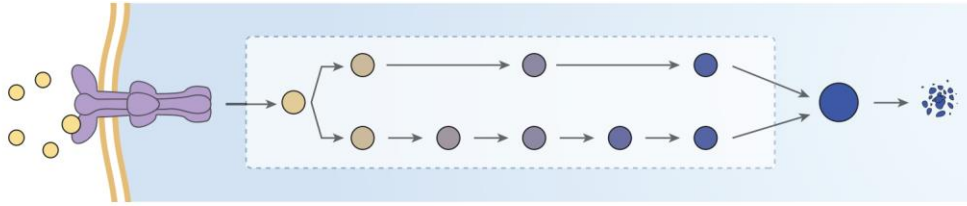
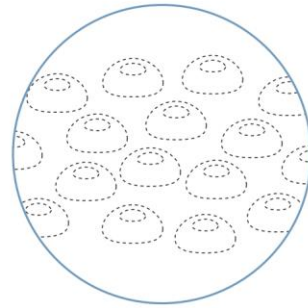
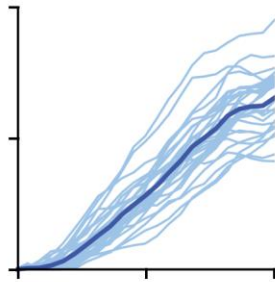


Multiple-timescale pathways reduce the cell-to-cell heterogeneity in response to external stress.

Multiple-timescale pathways

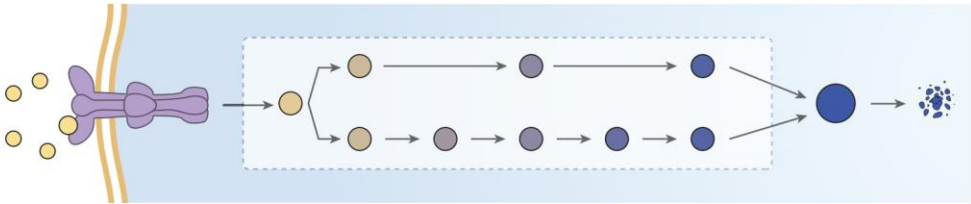


Small cell-to-cell heterogeneity

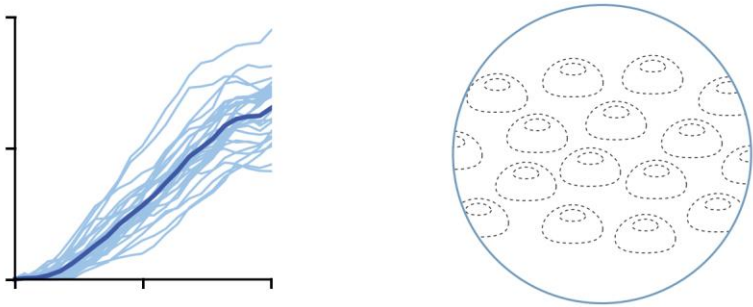


Multiple-timescale pathways reduce the cell-to-cell heterogeneity in response to external stress.

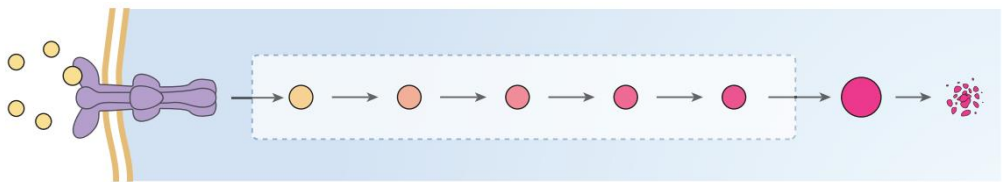
Multiple-timescale pathways



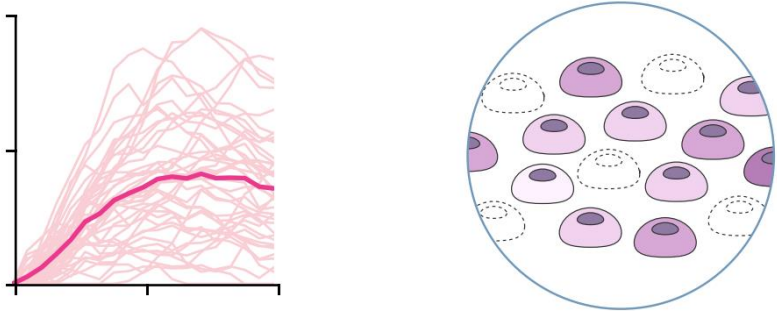
Small cell-to-cell heterogeneity



Single-timescale pathways



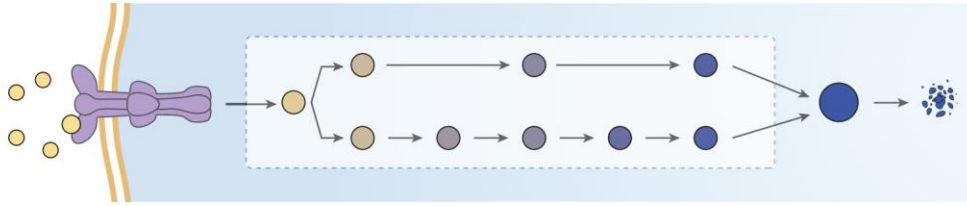
Large cell-to-cell heterogeneity



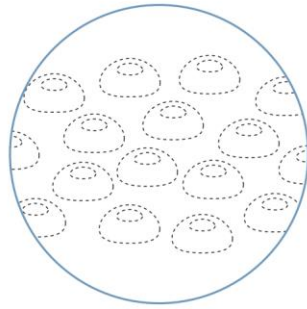
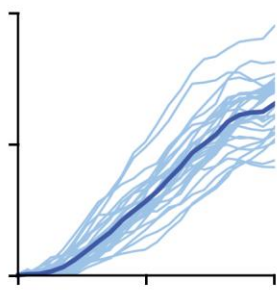
VS

Multiple-timescale pathways reduce the cell-to-cell heterogeneity in response to external stress.

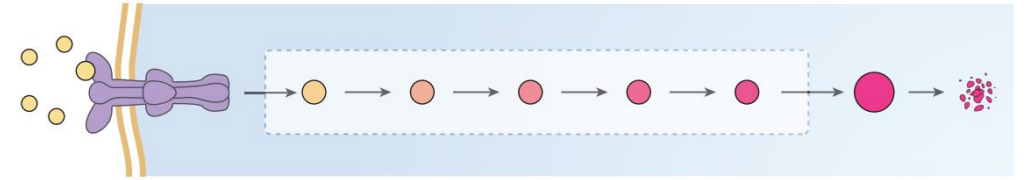
Multiple-timescale pathways



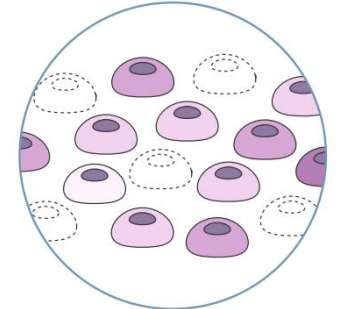
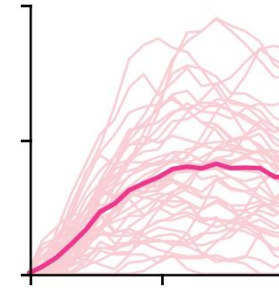
Small cell-to-cell heterogeneity



Single-timescale pathways



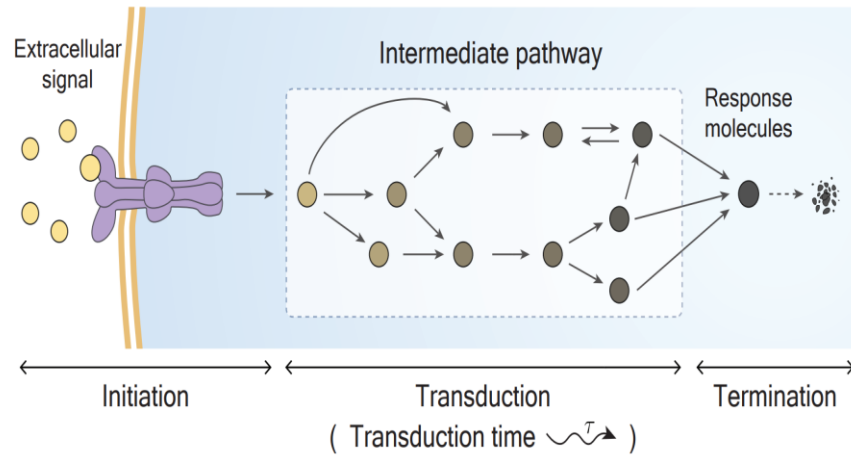
Large cell-to-cell heterogeneity



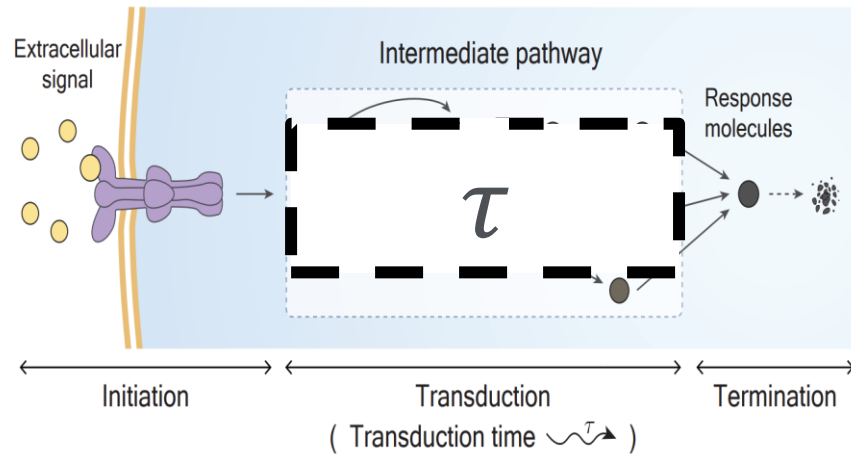
VS

Goal: Find why **multiple-timescale pathways** reduce the cell-to-cell heterogeneity.

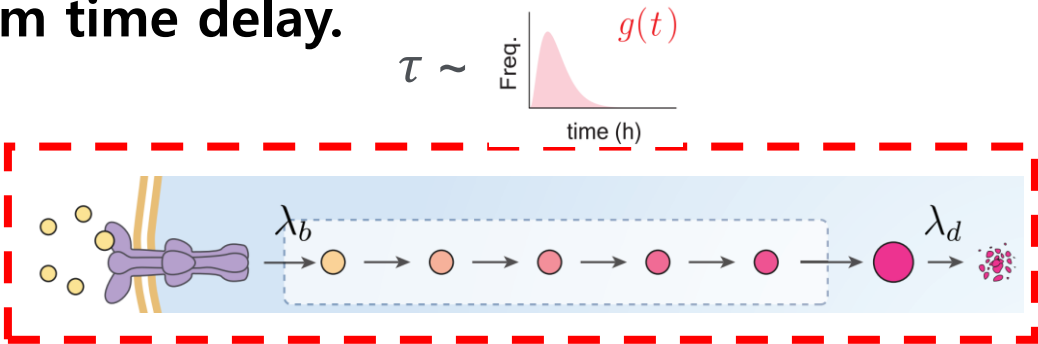
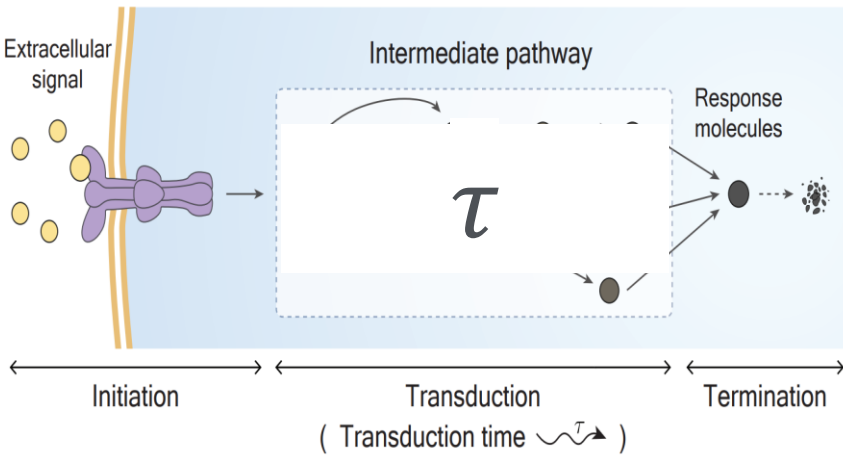
Signal transduction pathways with hidden reactions can be described with a stochastic delayed birth-death process with a single random time delay.



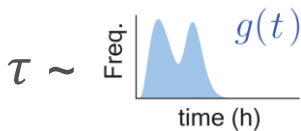
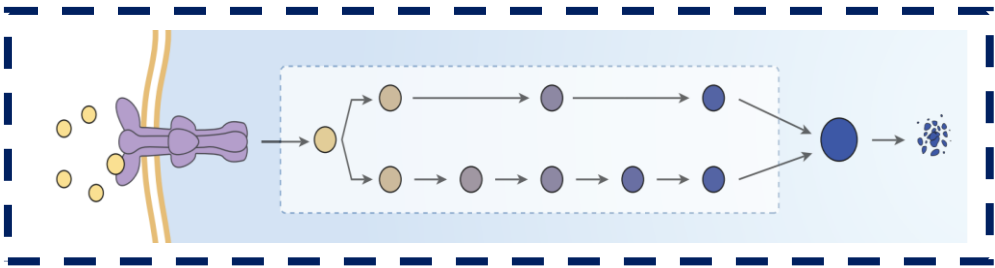
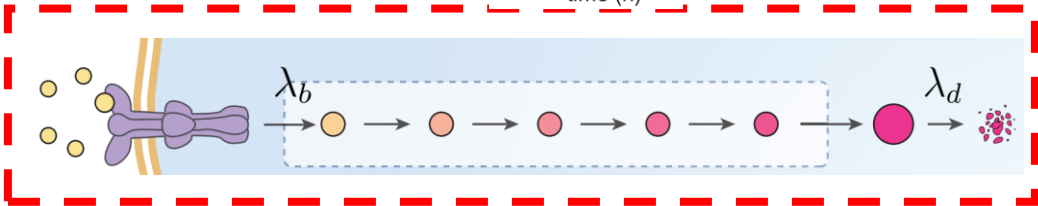
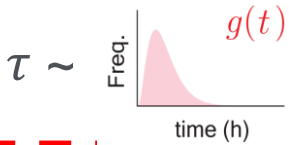
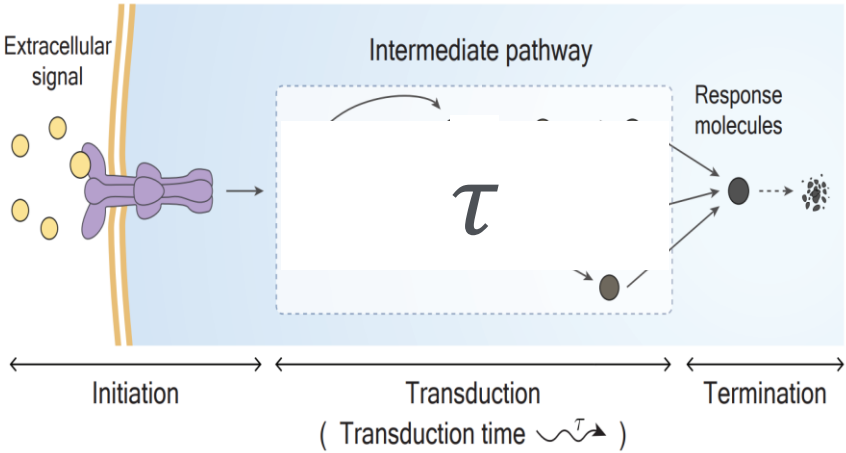
Signal transduction pathways with hidden reactions can be described with a stochastic delayed birth-death process with a single random time delay.



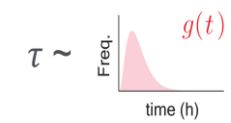
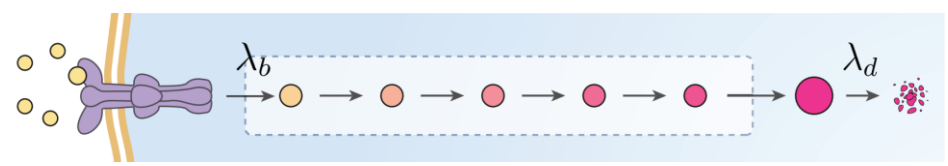
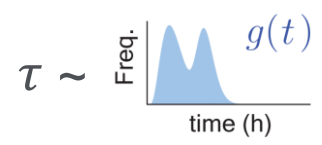
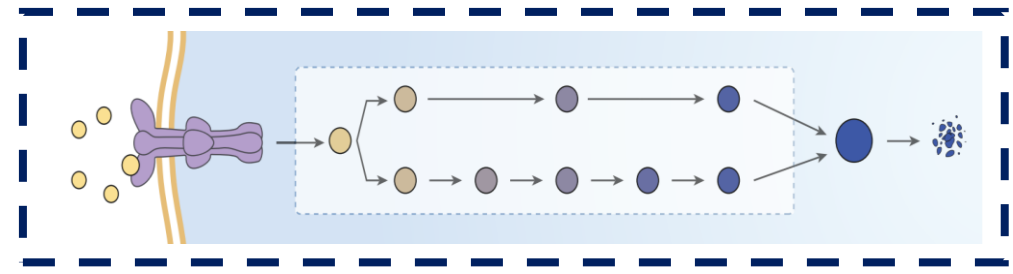
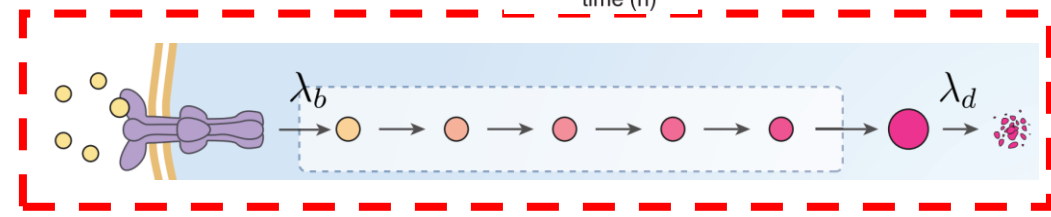
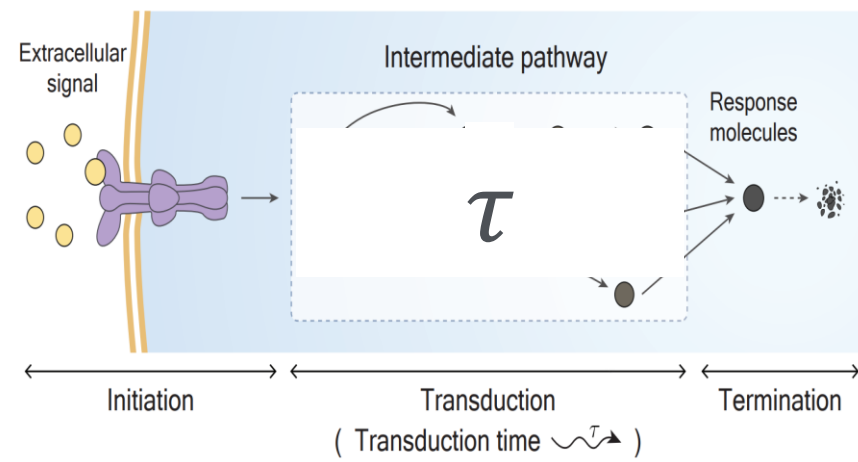
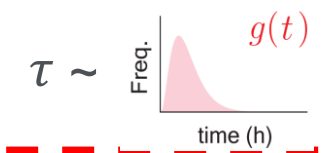
Signal transduction pathways with hidden reactions can be described with a stochastic delayed birth-death process with a single random time delay.



Signal transduction pathways with hidden reactions can be described with a stochastic delayed birth-death process with a single random time delay.

