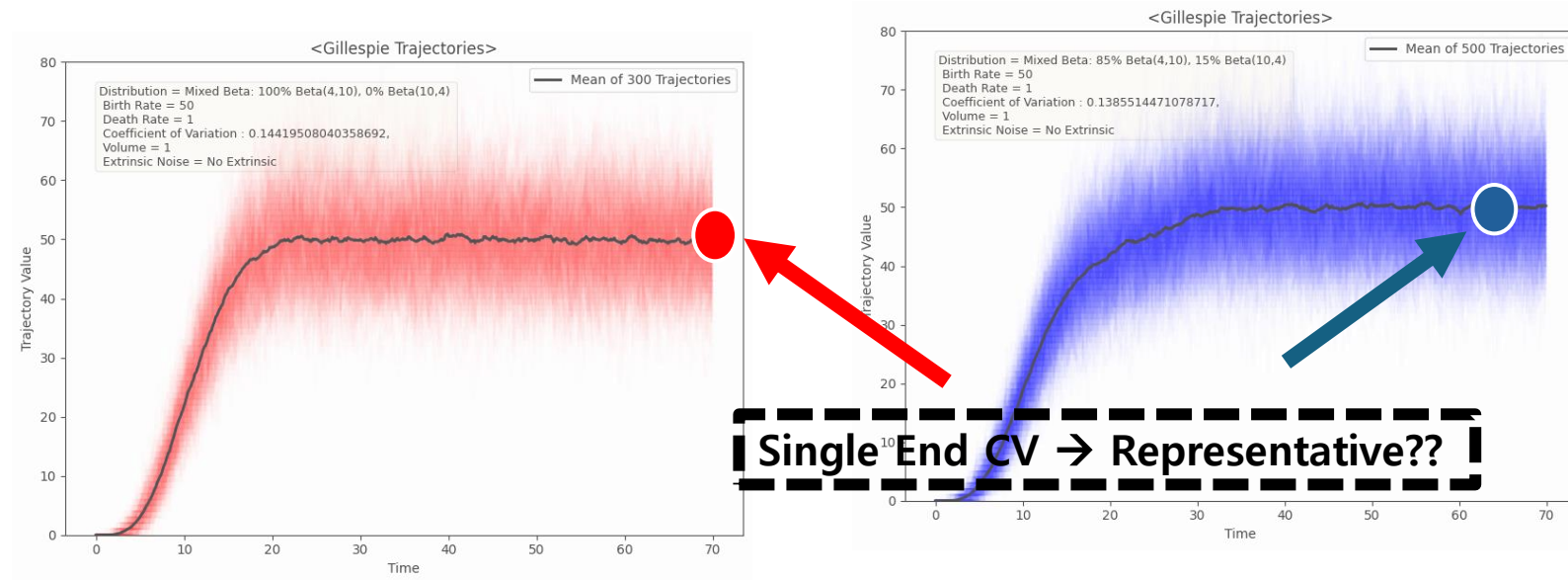


Original [Paper]

Let's Measure Many CVs / graph

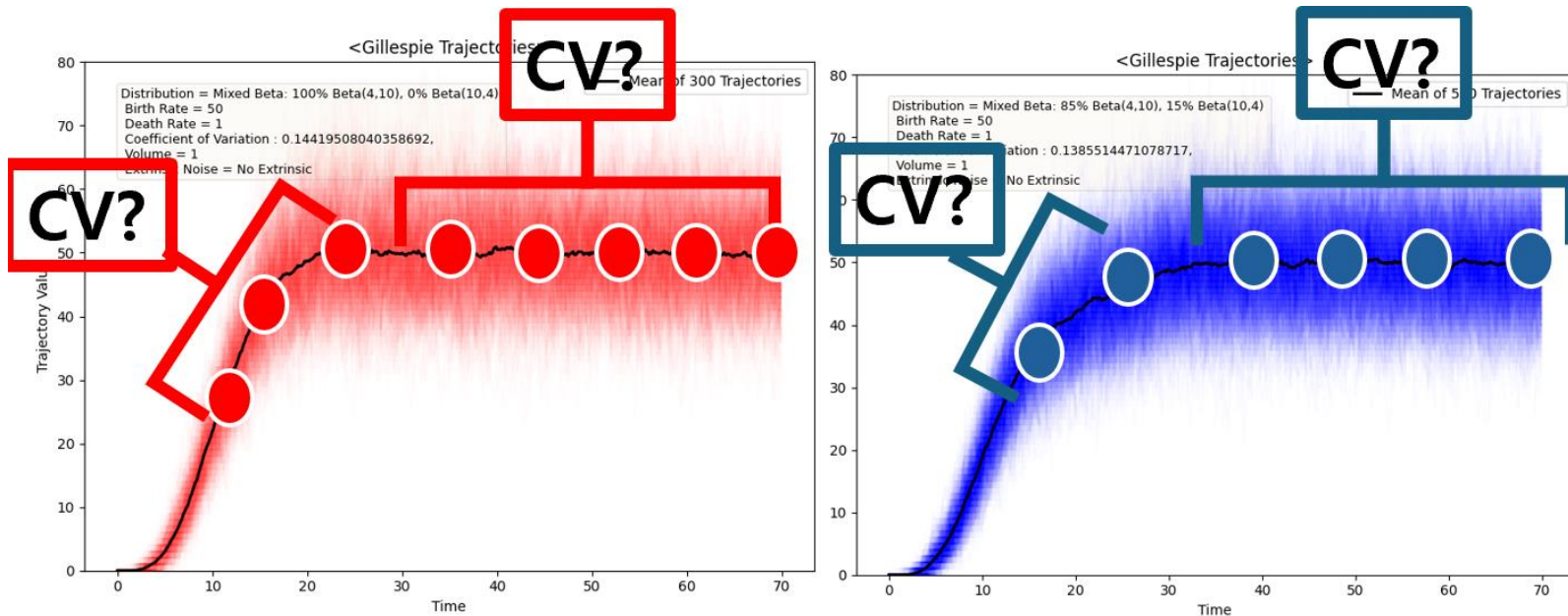
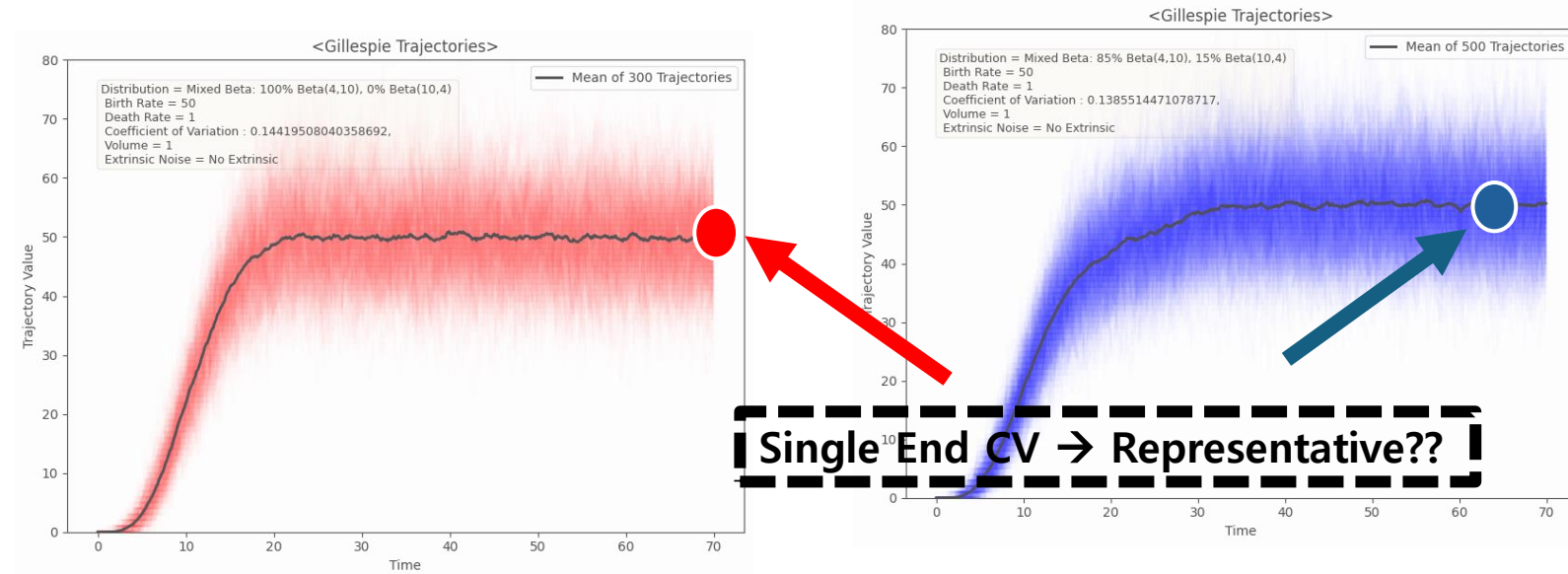


CV uniformly decreases to 0 as trajectories approach to steady-state.

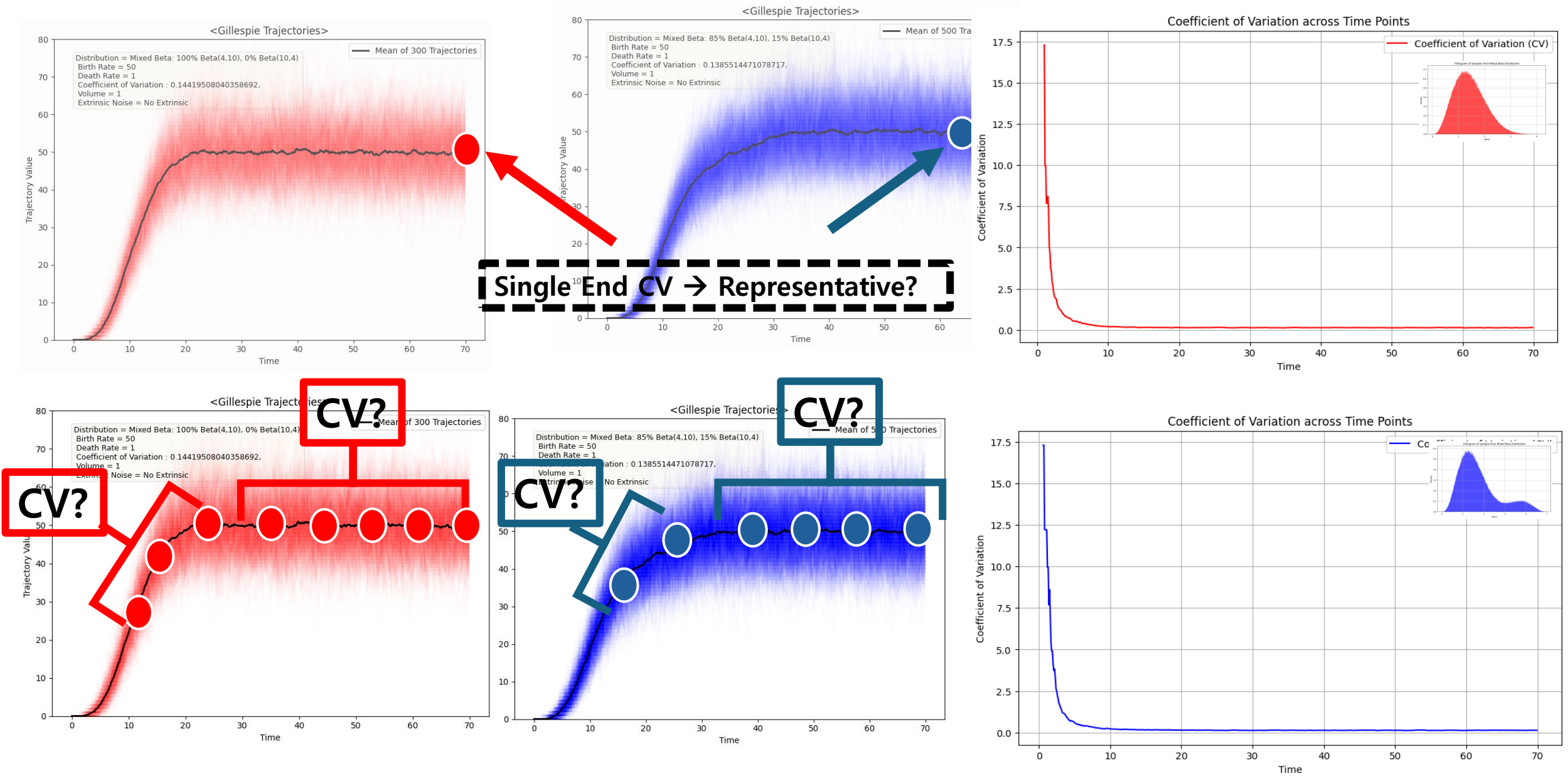




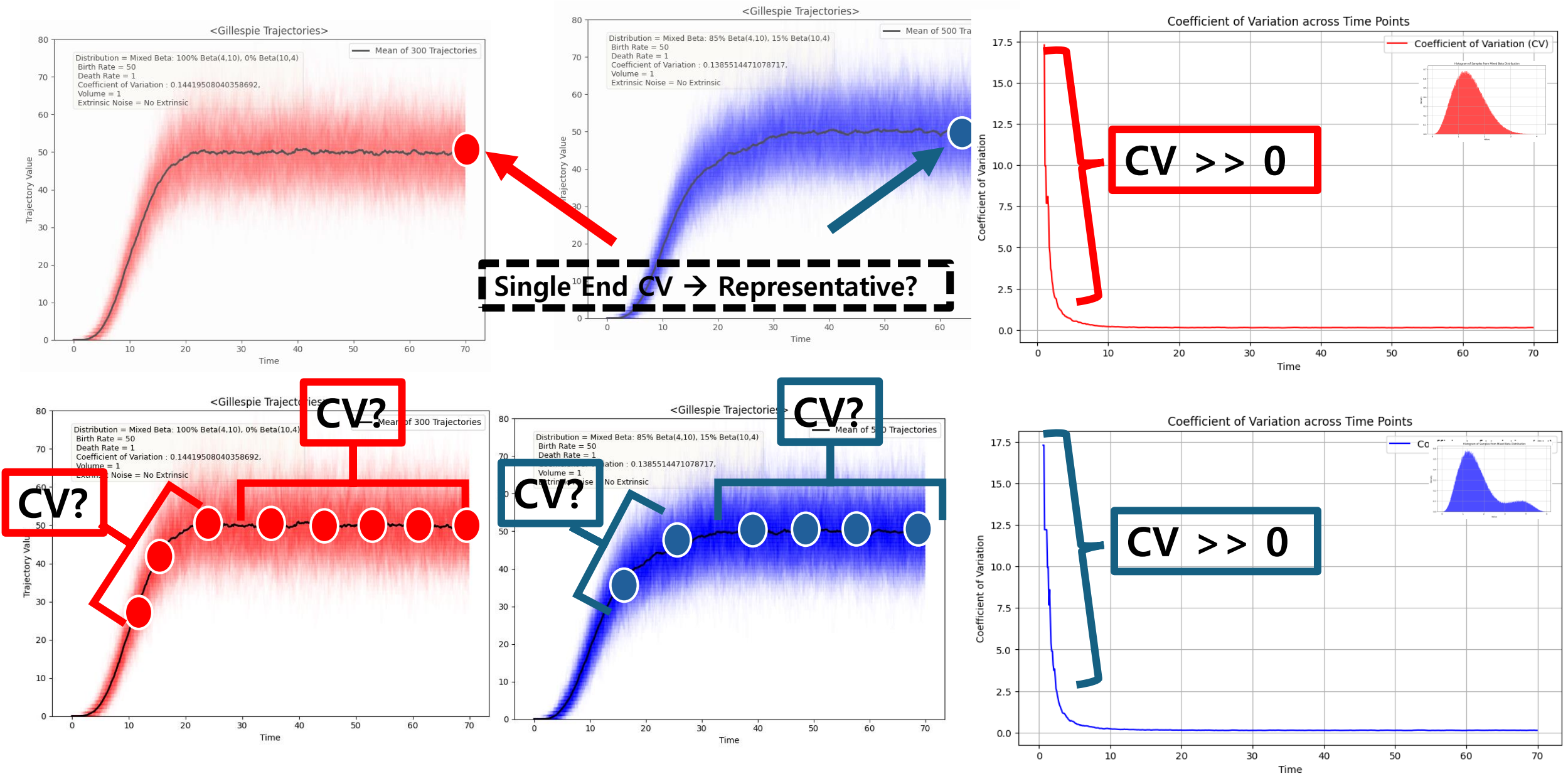
CV uniformly decreases to 0 as trajectories approach to steady-state.



# CV uniformly decreases to 0 as trajectories approach to steady-state.

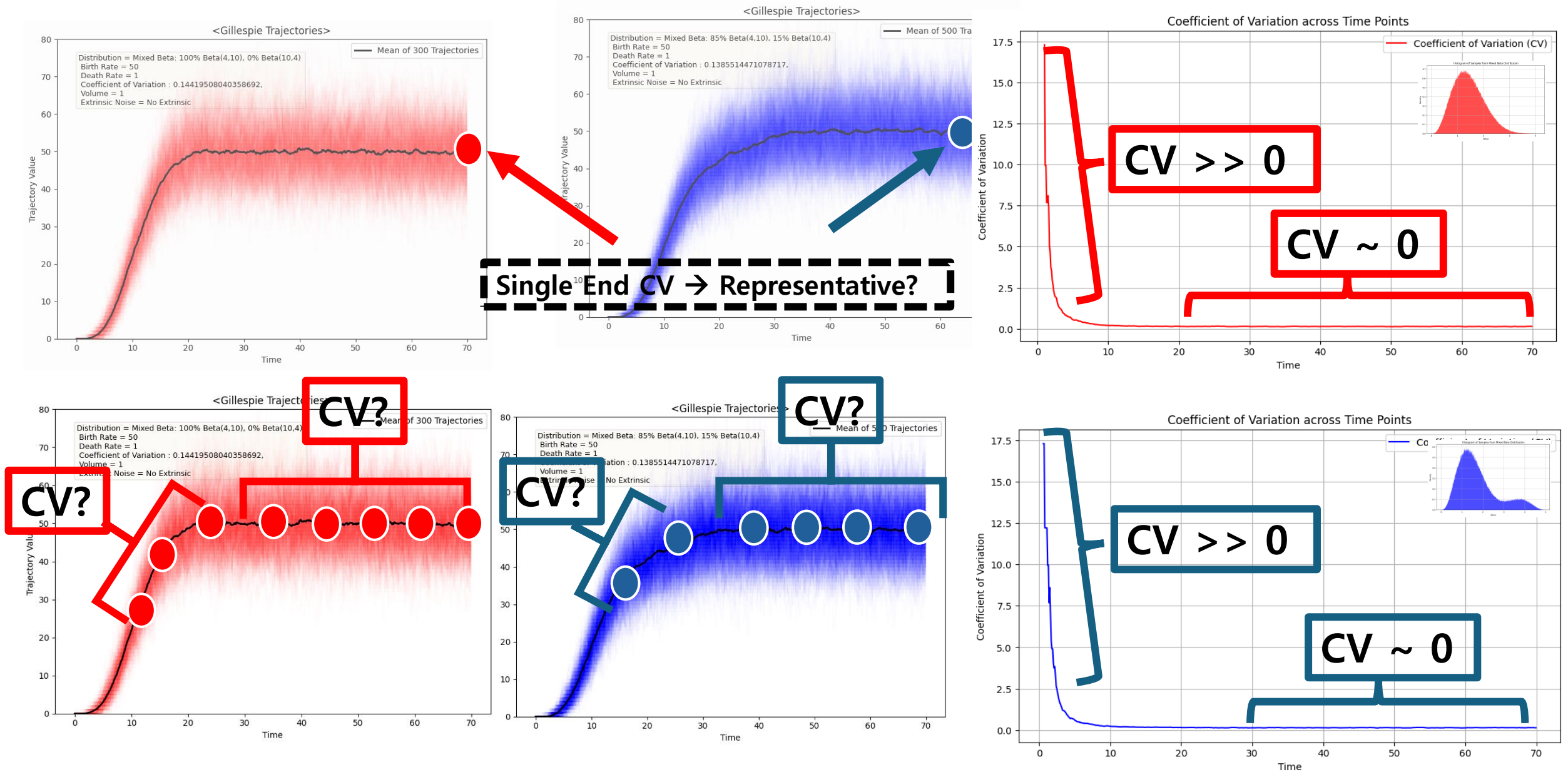


# CV uniformly decreases to 0 as trajectories approach to steady-state.

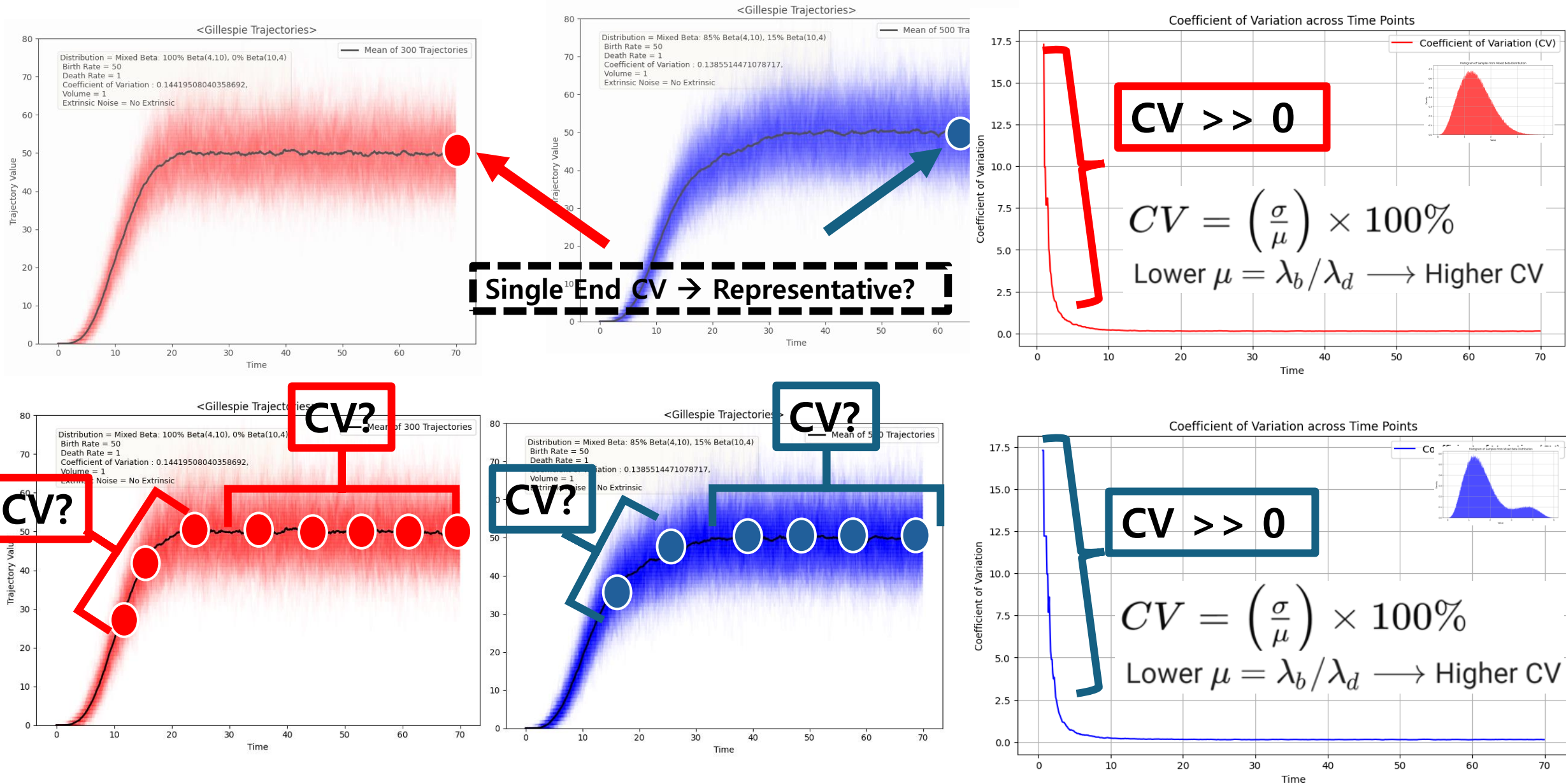




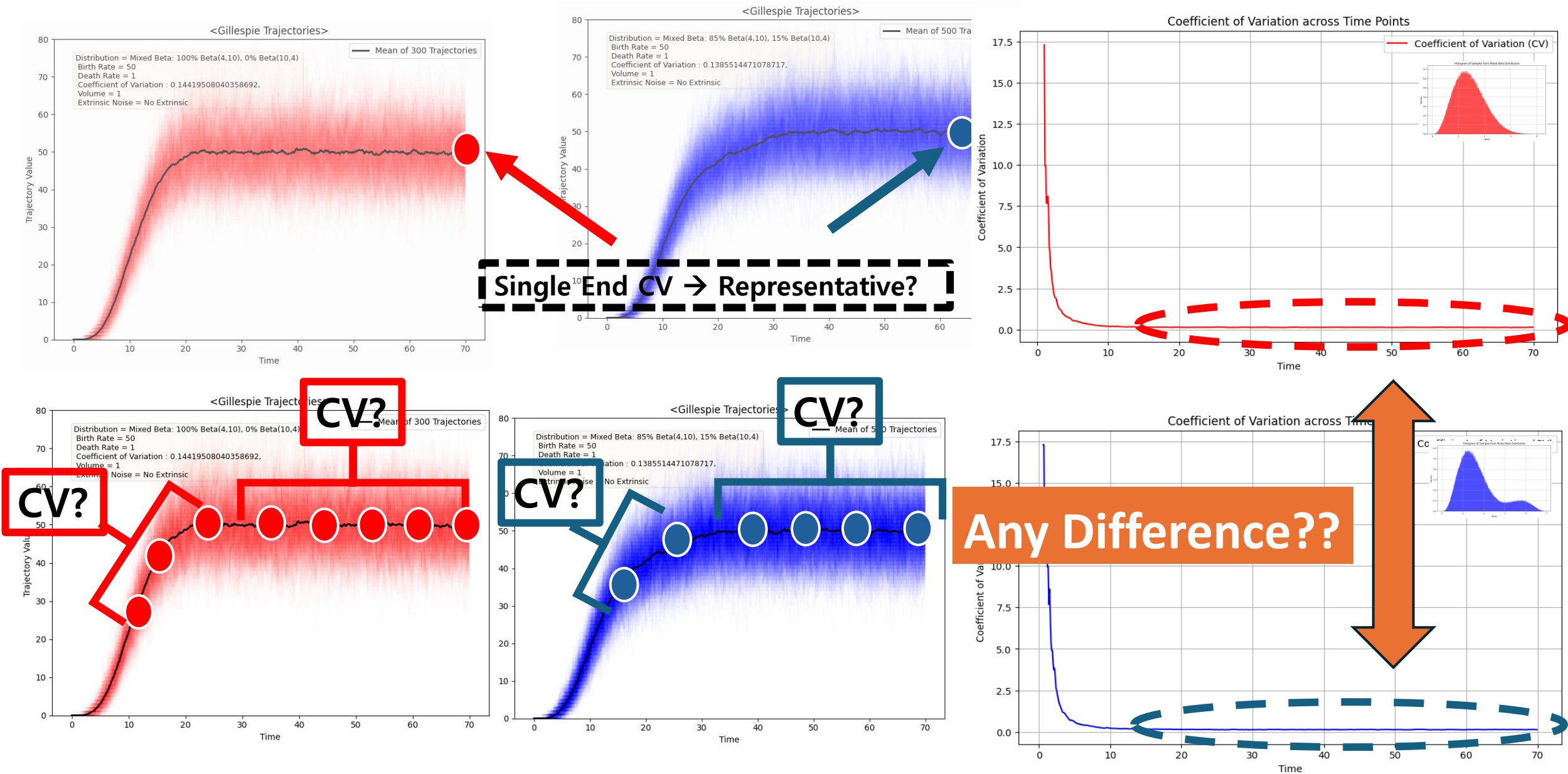
Both unimodal CV and bimodal CV uniformly decrease to 0 as trajectories approach to steady-state.



Both unimodal CV and bimodal CV uniformly decrease to 0 as trajectories approach to steady-state.

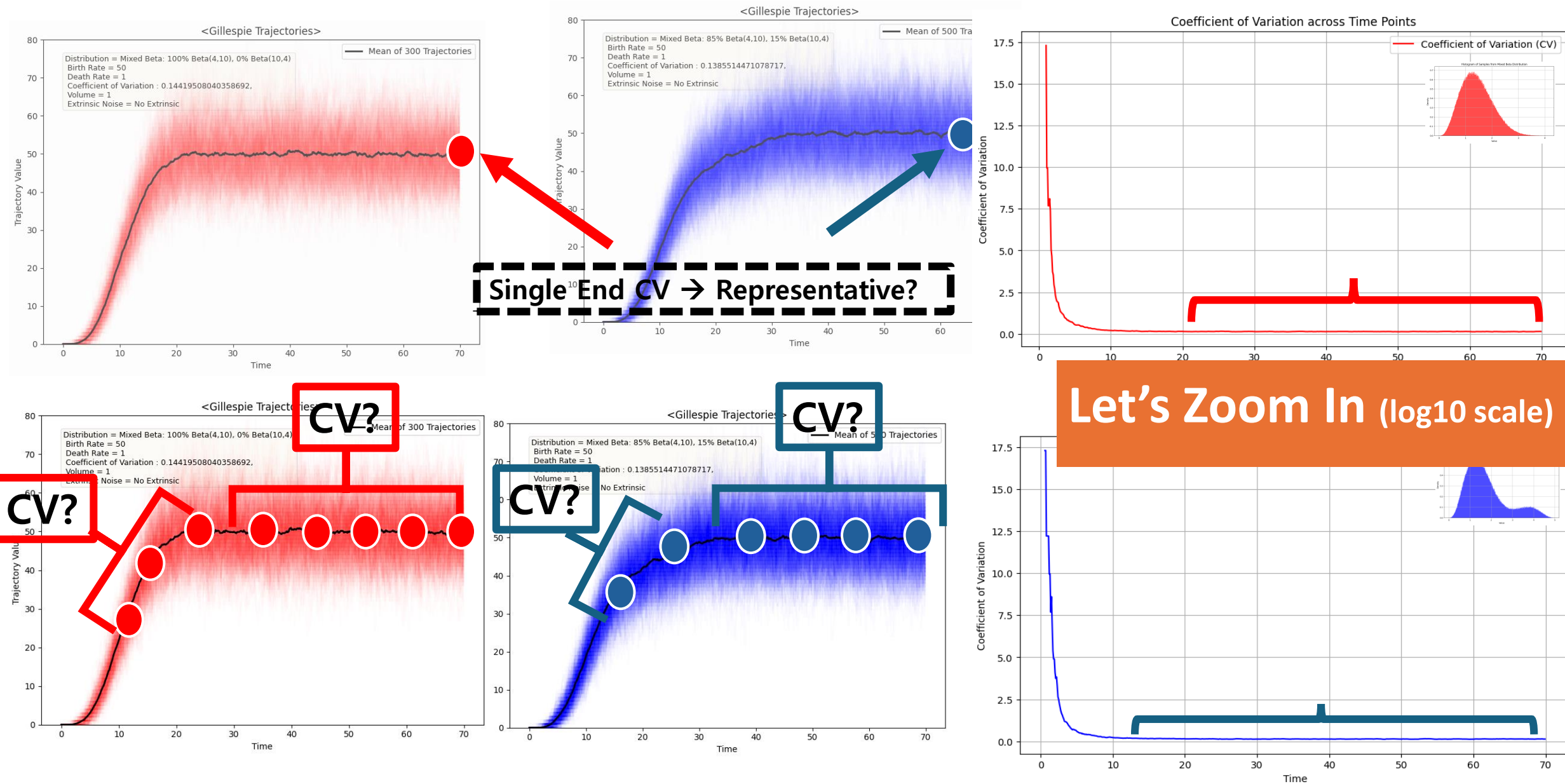


Both unimodal CV and bimodal CV uniformly decrease to 0 as trajectories approach to steady-state.