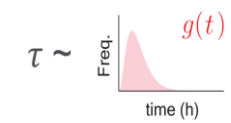
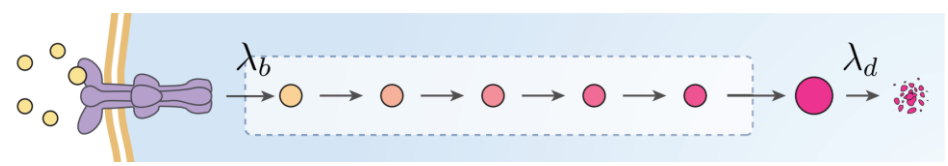
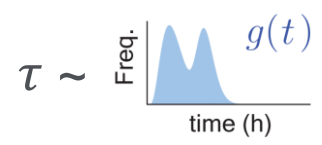
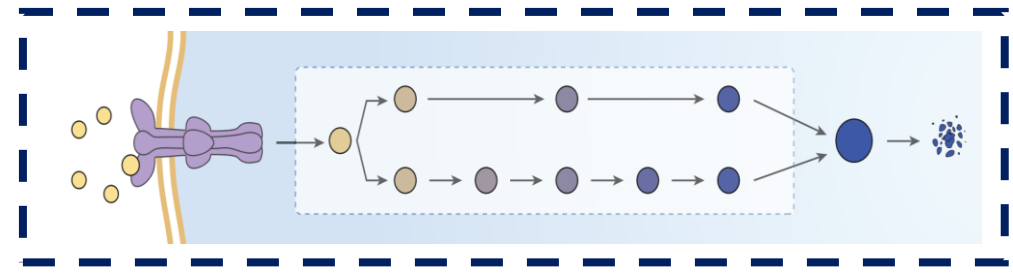
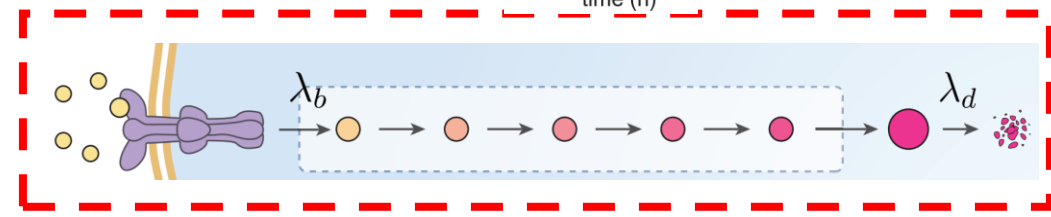
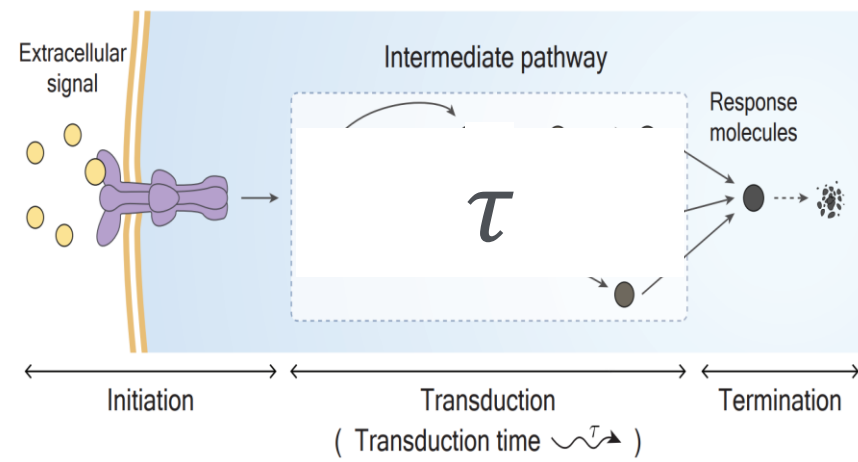
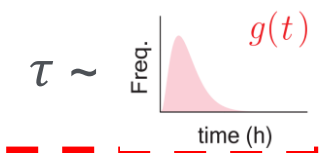


Signal transduction pathways with hidden reactions can be described with a stochastic delayed birth-death process with a single random time delay.

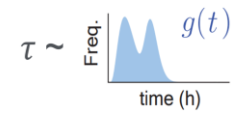
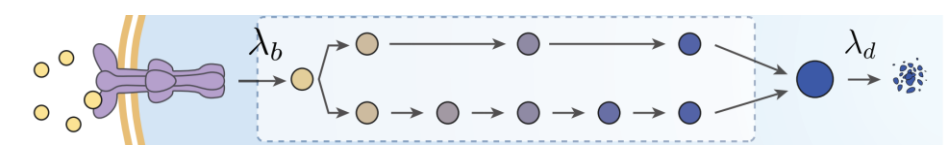


Delayed Gillespie with unimodal delay

Signal transduction pathways with hidden reactions can be described with a stochastic delayed birth-death process with a single random time delay.



Delayed Gillespie with unimodal delay



Delayed Gillespie with bimodal delay

Signal transduction pathways with hidden reactions can be described with a stochastic delayed birth-death process with a single random time delay.

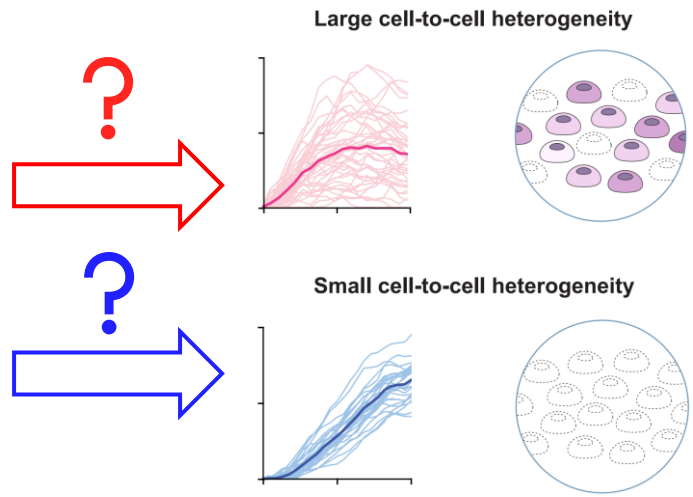
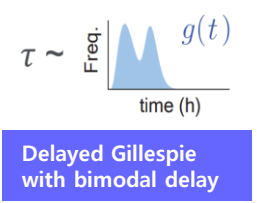
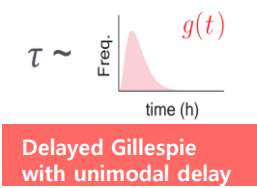
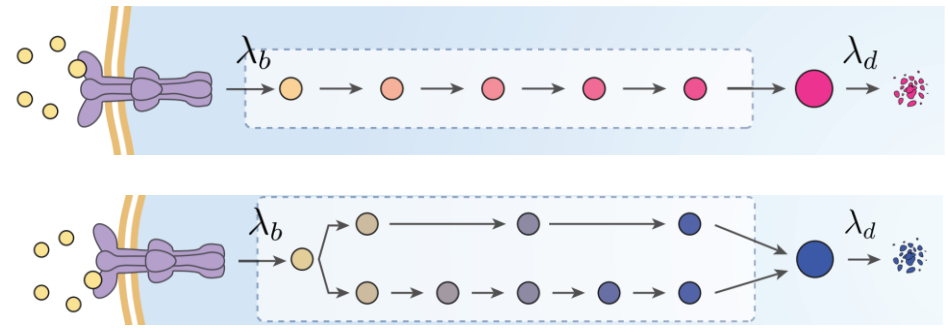
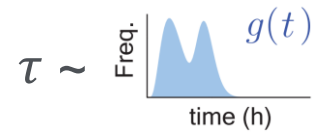
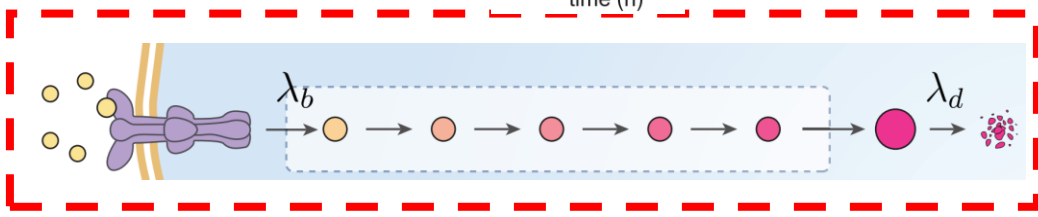
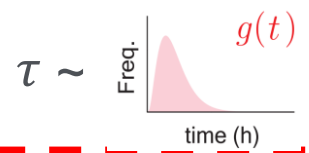
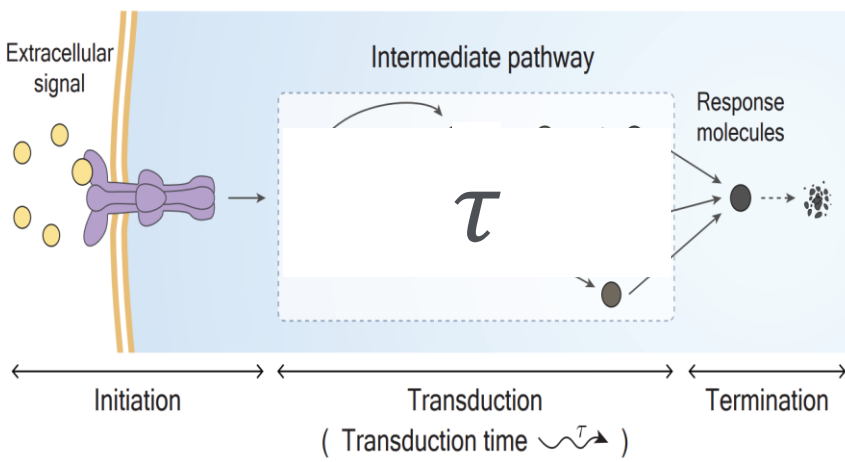


Fig.3.B time traces show no significant differences between **unimodal and **bimodal** (weak/strong) delay distributions.**

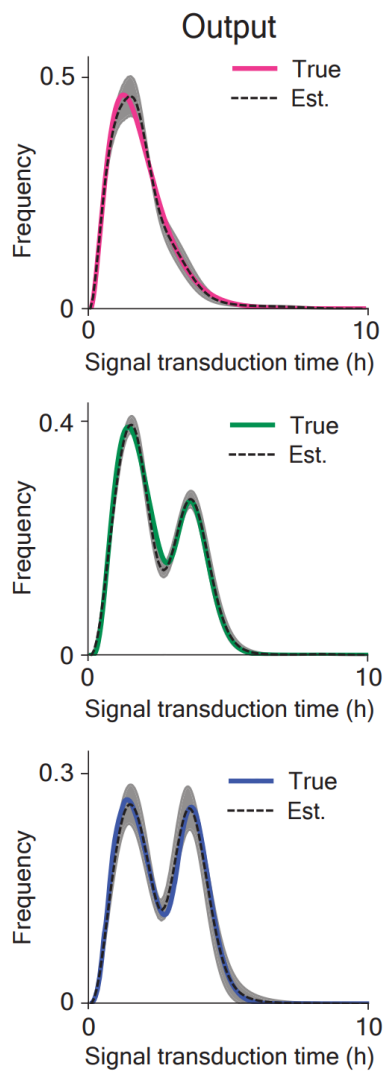
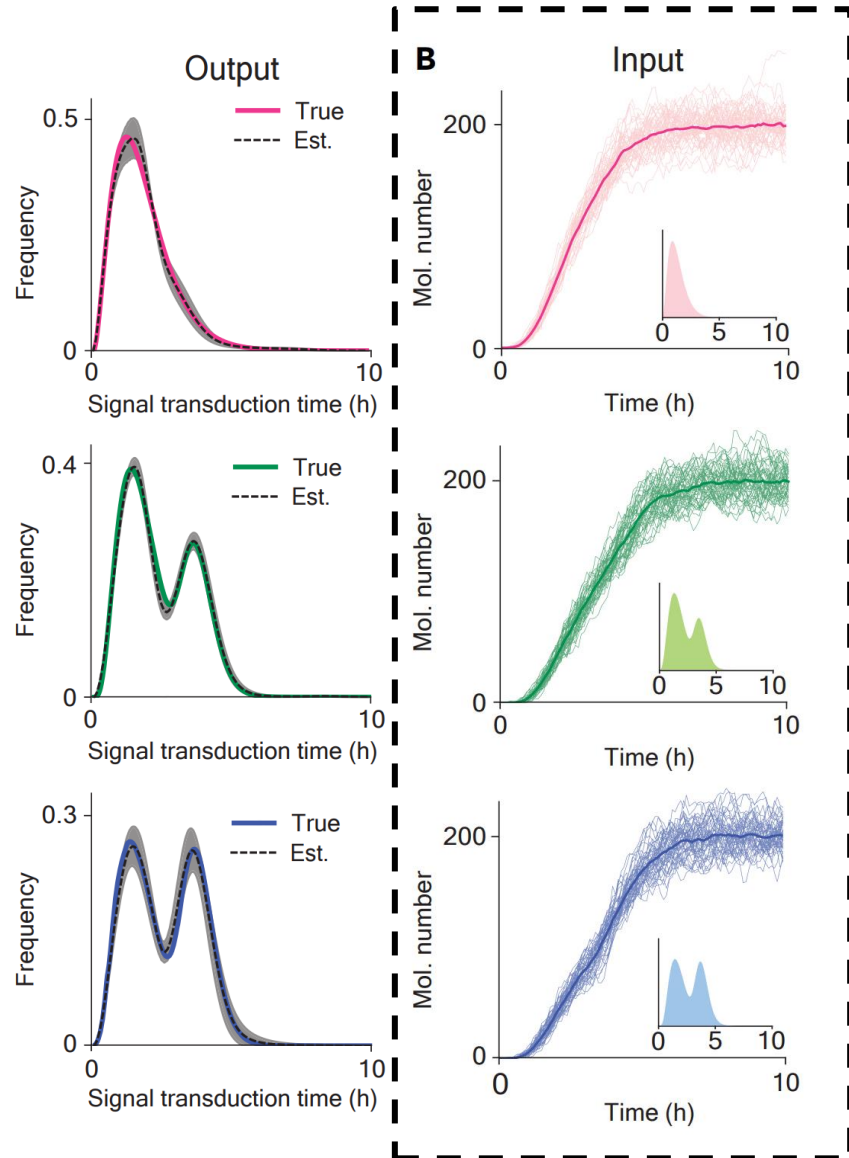
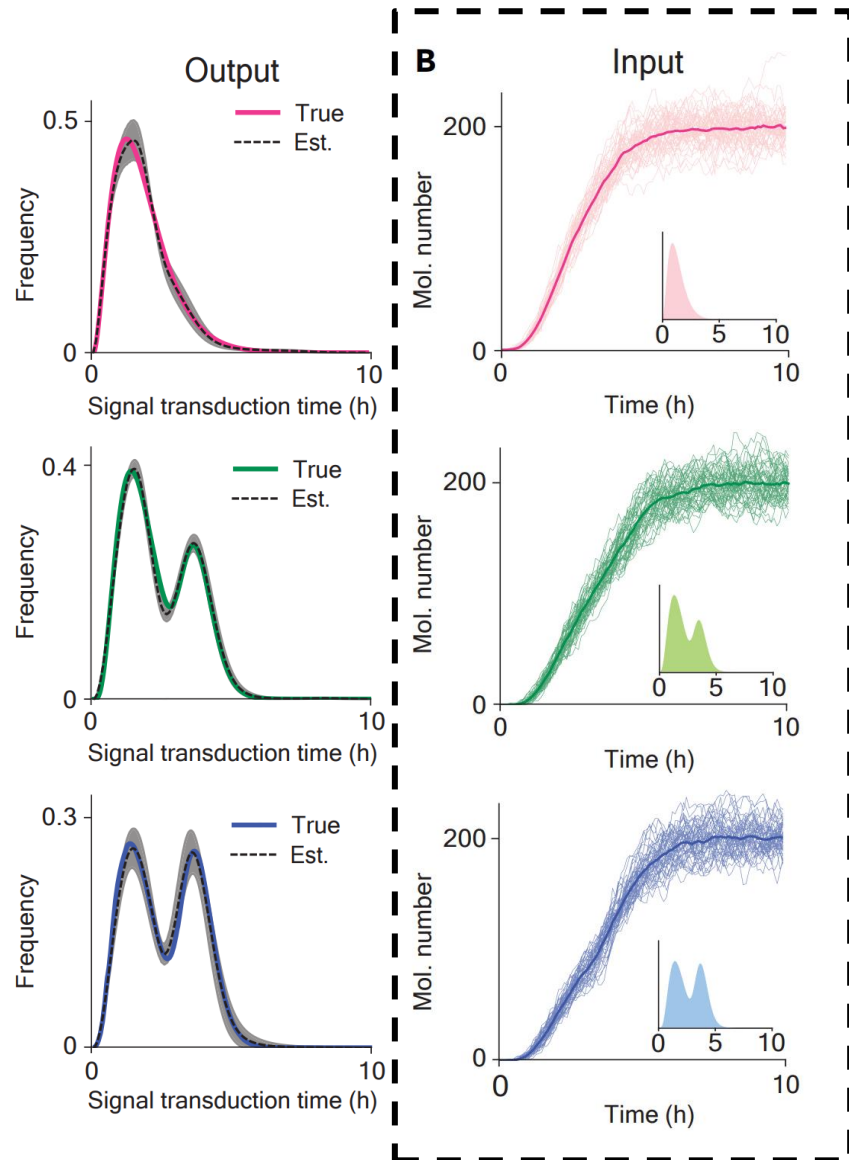


Fig.3.B time traces show no significant differences between **unimodal** and **bimodal** (weak/strong) delay distributions.



No visible difference.

Fig.3.B time traces show no significant differences between **unimodal** and **bimodal** (weak/strong) delay distributions.



No visible difference.

But Exact CV..??

$$CV = \left(\frac{\sigma}{\mu} \right) \times 100\%$$

Fig.3.B time traces show no significant differences between **unimodal** and **bimodal** (weak/strong) delay distributions.