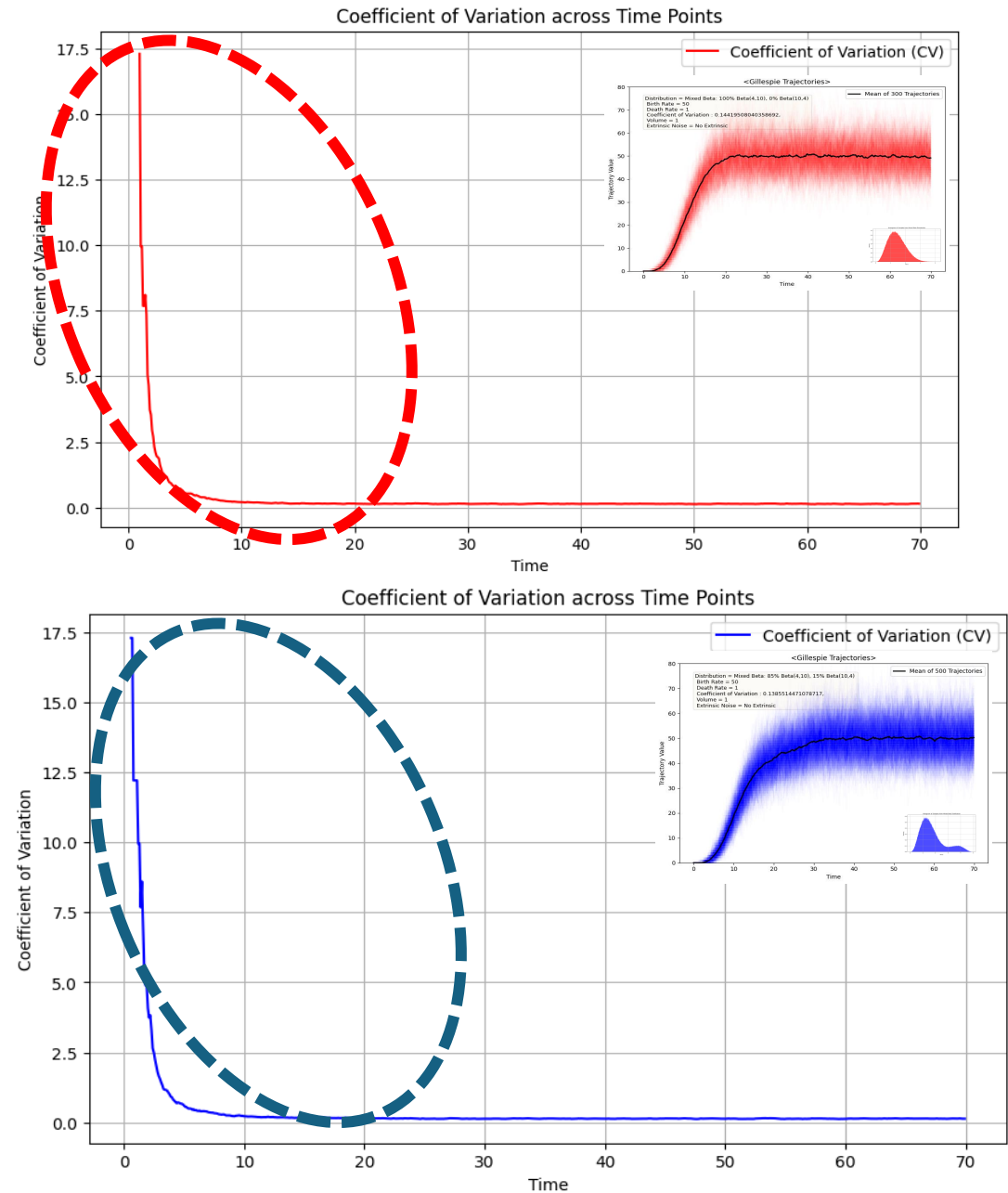




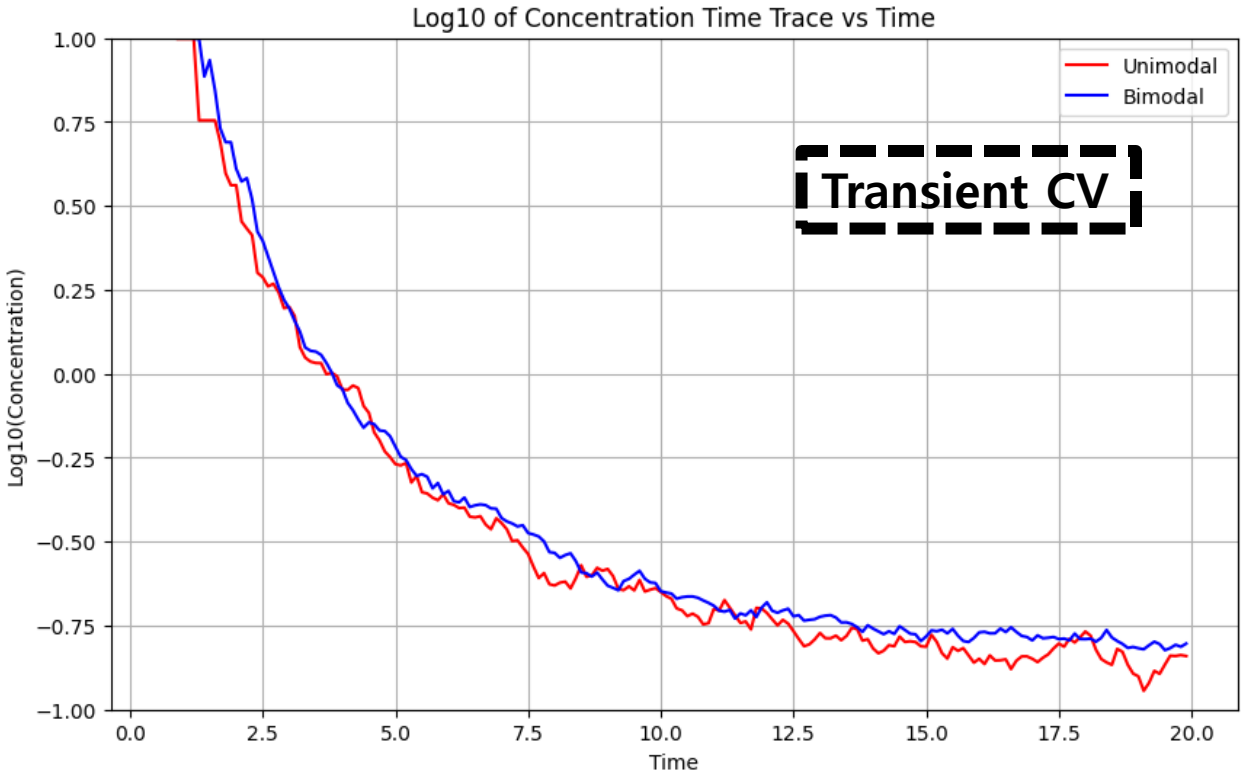
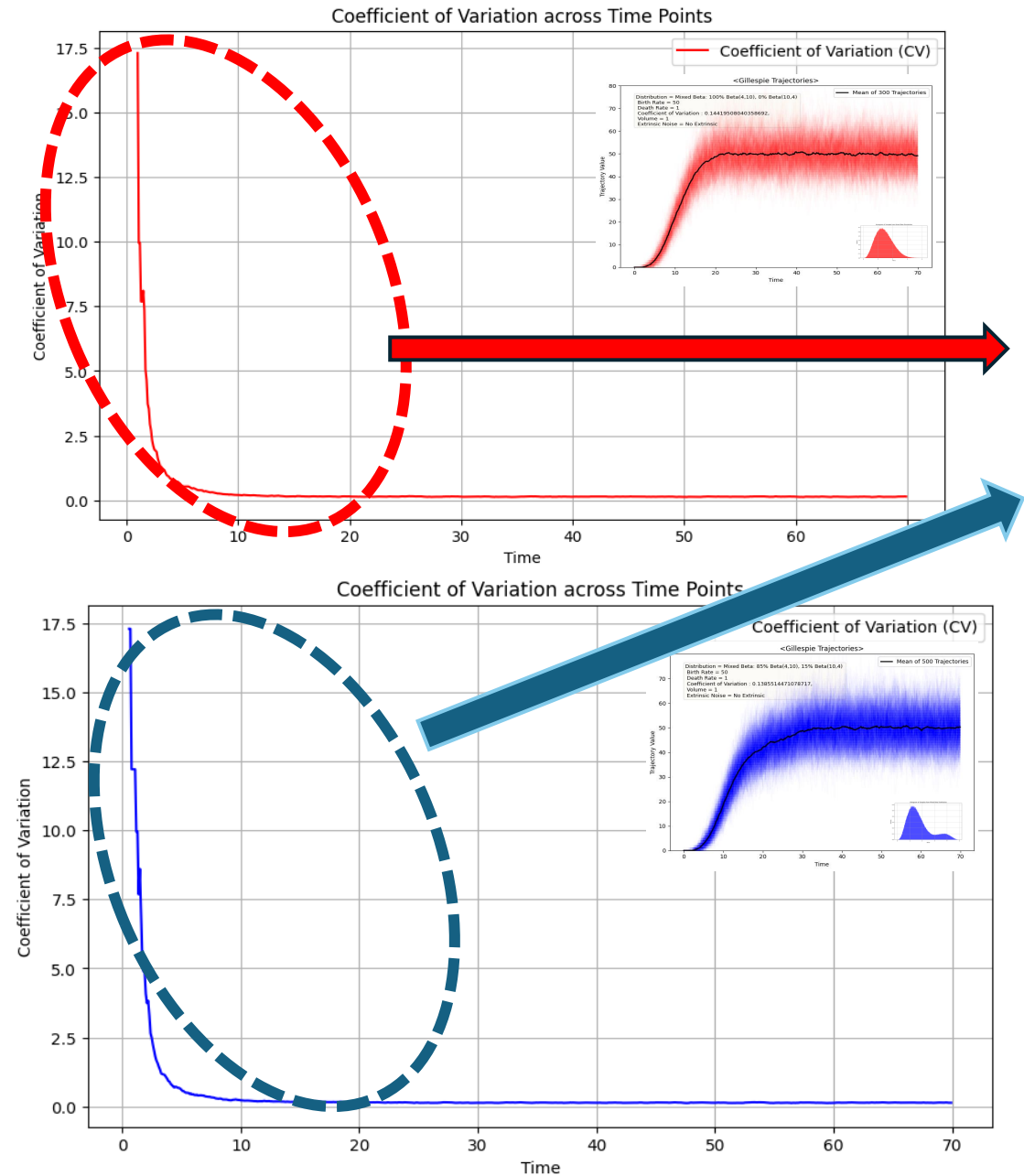
Both unimodal CV and bimodal CV uniformly decrease to 0 as trajectories approach to steady-state.



CV time trace repeatedly alternates between unimodal CV > bimodal CV, and unimodal CV < bimodal CV, in both transient and steady-state.

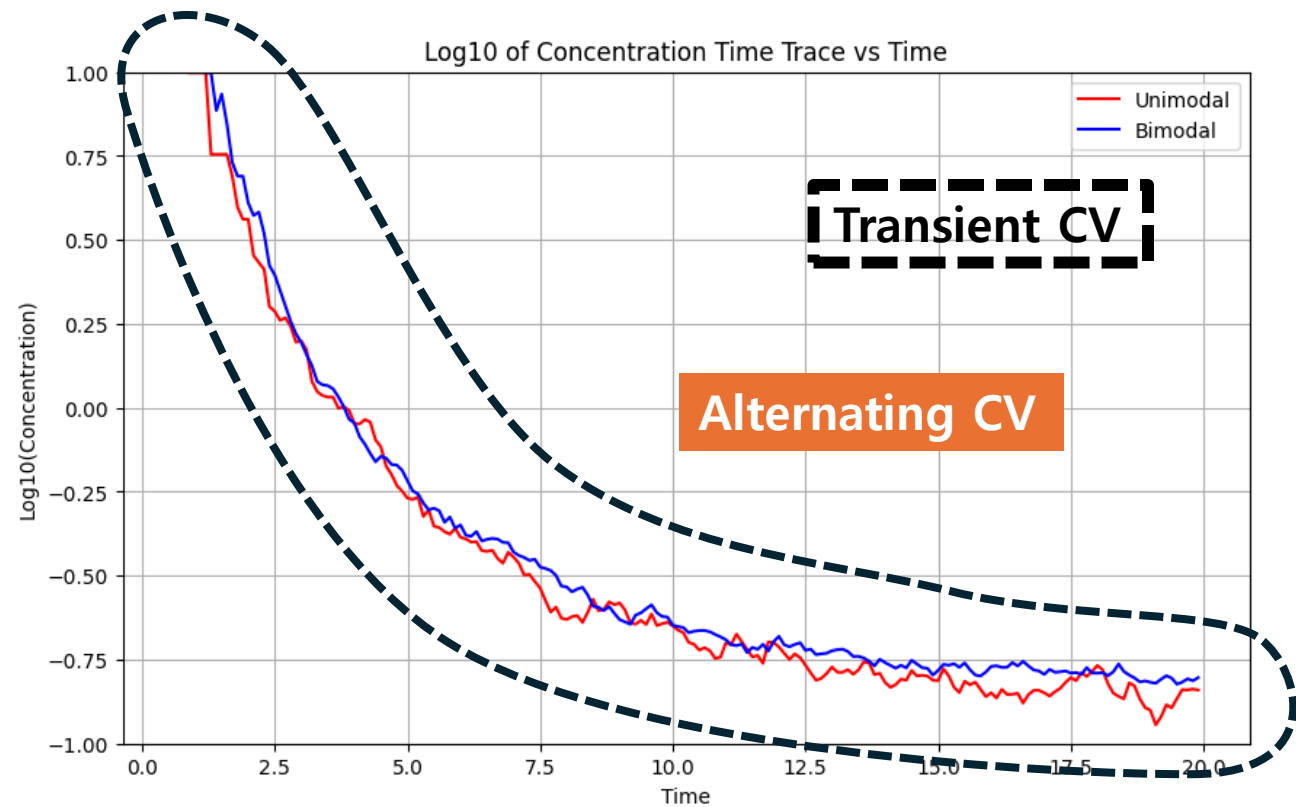
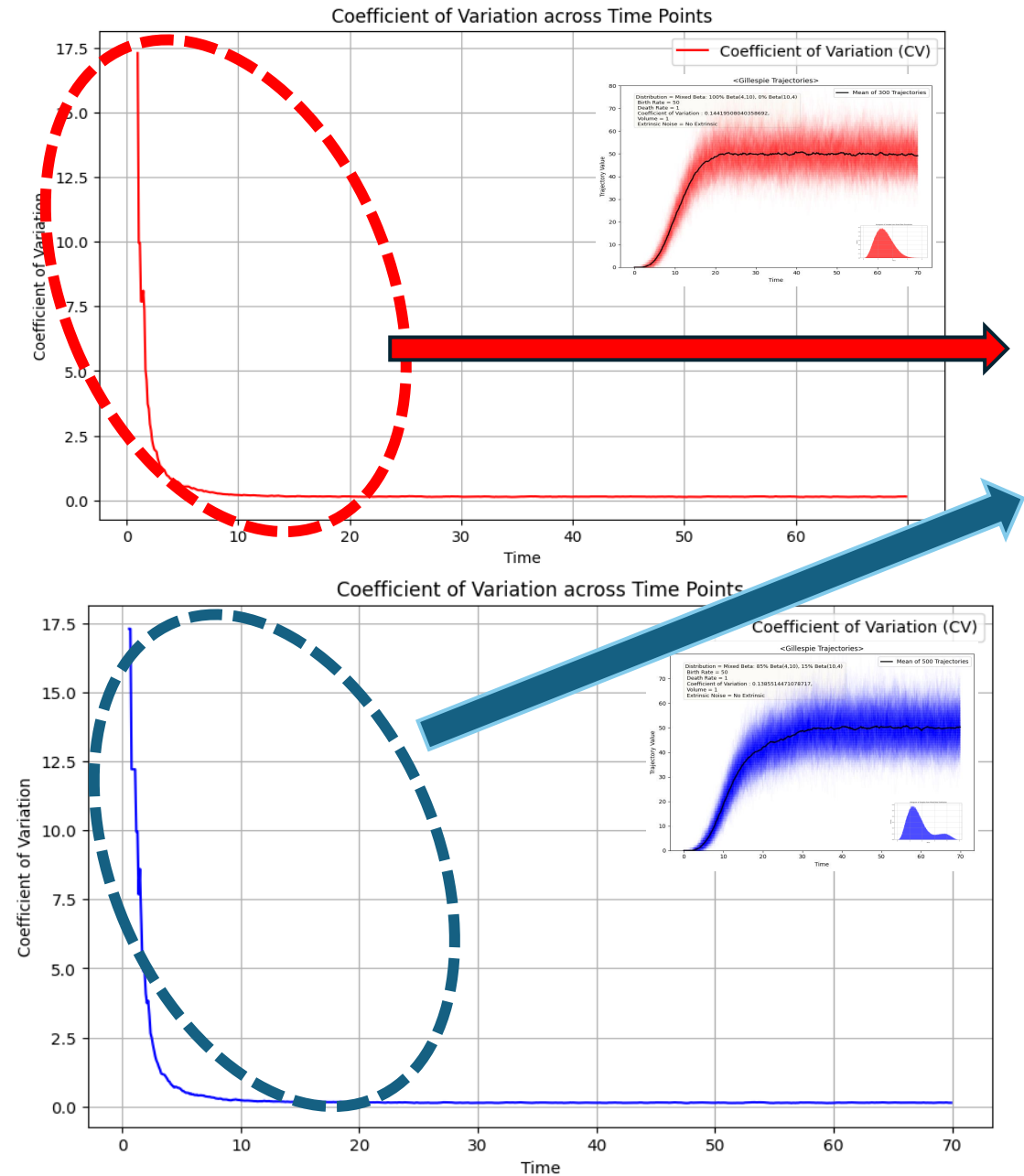


CV time trace repeatedly alternates between unimodal CV > bimodal CV, and unimodal CV < bimodal CV, in both transient and steady-state.

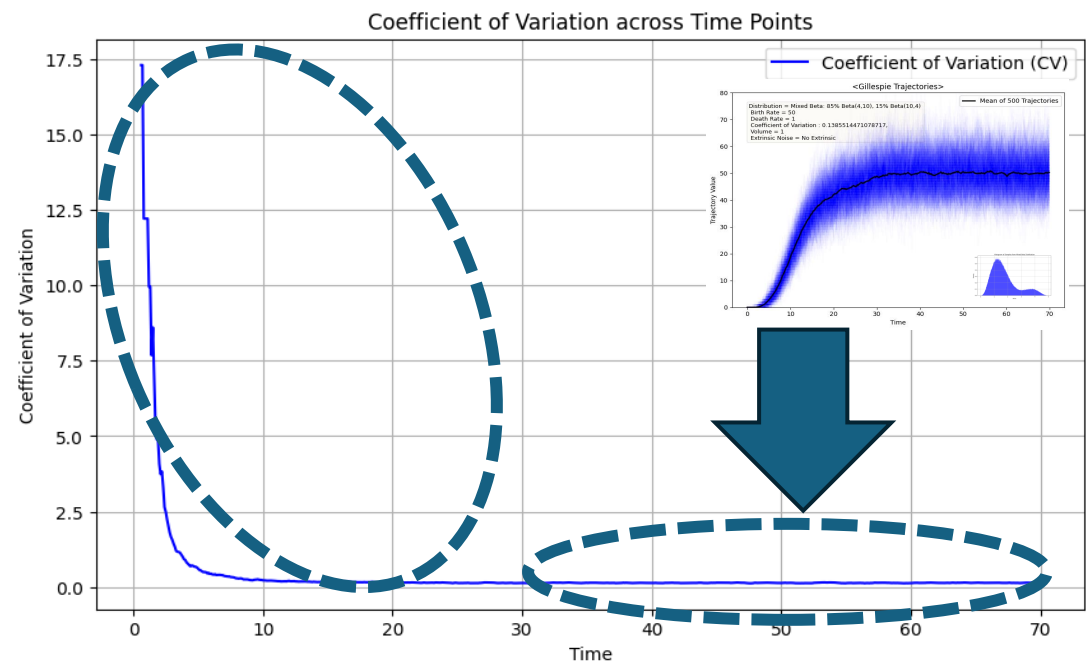
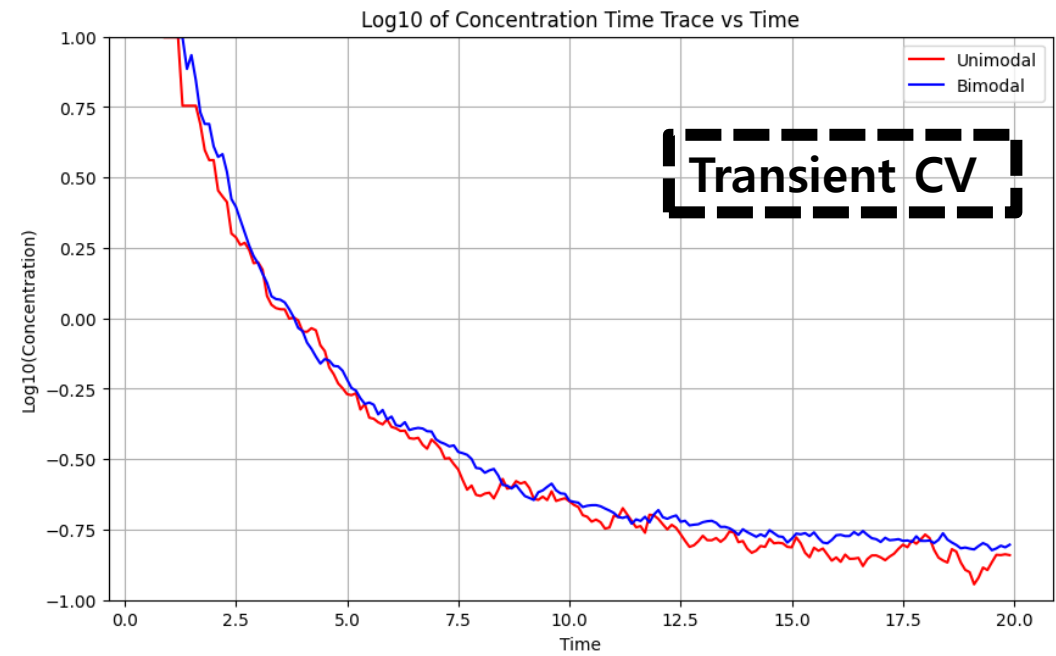
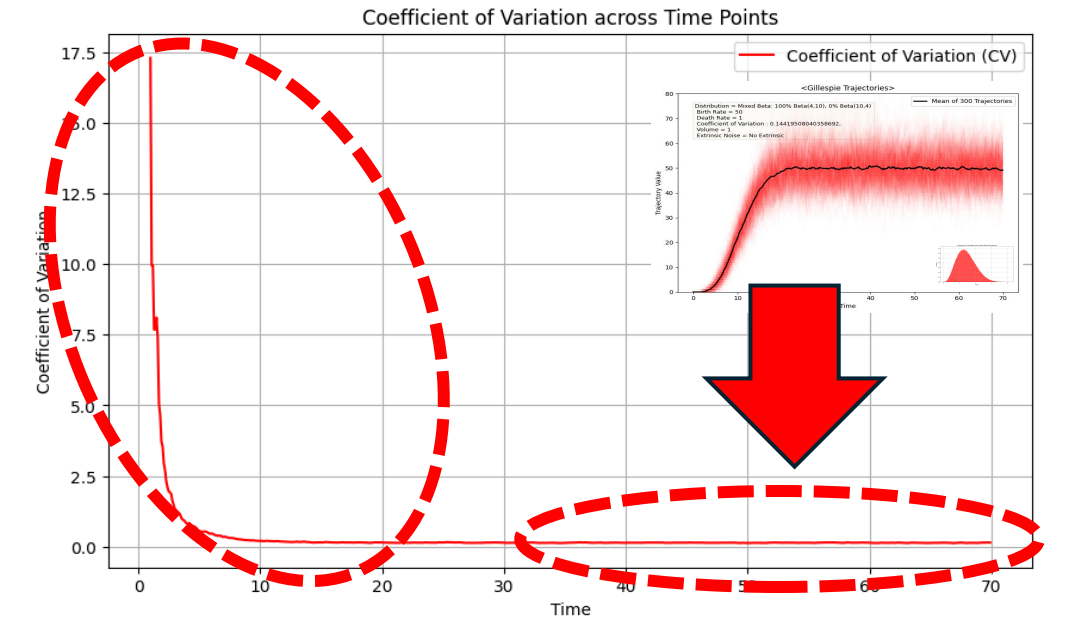




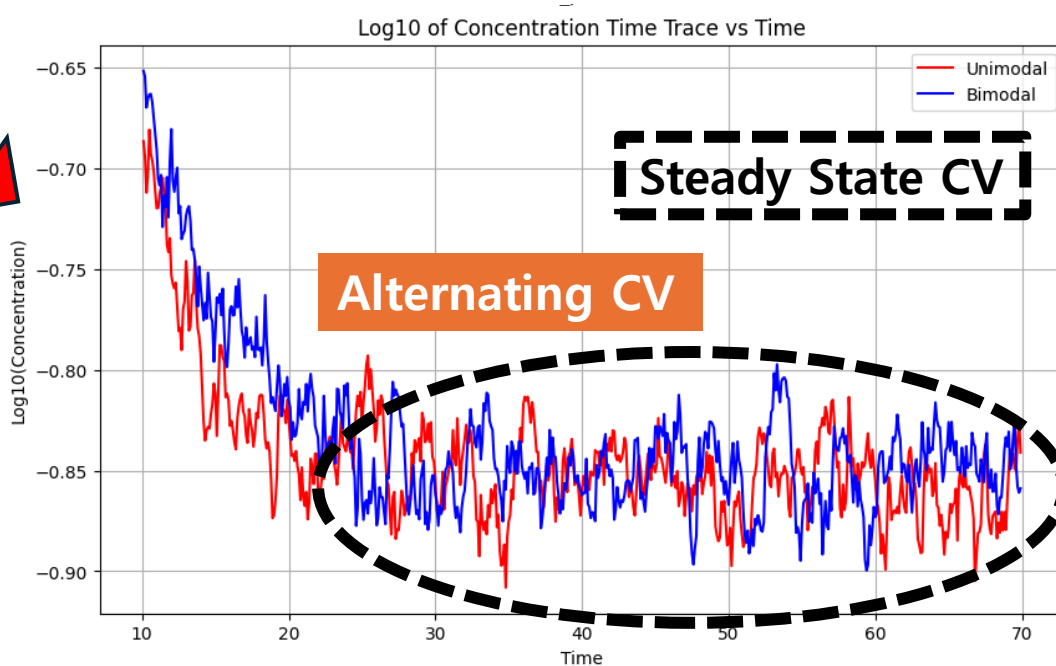
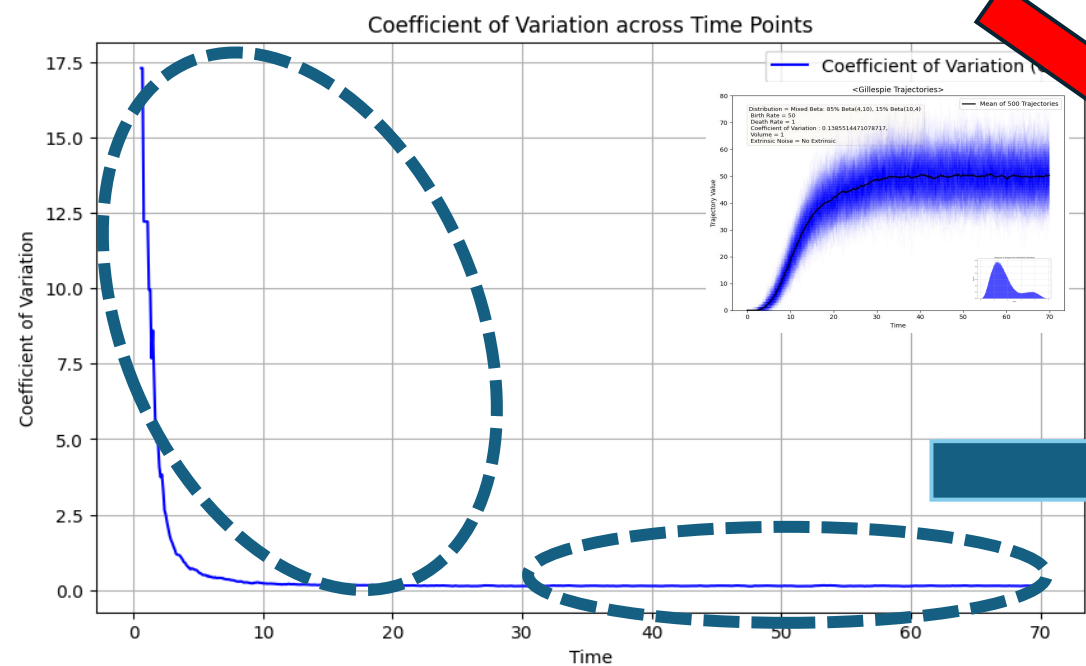
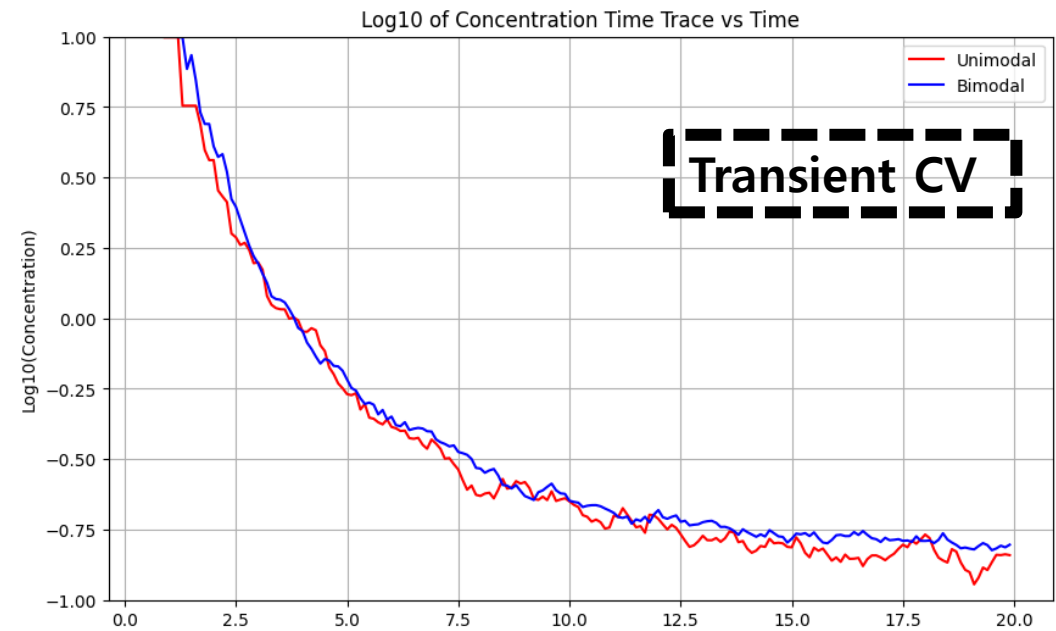
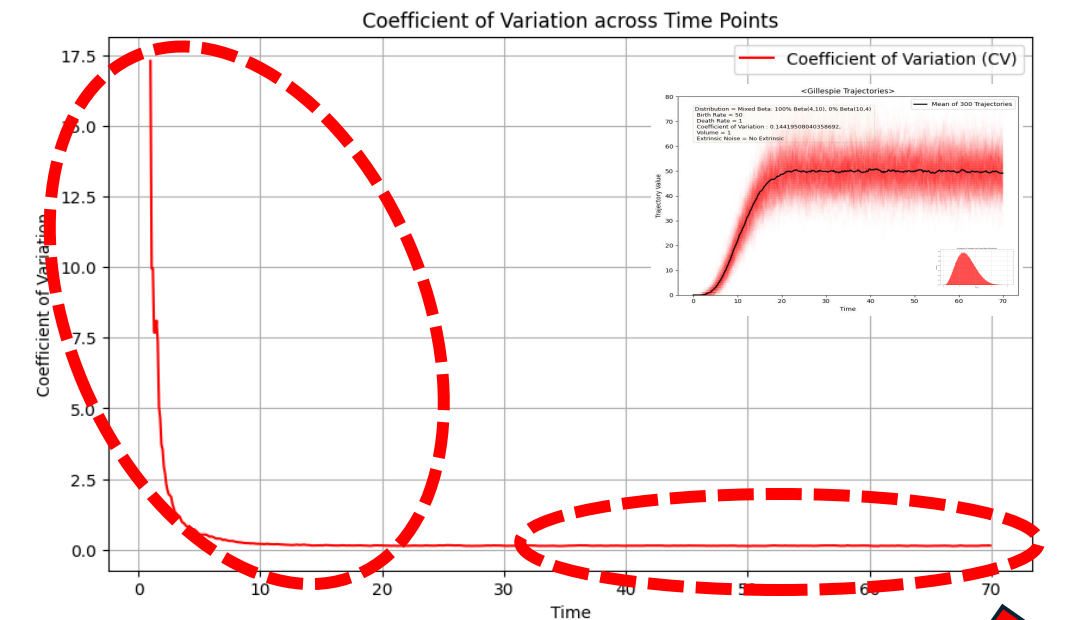
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CV time trace repeatedly alternates between unimodal CV > bimodal CV, and unimodal CV < bimodal CV, in both transient and steady-state.

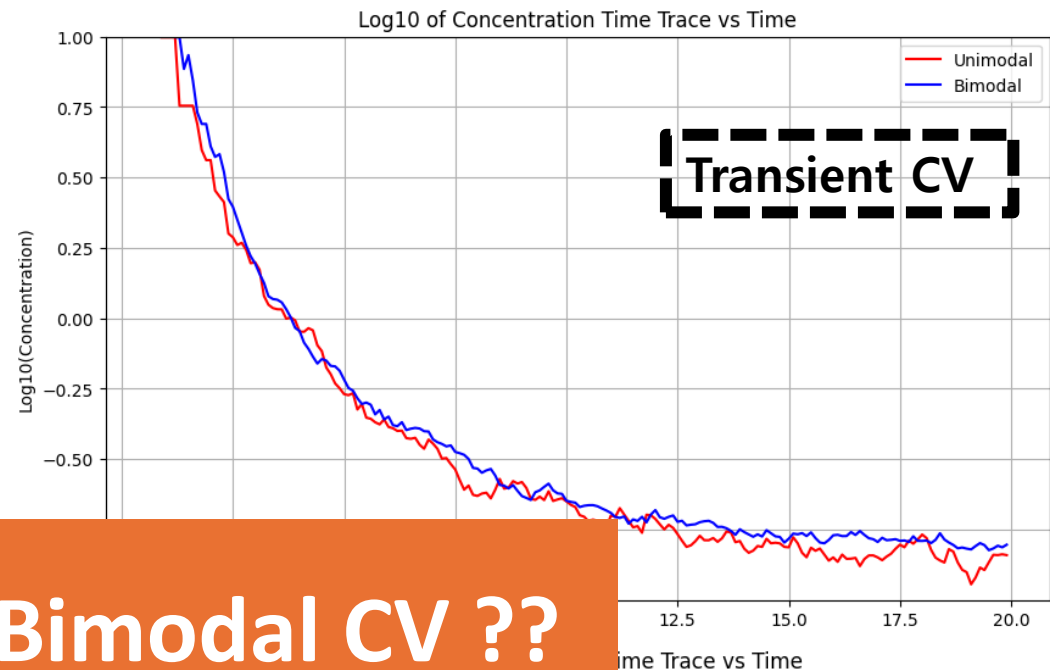
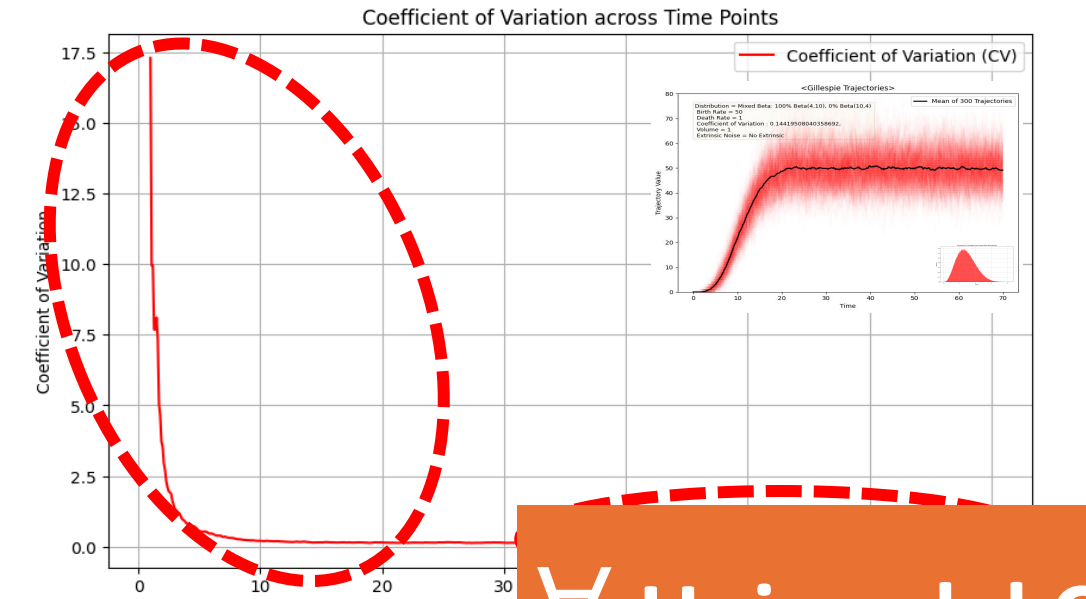


CV time trace repeatedly alternates between unimodal CV > bimodal CV, and unimodal CV < bimodal CV, in both transient and steady-state.

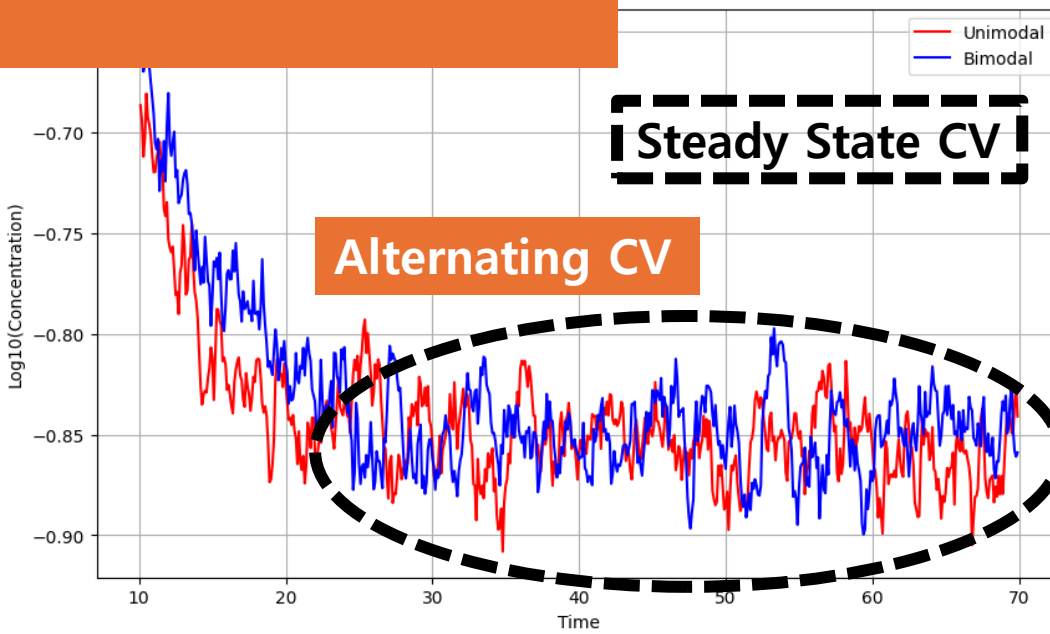
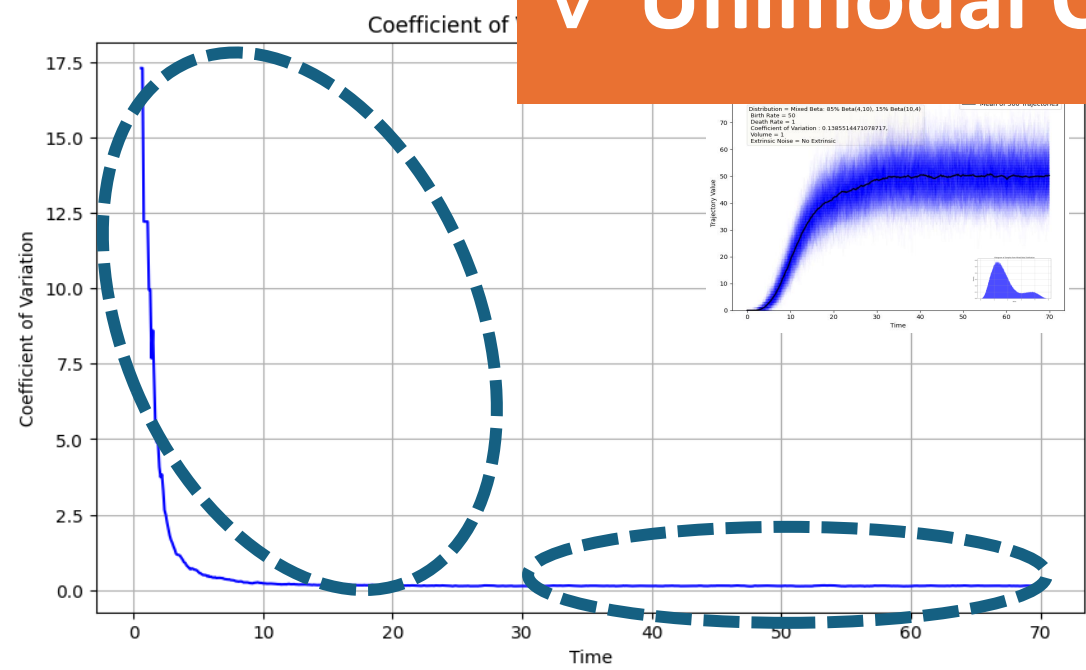




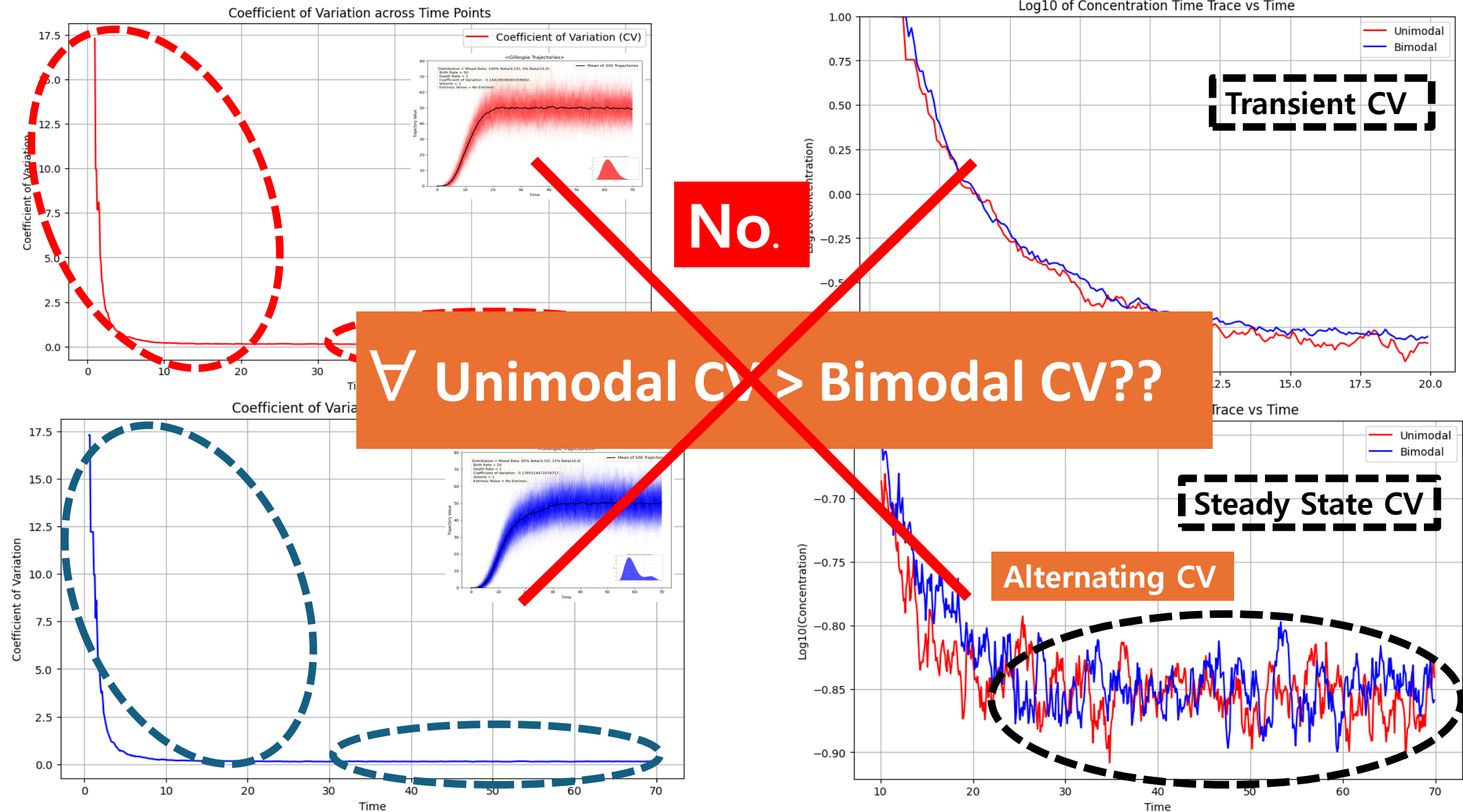
CV time trace repeatedly alternates between unimodal CV > bimodal CV, and unimodal CV < bimodal CV, in both transient and steady-state.



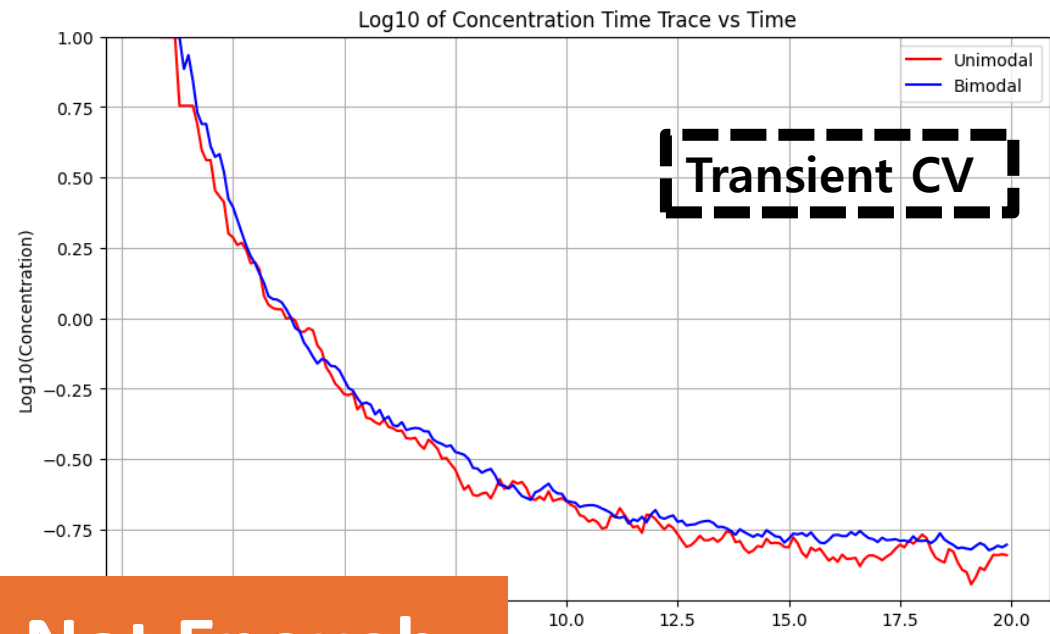
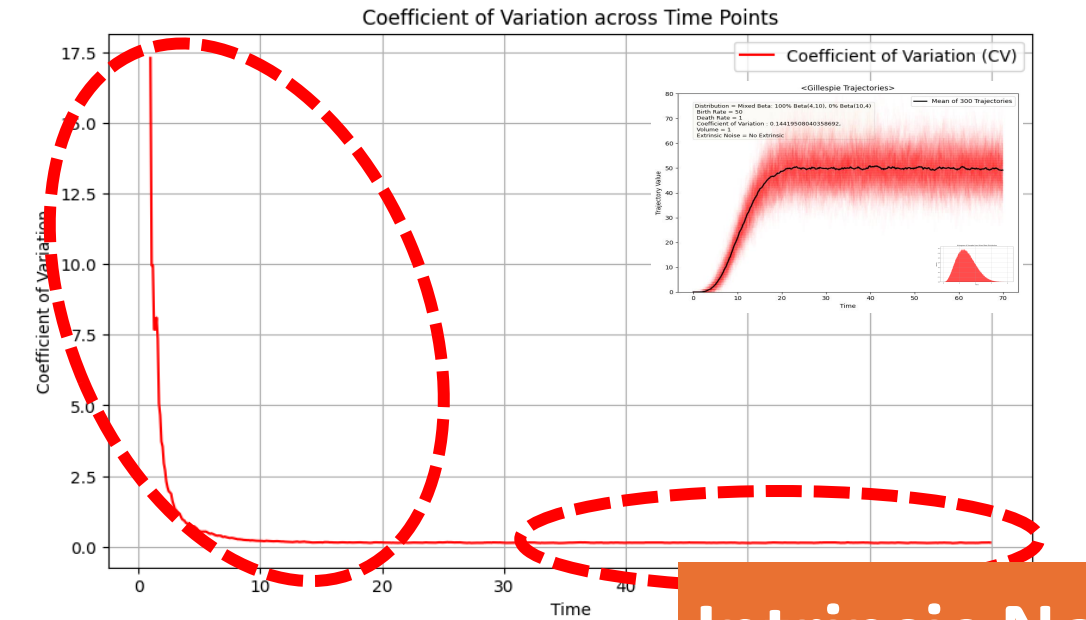
▽ Unimodal CV > Bimodal CV ??



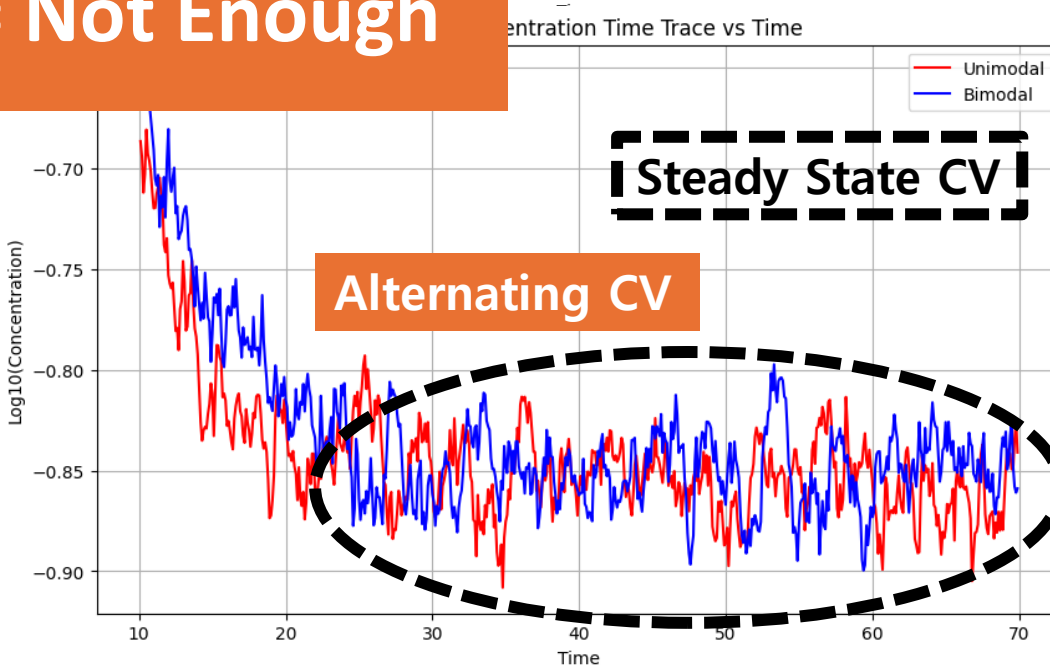
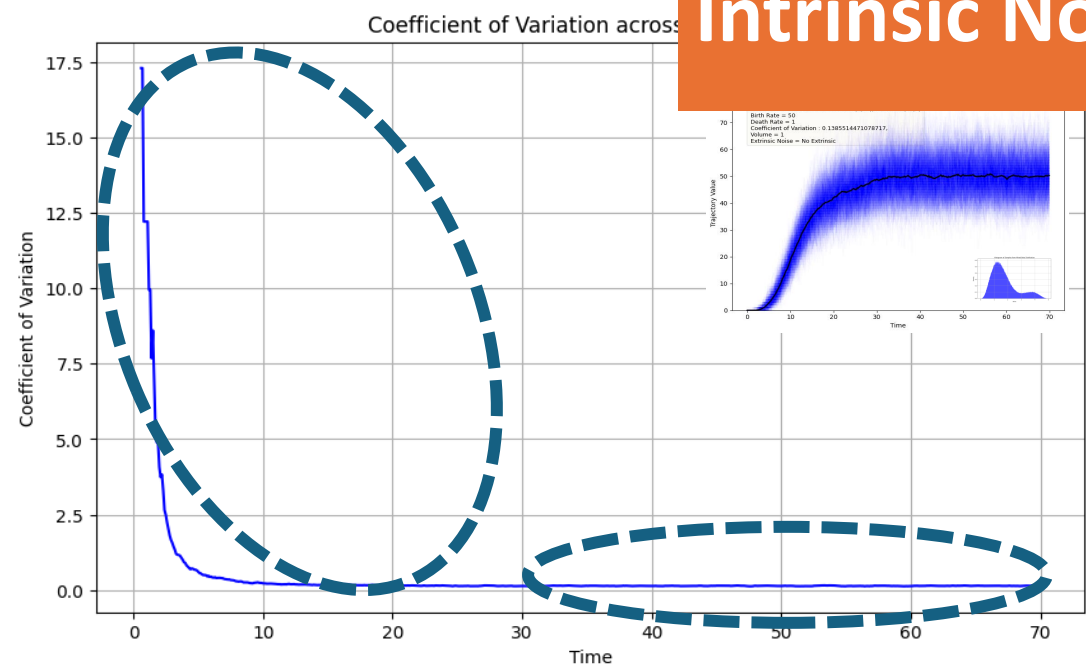
CV time trace repeatedly alternates between unimodal CV > bimodal CV, and unimodal CV < bimodal CV, in both transient and steady-state.



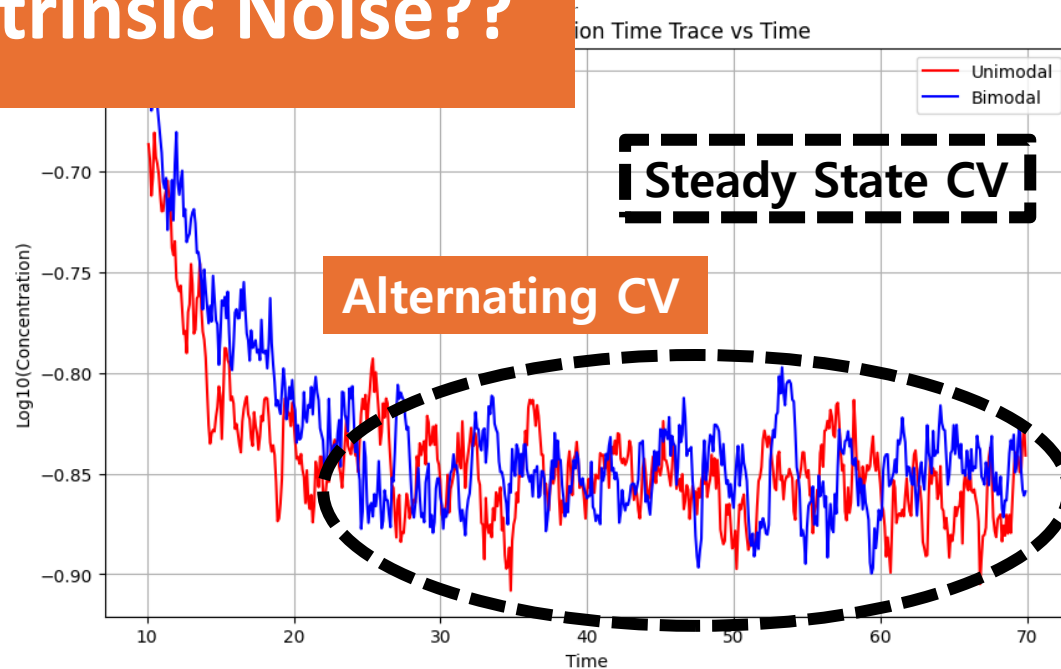
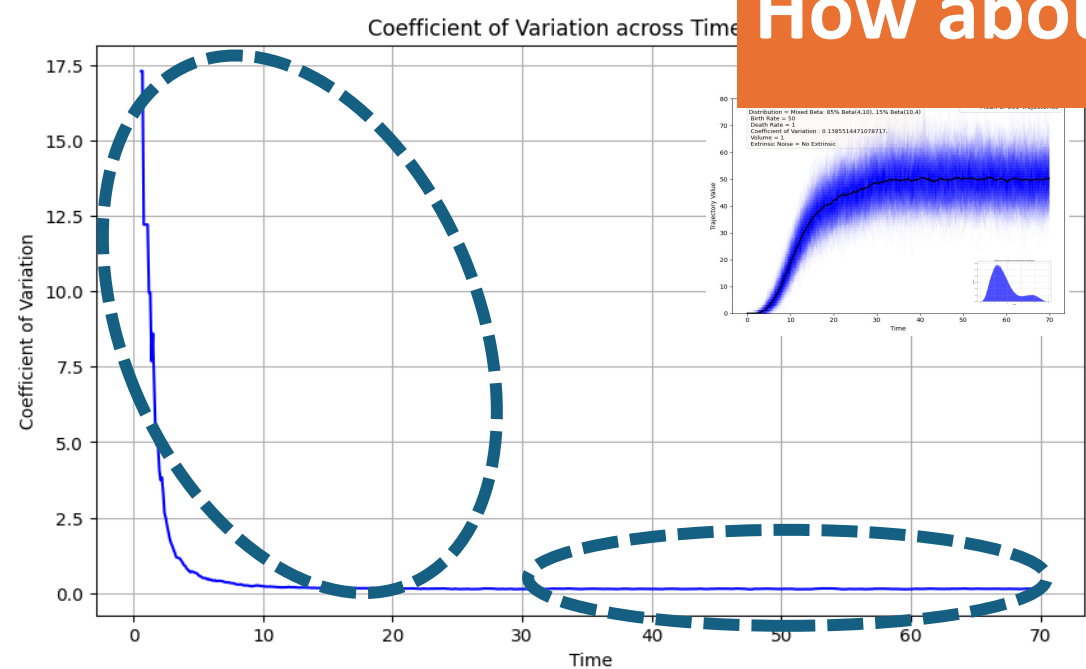
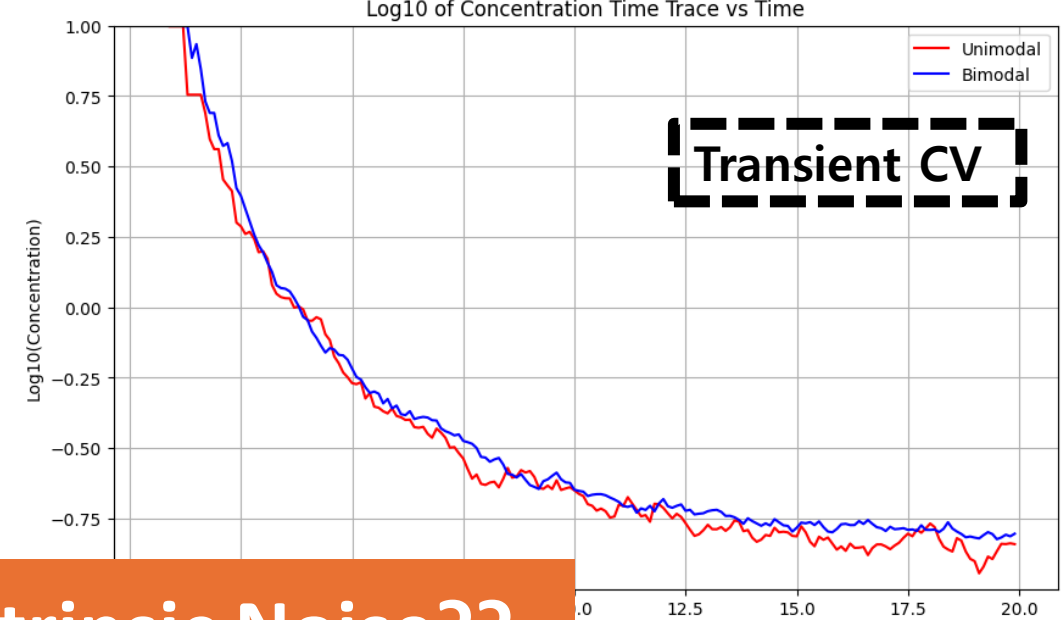
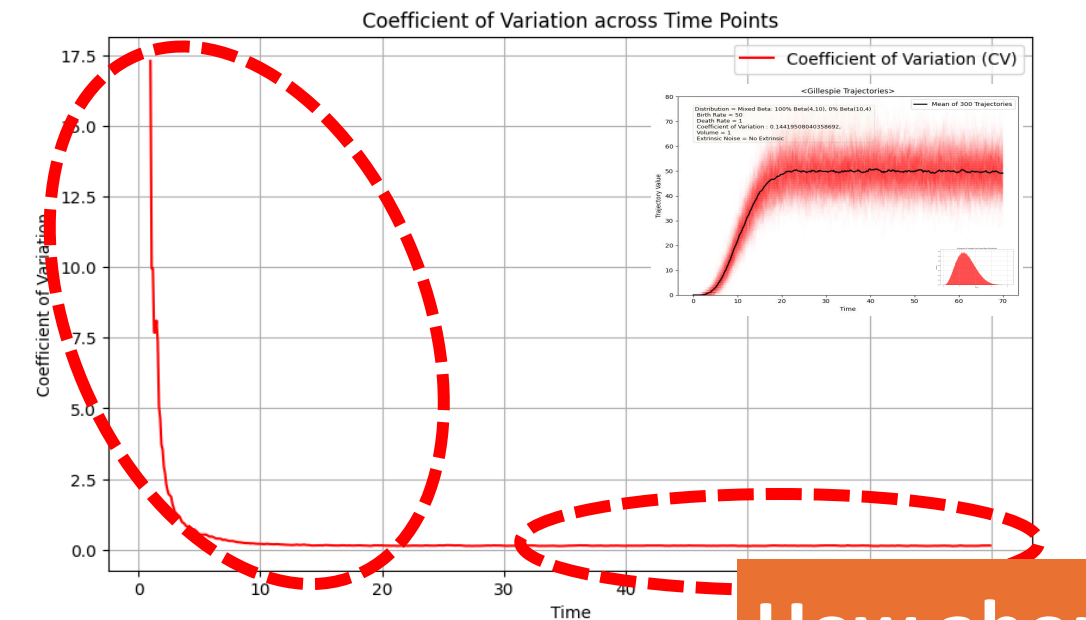
CV time trace repeatedly alternates between unimodal CV > bimodal CV, and unimodal CV < bimodal CV, in both transient and steady-state.