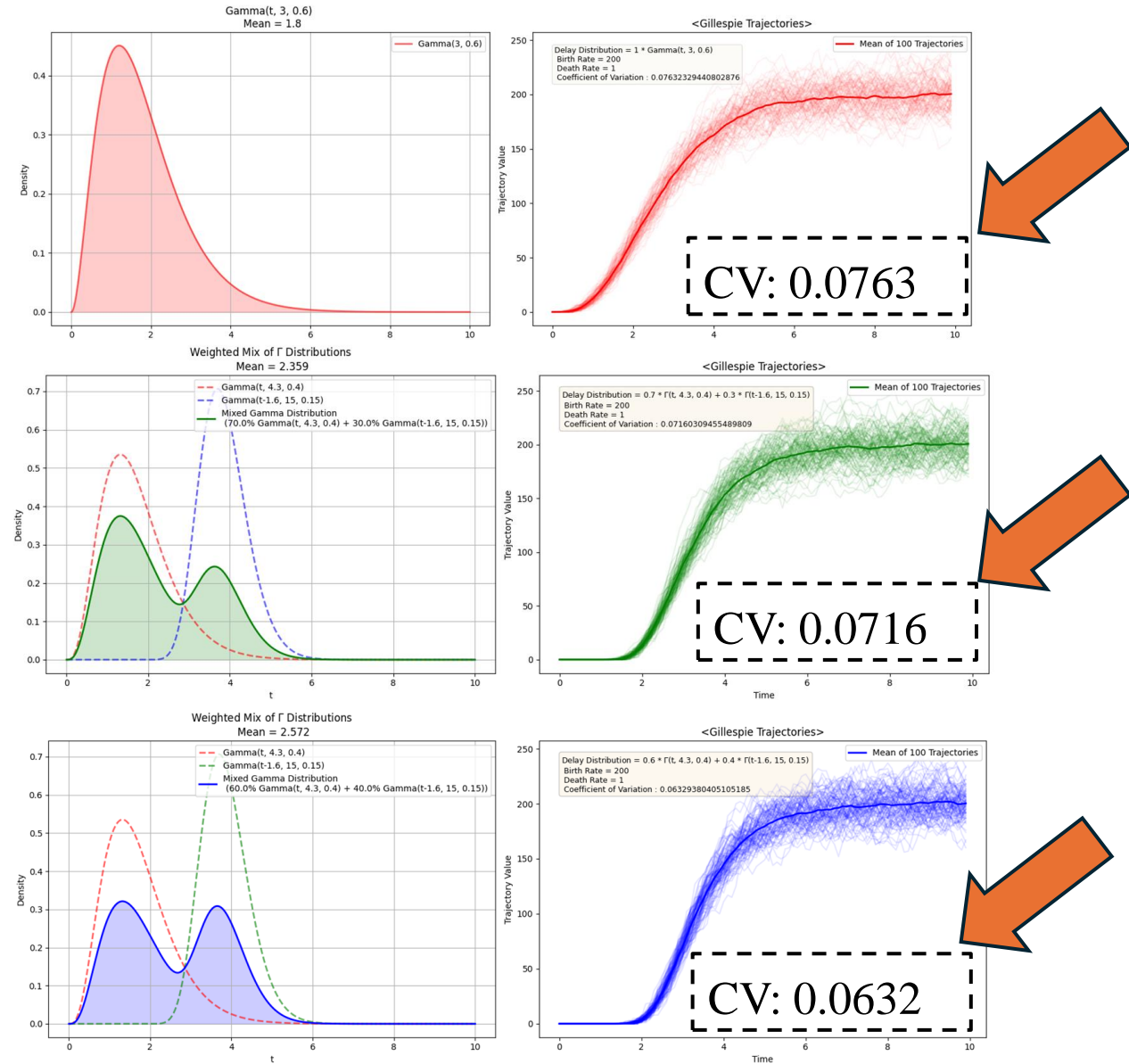
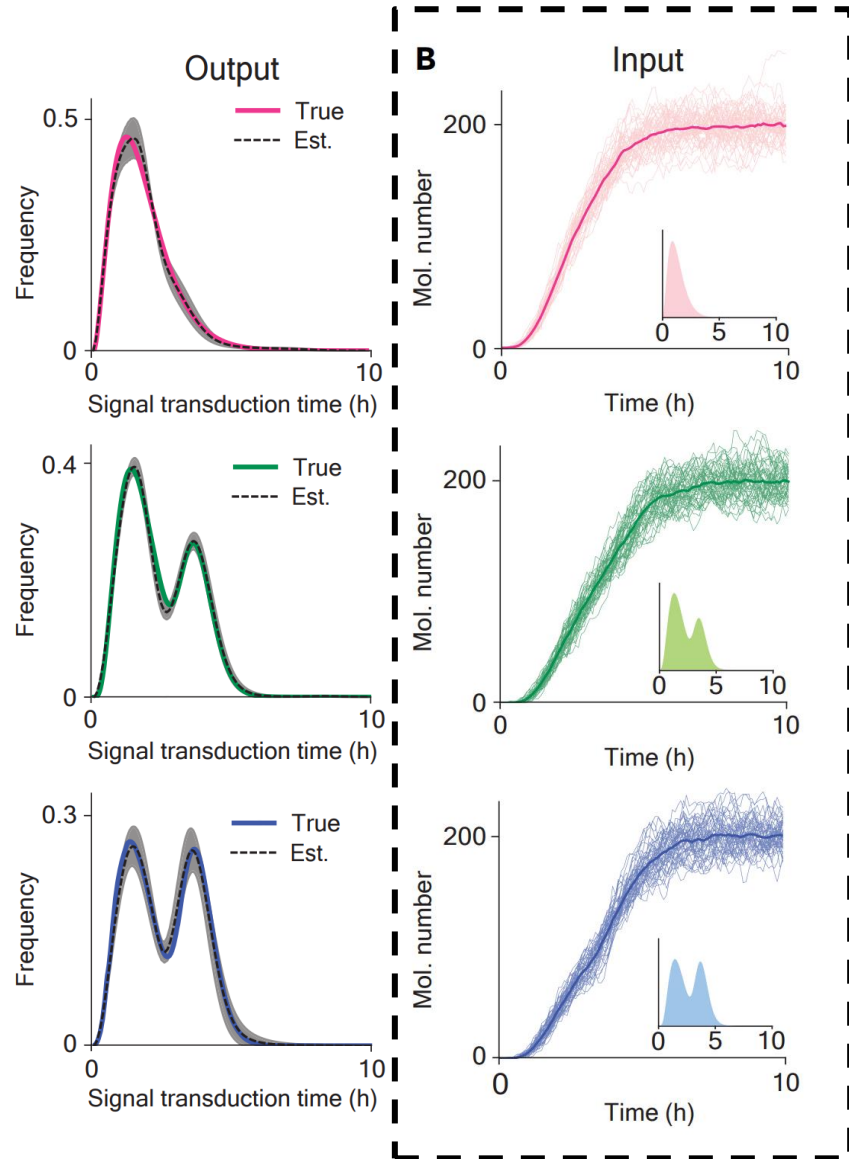
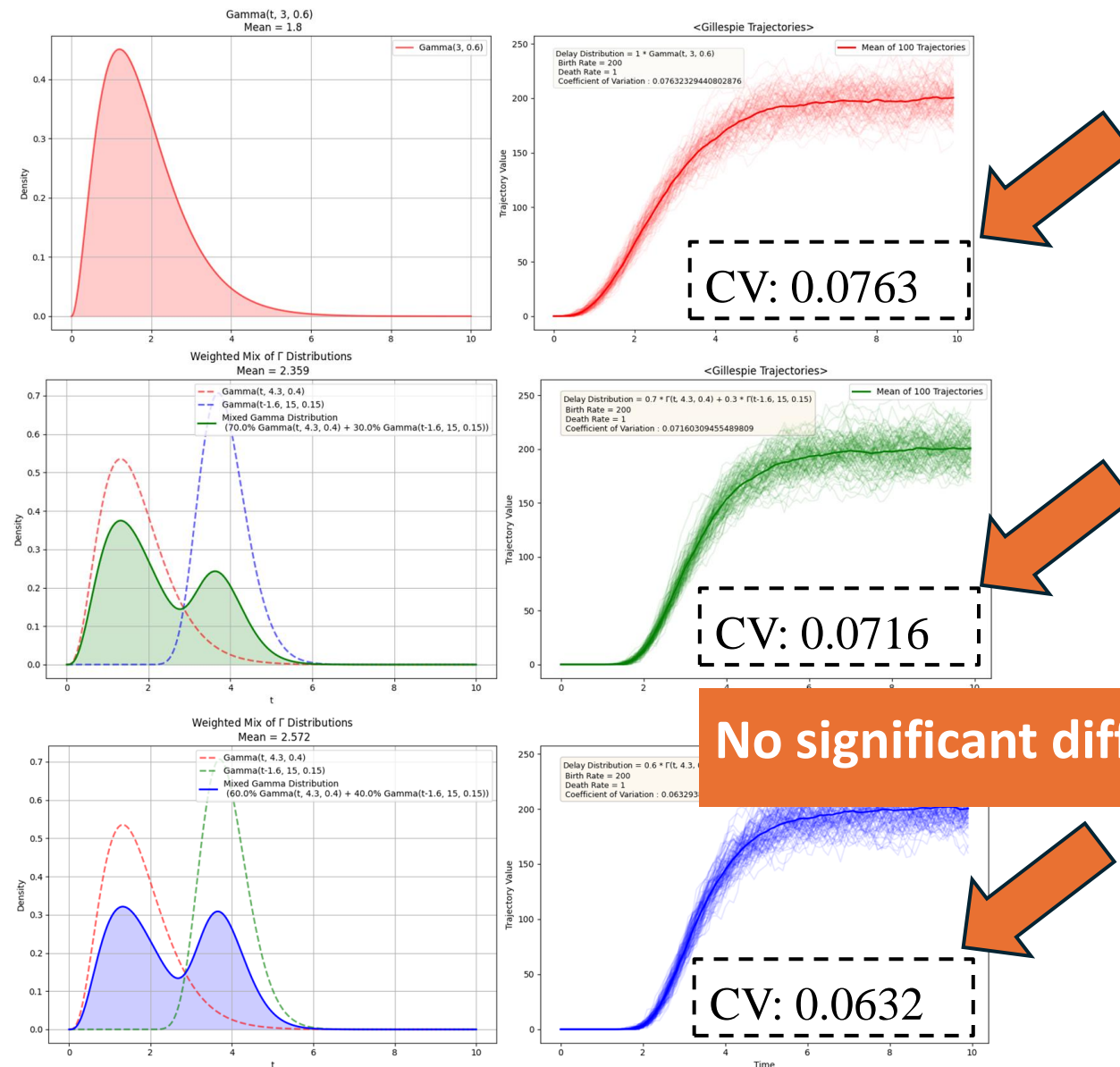
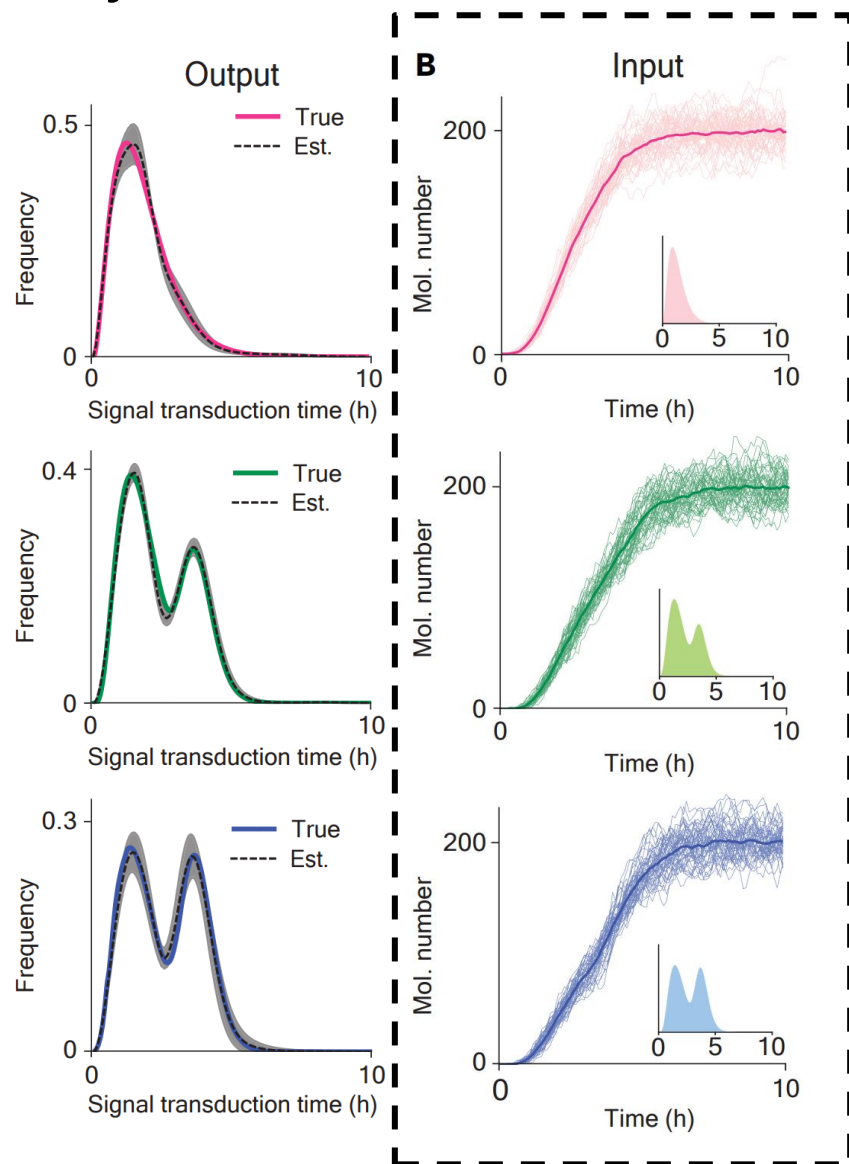
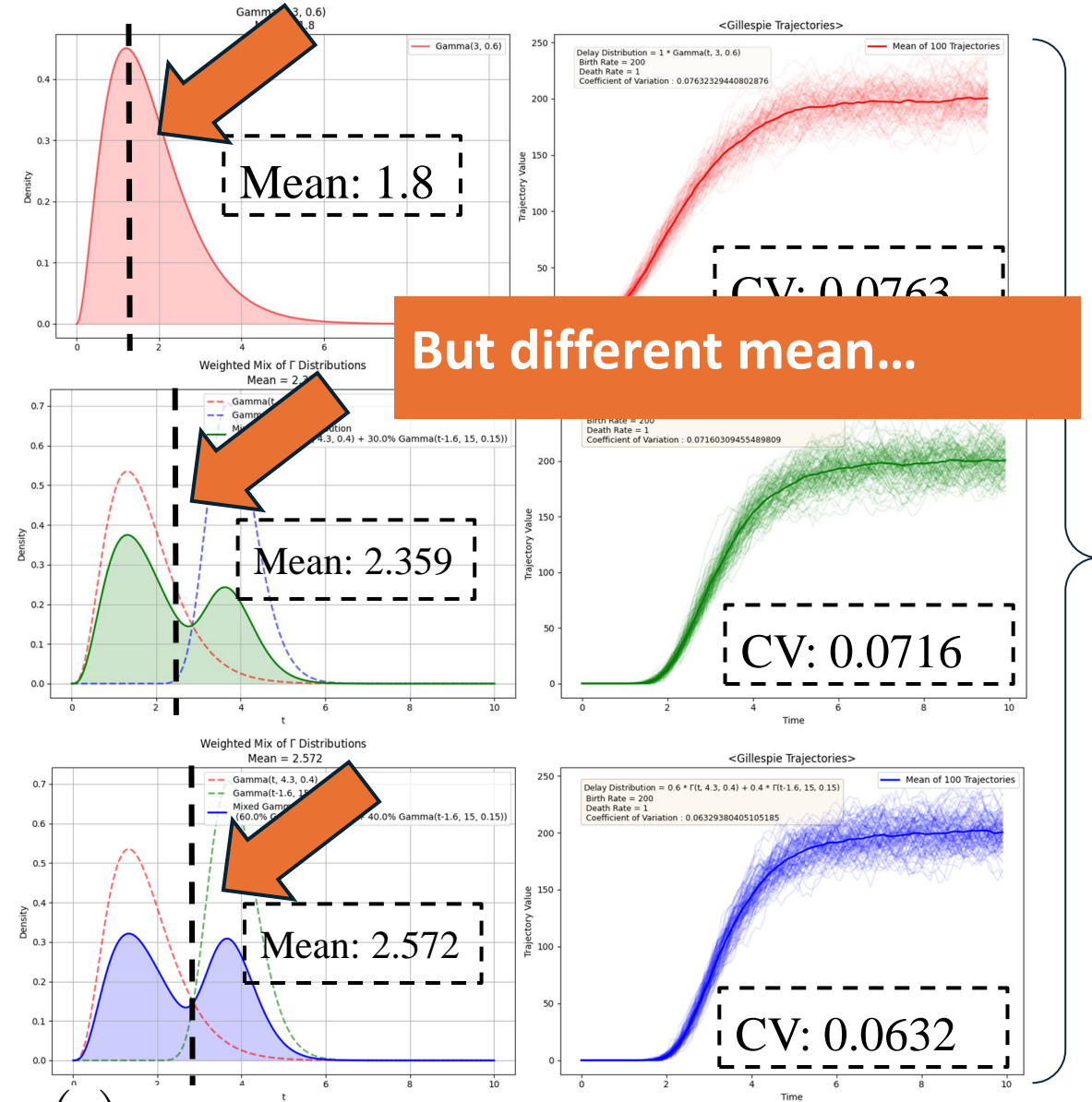
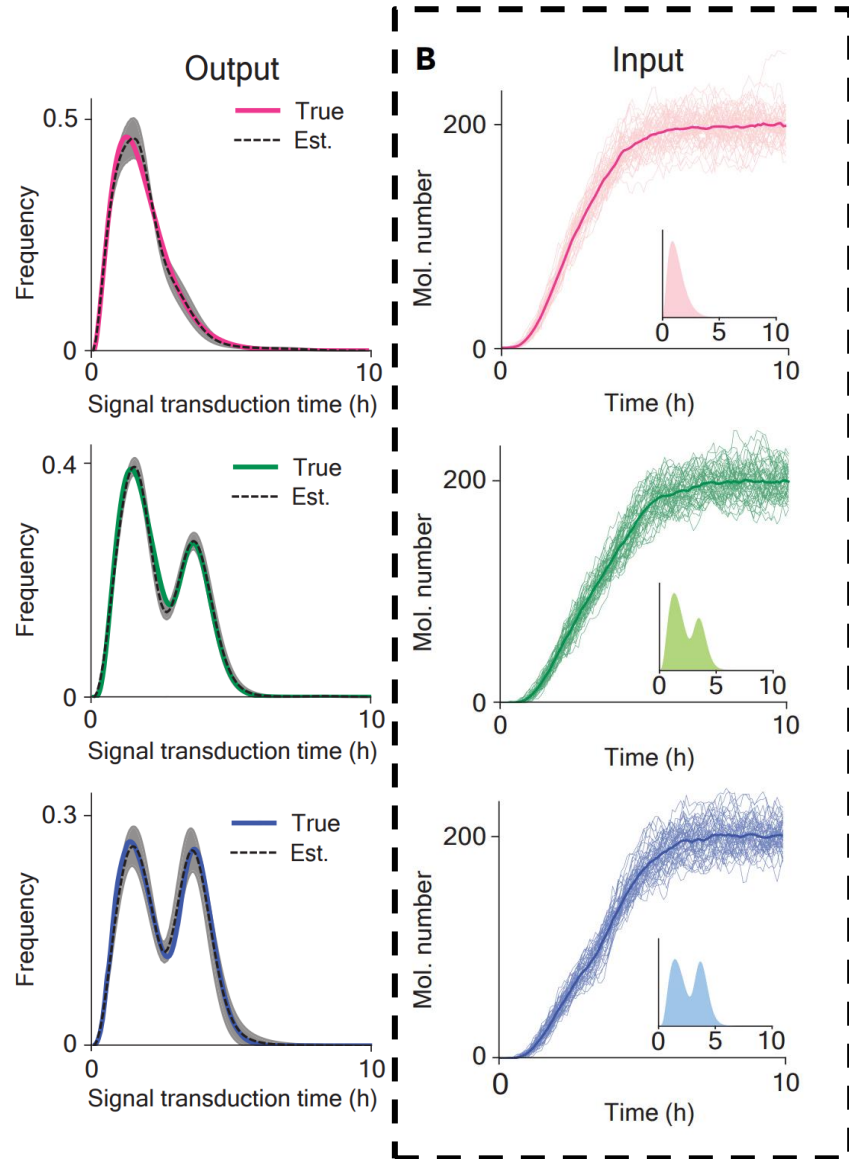


Fig.3.B time traces show no significant differences between **unimodal** and **bimodal** (weak/strong) delay distributions.