Intrinsic Noise = Not Enough



CV time trace repeatedly alternates between unimodal CV > bimodal CV, and unimodal CV < bimodal CV, in both transient and steady-state.

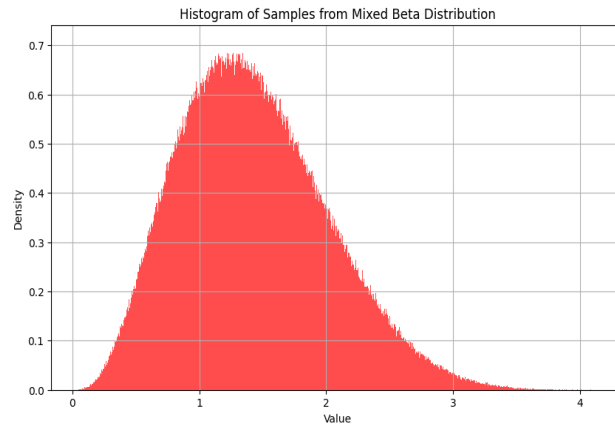


How about Extrinsic Noise??

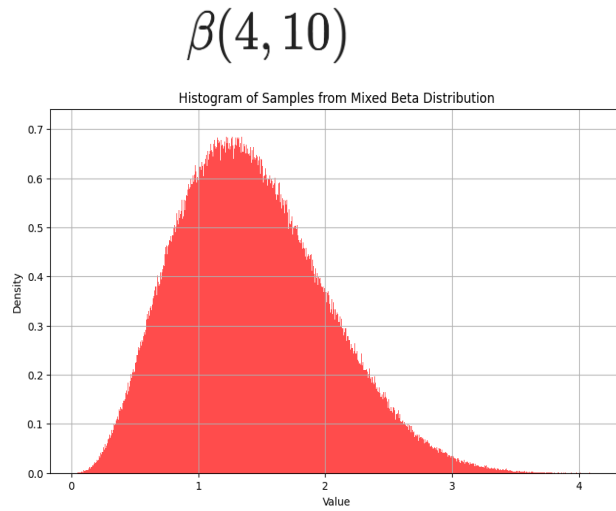


**Adding extrinsic noise in time delay can be done by sampling (beta) distribution parameter 'index' from normal distribution**

$$\beta(4, 10)$$



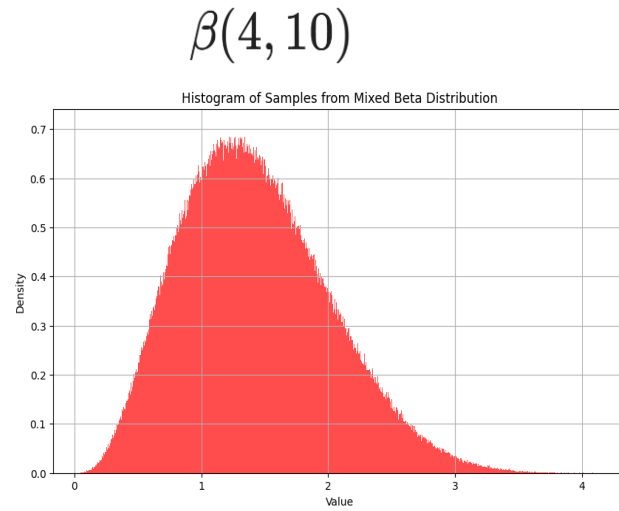
Adding extrinsic noise in time delay can be done by sampling (beta) distribution parameter 'index' from normal distribution



**<Intrinsic>**

All traj. ← **Single** delay distribution

# Adding extrinsic noise in time delay can be done by sampling (beta) distribution parameter 'index' from normal distribution

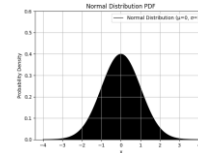


<Intrinsic>

For Each Trajectory,

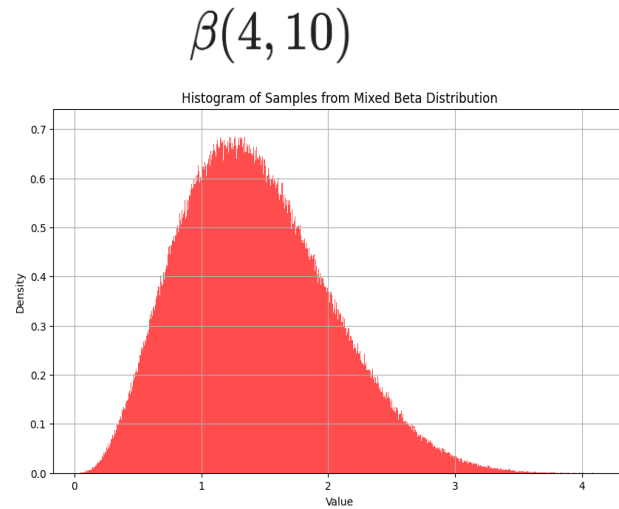


$index \sim Normal(0, 1)$



All traj.  $\leftarrow$  **Single** delay distribution

# Adding extrinsic noise in time delay can be done by sampling (beta) distribution parameter 'index' from normal distribution



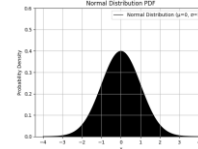
<Intrinsic>

All traj.  $\leftarrow$  **Single** delay distribution

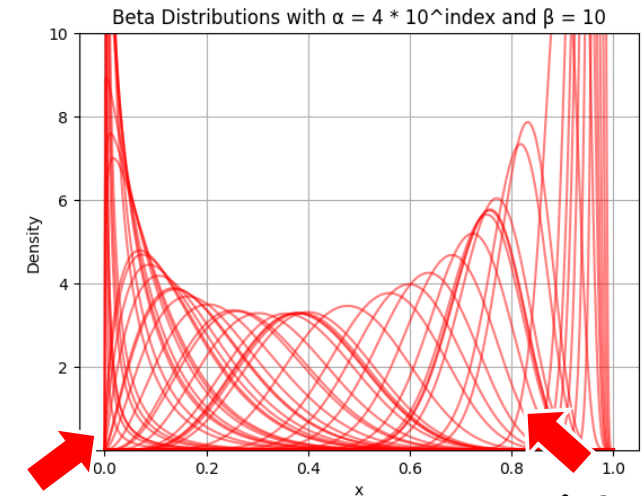
For Each Trajectory,



$index \sim Normal(0, 1)$



$$\beta(4 \cdot (10^{index}), 10)$$

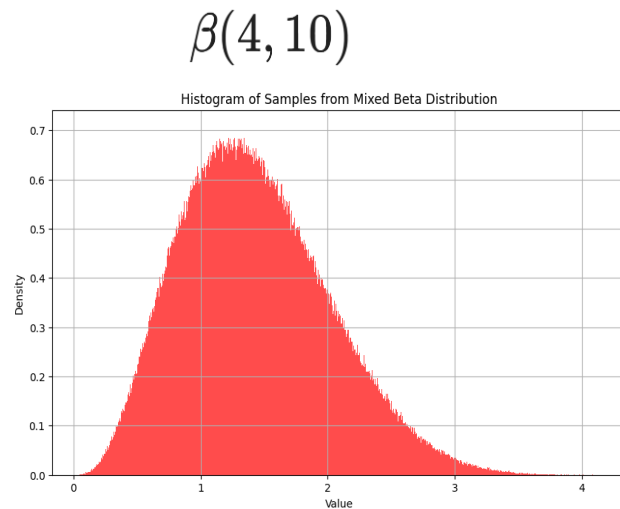


Traj. 1 (small index)

Traj. 2: (big index)



# Adding extrinsic noise in time delay can be done by sampling (beta) distribution parameter 'index' from normal distribution



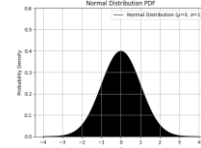
<Intrinsic>

All traj. ← **Single** delay distribution

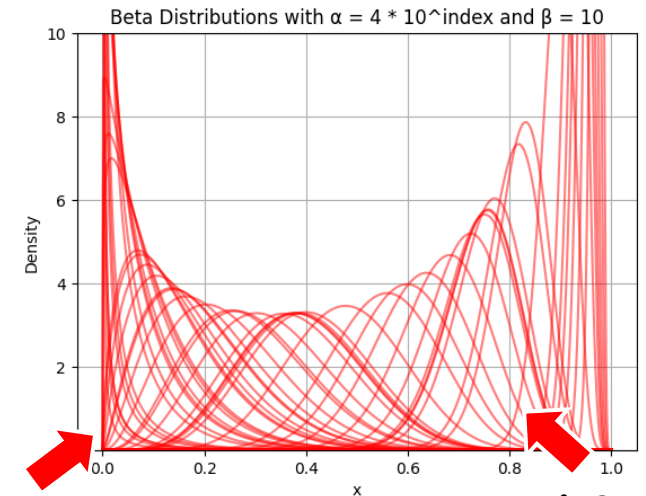
For Each Trajectory,



$index \sim Normal(0, 1)$



$$\beta(4 \cdot (10^{index}), 10)$$



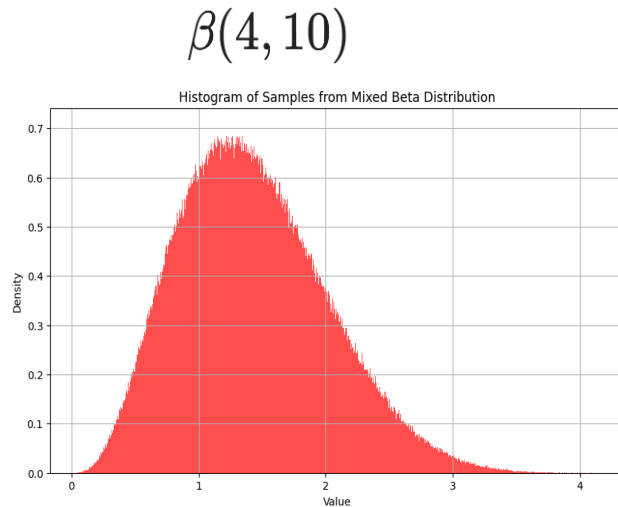
Traj. 1 (small index)

Traj. 2: (big index)

<Extrinsic>

All traj. ← **Different** delay distribution

# Adding extrinsic noise in time delay can be done by sampling (beta) distribution parameter 'index' from normal distribution



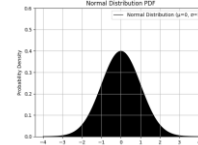
<Intrinsic>

All traj.  $\leftarrow$  **Single** delay distribution

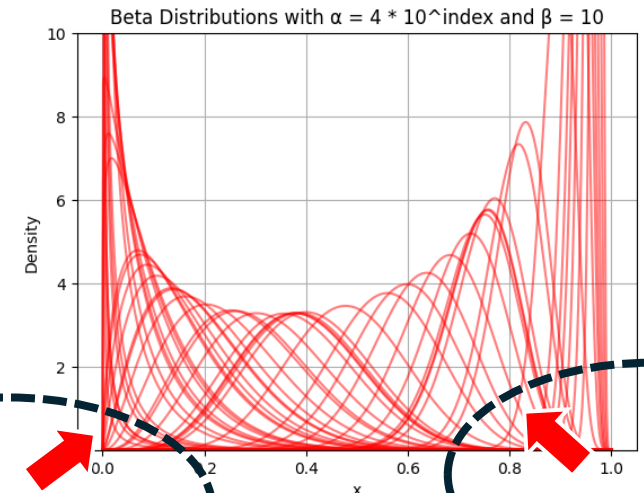
For Each Trajectory,



$index \sim Normal(0, 1)$



$$\beta(4 \cdot (10^{index}), 10)$$



Traj. 1 (small index)

Small Delay

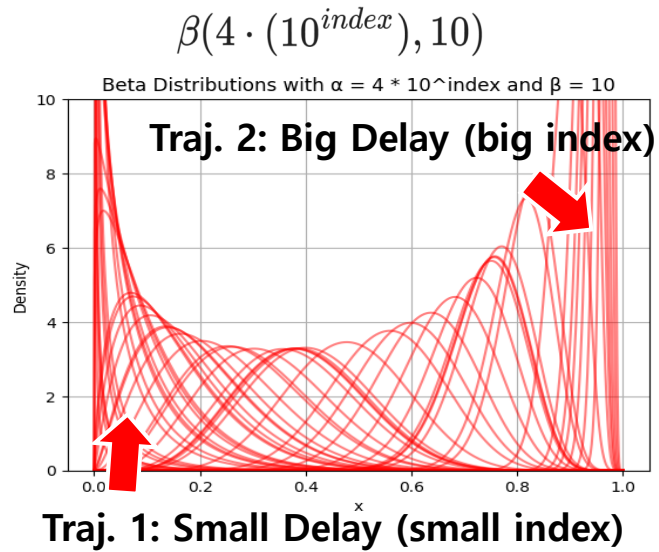
Traj. 2: (big index)

Long Delay

<Extrinsic>

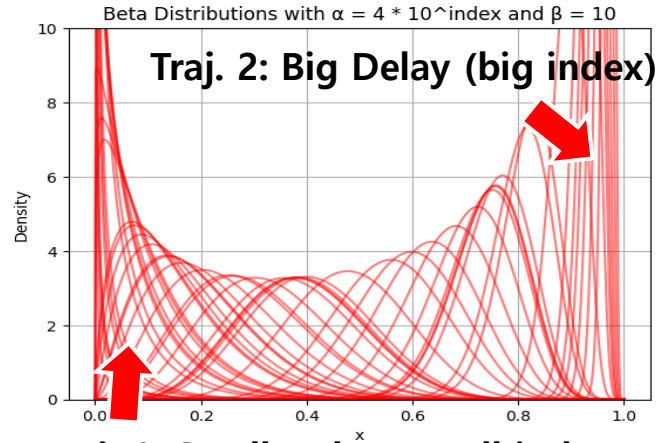
All traj.  $\leftarrow$  **Different** delay distribution

**Adding extrinsic noise in time delay increases CV in transient, but ultimately, no significant difference in steady state CV.**



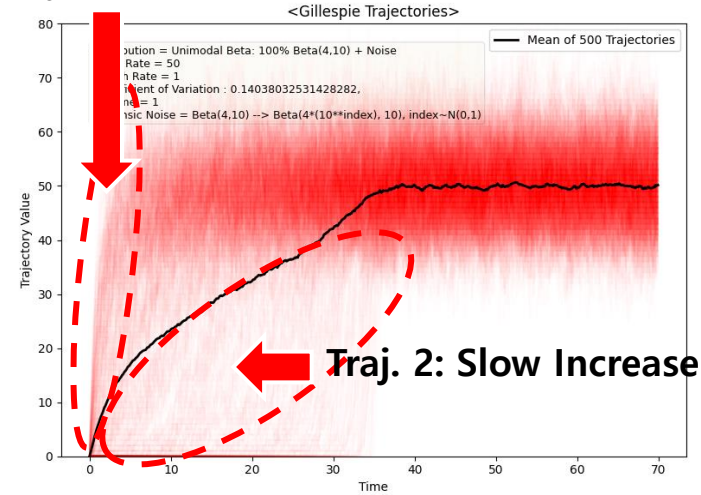
**Adding extrinsic noise in time delay increases CV in transient, but ultimately, no significant difference in steady state CV.**

$$\beta(4 \cdot (10^{\text{index}}), 10)$$



**Traj. 1: Small Delay (small index)**

**Traj. 1: Fast Increase**

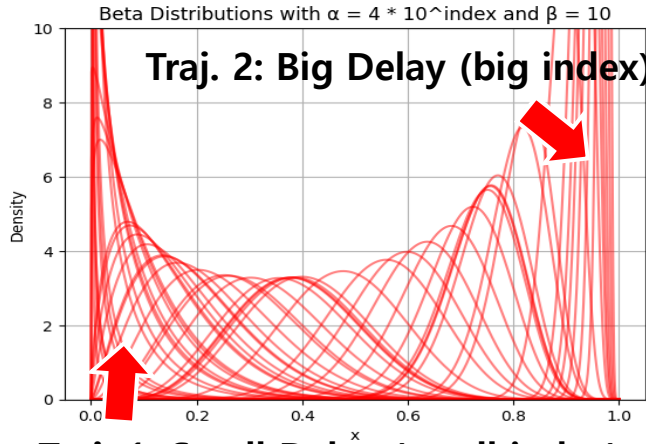


**Traj. 2: Slow Increase**

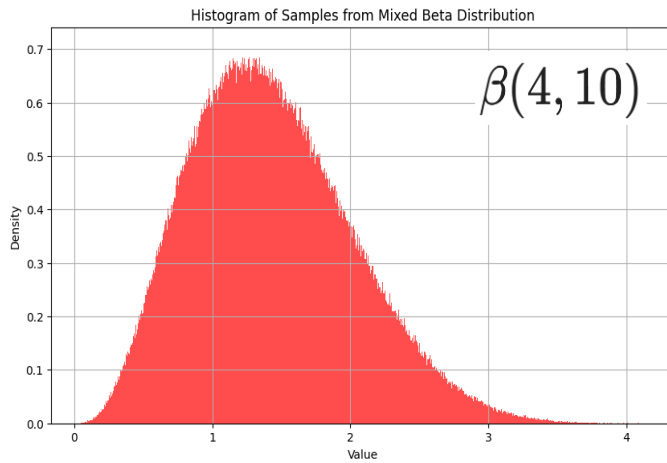


**Adding extrinsic noise in time delay increases CV in transient, but ultimately, no significant difference in steady state CV.**

$$\beta(4 \cdot (10^{\text{index}}), 10)$$

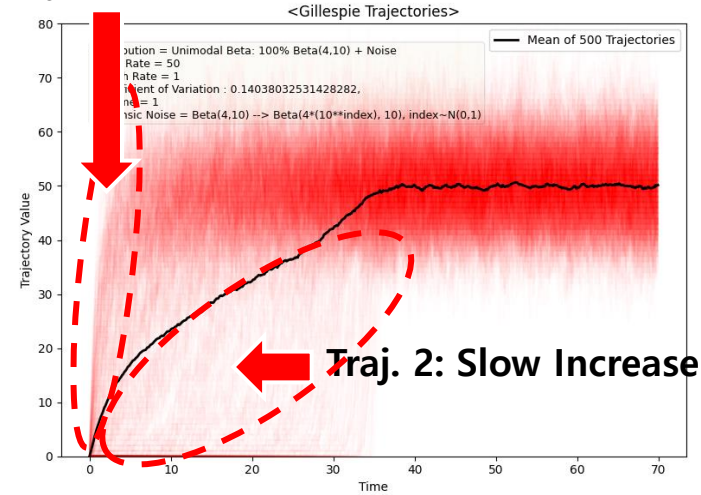


**Traj. 1: Small Delay (small index)**



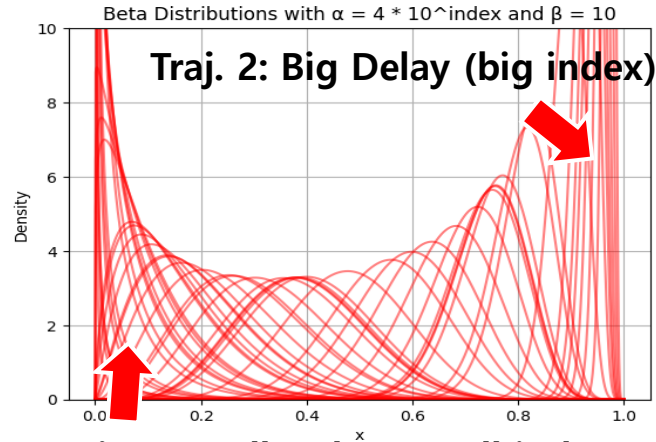
**∀ Trajectory: Same Distribution**

**Traj. 1: Fast Increase**

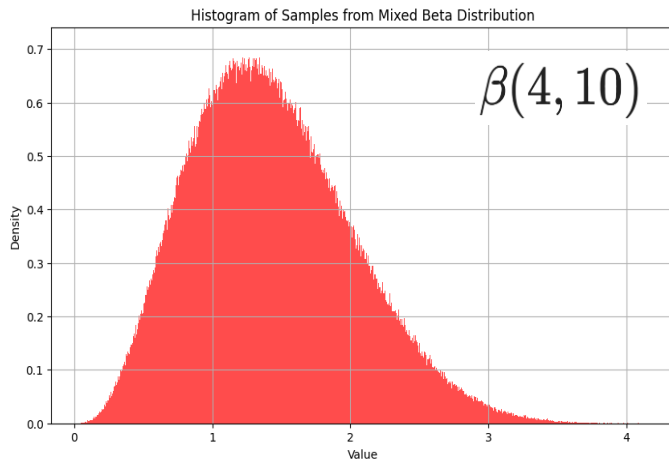


**Adding extrinsic noise in time delay increases CV in transient, but ultimately, no significant difference in steady state CV.**

$$\beta(4 \cdot (10^{\text{index}}), 10)$$

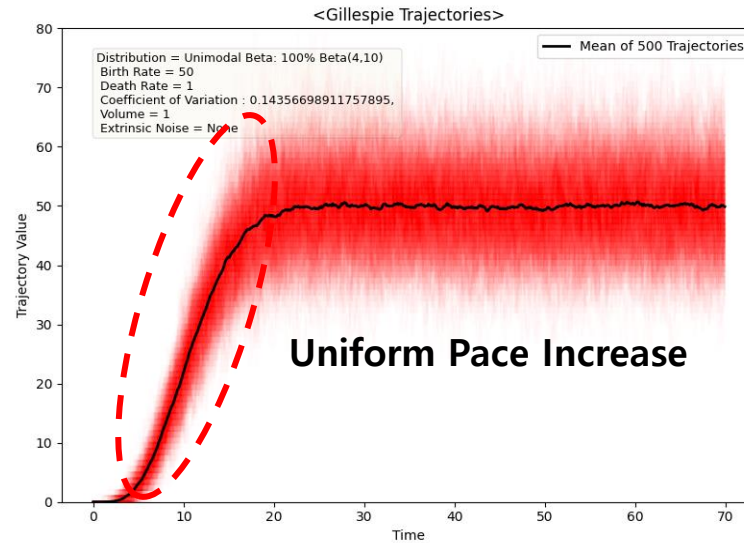
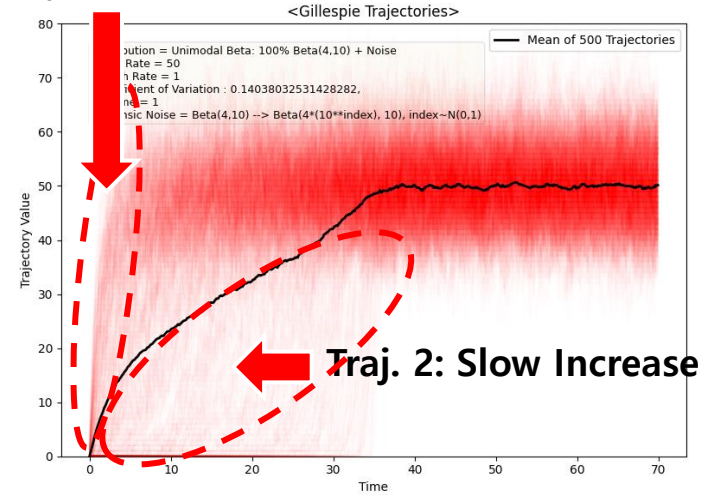


**Traj. 1: Small Delay (small index)**



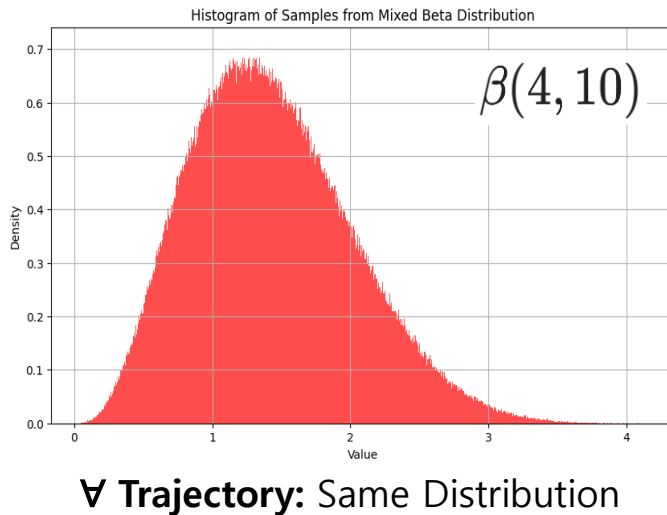
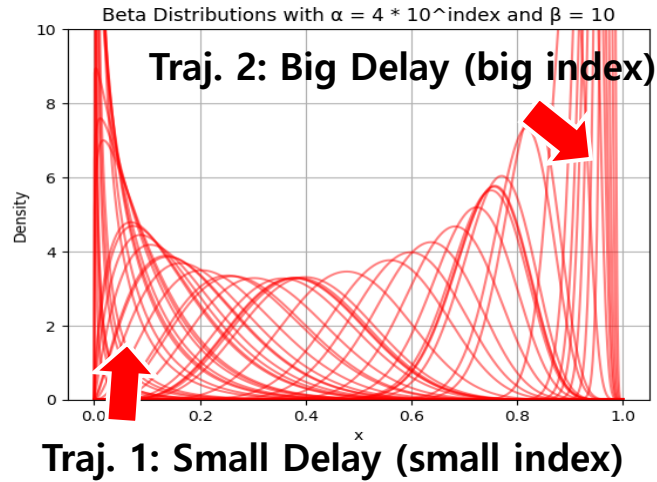
**∀ Trajectory: Same Distribution**

**Traj. 1: Fast Increase**

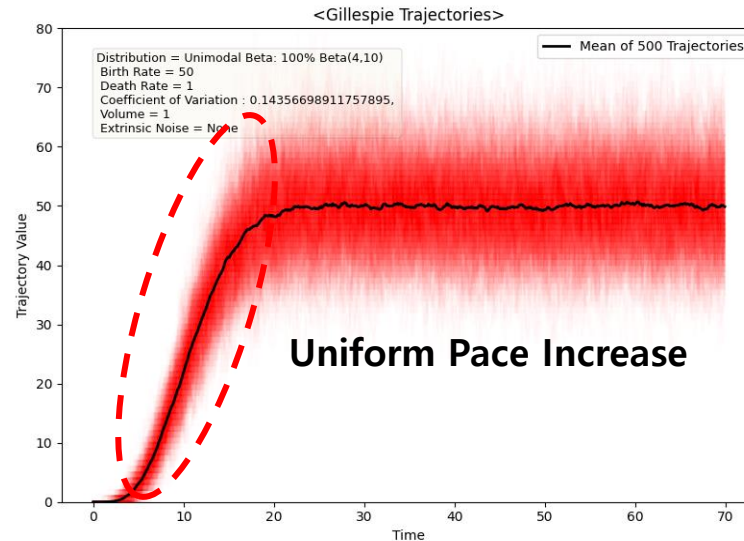
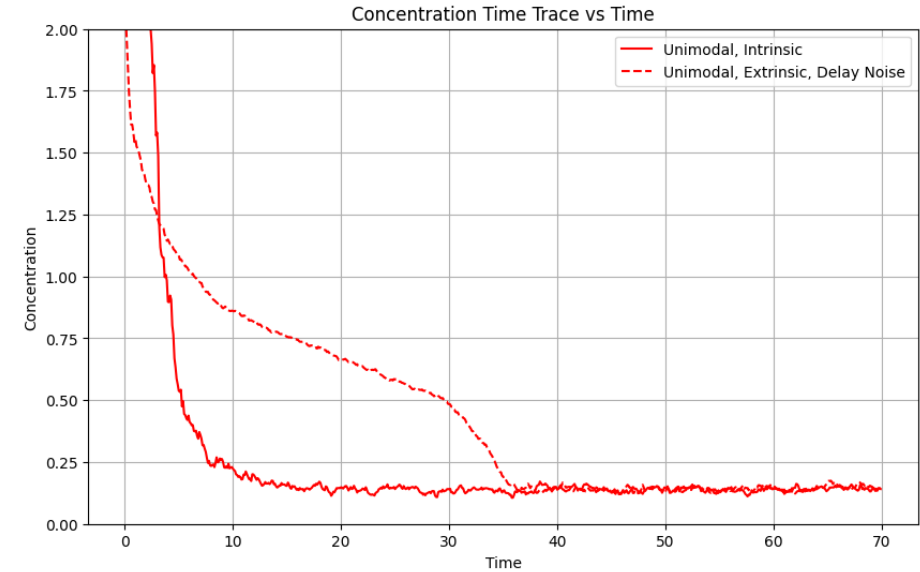
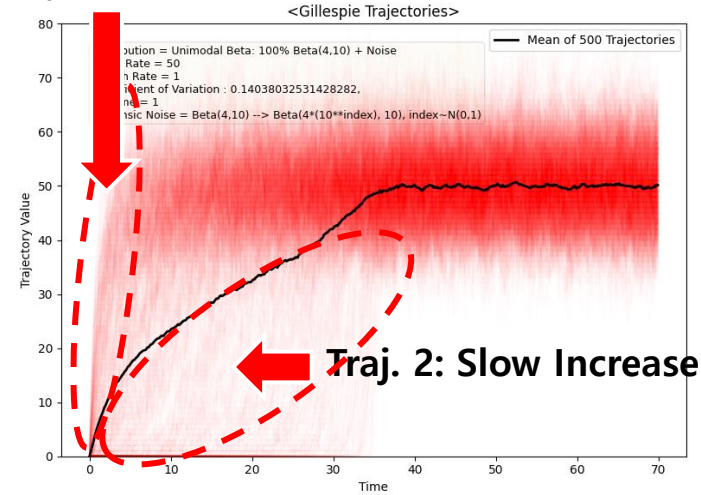


# Adding extrinsic noise in time delay increases CV in transient, but ultimately, no significant difference in steady state CV.

$$\beta(4 \cdot (10^{\text{index}}), 10)$$

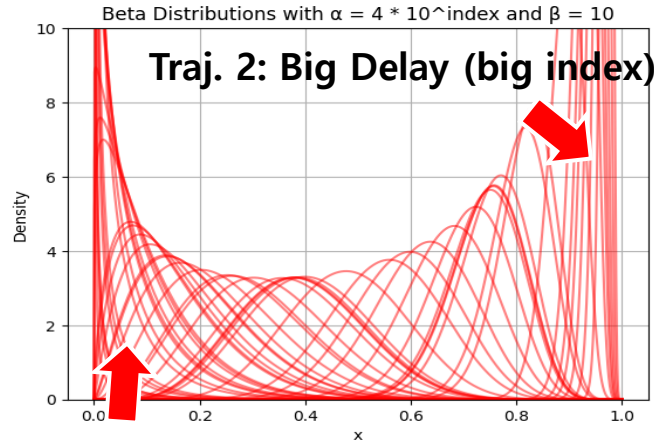


**Traj. 1: Fast Increase**

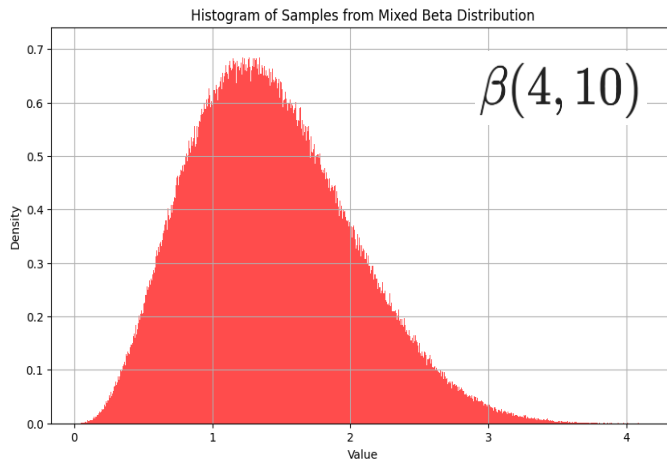


# Adding extrinsic noise in time delay increases CV in transient, but ultimately, no significant difference in steady state CV.

$$\beta(4 \cdot (10^{\text{index}}), 10)$$

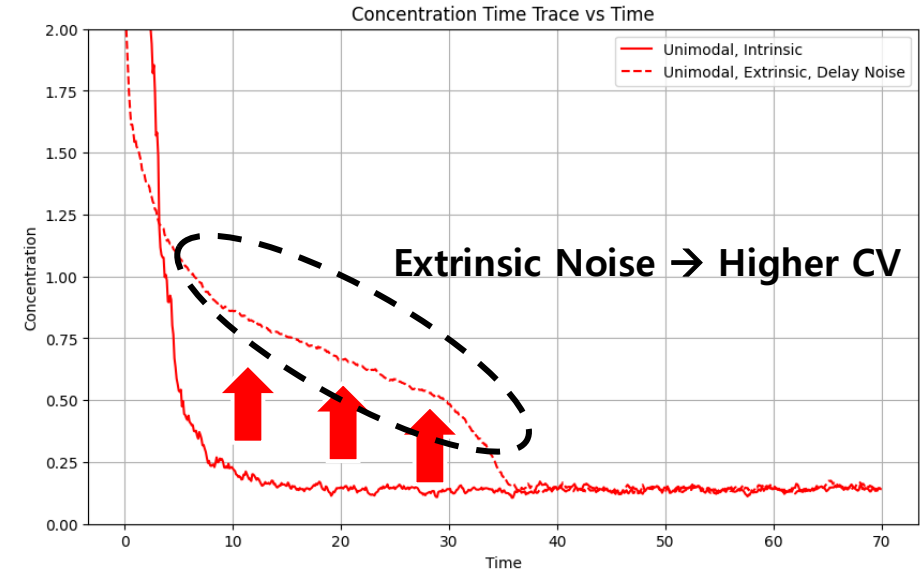
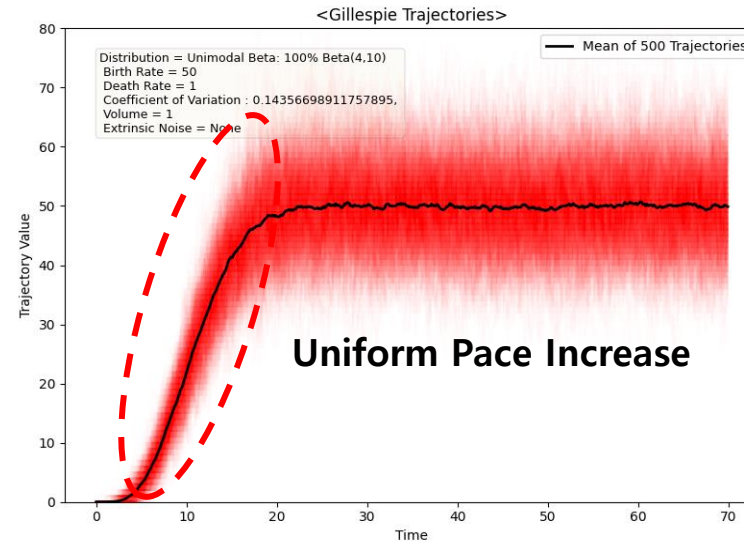
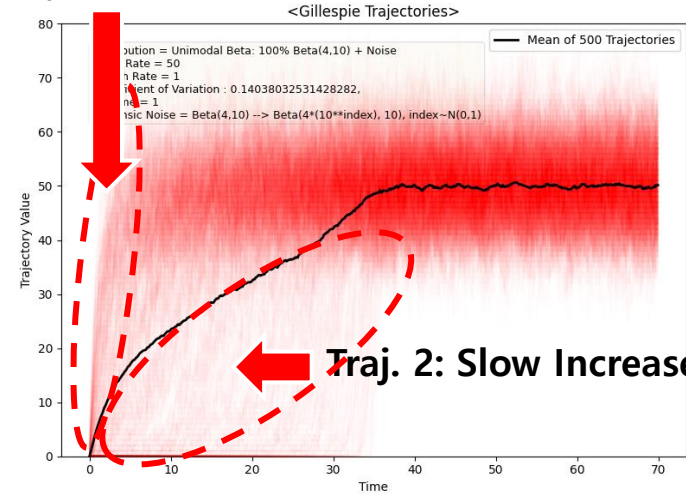


**Traj. 1: Small Delay (small index)**



∀ Trajectory: Same Distribution

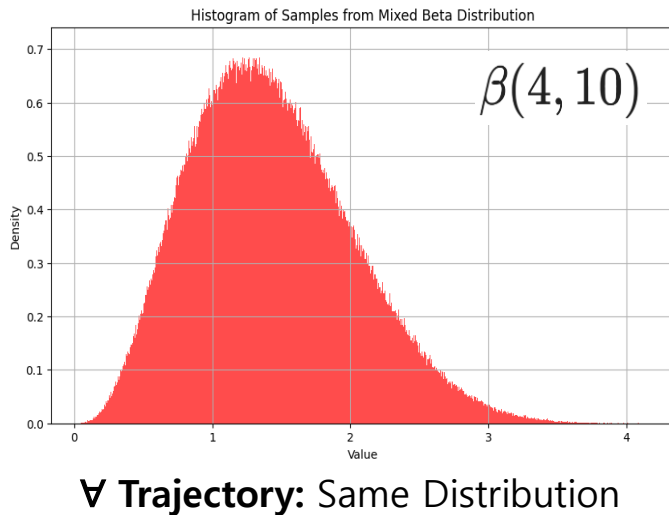
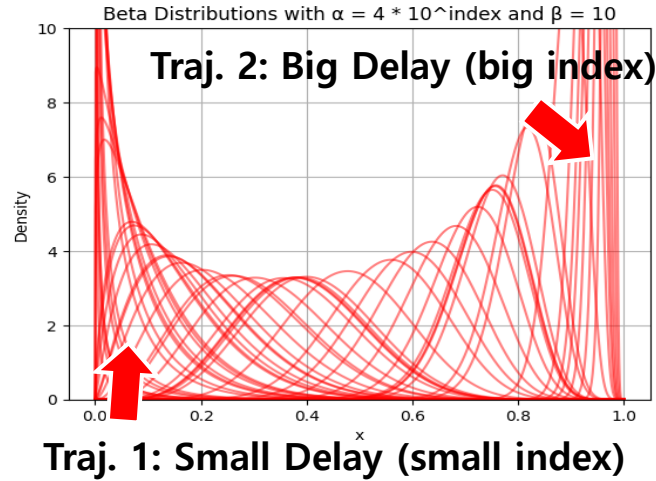
**Traj. 1: Fast Increase**



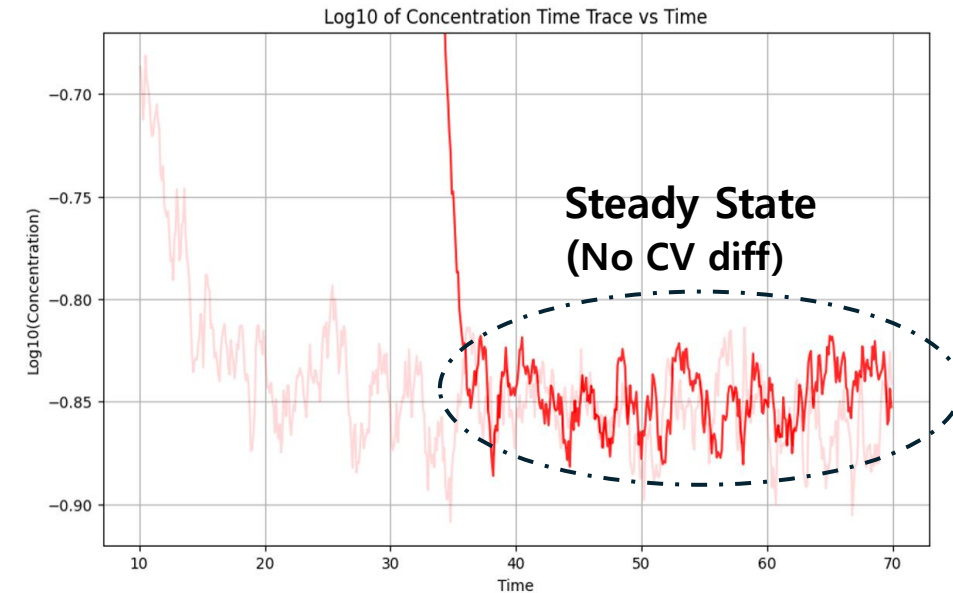
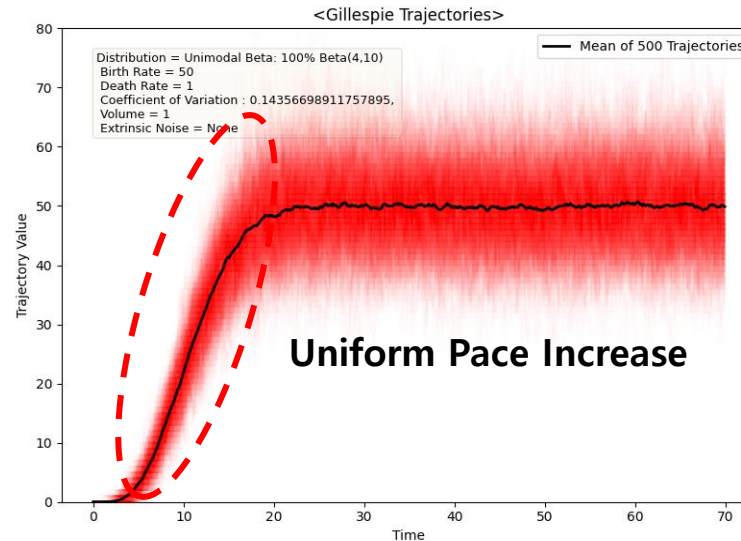
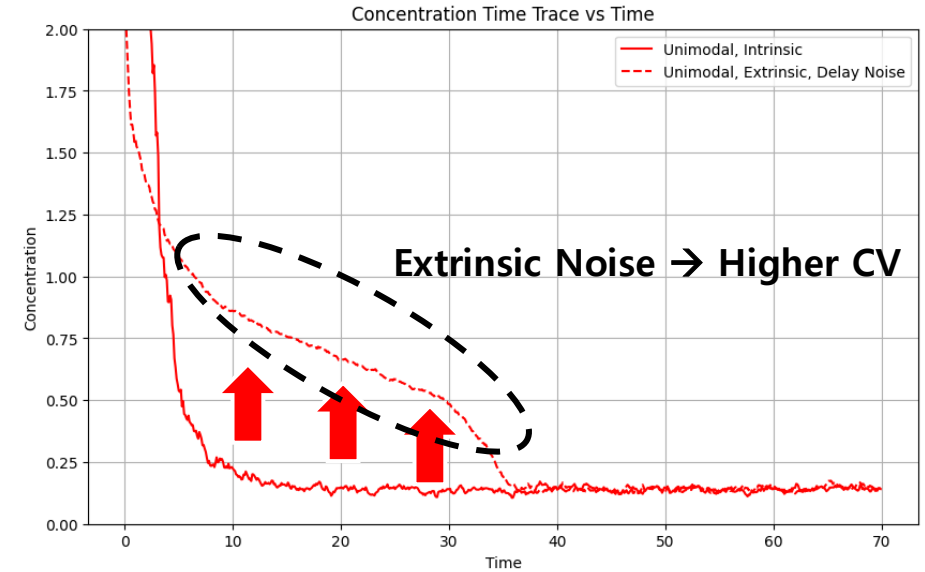
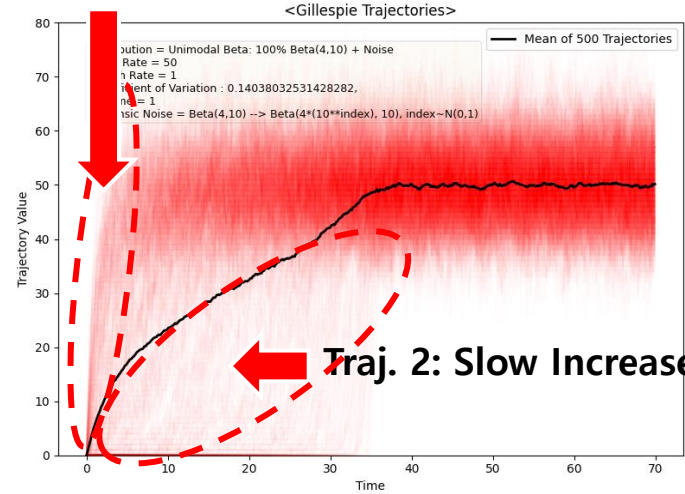


Adding extrinsic noise in time delay increases CV in transient, but ultimately, no significant difference in steady state CV.

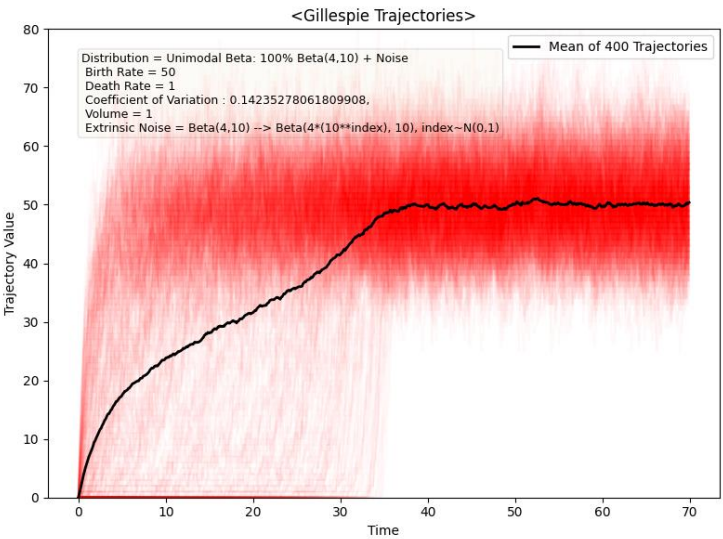
$$\beta(4 \cdot (10^{\text{index}}), 10)$$



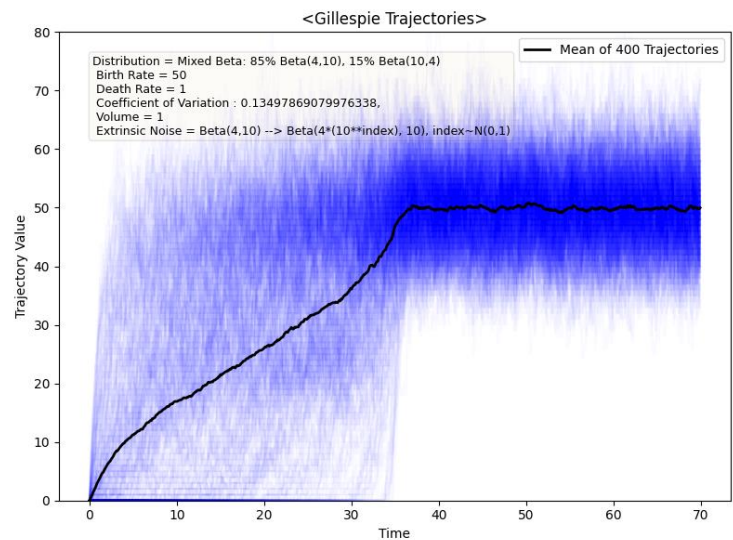
**Traj. 1: Fast Increase**



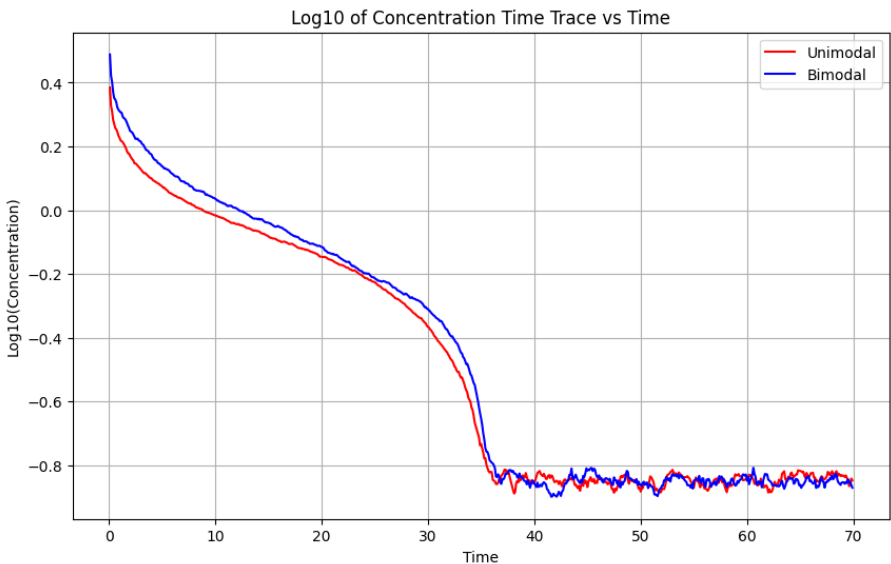
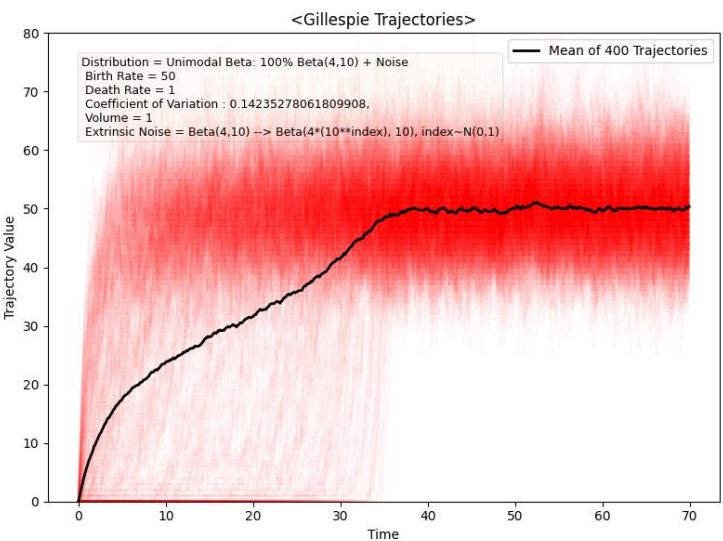
**CV time trace repeatedly alternates between unimodal CV > bimodal CV, and unimodal CV < bimodal CV, in steady-state, and shows unimodal CV < bimodal CV in transient.**



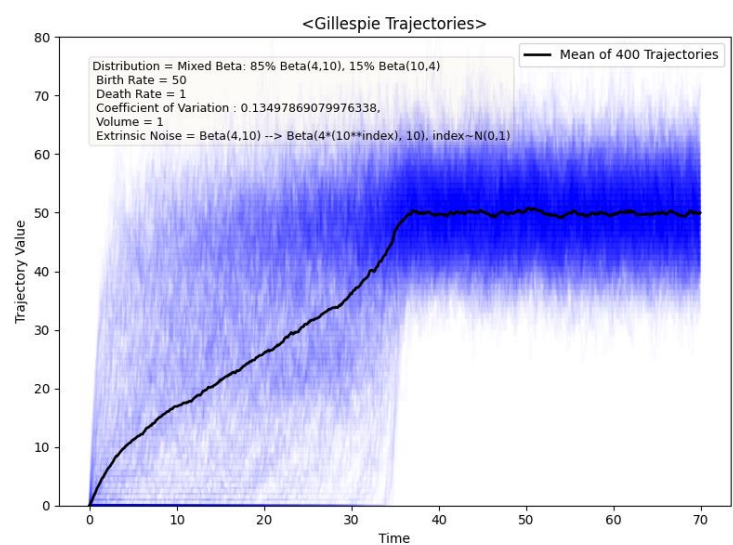
Same Extrinsic Noise (delay)



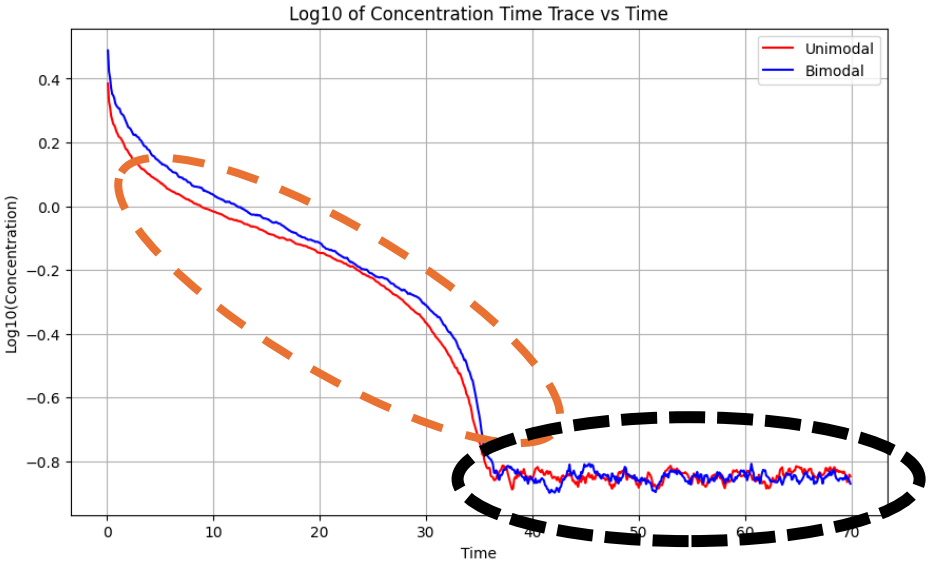
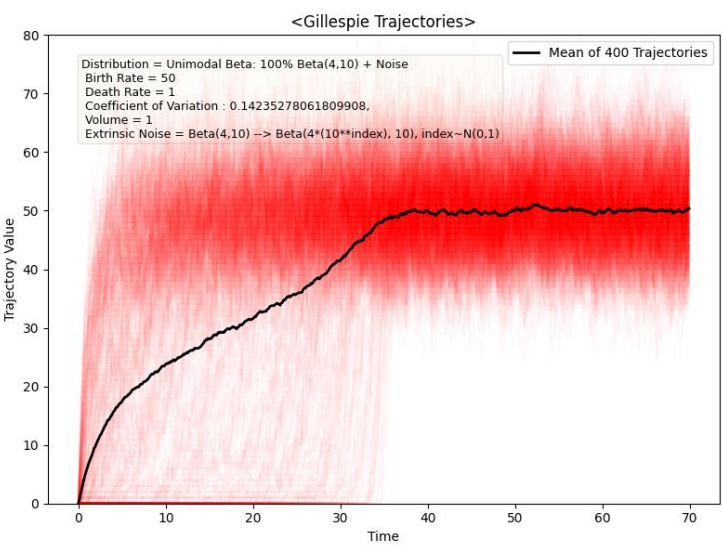
CV time trace repeatedly alternates between unimodal CV > bimodal CV, and unimodal CV < bimodal CV, in steady-state, and shows unimodal CV < bimodal CV in transient.



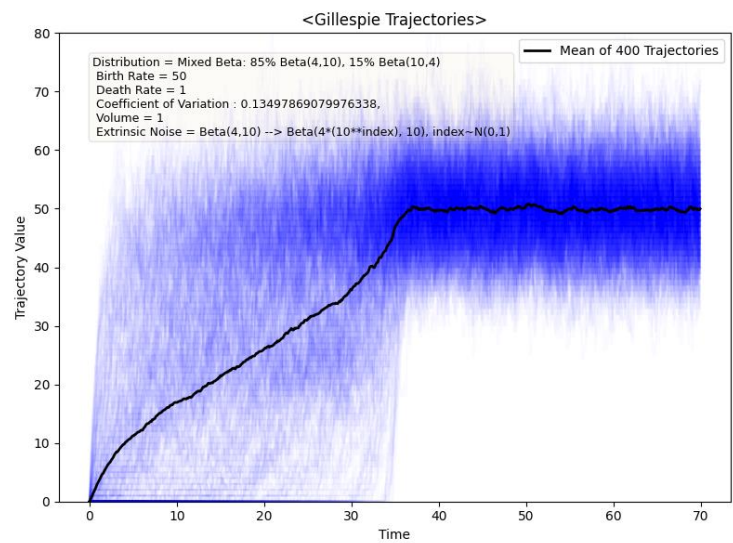
Same Extrinsic Noise (delay)



CV time trace repeatedly alternates between unimodal CV > bimodal CV, and unimodal CV < bimodal CV, in steady-state, and shows unimodal CV < bimodal CV in transient.

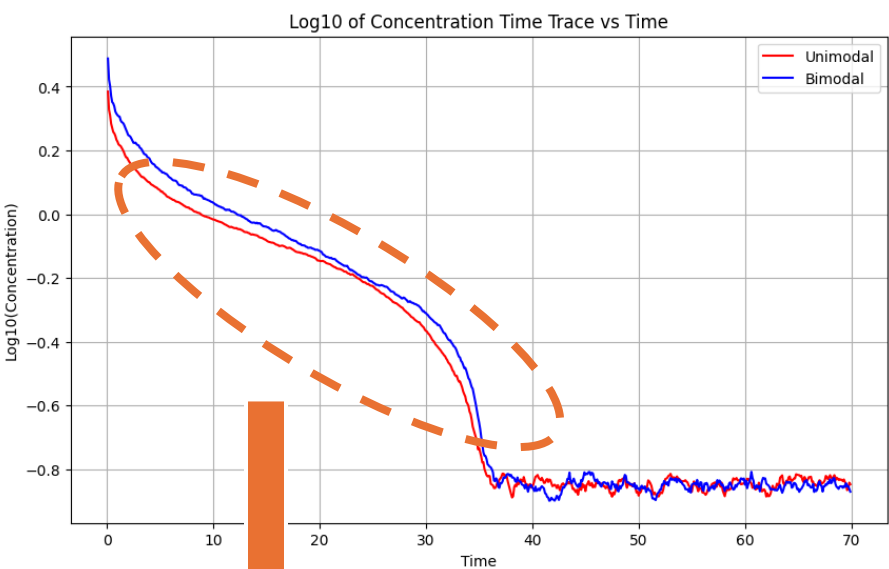
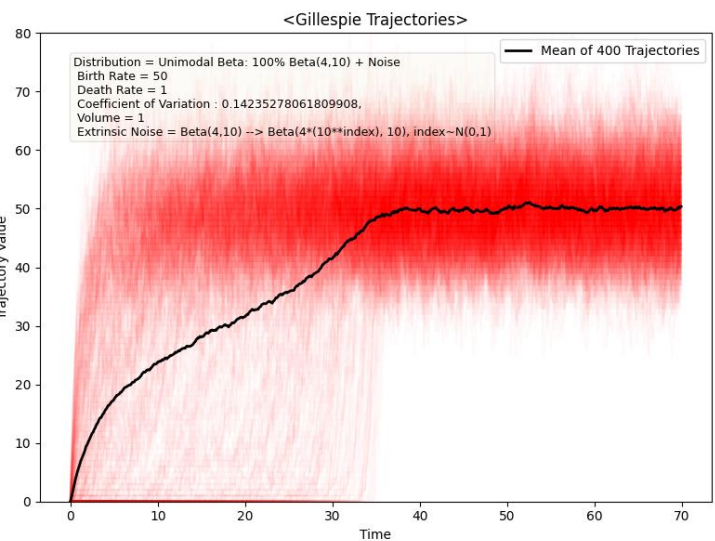


Same Extrinsic Noise (delay)

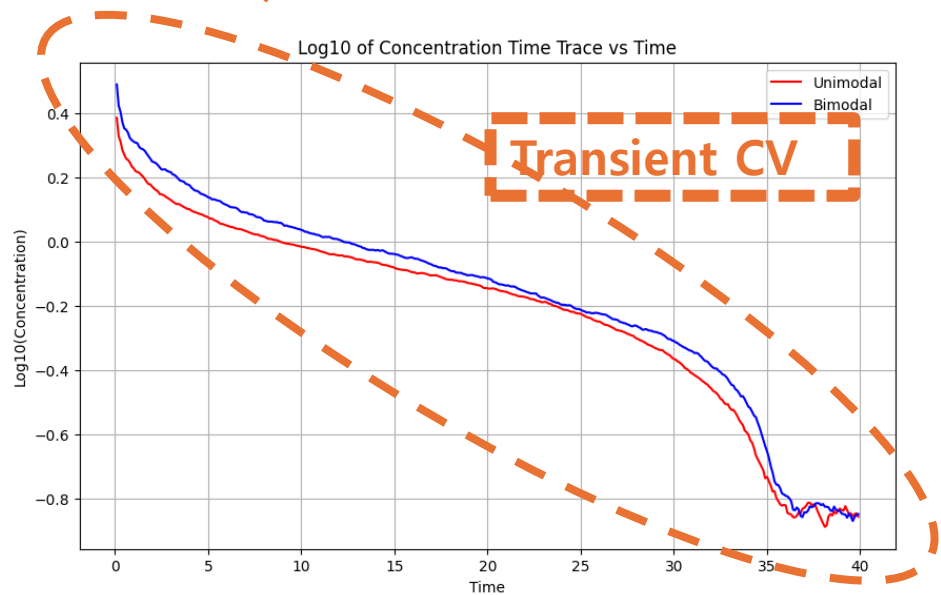
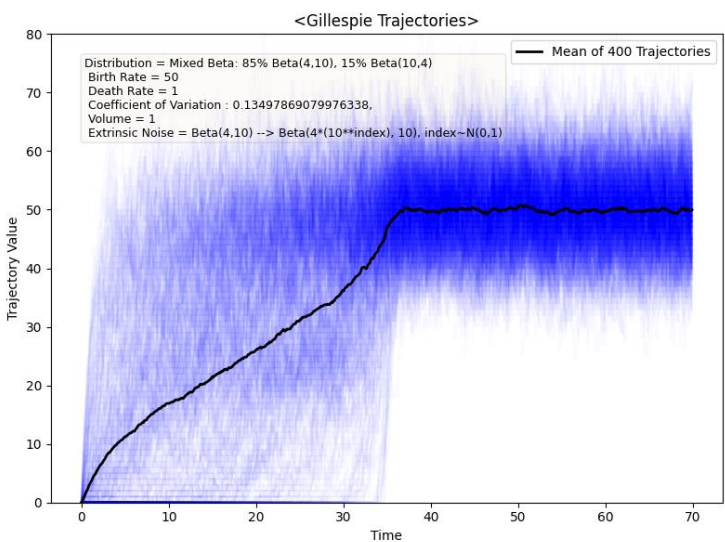




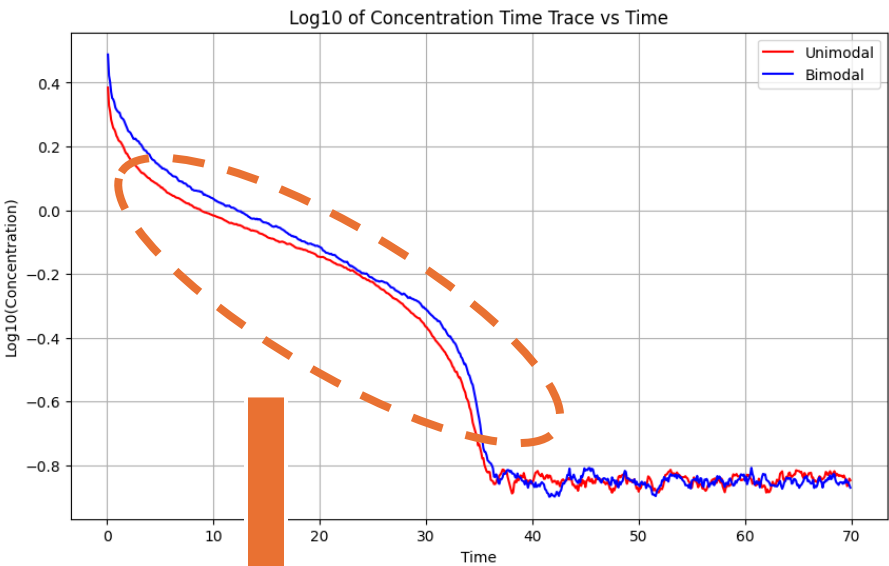
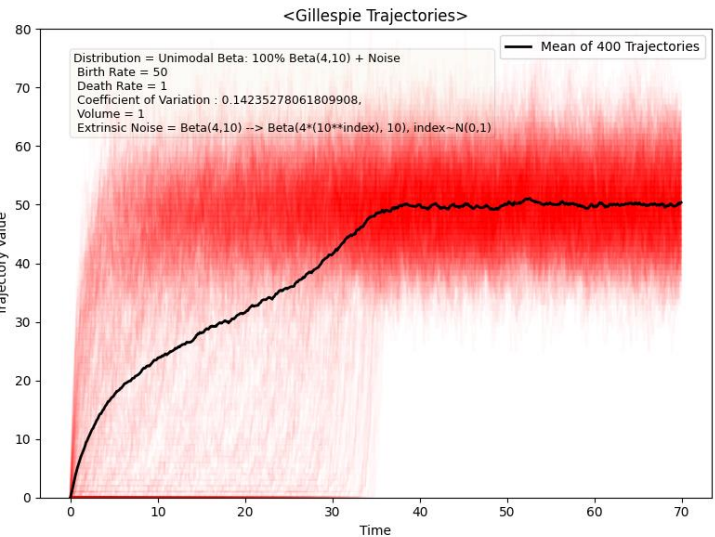
CV time trace repeatedly alternates between unimodal CV > bimodal CV, and unimodal CV < bimodal CV, in steady-state, and shows unimodal CV < bimodal CV in transient.