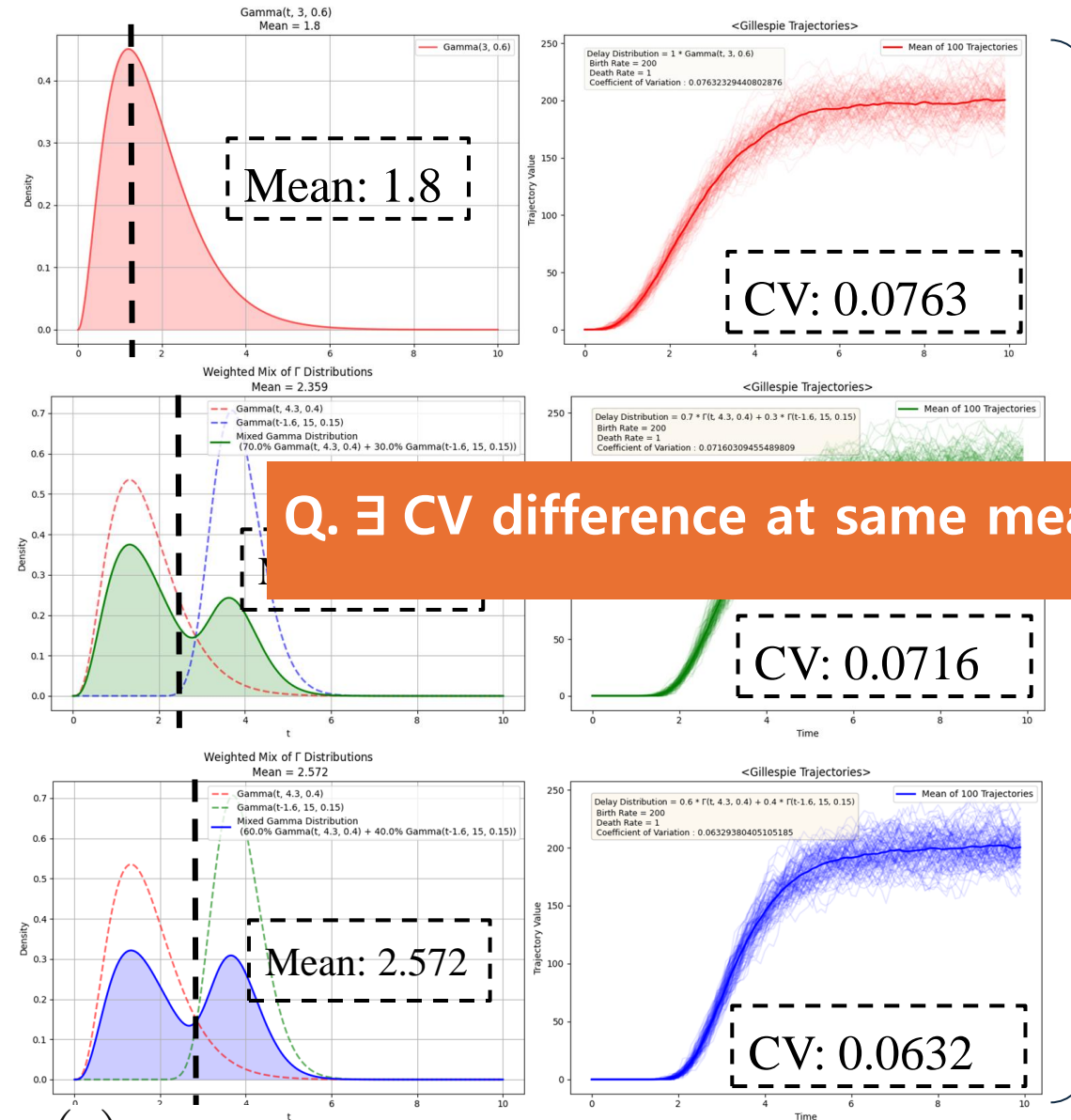
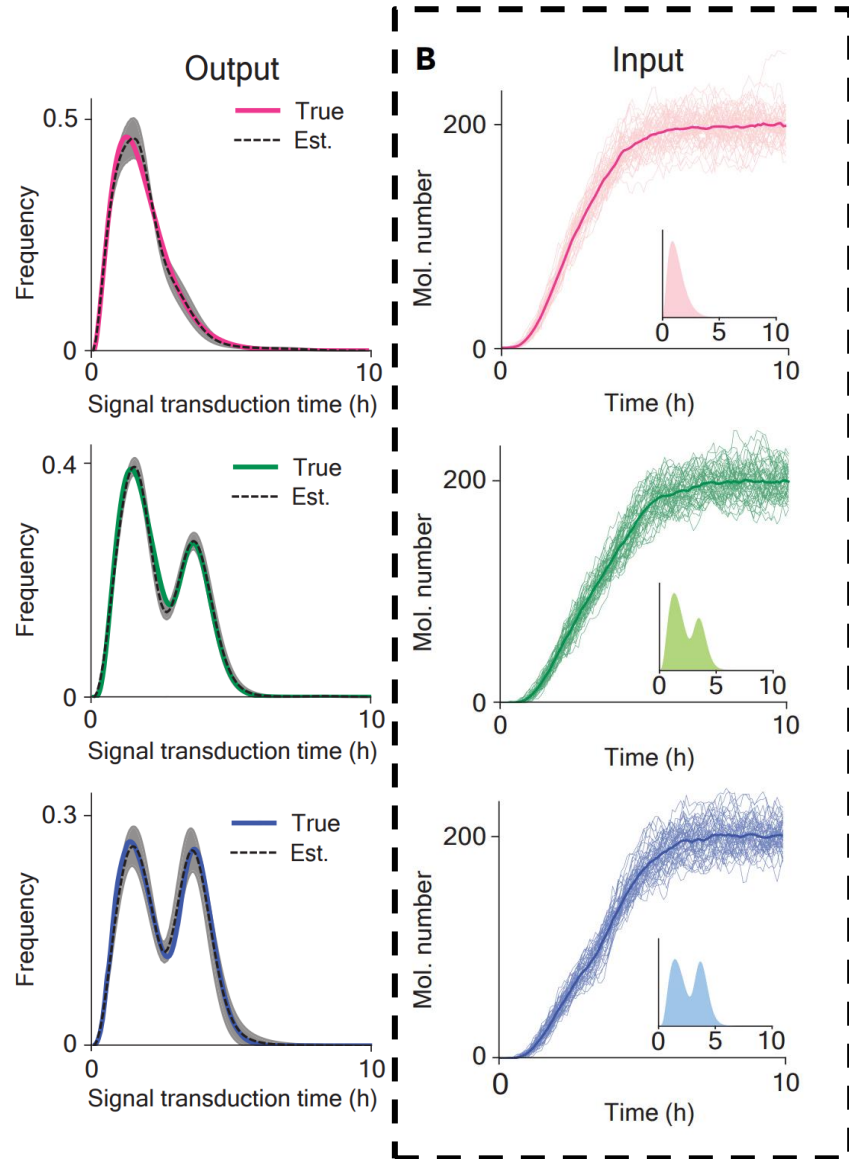
Exact Coefficient of Variation (CV)? $CV = \left(\frac{\sigma}{\mu} \right) \times 100\%$

Fig.3.B time traces show no significant differences between **unimodal** and **bimodal** (weak/strong) delay distributions.



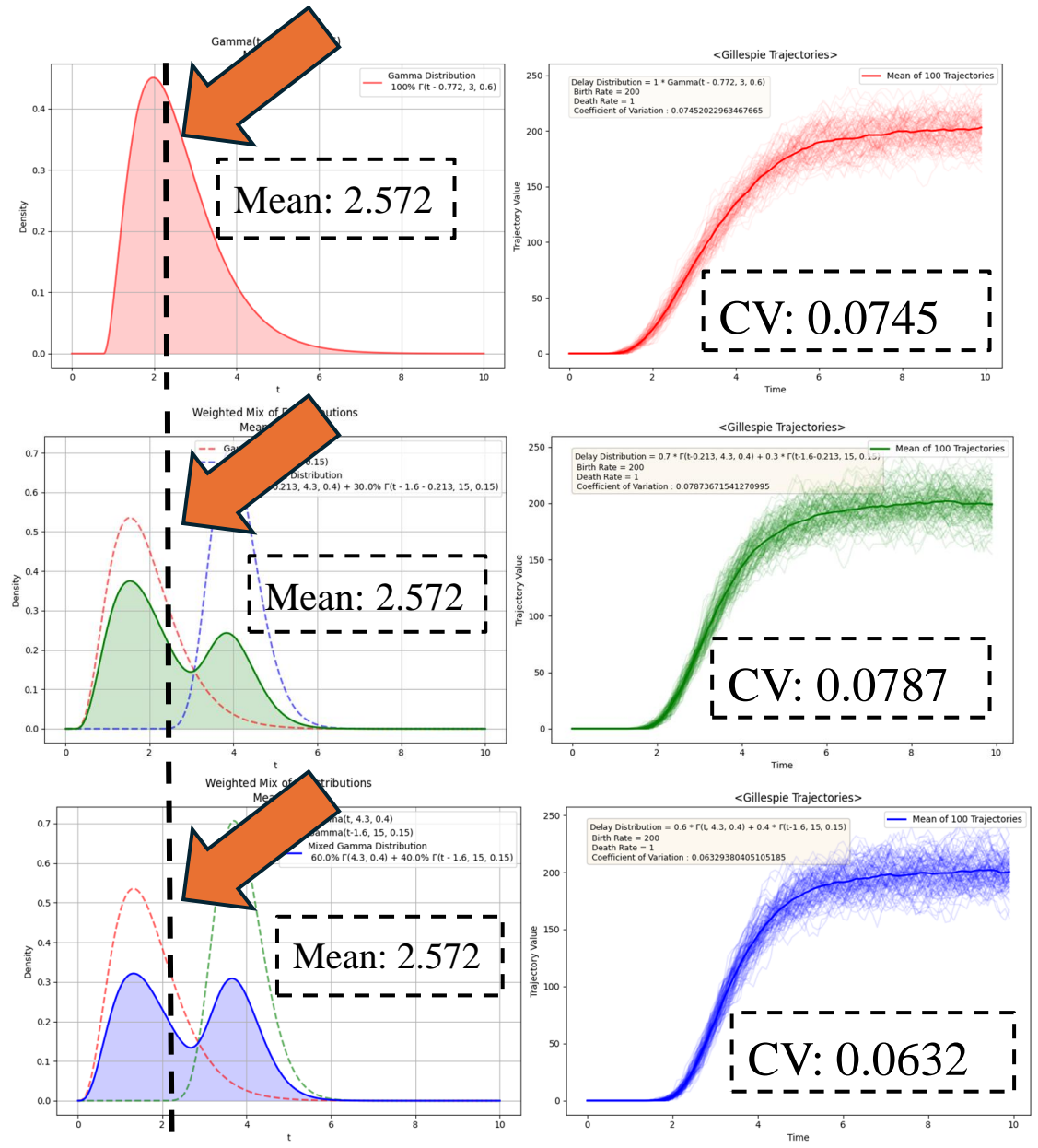
Exact Coefficient of Variation (CV)? $CV = \left(\frac{\sigma}{\mu} \right) \times 100\%$

Fig.3.B time traces show no significant differences between **unimodal** and **bimodal** (weak/strong) delay distributions.

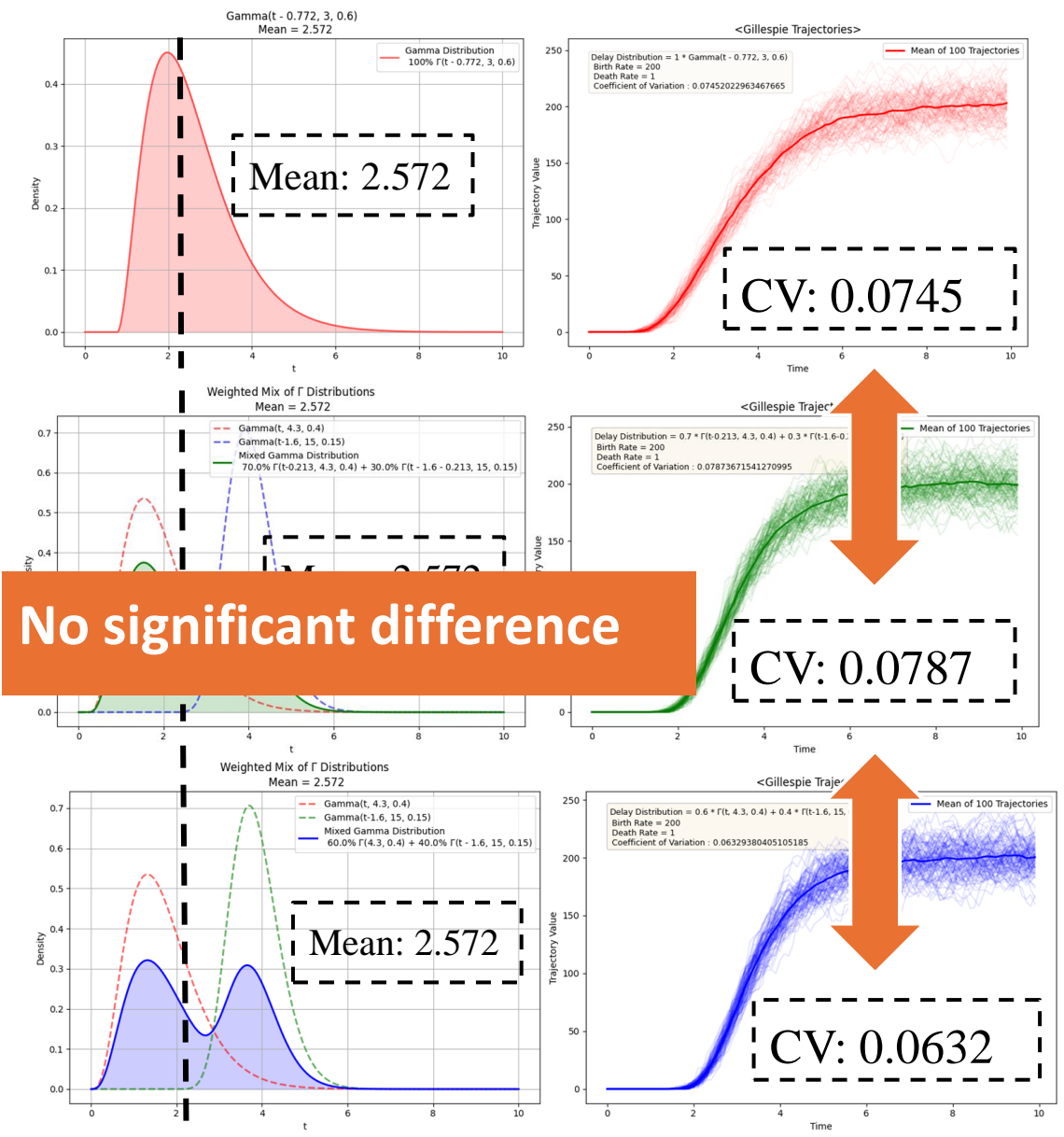


Exact Coefficient of Variation (CV)? $CV = \left(\frac{\sigma}{\mu} \right) \times 100\%$

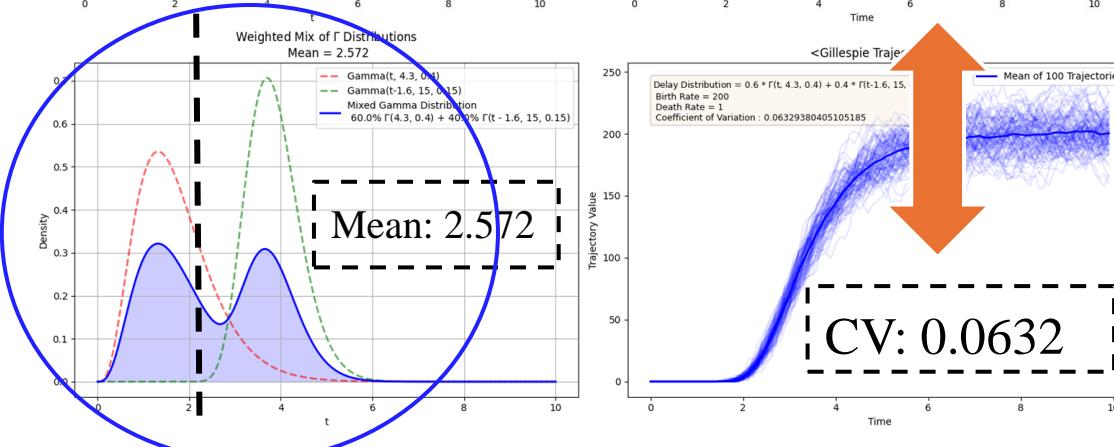
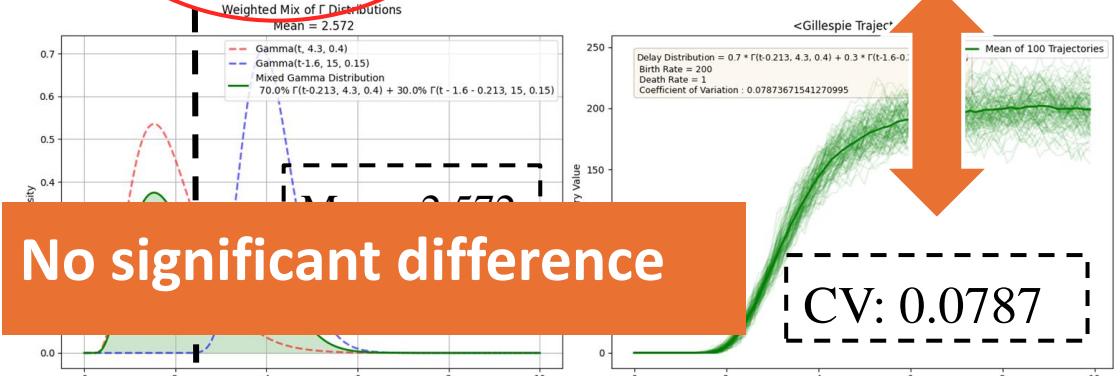
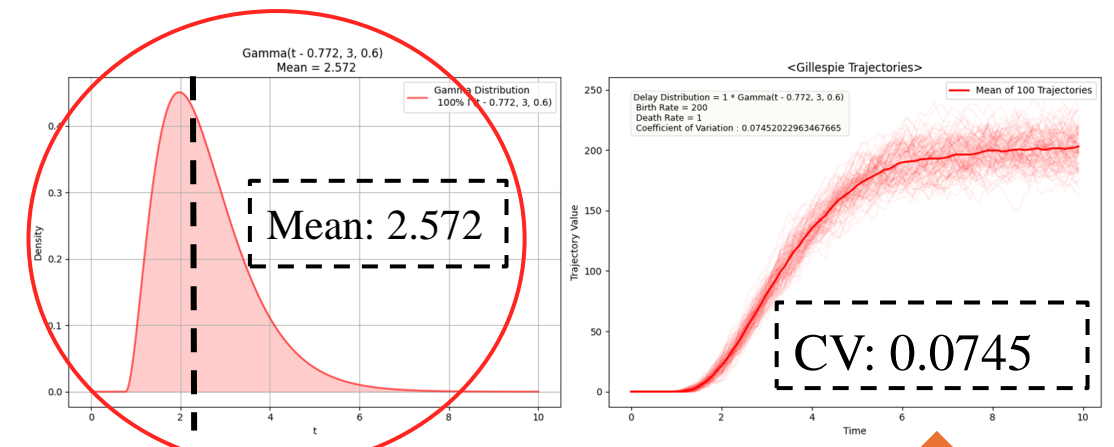
Unimodal and **bimodal** (**weak/strong**) delay distributions with the same mean show no significant differences in final time trace heterogeneity.



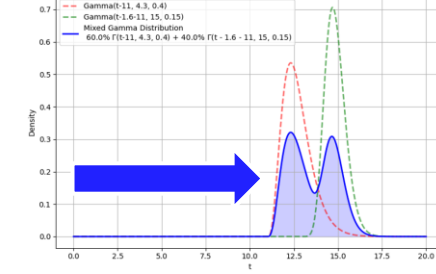
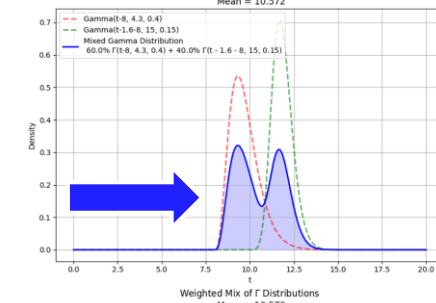
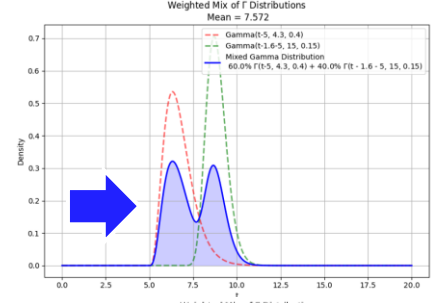
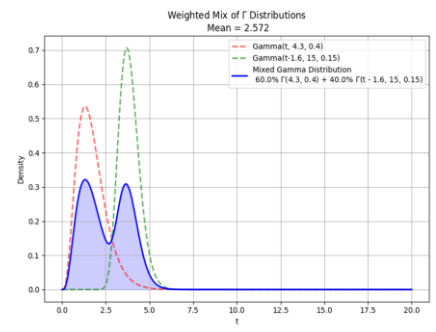
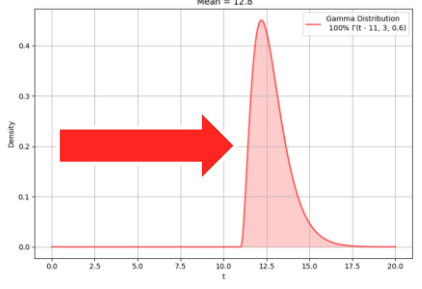
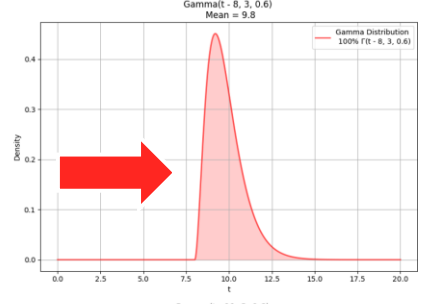
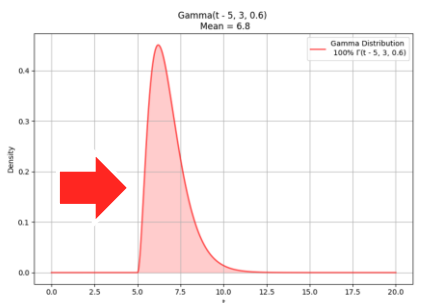
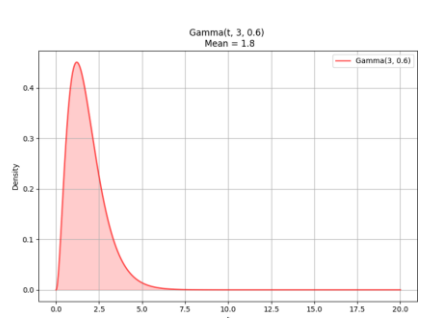
Unimodal and **bimodal** (**weak/strong**) delay distributions with the same mean show no significant differences in final time trace heterogeneity.



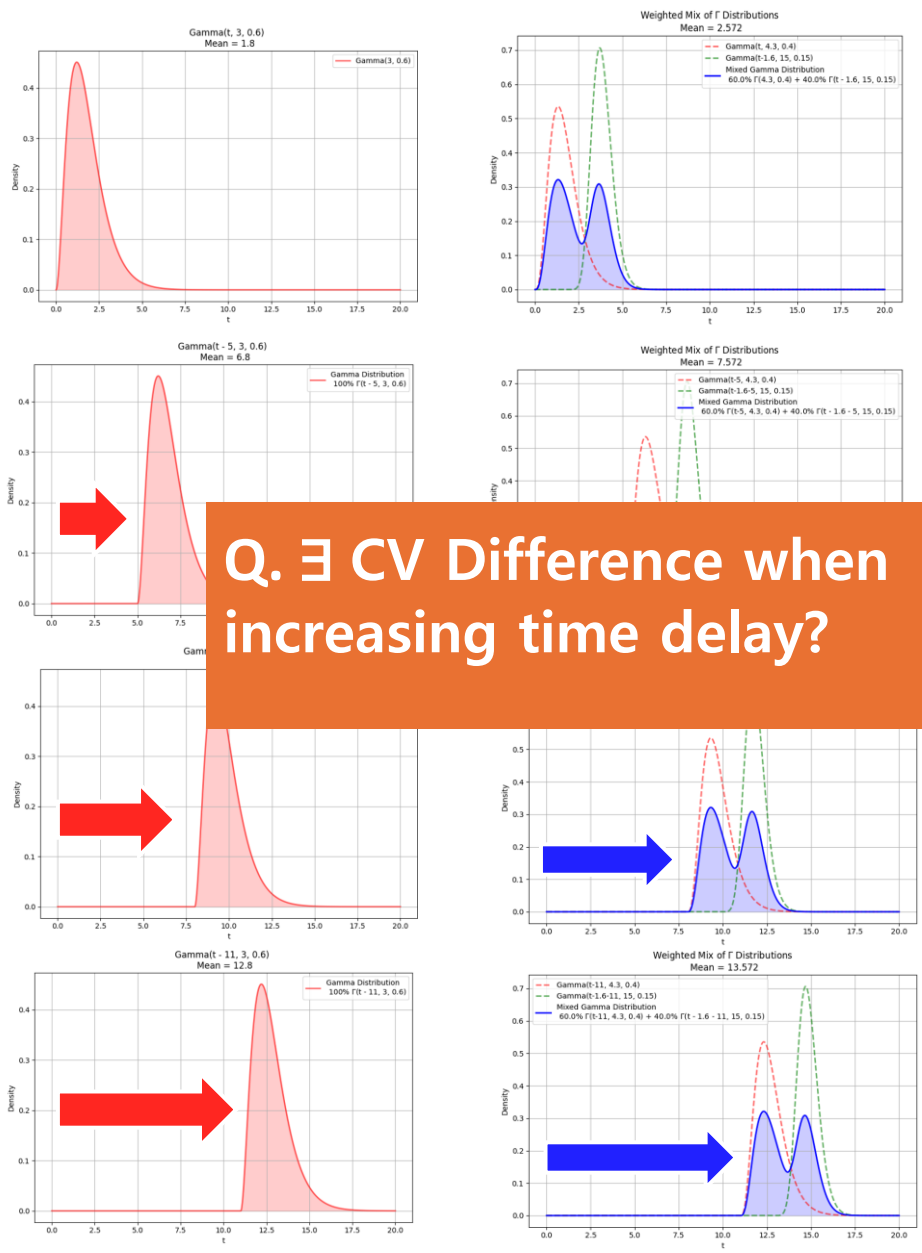
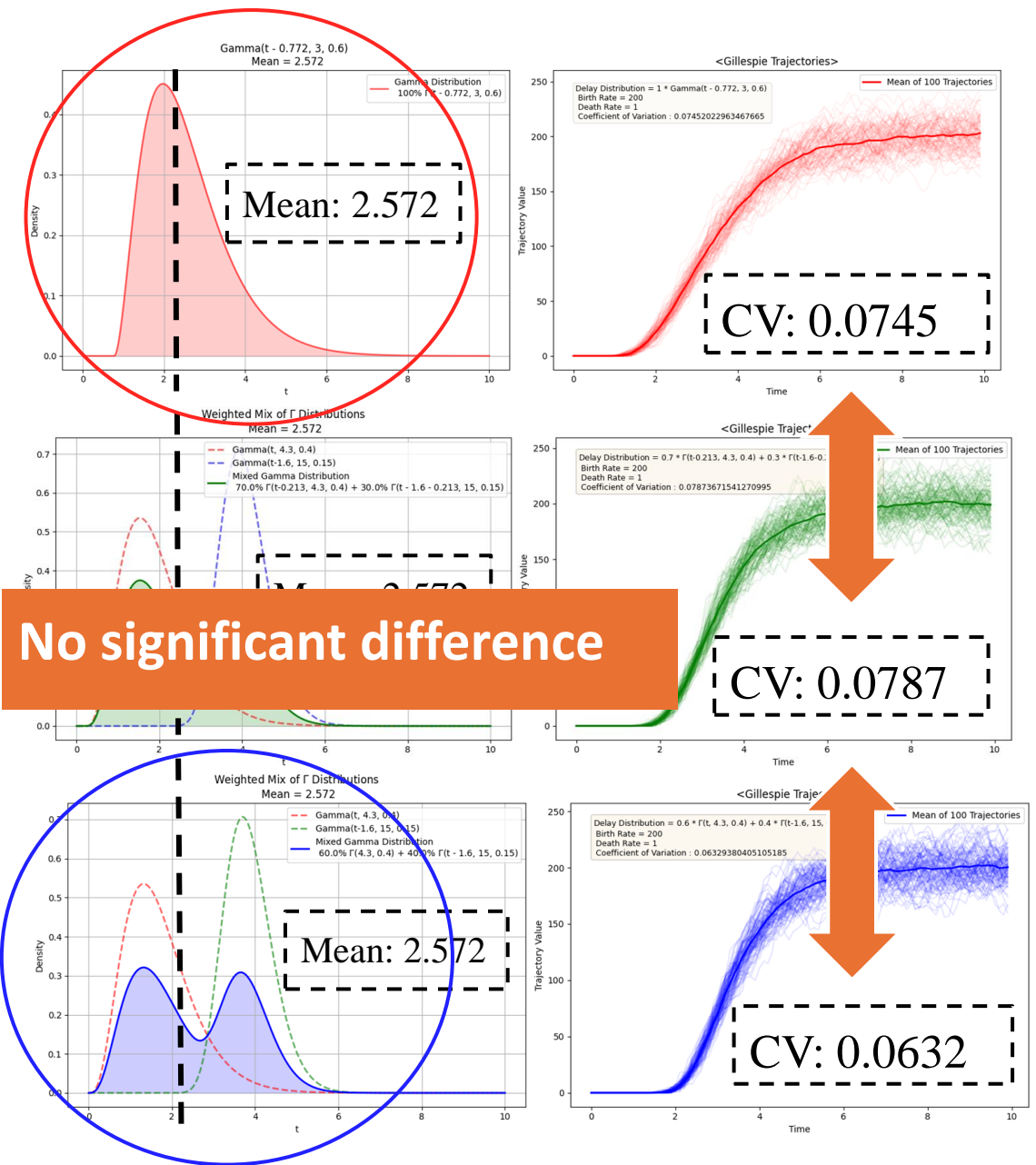
Unimodal and bimodal (weak/strong) delay distributions with the same mean show no significant differences in final time trace heterogeneity.



No significant difference

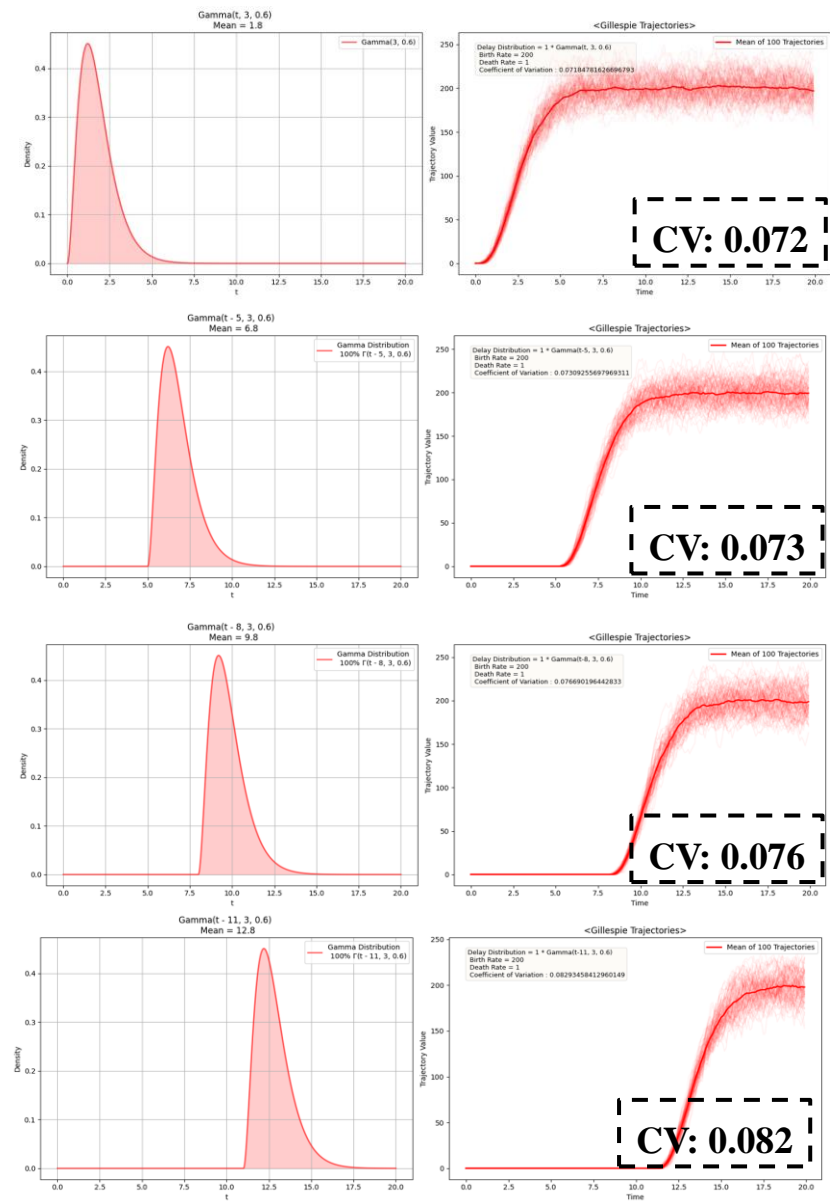


Unimodal and bimodal (weak/strong) delay distributions with the same mean show no significant differences in final time trace heterogeneity.

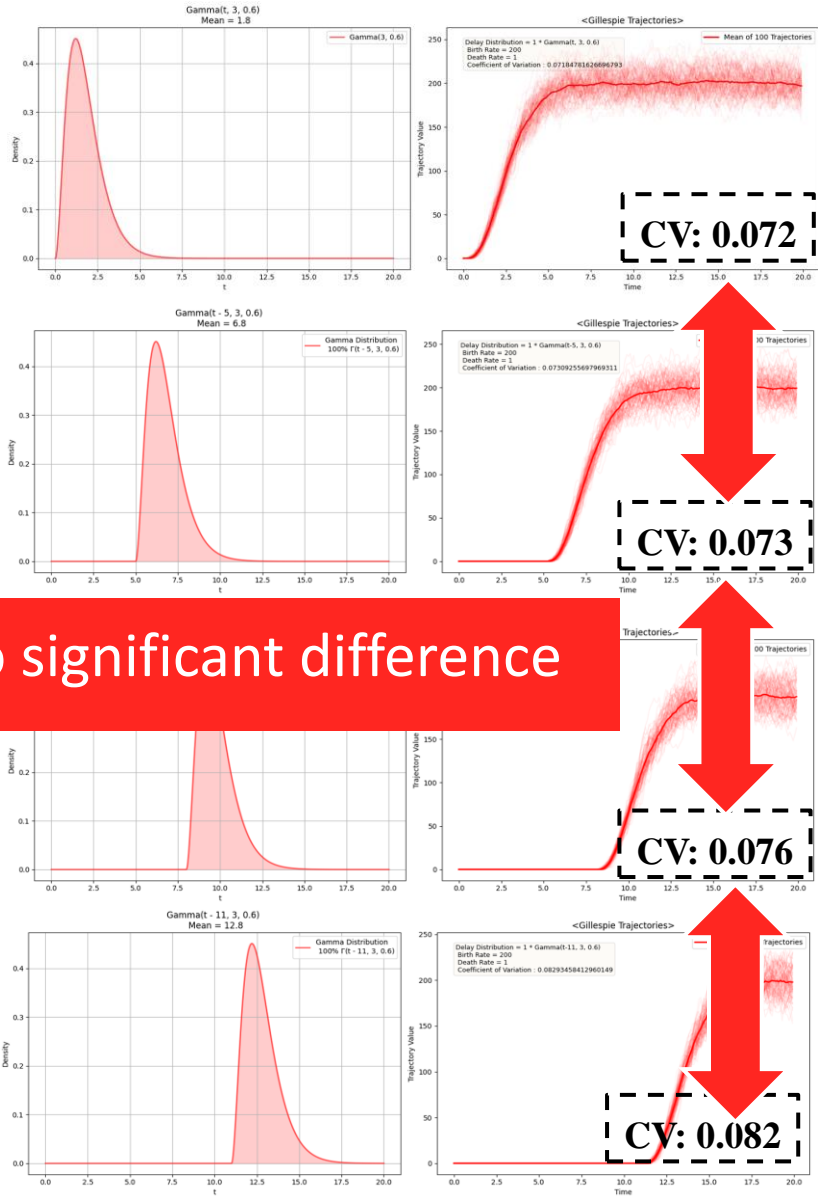


Q. \exists CV Difference when increasing time delay?

Increasing time delay by horizontal shift slightly increases **unimodal heterogeneity** and slightly decreases **bimodal heterogeneity**, but no significant differences in final time trace heterogeneity.

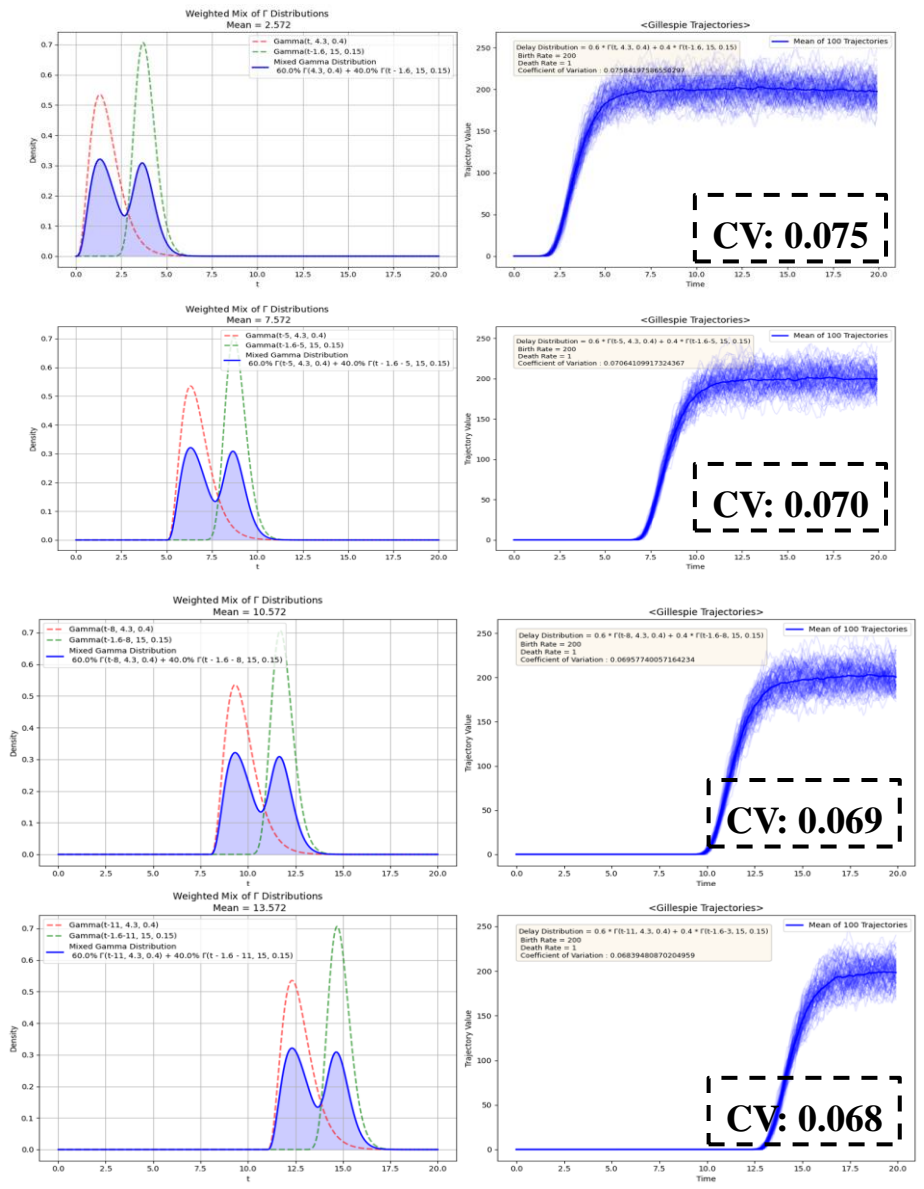
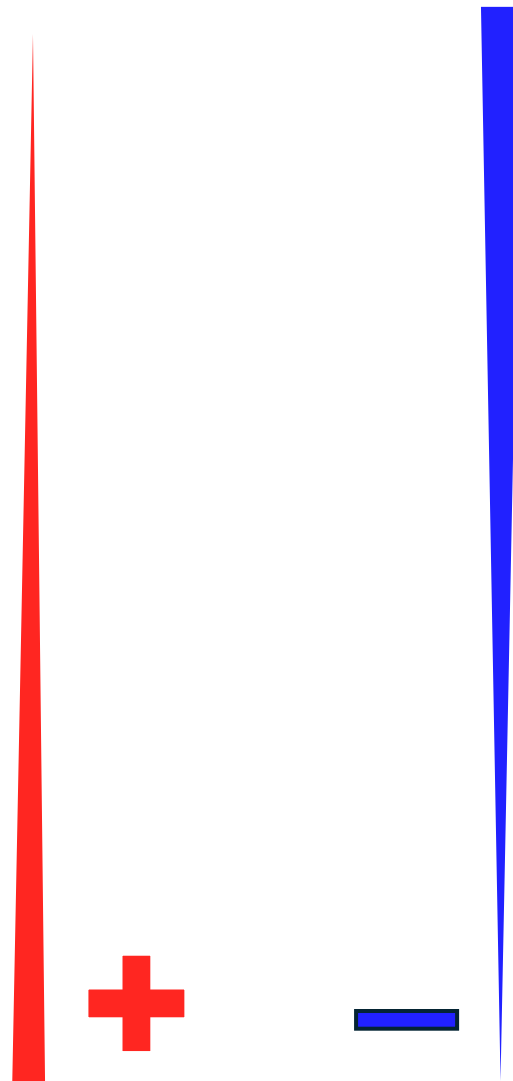
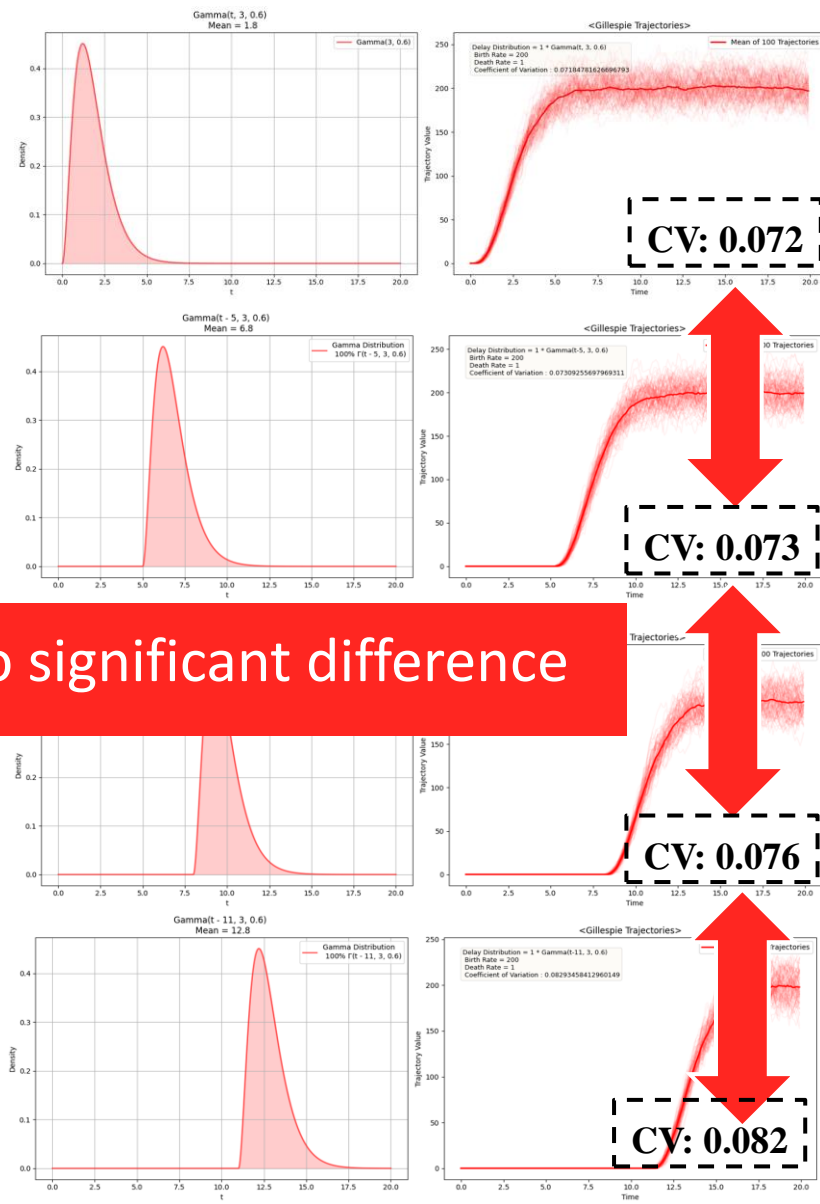


Increasing time delay by horizontal shift slightly increases **unimodal heterogeneity** and slightly decreases **bimodal heterogeneity**, but no significant differences in final time trace heterogeneity.