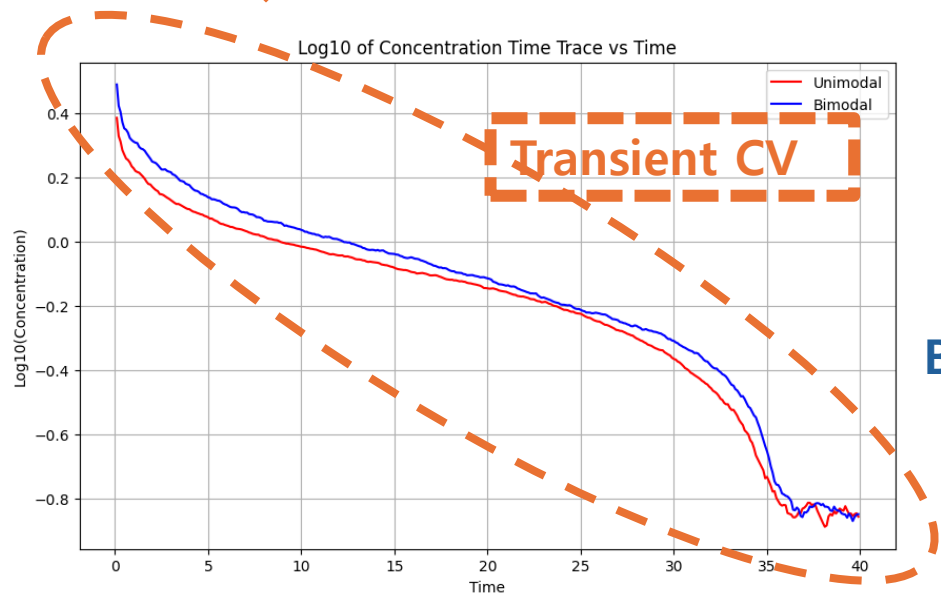
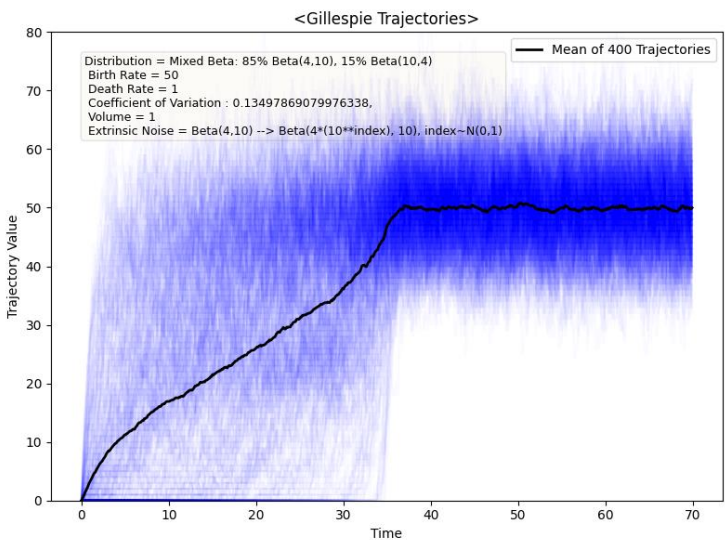
Same Extrinsic Noise (delay)



CV time trace repeatedly alternates between unimodal CV > bimodal CV, and unimodal CV < bimodal CV, in steady-state, and shows unimodal CV < bimodal CV in transient.

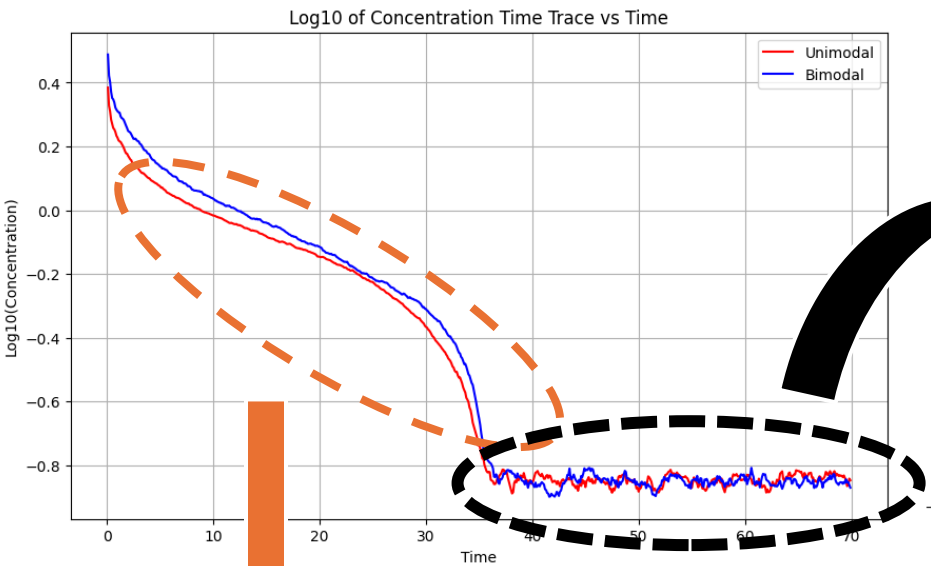
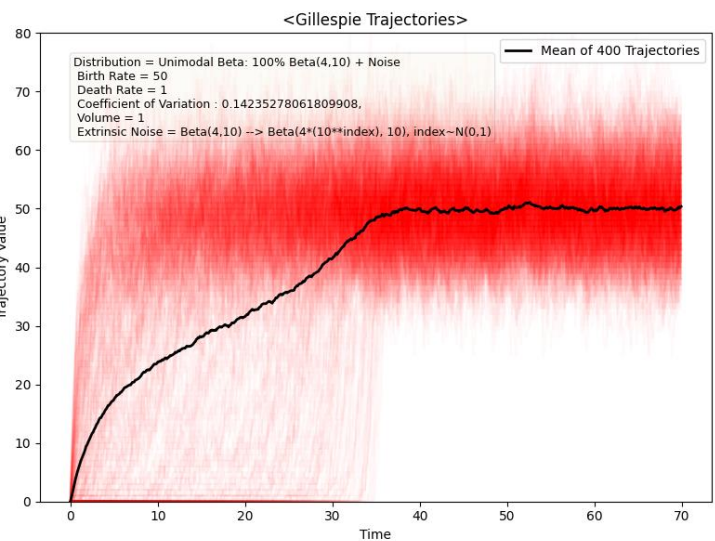


Same Extrinsic Noise (delay)

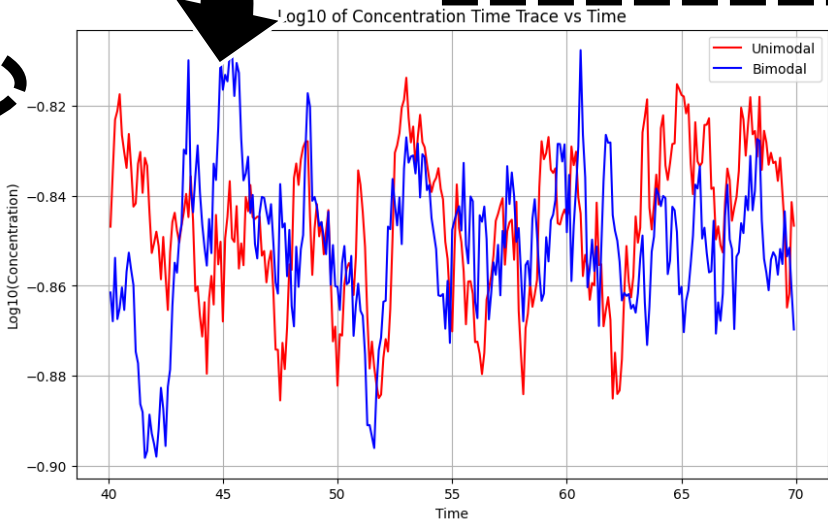


Bimodal CV > Unimodal CV (??)

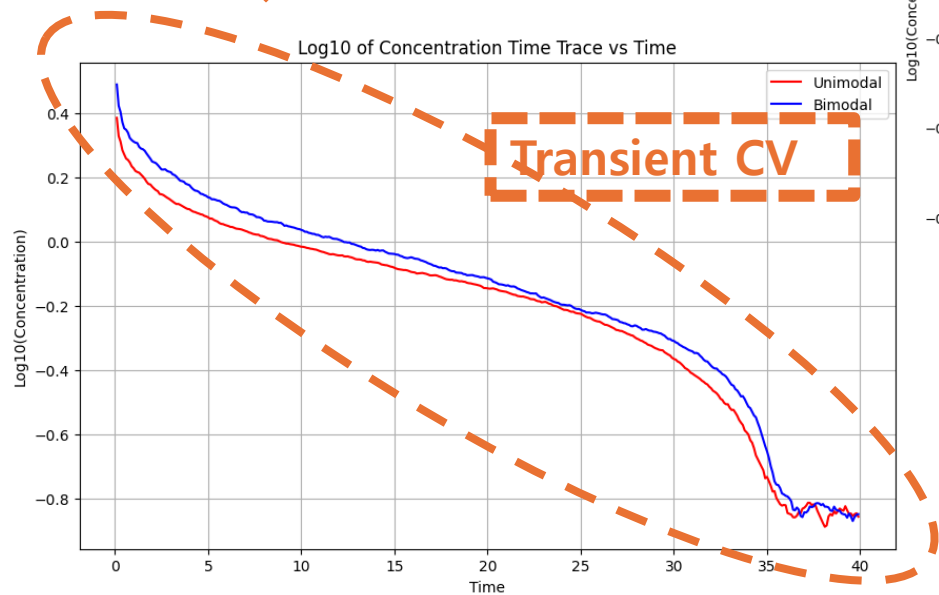
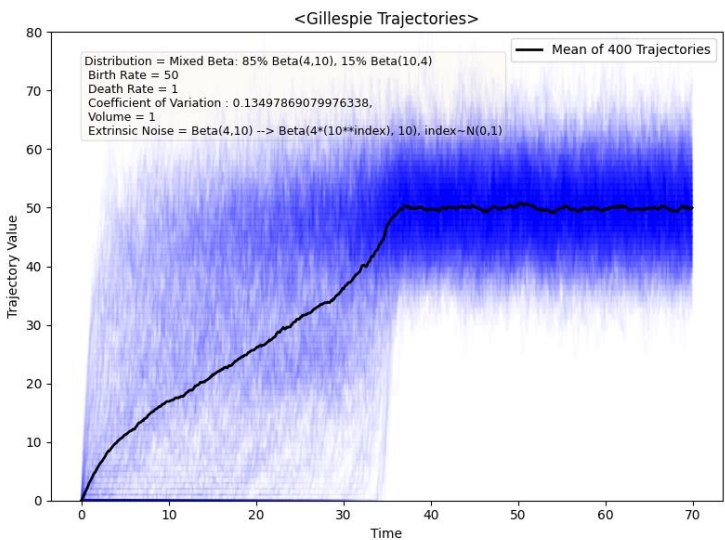
CV time trace repeatedly alternates between unimodal CV > bimodal CV, and unimodal CV < bimodal CV, in steady-state, and shows unimodal CV < bimodal CV in transient.



Steady State CV



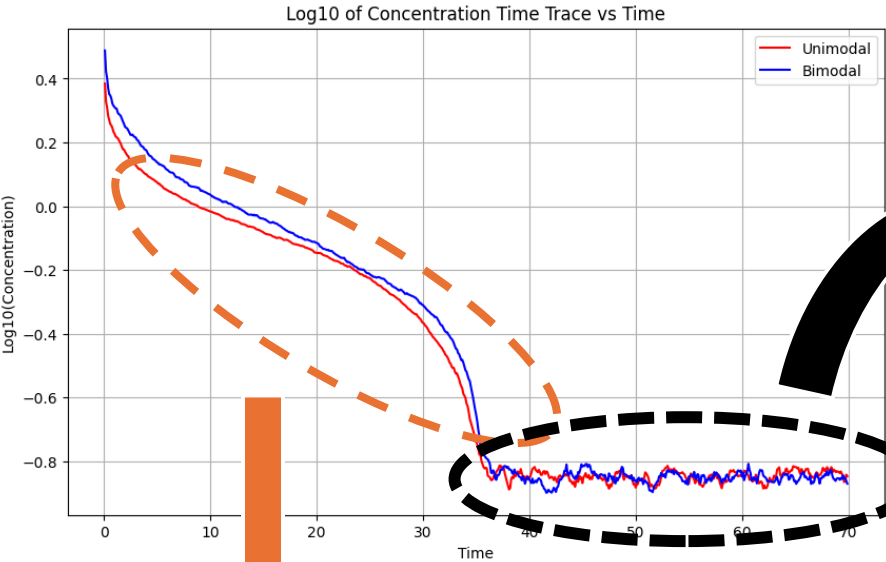
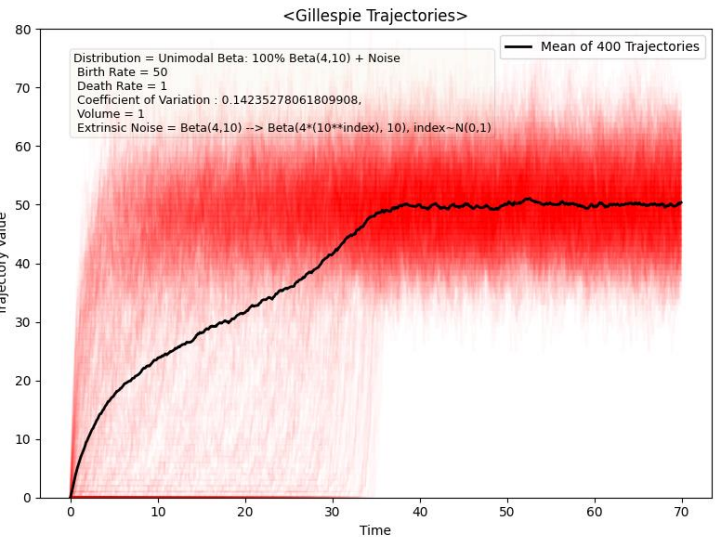
Same Extrinsic Noise (delay)



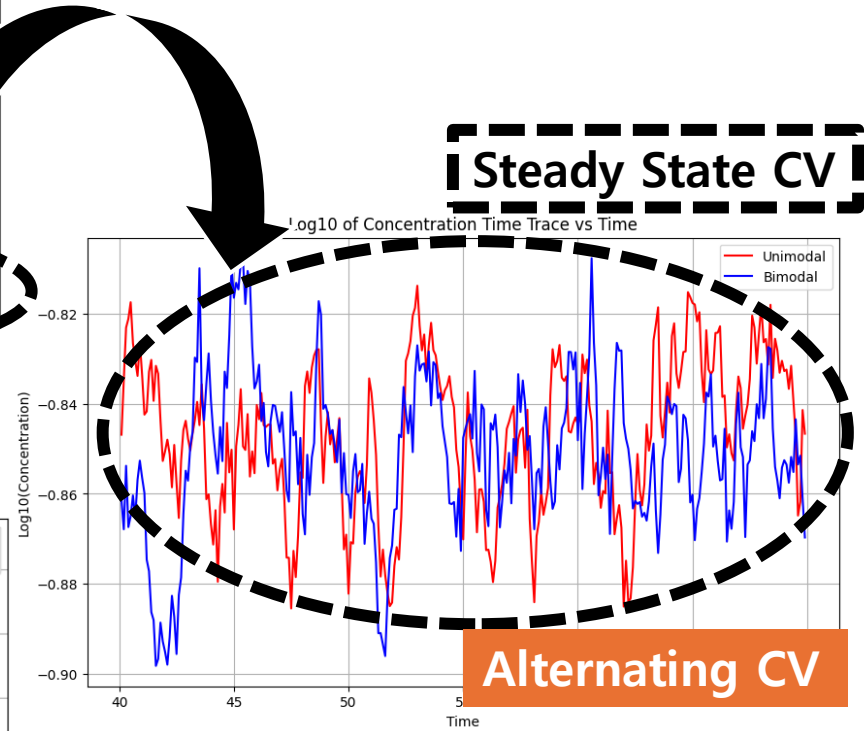
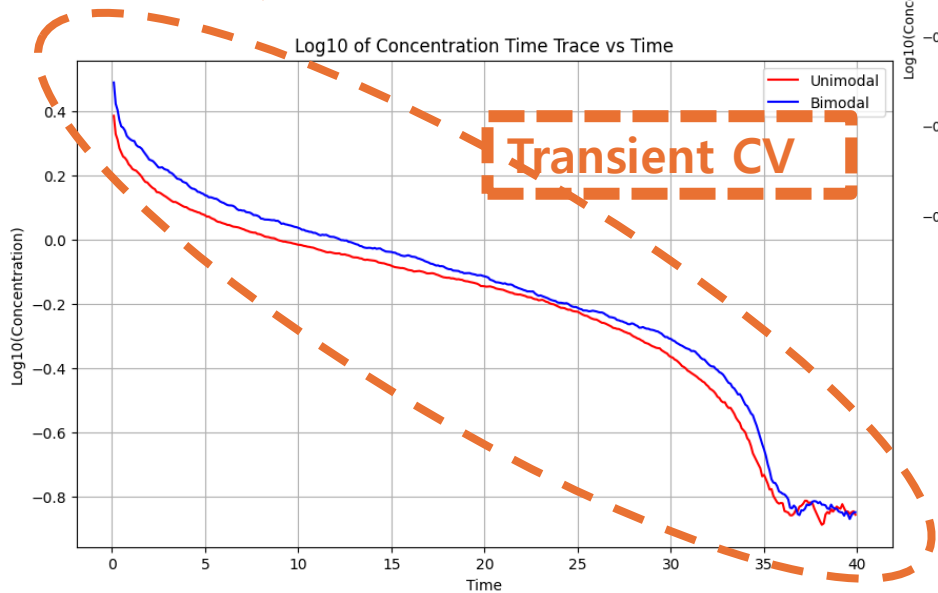
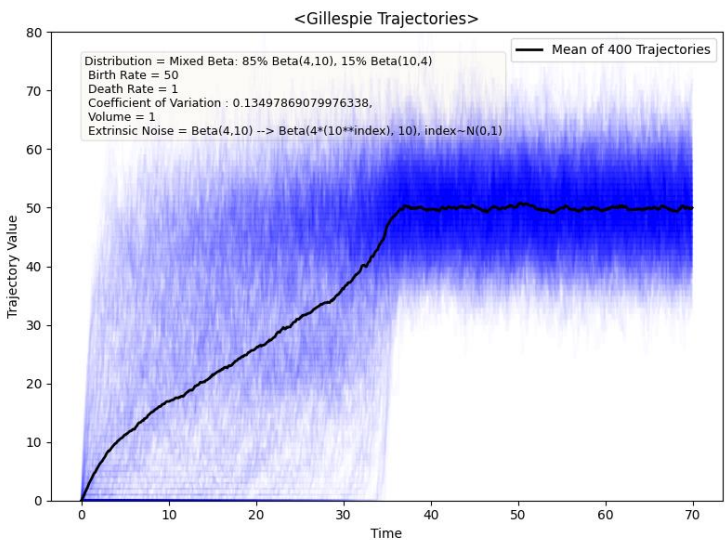
Transient CV



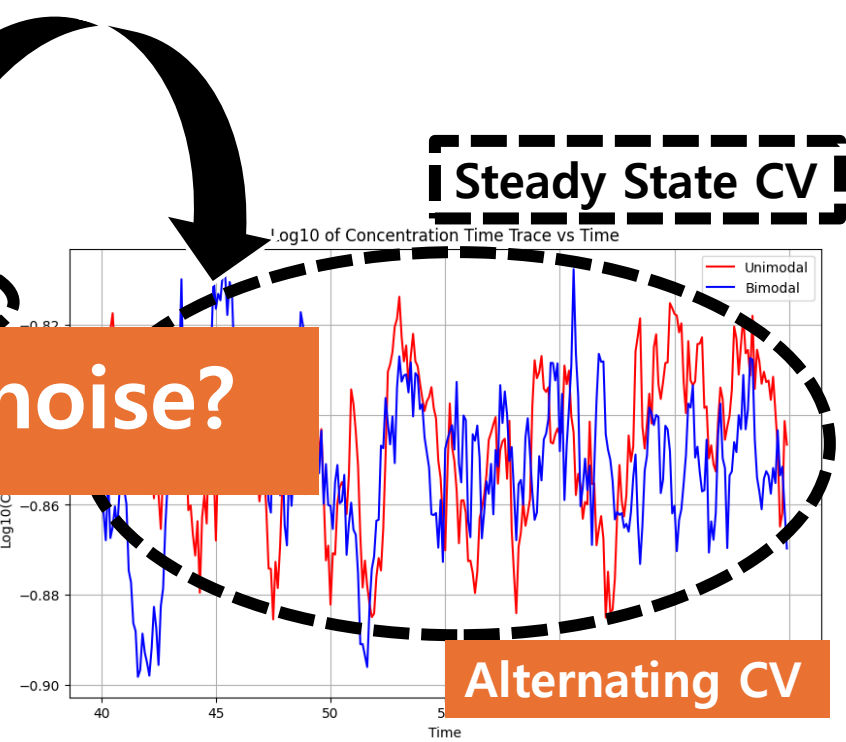
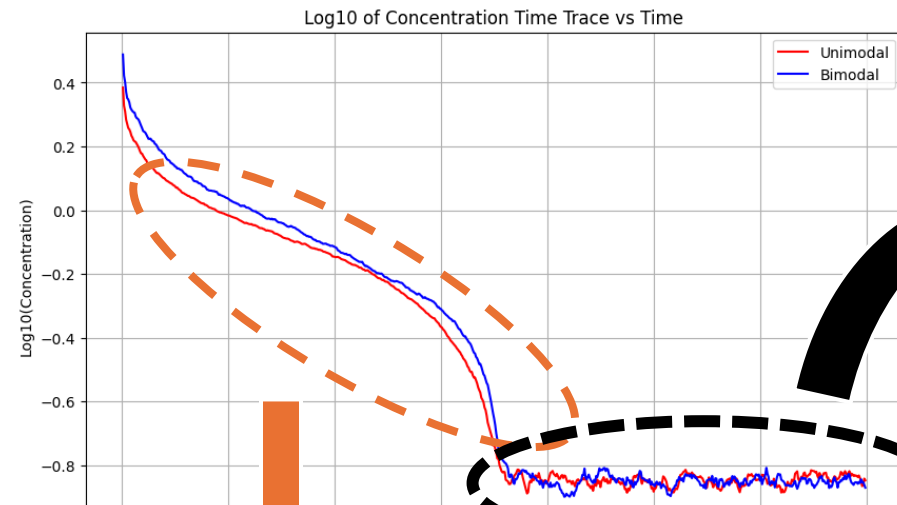
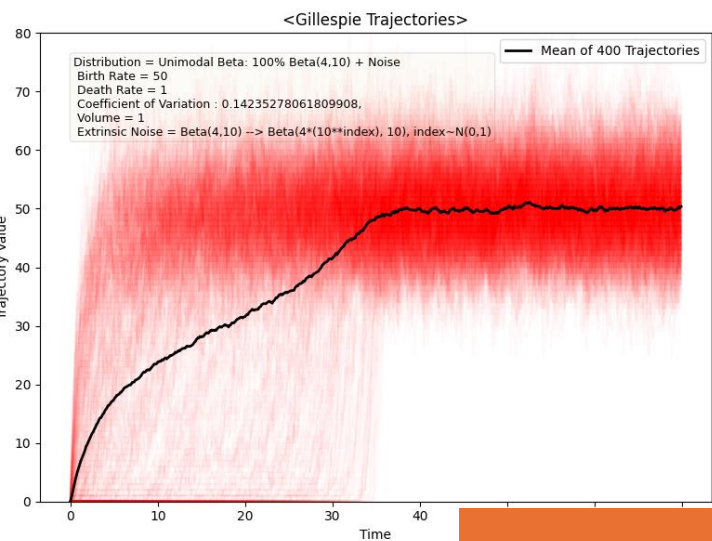
CV time trace repeatedly alternates between unimodal CV > bimodal CV, and unimodal CV < bimodal CV, in steady-state, and shows unimodal CV < bimodal CV in transient.



Same Extrinsic Noise (delay)

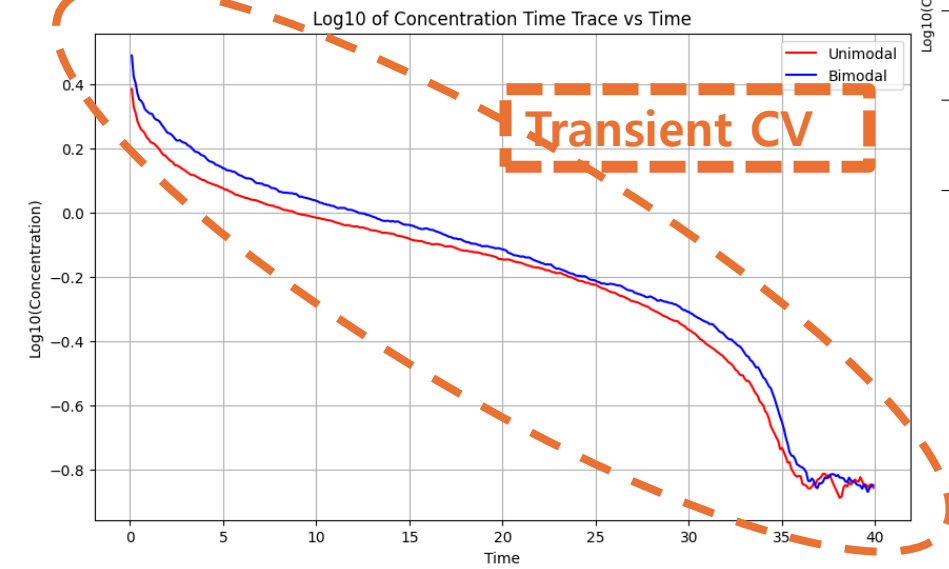
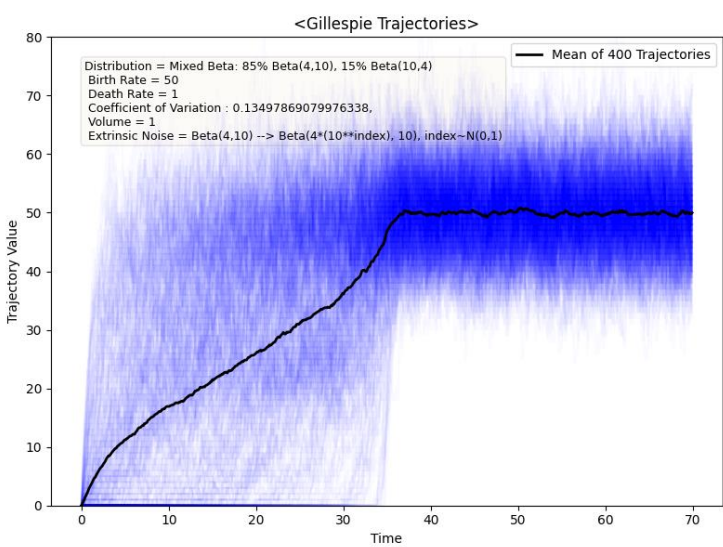


CV time trace repeatedly alternates between unimodal CV > bimodal CV, and unimodal CV < bimodal CV, in steady-state, and shows unimodal CV < bimodal CV in transient.



How about birth/death rate noise?