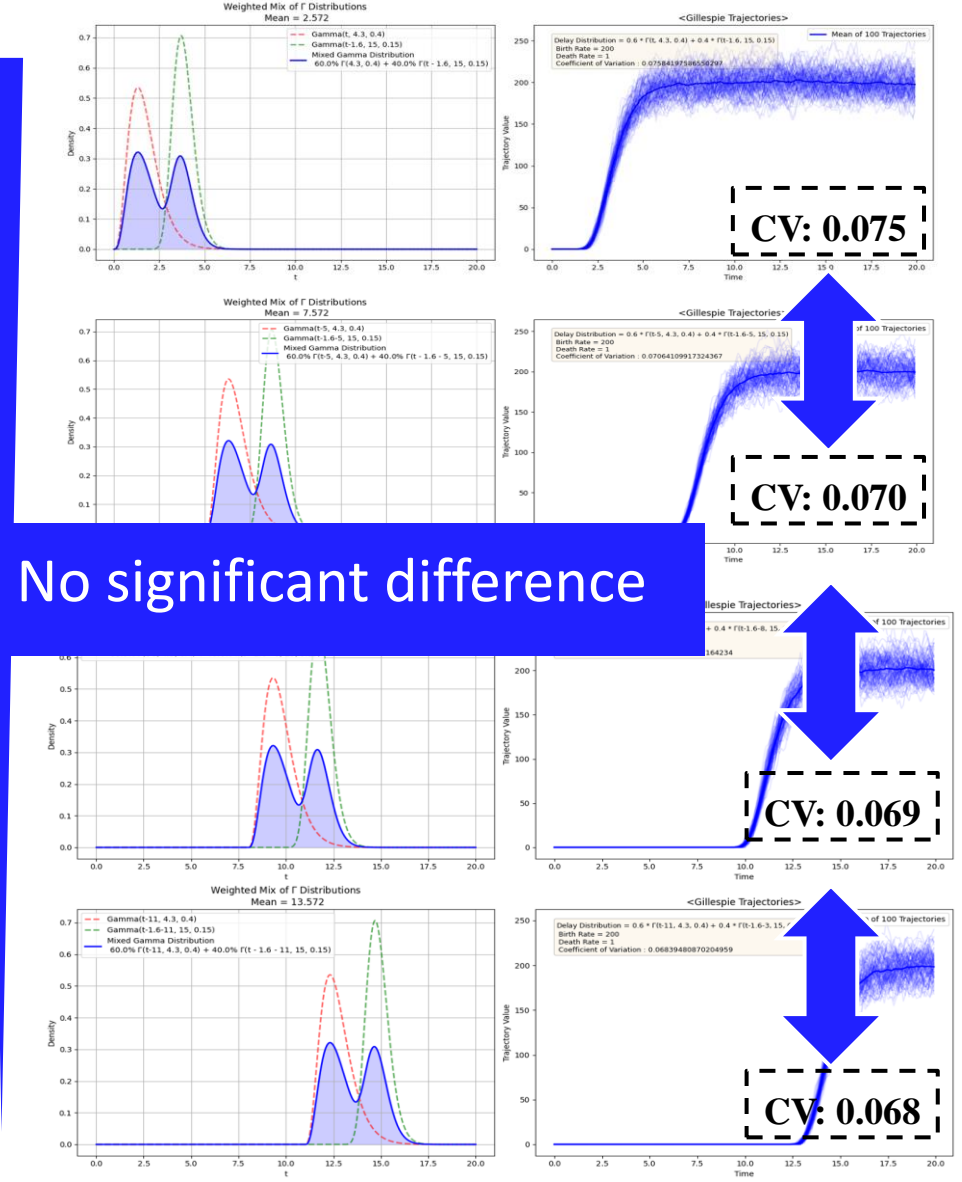
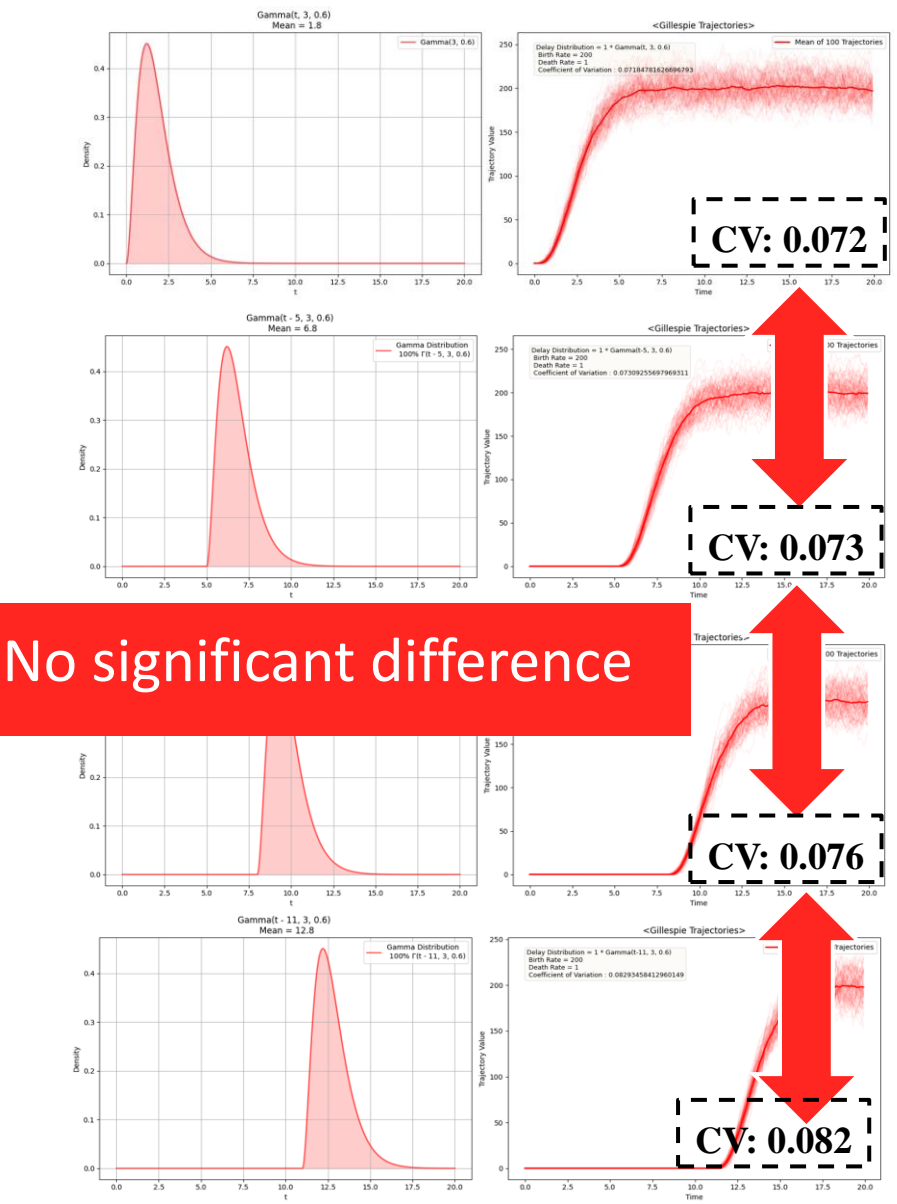


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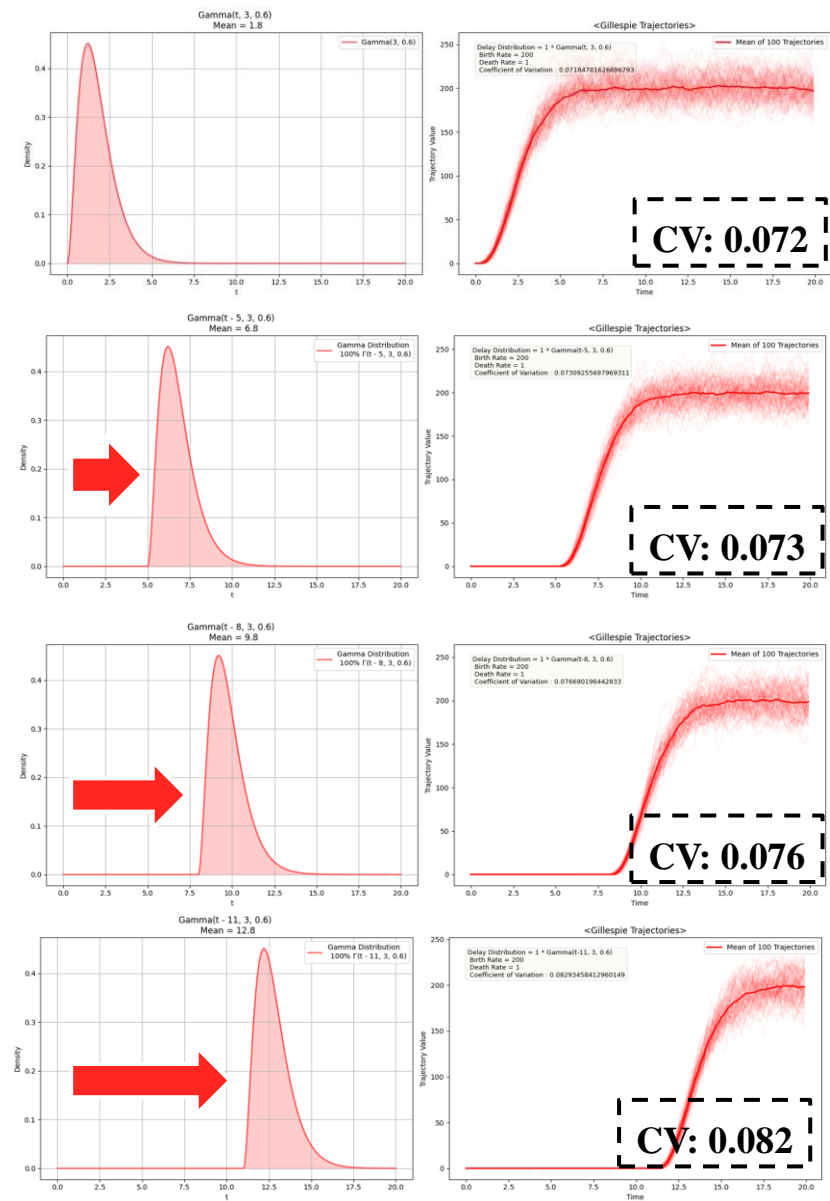
No significant difference



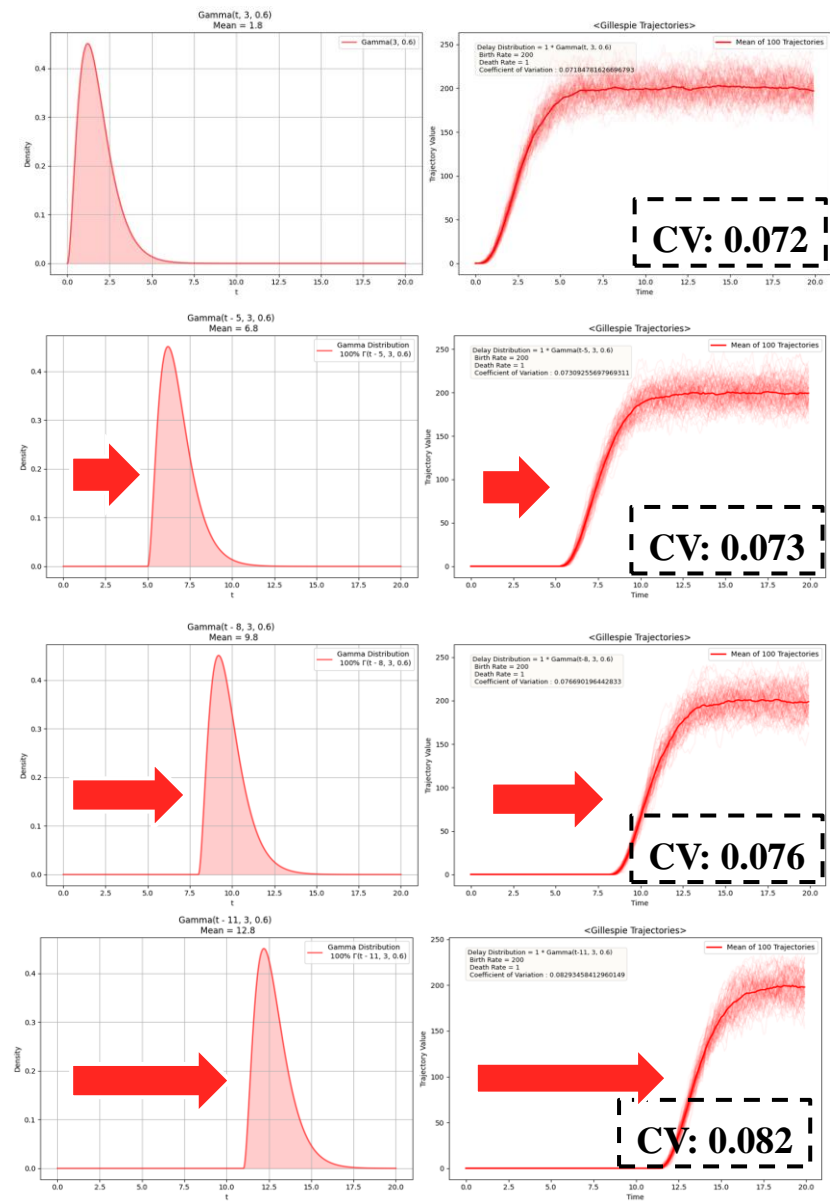
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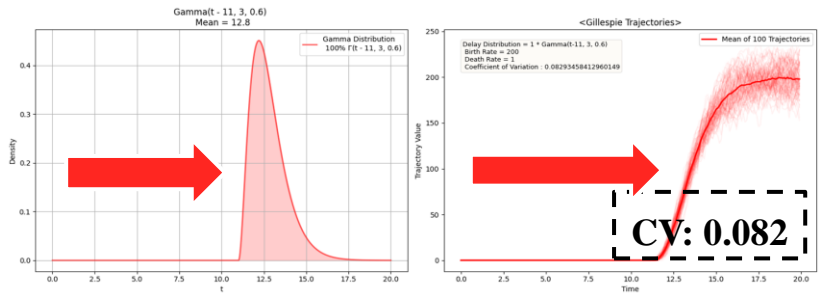
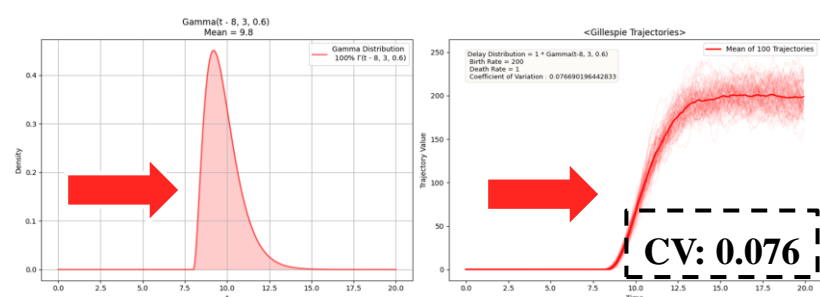
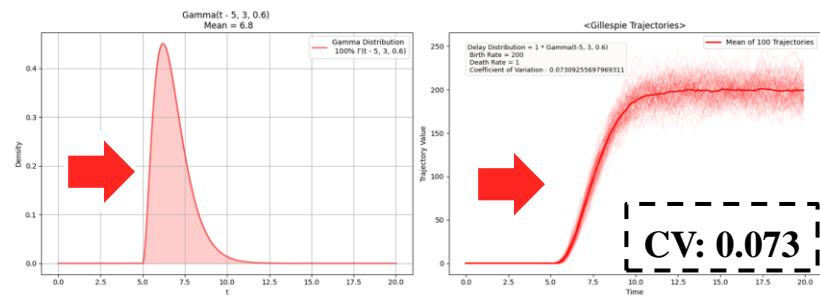
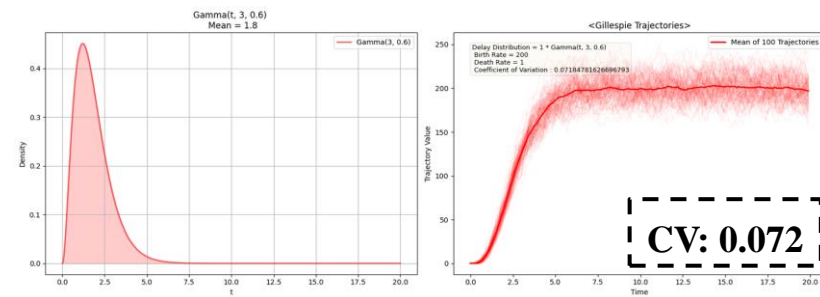
Increasing time delay by horizontal shift slightly increases **unimodal heterogeneity** and slightly decreases **bimodal heterogeneity**, but no significant differences in final time trace heterogeneity.