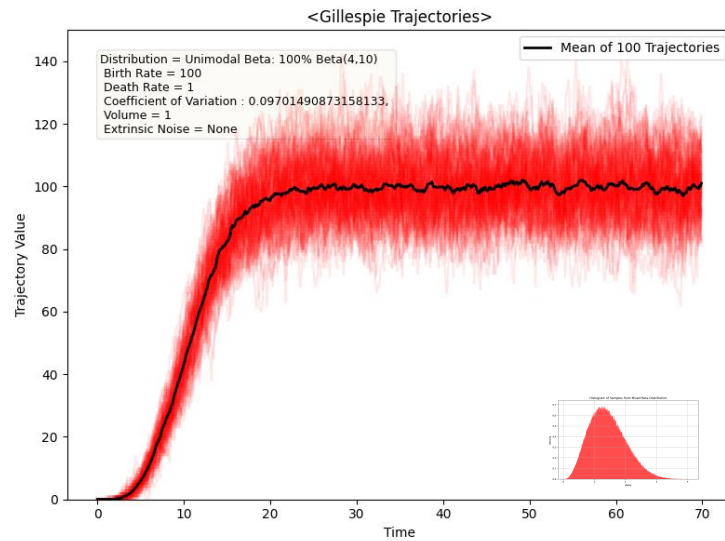
Same Extrinsic Noise





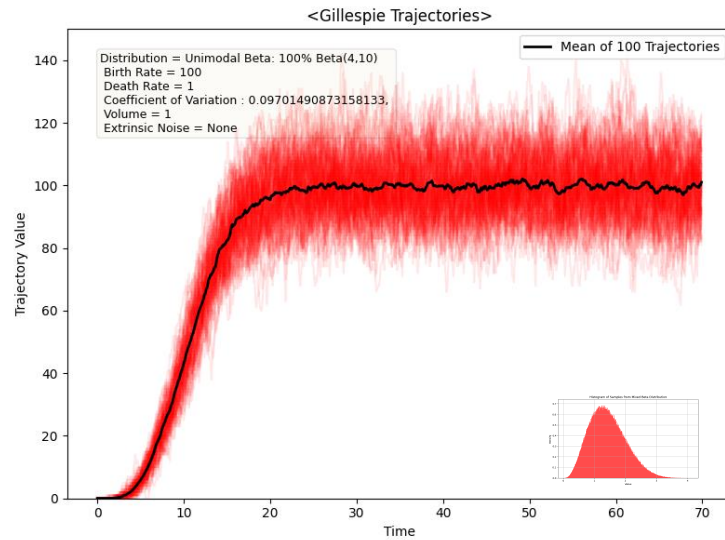
**Adding extrinsic noise in birth/death rates increases CV in both transient and steady states.**

$$\lambda_b = 100, \lambda_d = 1$$



# Adding extrinsic noise in birth/death rates increases CV in both transient and steady states.

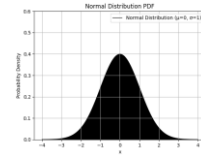
$$\lambda_b = 100, \lambda_d = 1$$



For Each Trajectory,

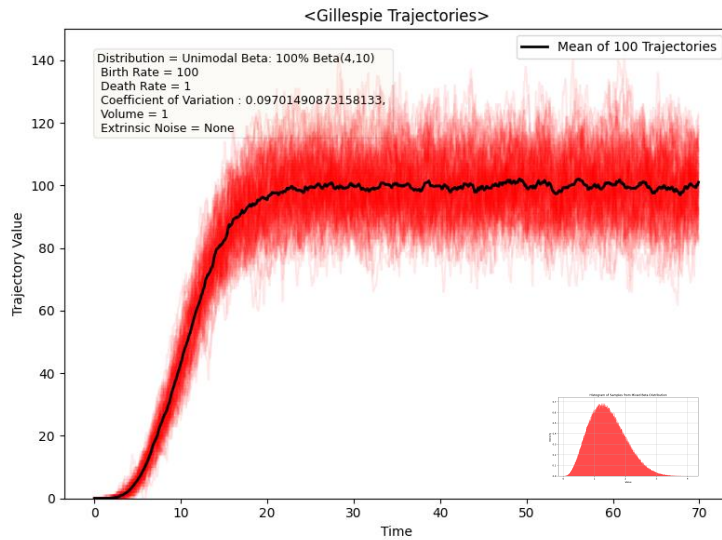


$$k \sim N(0, 1)$$



# Adding extrinsic noise in birth/death rates increases CV in both transient and steady states.

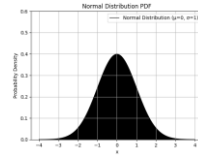
$$\lambda_b = 100, \quad \lambda_d = 1$$



For Each Trajectory,



$$k \sim N(0, 1)$$

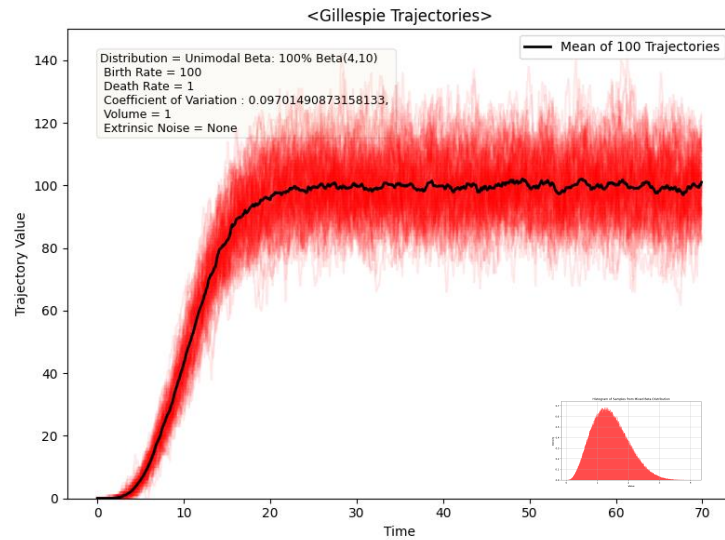


$$\lambda_b = 100 + 20 \cdot k, \quad k \sim N(0, 1)$$

$$\lambda_d = 1$$

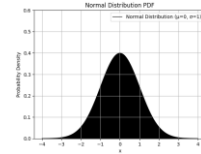
# Adding extrinsic noise in birth/death rates increases CV in both transient and steady states.

$$\lambda_b = 100, \quad \lambda_d = 1$$

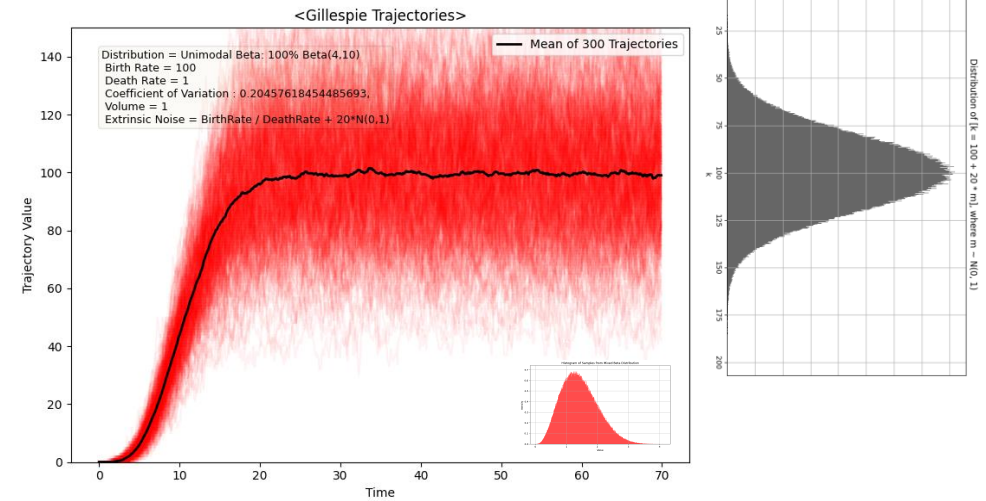


For Each Trajectory,

$$k \sim N(0, 1)$$

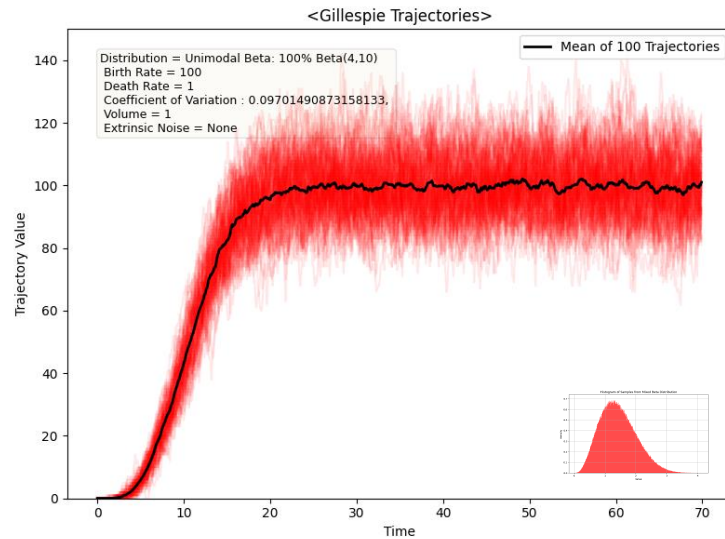


$$\lambda_b = 100 + 20 \cdot k, \quad k \sim N(0, 1)$$
$$\lambda_d = 1$$



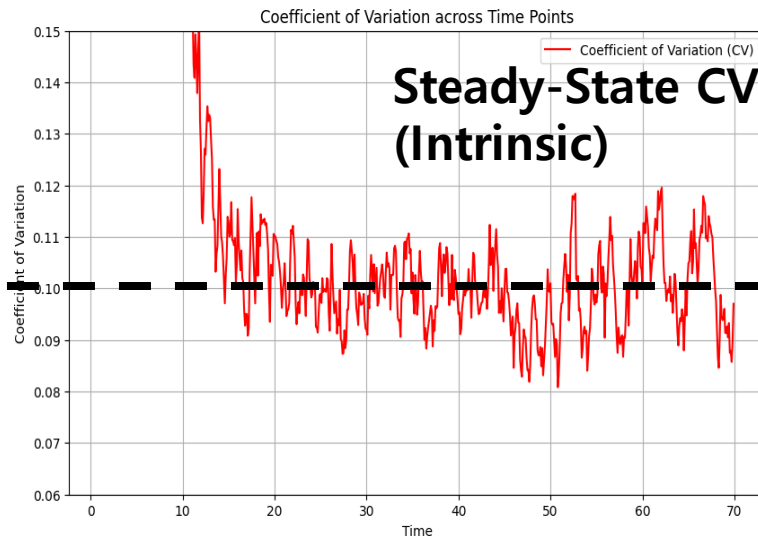
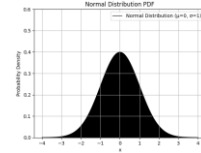
# Adding extrinsic noise in birth/death rates increases CV in both transient and steady states.

$$\lambda_b = 100, \quad \lambda_d = 1$$



For Each Trajectory,

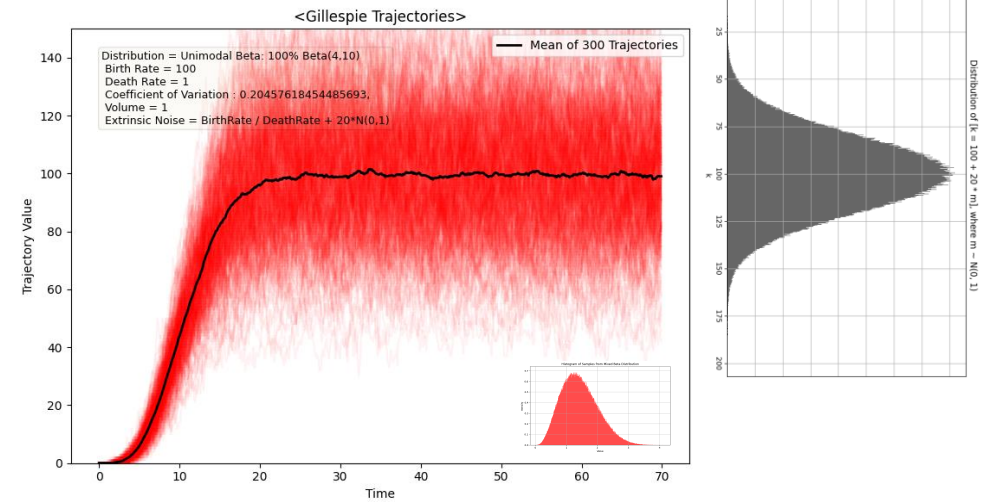
$$k \sim N(0, 1)$$



--- → CV ~ 0.10

$$\lambda_b = 100 + 20 \cdot k, \quad k \sim N(0, 1)$$

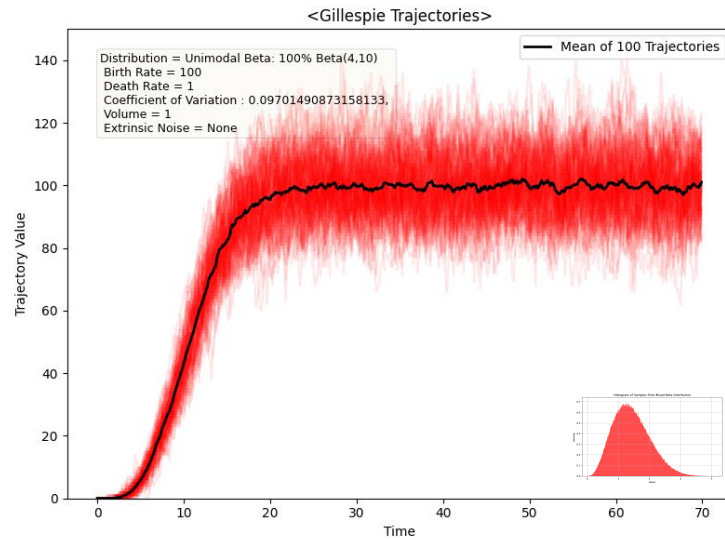
$$\lambda_d = 1$$





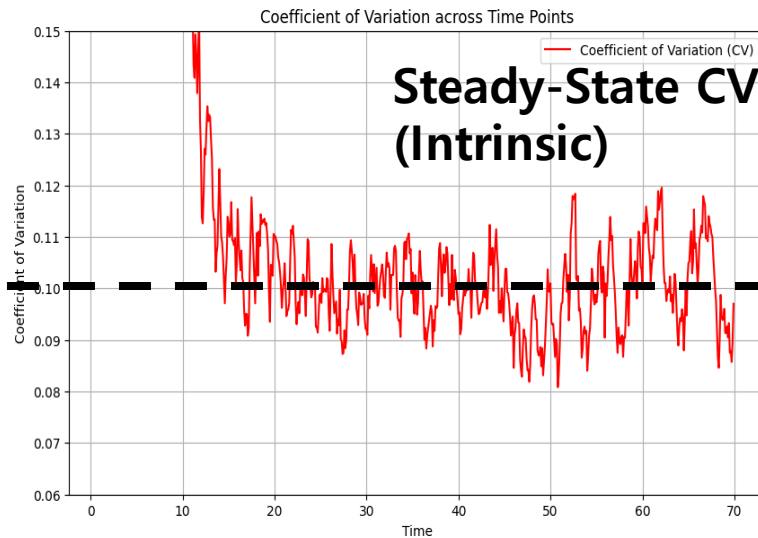
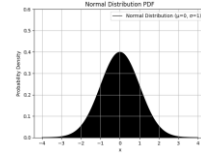
# Adding extrinsic noise in birth/death rates increases CV in both transient and steady states.

$$\lambda_b = 100, \quad \lambda_d = 1$$



For Each Trajectory,

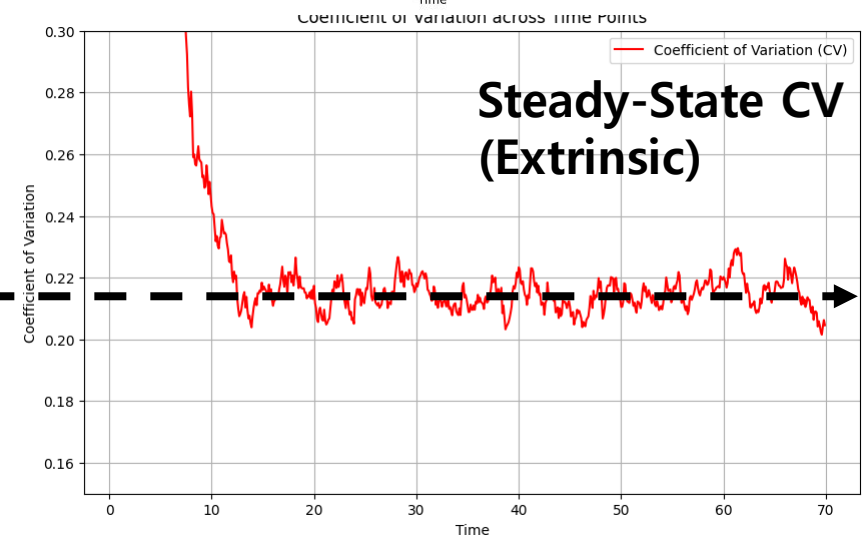
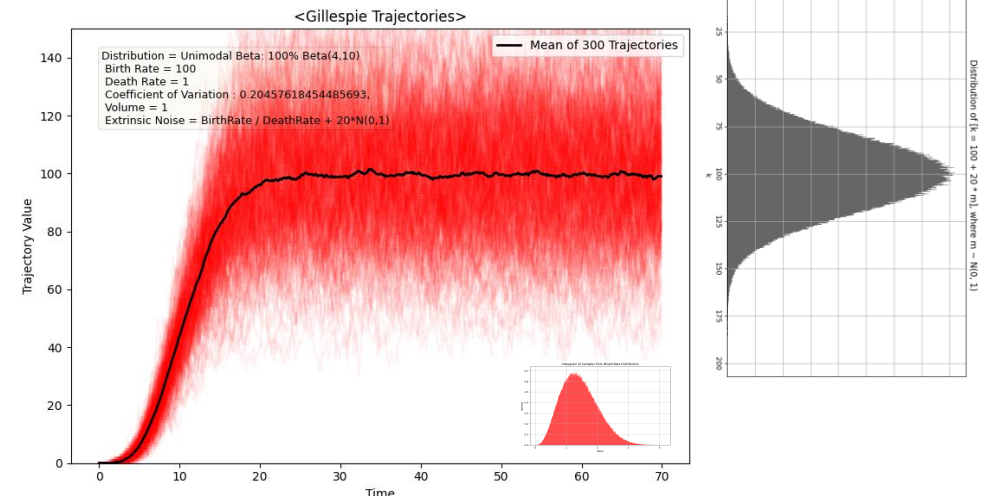
$$k \sim N(0, 1)$$



CV ~ 0.10

$$\lambda_b = 100 + 20 \cdot k, \quad k \sim N(0, 1)$$

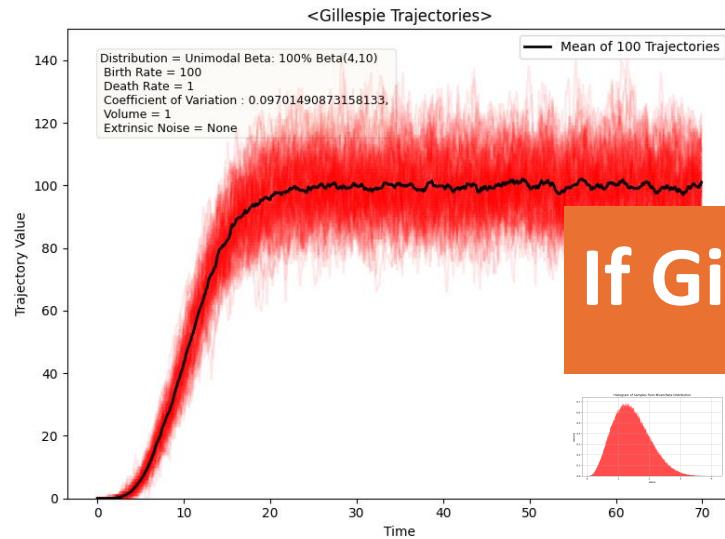
$$\lambda_d = 1$$



CV ~ 0.21

# Adding extrinsic noise in birth/death rates increases CV in both transient and steady states.

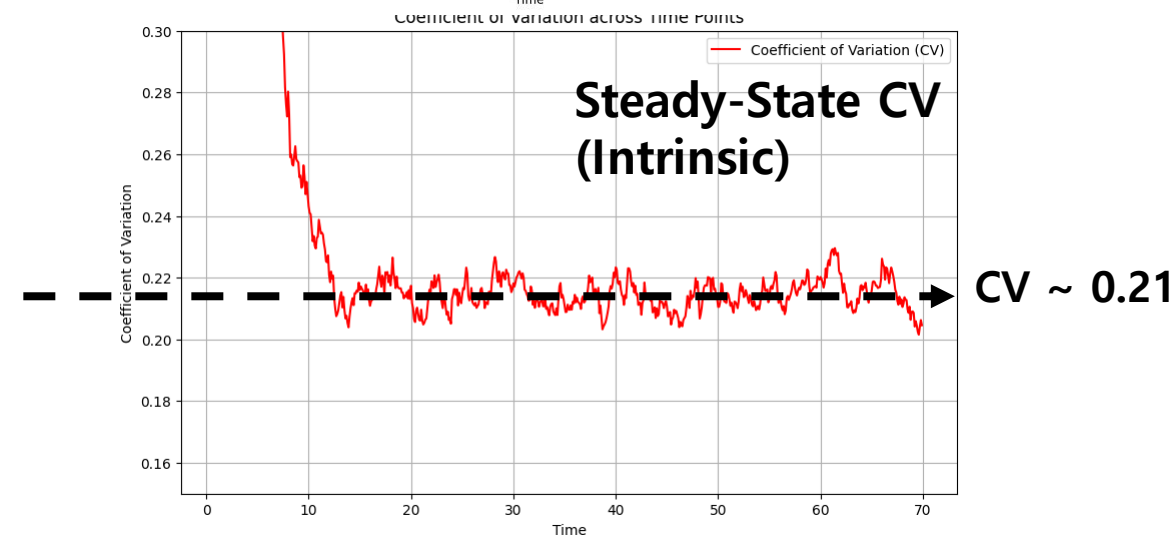
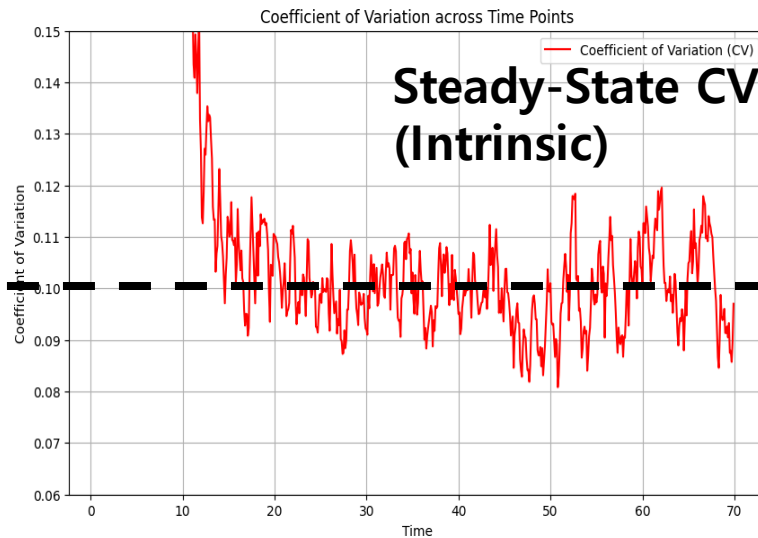
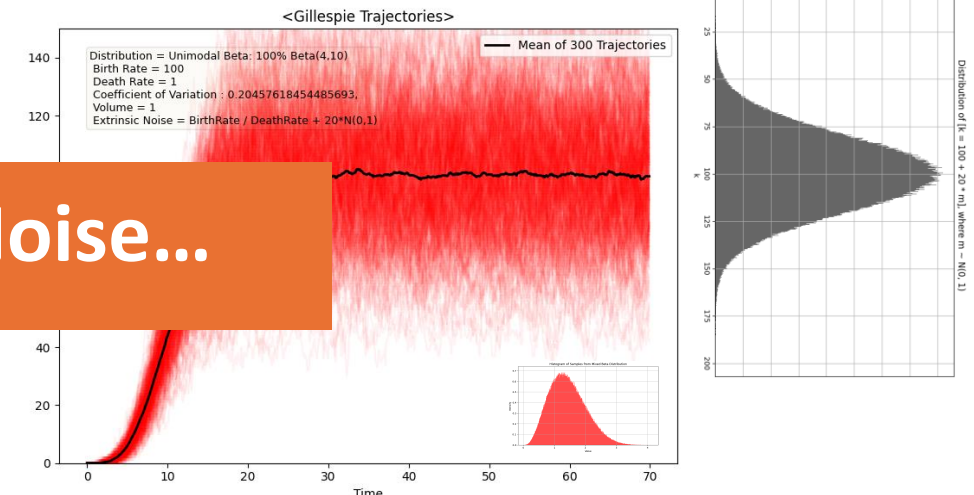
$$\lambda_b = 100, \quad \lambda_d = 1$$



If Given Same Extrinsic Noise...

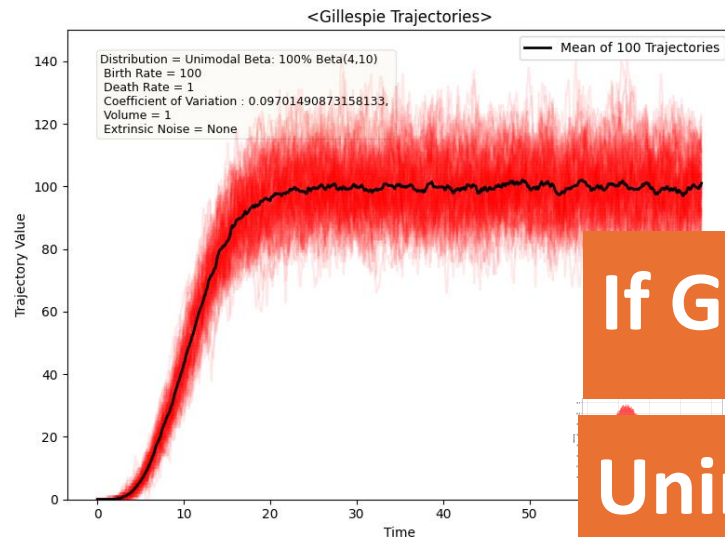
$$\lambda_b = 100 + 20 \cdot k, \quad k \sim N(0, 1)$$

$$\lambda_d = 1$$



Adding extrinsic noise in birth/death rates increases CV in both transient and steady states.

$$\lambda_b = 100, \lambda_d = 1$$

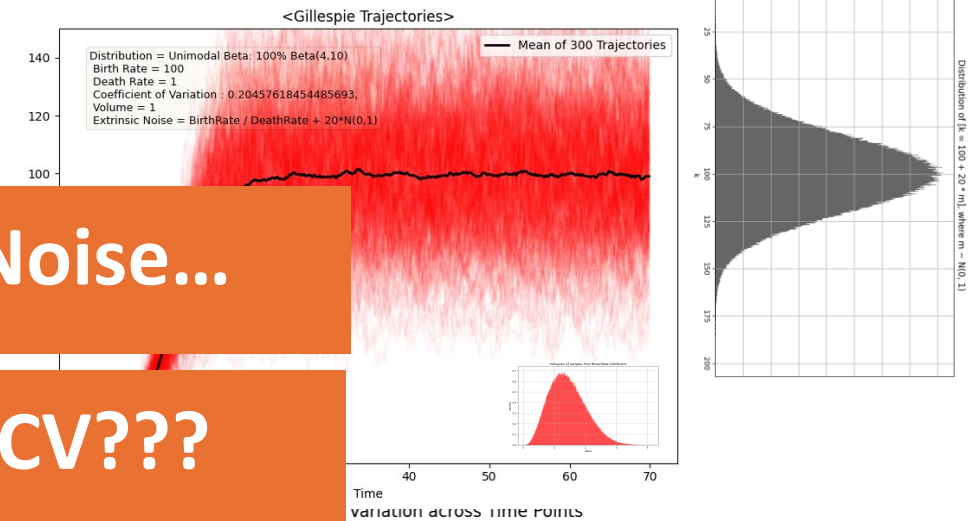


For Each Trajectory,



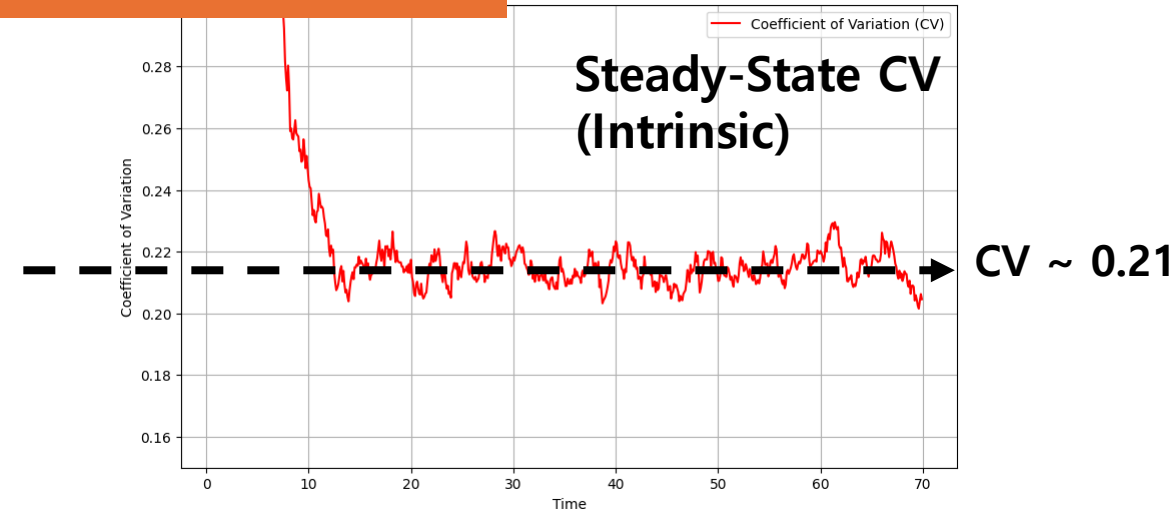
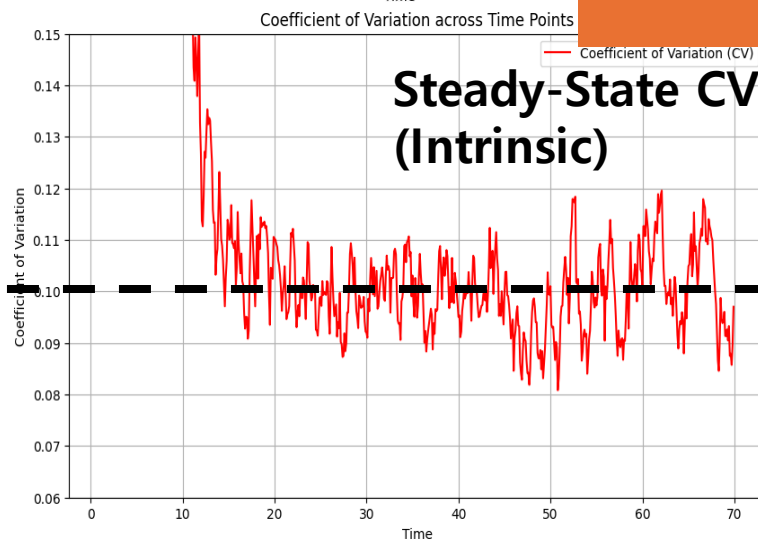
$$\lambda_b = 100 + 20 \cdot k, \quad k \sim N(0, 1)$$

$$\lambda_d = 1$$



If Given Same Extrinsic Noise...

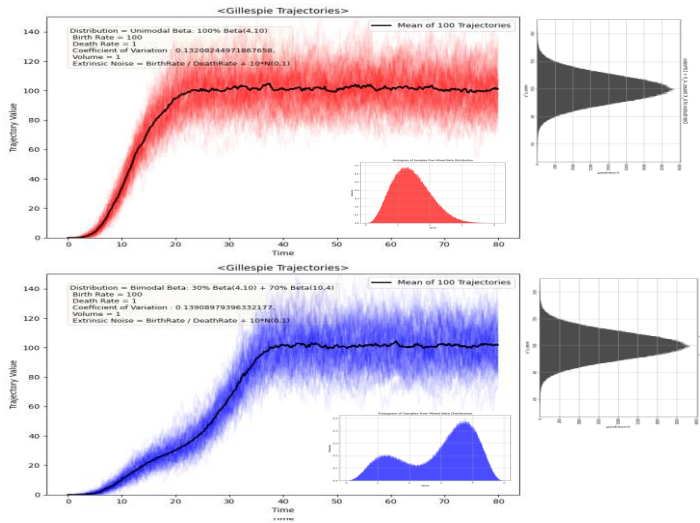
Unimodal CV > Bimodal CV???