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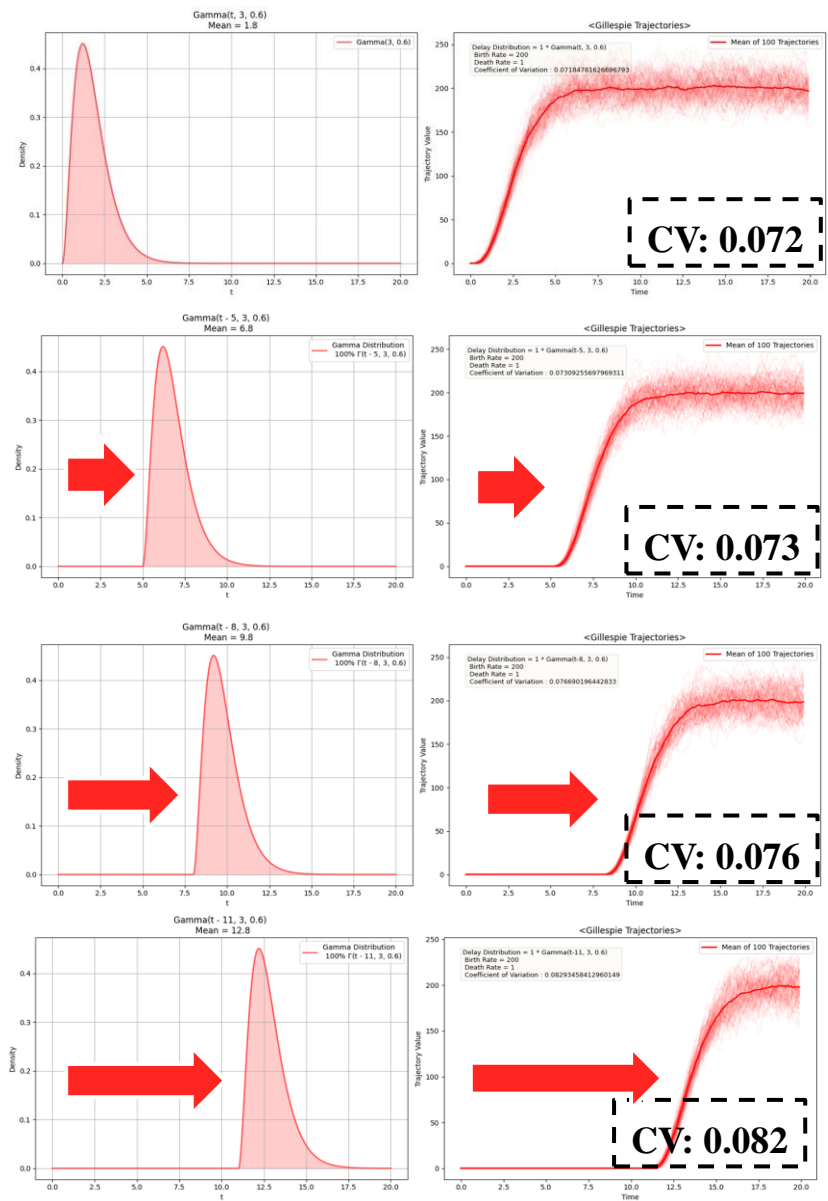
} Size Change X

}

} Size Change X

Intuitively, Shift ~~→~~ Change in Variance

Increasing time delay by horizontal shift slightly increases **unimodal heterogeneity** and slightly decreases **bimodal heterogeneity**, but no significant differences in final time trace heterogeneity.



Intuitively, Shift \nrightarrow Change in Variance

Q. What about changing birth/death rate??
∃ CV Difference??

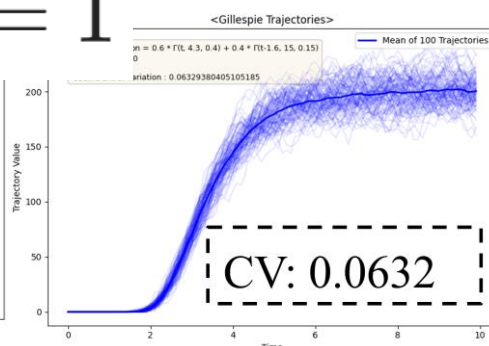
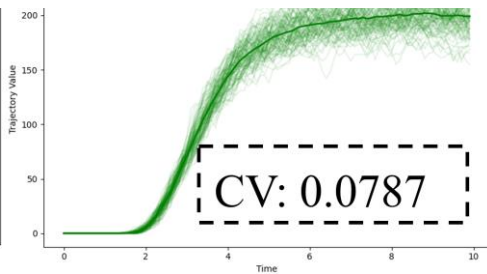
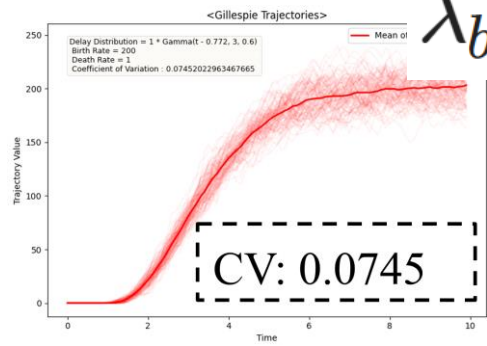
At lower birth/death ratio, there is no significant difference in the final time trace heterogeneity, (although the CV value increases with fixed distribution).

$$\lambda_b = 200, \quad \lambda_d = 1$$

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$\lambda_b = 200, \lambda_d = 1$

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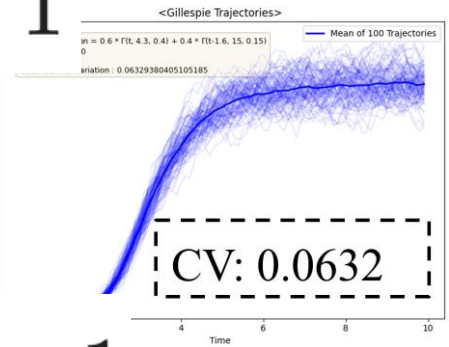
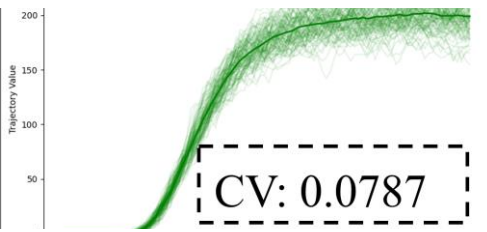
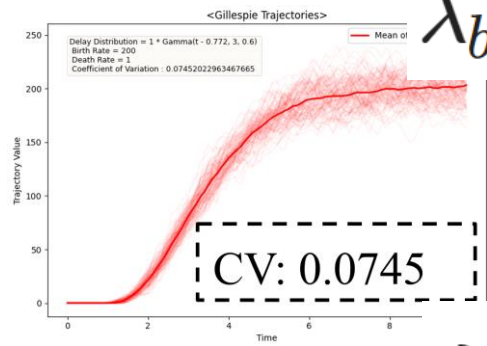


} Little CV difference (200:1)

At lower birth/death ratio, there is no significant difference in the final time trace heterogeneity, (although the CV value increases with fixed distribution).

$\lambda_b = 200, \lambda_d = 1$

$\lambda_b = 200, \lambda_d = 1$



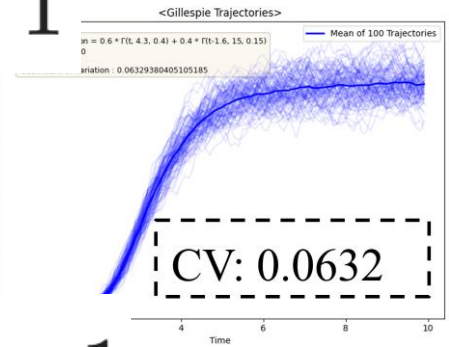
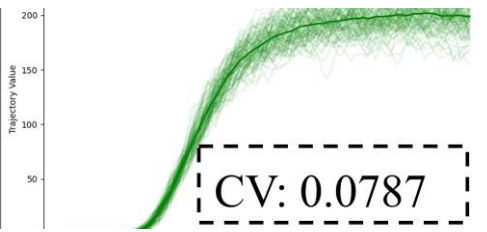
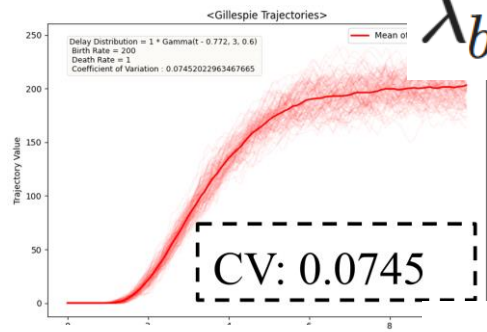
$\lambda_b = 100, \lambda_d = 1$

} Little CV difference (200:1)

At lower birth/death ratio, there is no significant difference in the final time trace heterogeneity, (although the CV value increases with fixed distribution).

$\lambda_b = 200, \lambda_d = 1$

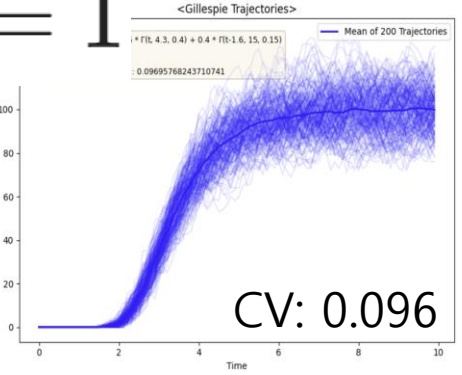
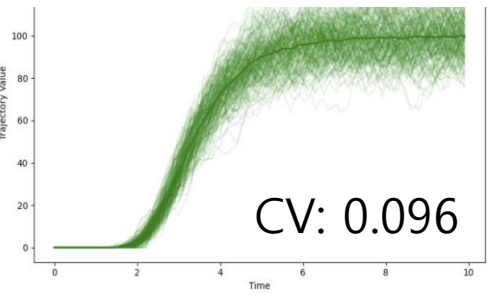
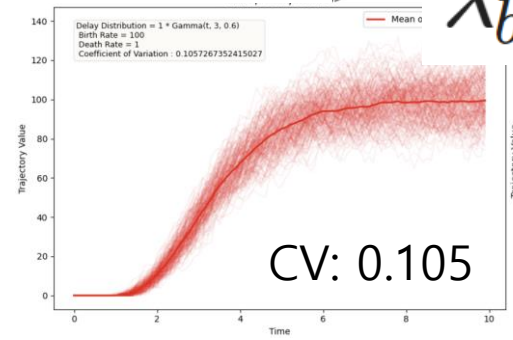
$\lambda_b = 200, \lambda_d = 1$



} Little CV difference (200:1)

$\lambda_b = 100, \lambda_d = 1$

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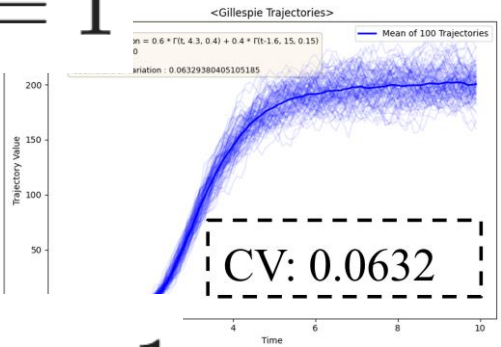
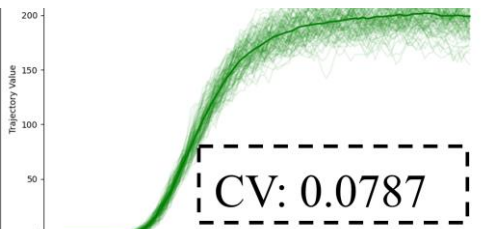
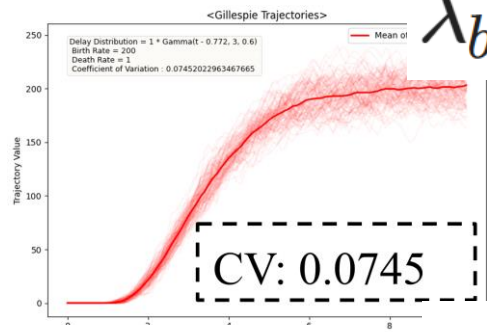


} Still little CV difference (100:1)

At lower birth/death ratio, there is no significant difference in the final time trace heterogeneity, (although the CV value increases with fixed distribution).

$\lambda_b = 200, \lambda_d = 1$

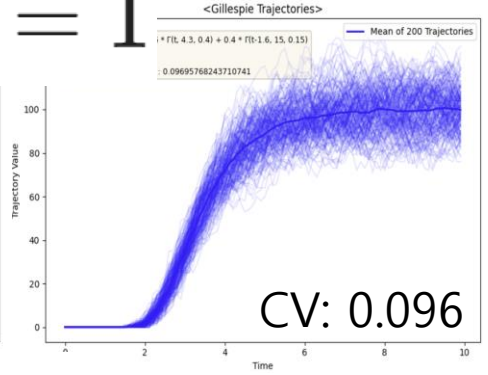
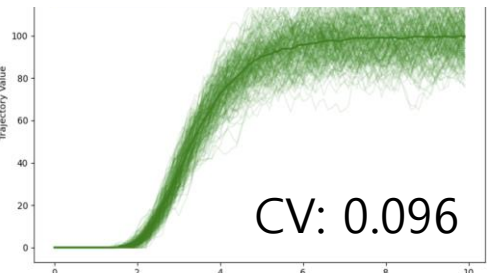
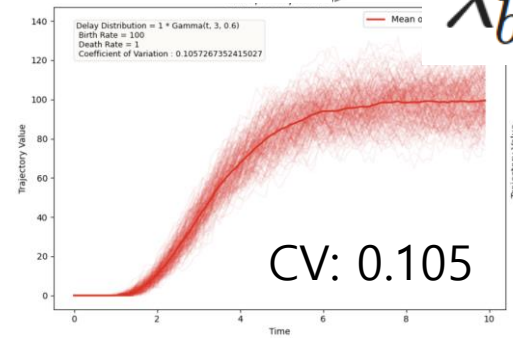
$\lambda_b = 200, \lambda_d = 1$



} Little CV difference (200:1)

$\lambda_b = 100, \lambda_d = 1$

$\lambda_b = 100, \lambda_d = 1$

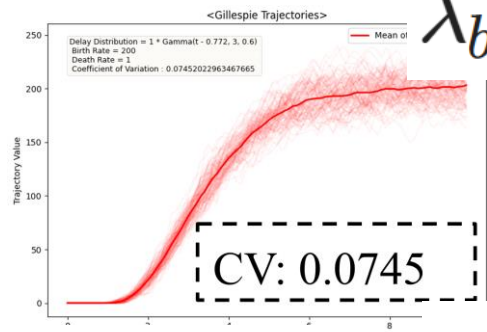


} Still little CV difference (100:1)

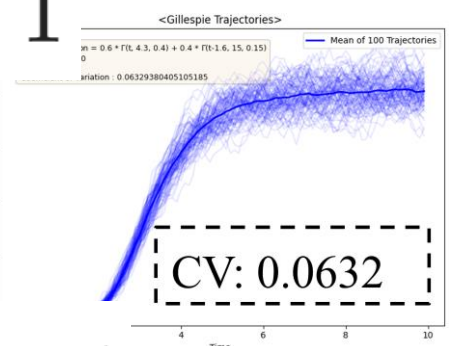
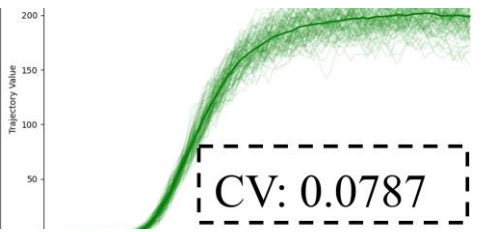
$\lambda_b = 4, \lambda_d = 1$

At lower birth/death ratio, there is no significant difference in the final time trace heterogeneity, (although the CV value increases with fixed distribution).

$\lambda_b = 200, \lambda_d = 1$

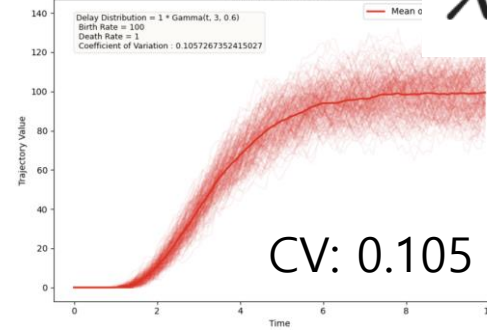


$\lambda_b = 200, \lambda_d = 1$

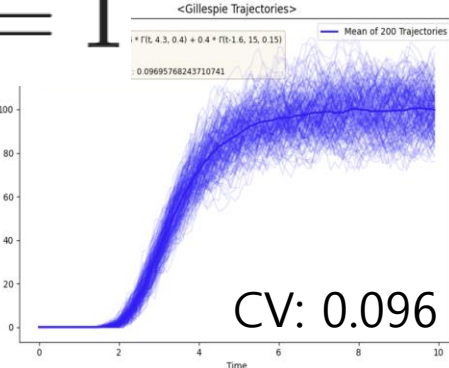
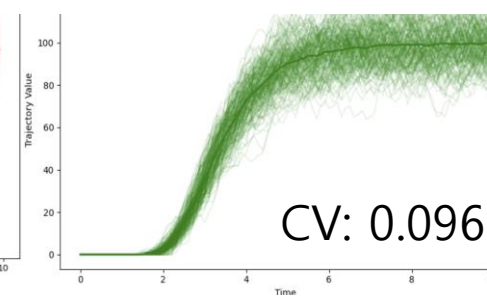


Little CV difference (200:1)

$\lambda_b = 100, \lambda_d = 1$

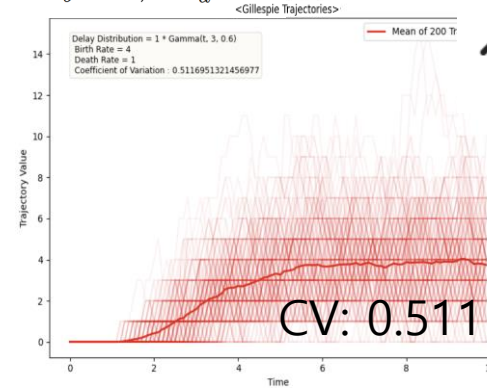


$\lambda_b = 100, \lambda_d = 1$

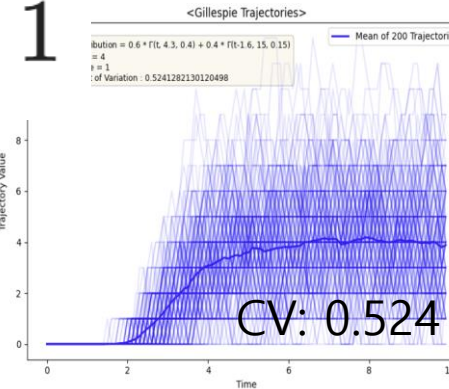
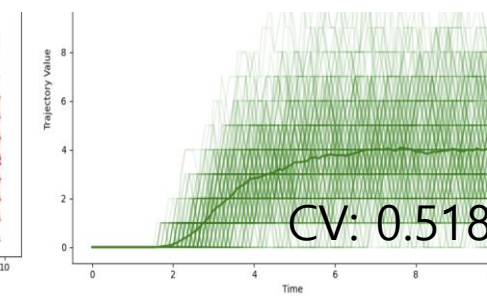


Still little CV difference (100:1)

$\lambda_b = 4, \lambda_d = 1$



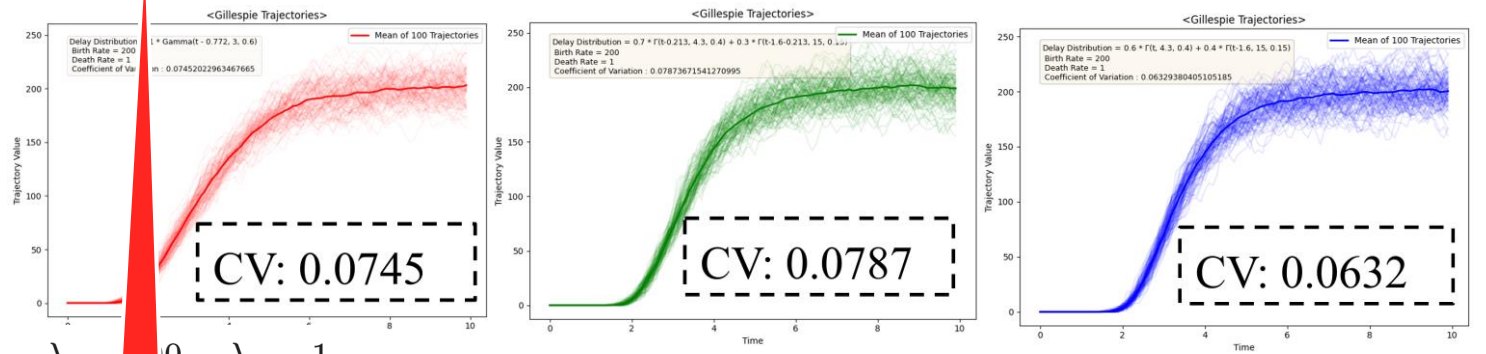
$\lambda_b = 4, \lambda_d = 1$



Still very little CV difference (4:1)

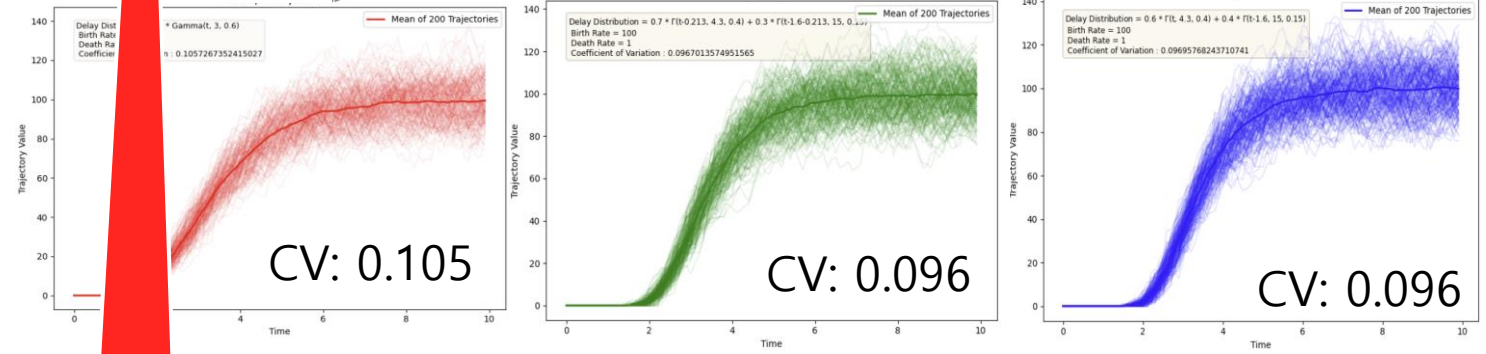
At lower birth/death ratio, there is no significant difference in the final time trace heterogeneity, (although the CV value increases with fixed distribution).