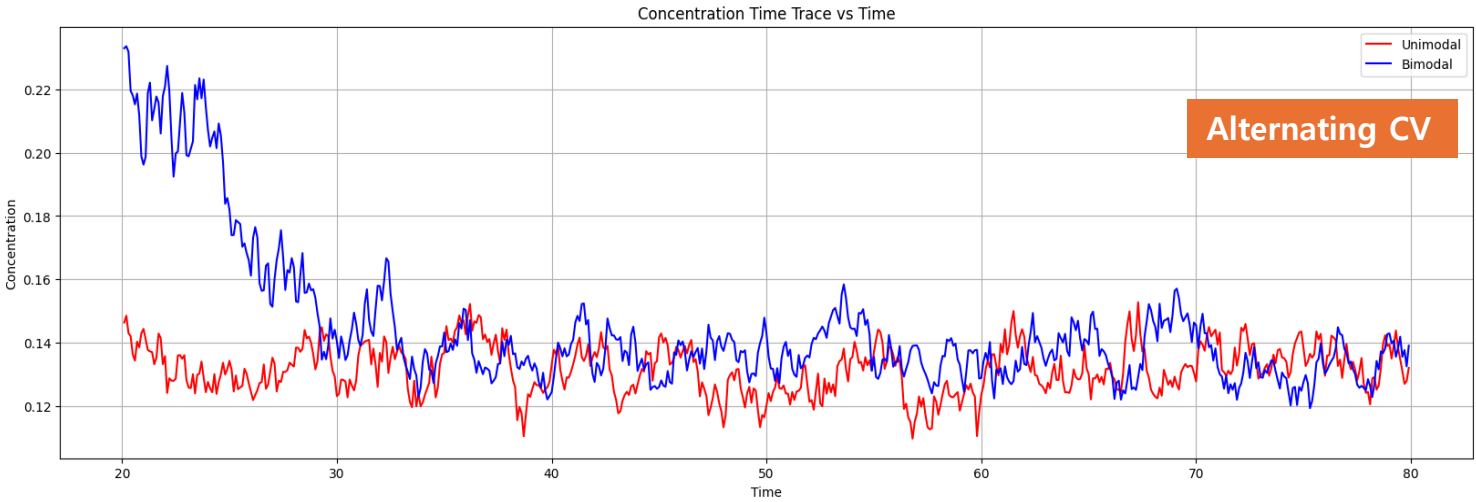
# Alternating CV relationship between unimodal and bimodal is observed despite various extrinsic noise settings.

$$\lambda_b = 100 + 10 \cdot k, \quad k \sim N(0, 1)$$

$$\lambda_d = 1$$



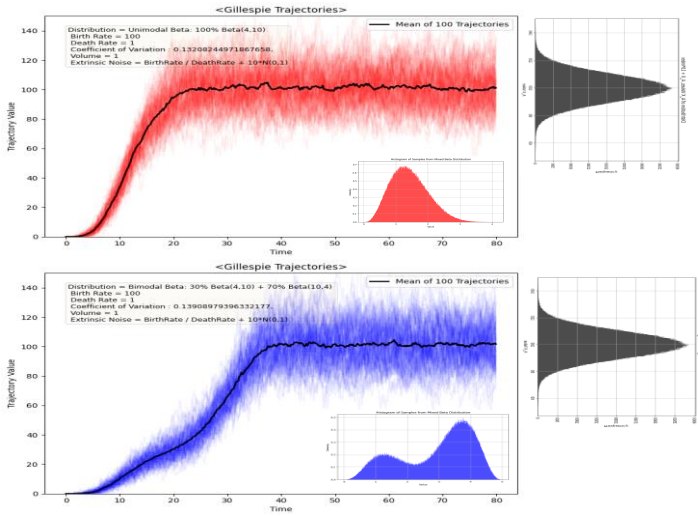
## Steady-State:



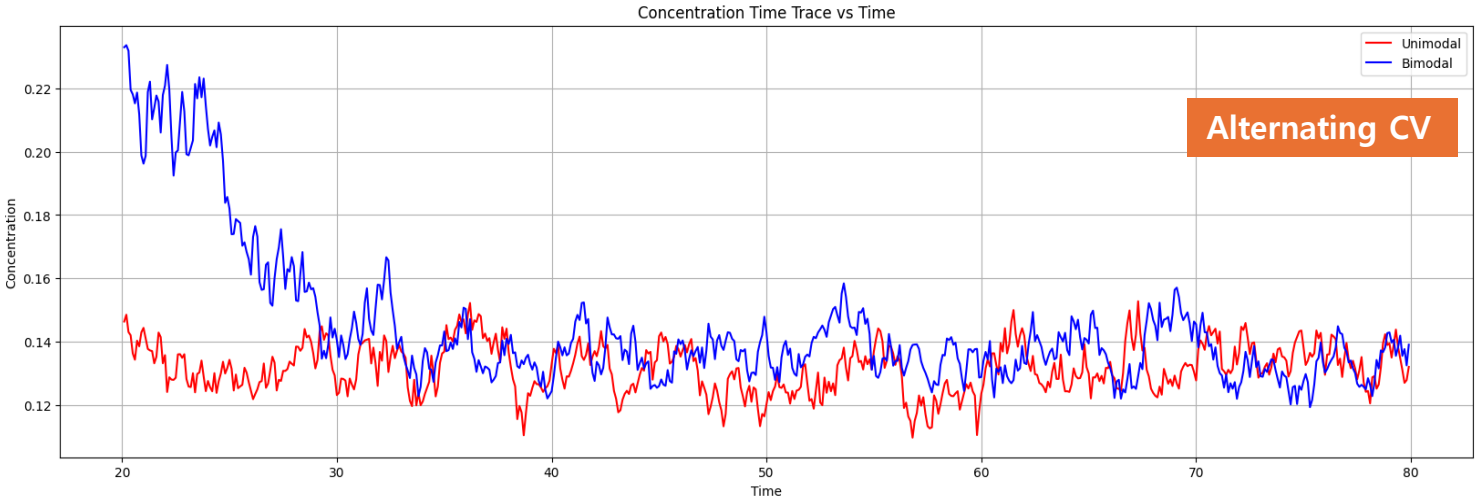
Alternating CV relationship between unimodal and bimodal is observed despite various extrinsic noise settings.

$\lambda_b = 100 + 10 \cdot k, \quad k \sim N(0, 1)$

$\lambda_d = 1$

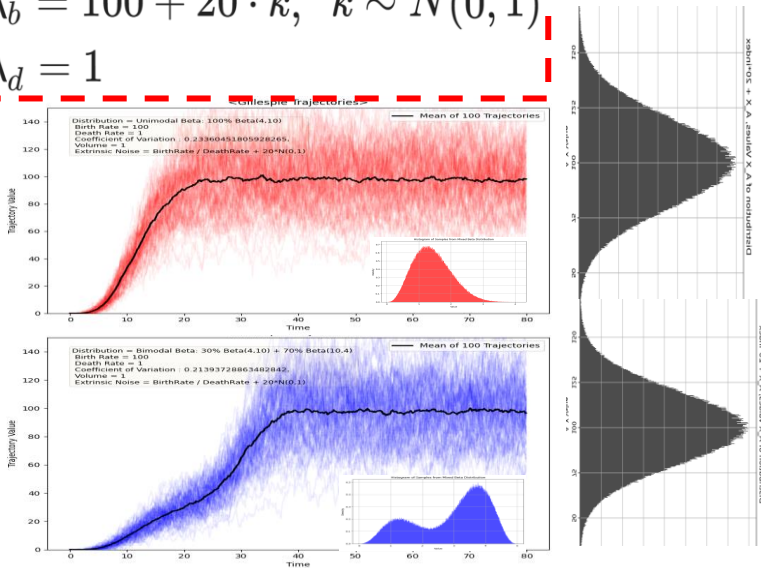


Steady-State:

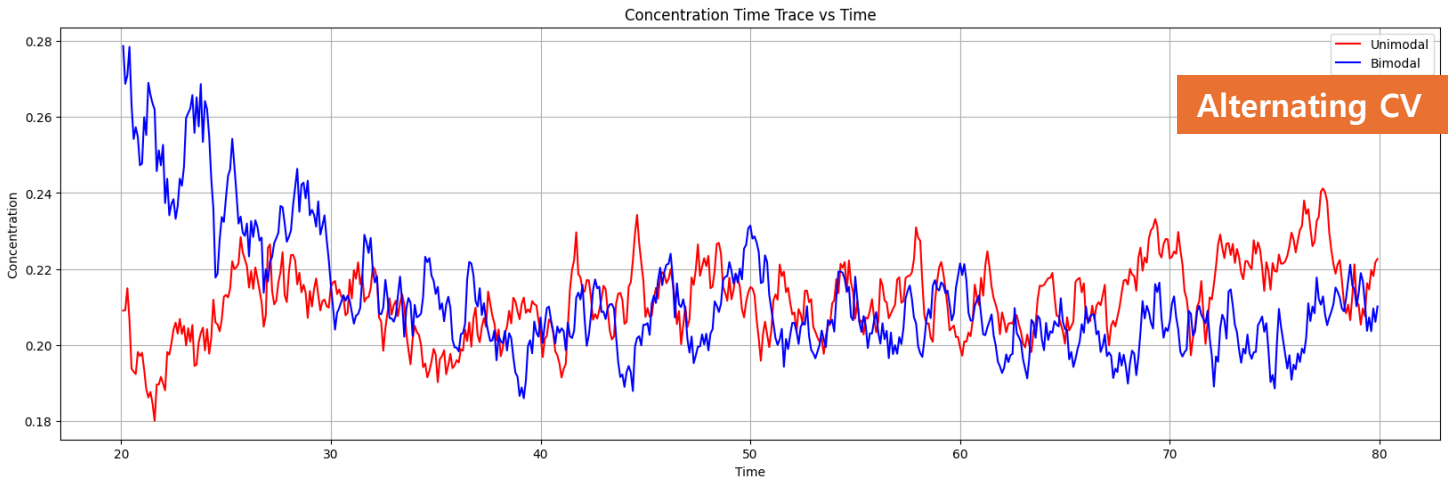


$\lambda_b = 100 + 20 \cdot k, \quad k \sim N(0, 1)$

$\lambda_d = 1$



Steady-State:



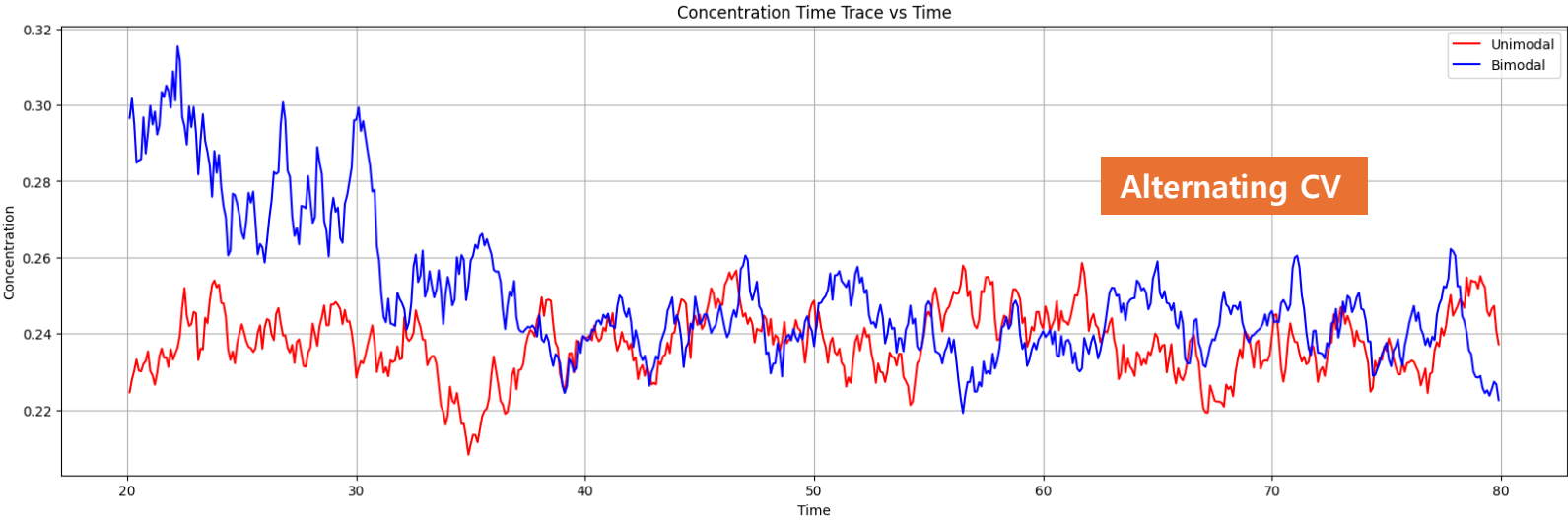
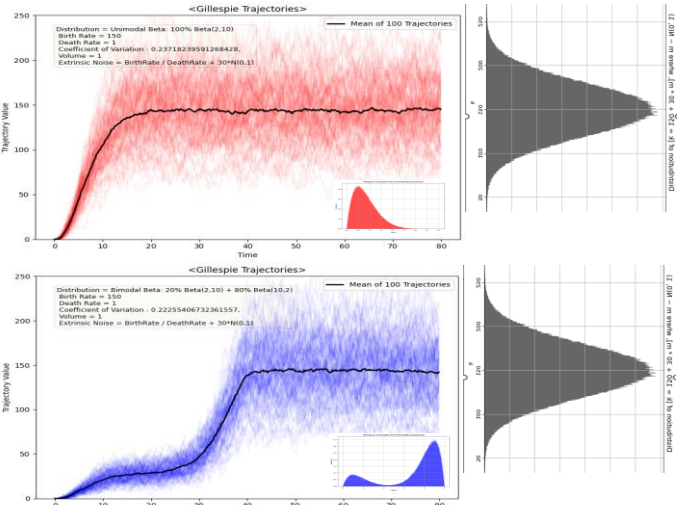


Alternating CV relationship between unimodal and bimodal is observed despite various extrinsic noise settings.

$\lambda_b = 150 + 30 \cdot k, \quad k \sim N(0,1)$

$\lambda_d = 1$

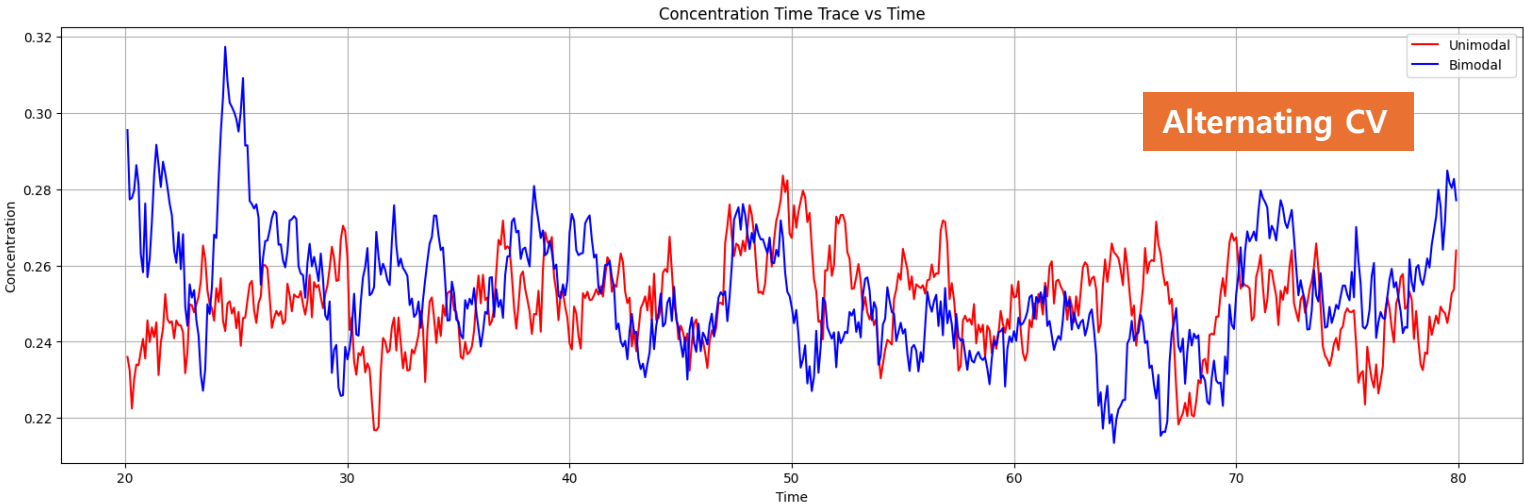
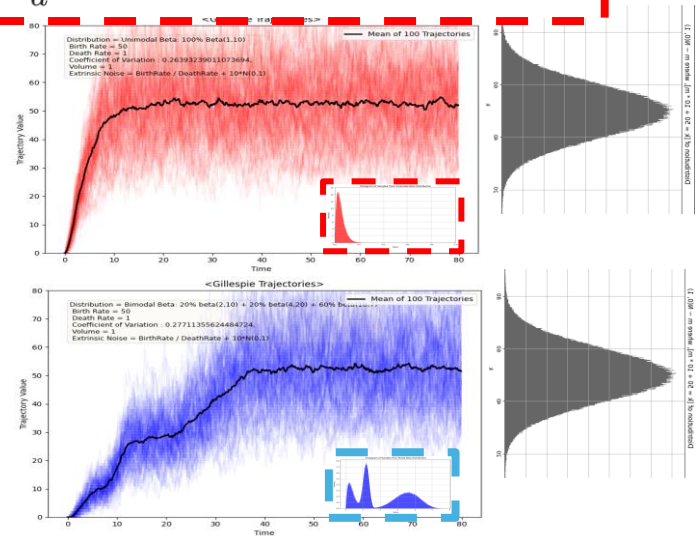
Steady-State:



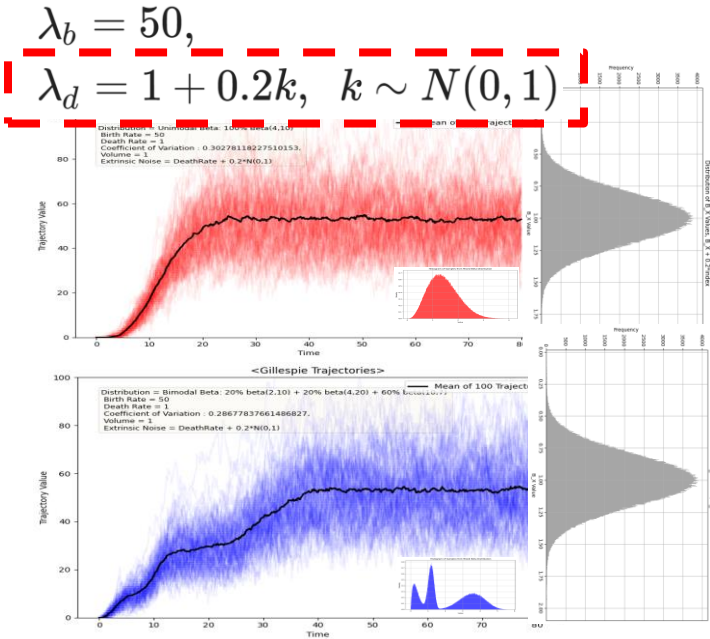
$\lambda_b = 50 + 10 \cdot k, \quad k \sim N(0,1)$

$\lambda_d = 1$

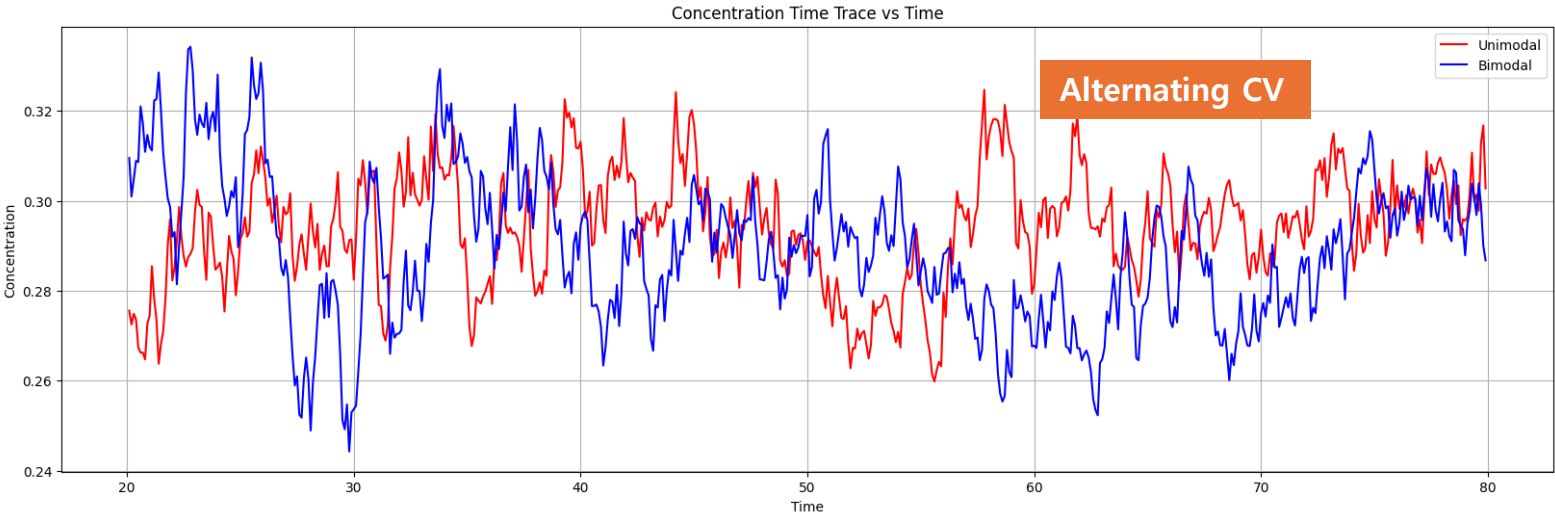
Steady-State:



Alternating CV relationship between unimodal and bimodal is observed despite various extrinsic noise settings.



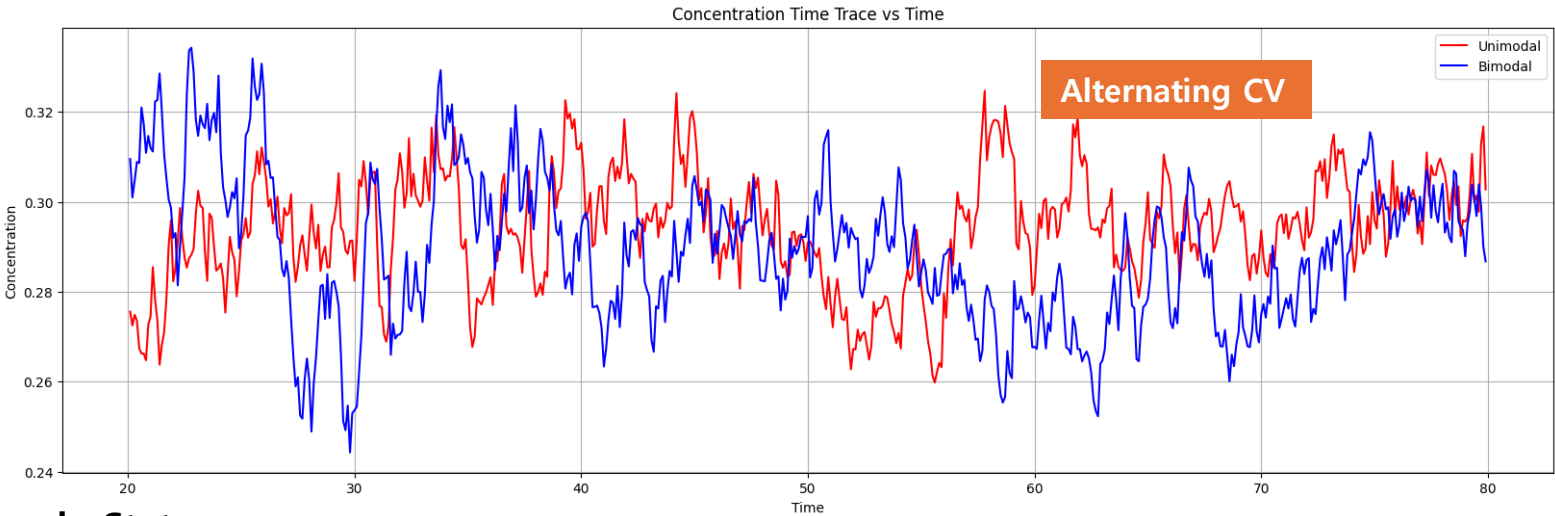
Steady-State:



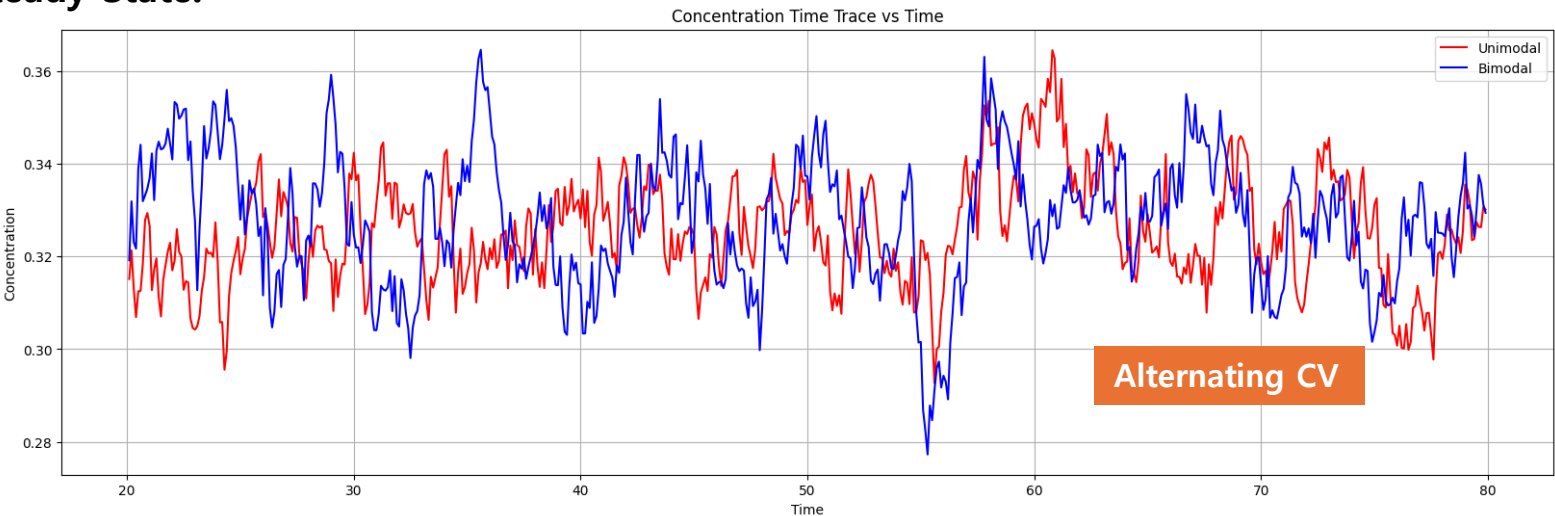
Alternating CV relationship between unimodal and bimodal is observed despite various extrinsic noise settings.

$$\lambda_b = 50,$$
$$\lambda_d = 1 + 0.2k, \quad k \sim N(0,1)$$

Steady-State:

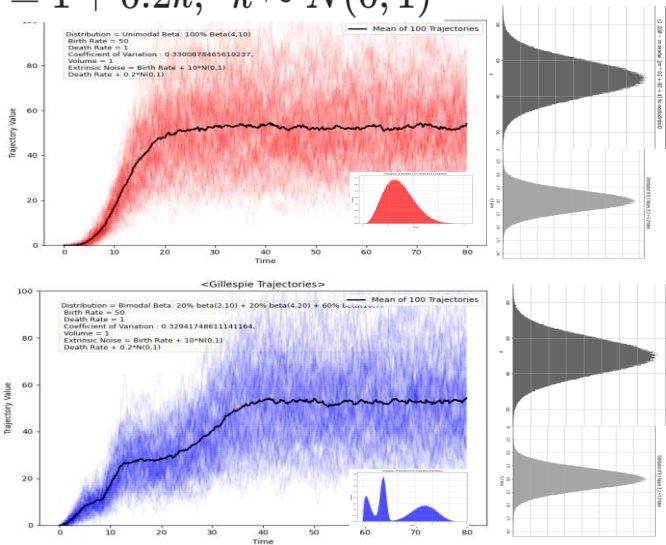


Steady-State:



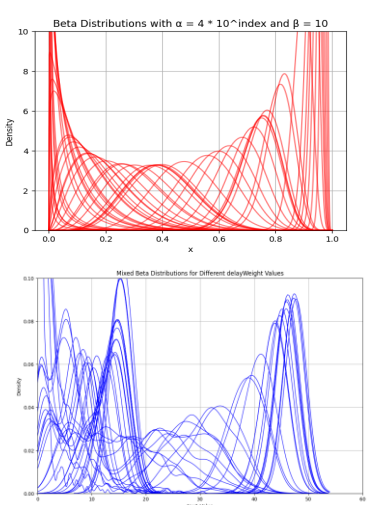
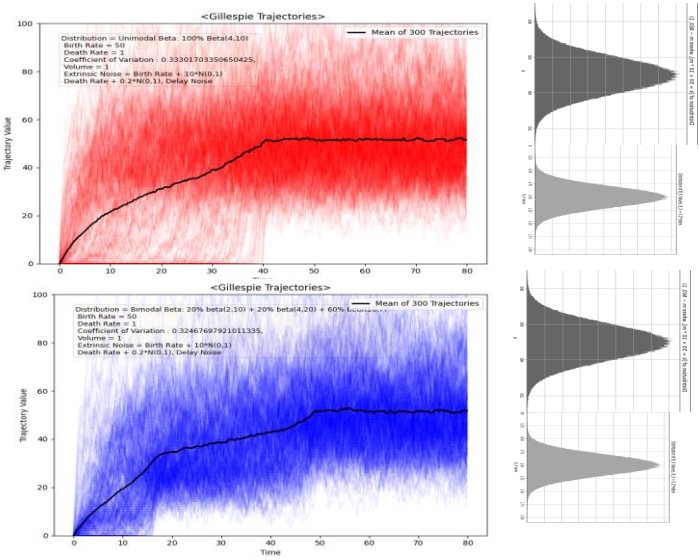
$$\lambda_b = 50 + 10k, \quad k \sim N(0,1)$$

$$\lambda_d = 1 + 0.2k, \quad k \sim N(0,1)$$

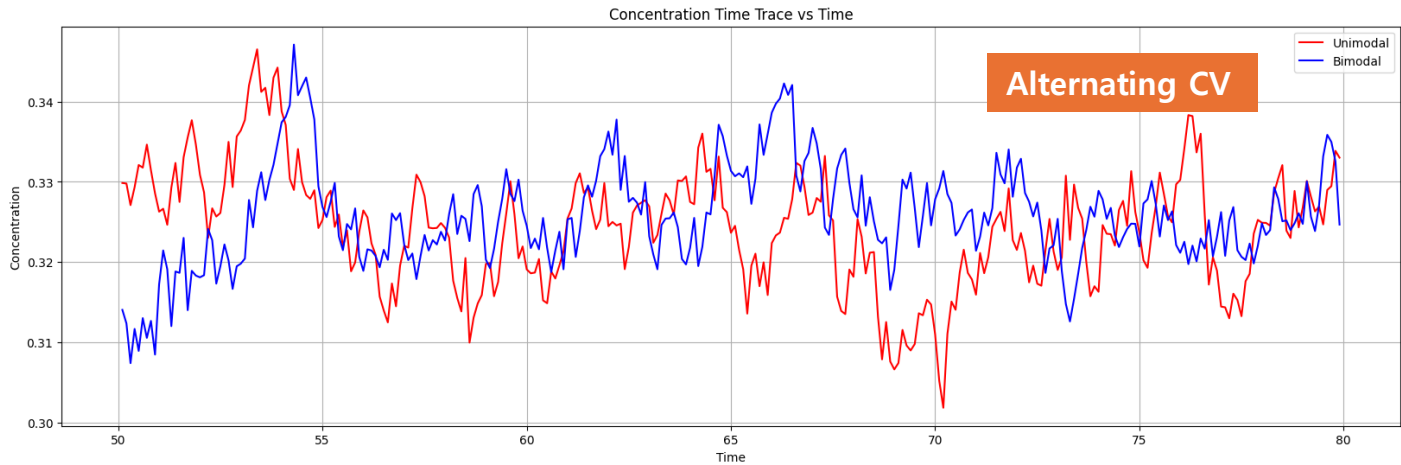


# Alternating CV relationship between unimodal and bimodal is observed despite various extrinsic noise settings.

$$\beta(4 \cdot (10^x)), \quad x \sim N(0, 1)$$
$$\lambda_b = 50 + 10k, \quad k \sim N(0, 1)$$
$$\lambda_d = 1 + 0.2k, \quad k \sim N(0, 1)$$



## Steady-State:





Summary: No clear CV relationship between unimodal and bimodal is observed despite various extrinsic noise settings.