$\lambda_b = 200, \lambda_d = 1$



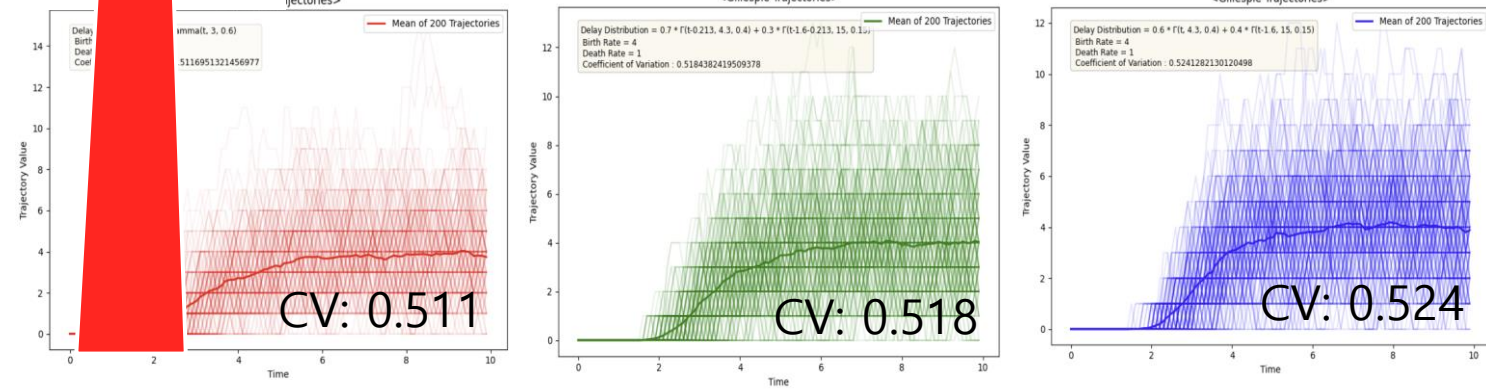
Very little CV difference (200:1)

$\lambda_b = 100, \lambda_d = 1$



Very little CV difference (100:1)

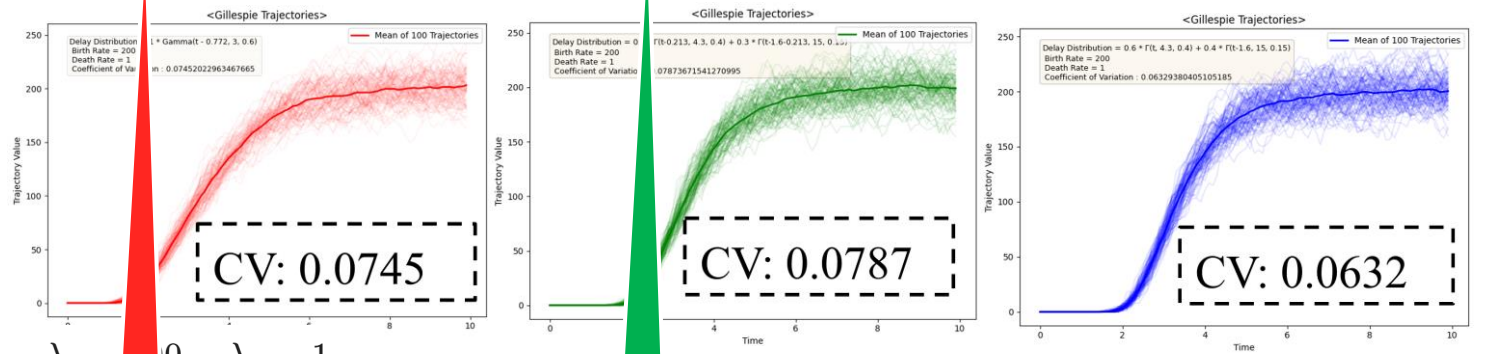
$\lambda_b = 4, \lambda_d = 1$



Very little CV difference (4:1)

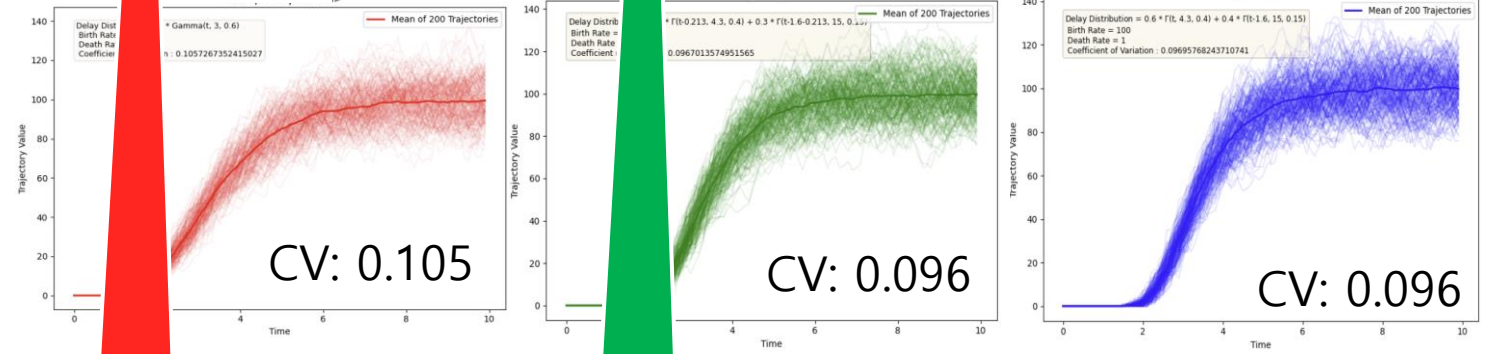
At lower birth/death ratio, there is no significant difference in the final time trace heterogeneity, (although the CV value increases with fixed distribution).

$\lambda_b = 200, \lambda_d = 1$



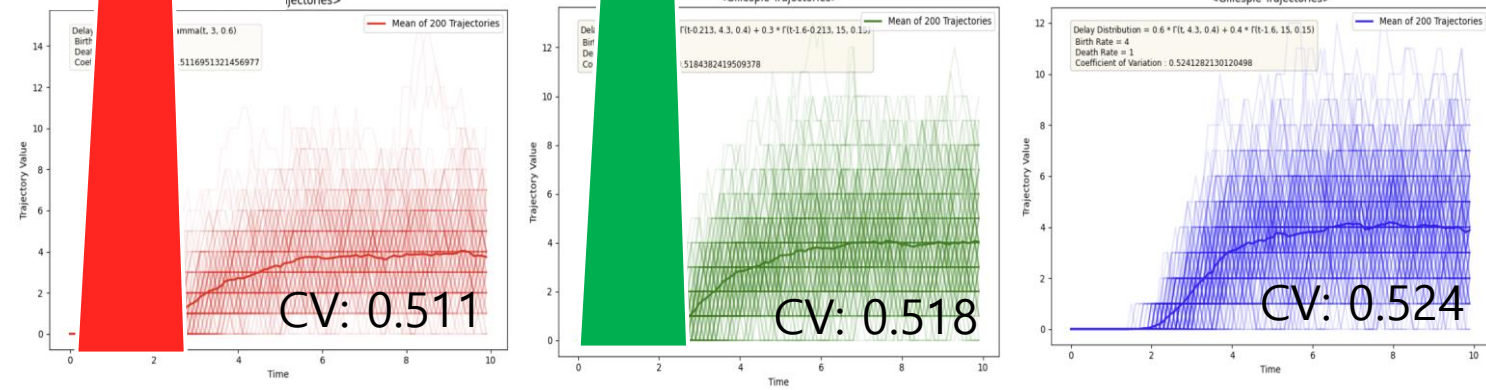
Very little CV difference (200:1)

$\lambda_b = 100, \lambda_d = 1$



Very little CV difference (100:1)

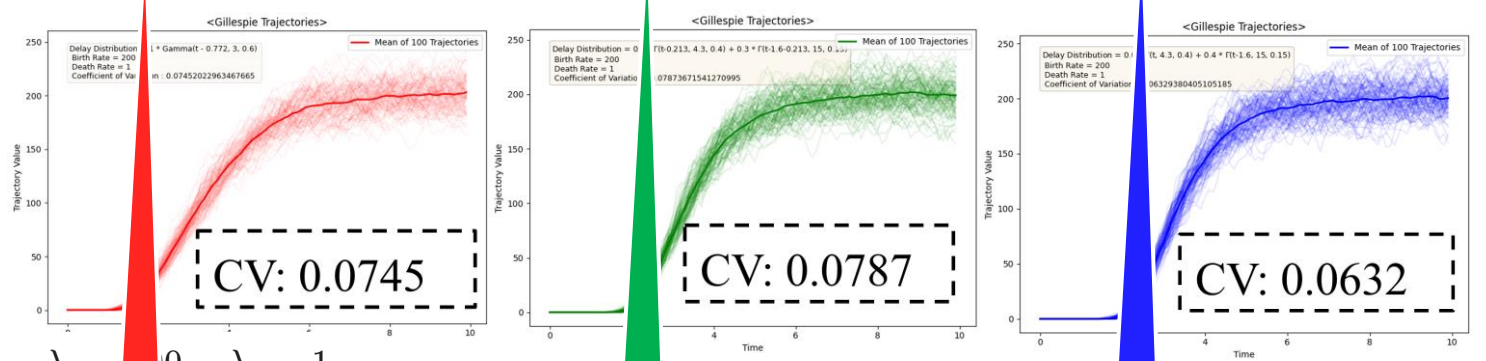
$\lambda_b = 4, \lambda_d = 1$



Very little CV difference (4:1)

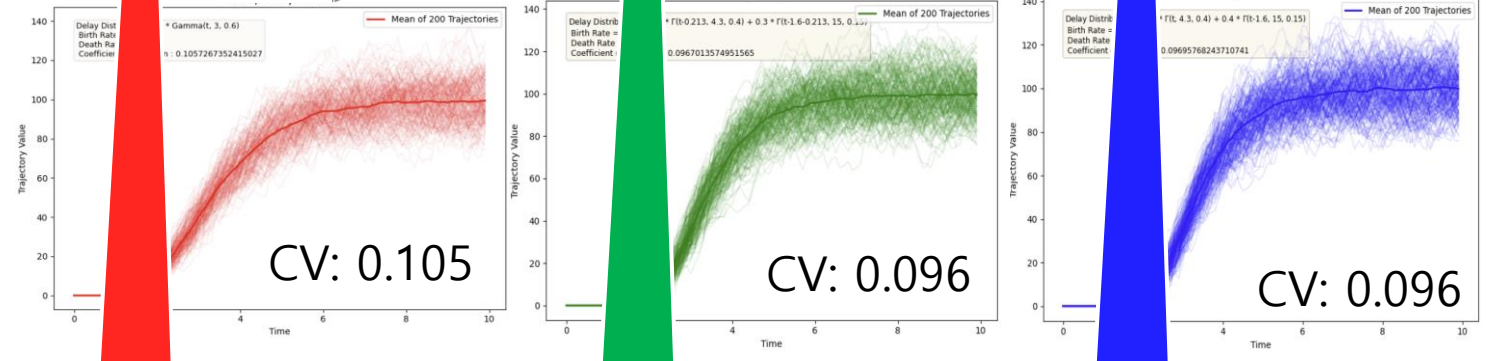
At lower birth/death ratio, there is no significant difference in the final time trace heterogeneity, (although the CV value increases with fixed distribution).

$\lambda_b = 200, \lambda_d = 1$



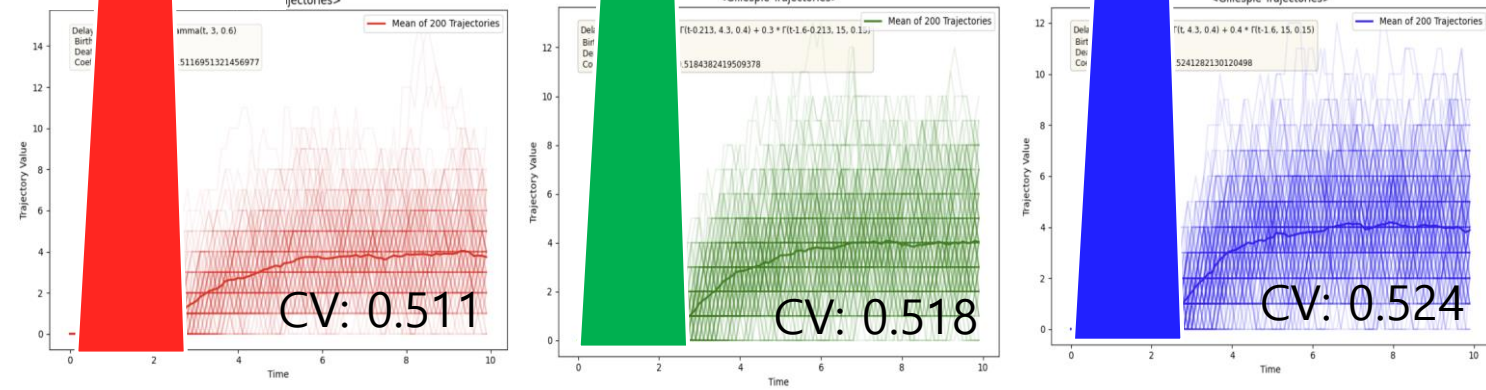
Very little CV difference (200:1)

$\lambda_b = 10, \lambda_d = 1$



Very little CV difference (100:1)

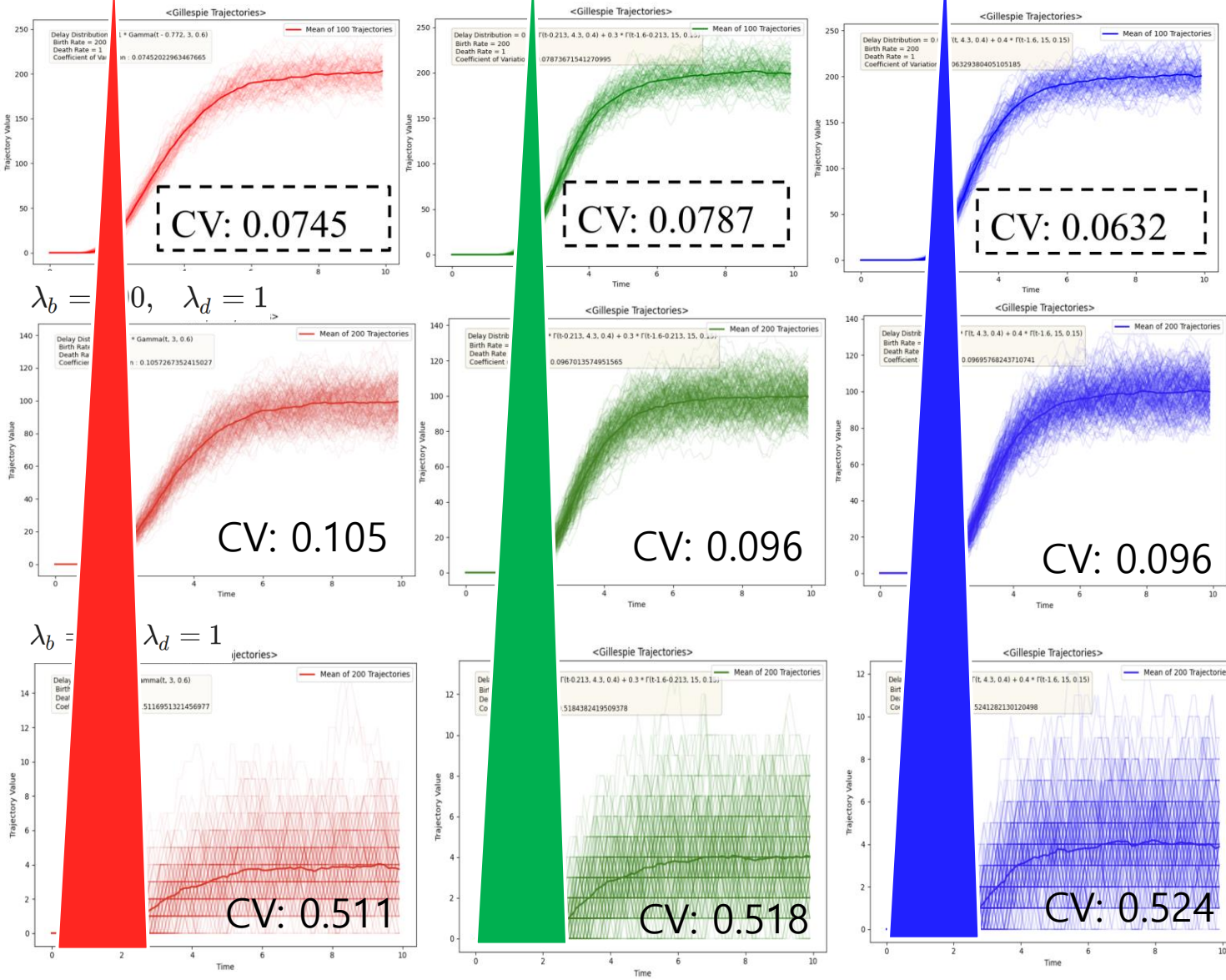
$\lambda_b = 4, \lambda_d = 1$



Very little CV difference (4:1)

At lower birth/death ratio, there is no significant difference in the final time trace heterogeneity, (although the CV value increases with fixed distribution).

$$\lambda_b = 200, \quad \lambda_d = 1$$

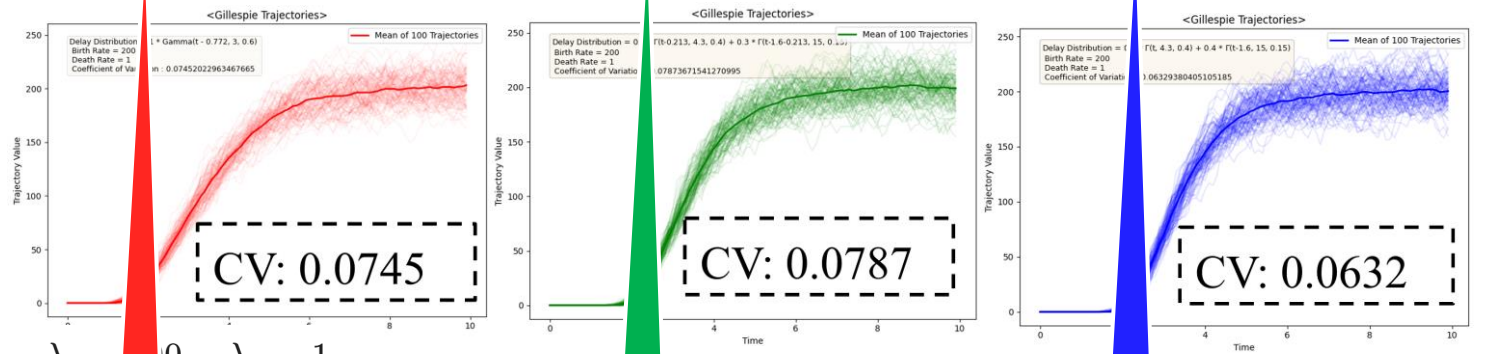


$$CV = \left(\frac{\sigma}{\mu} \right) \times 100\%$$

Lower $\mu = \lambda_b / \lambda_d \longrightarrow$ Higher CV

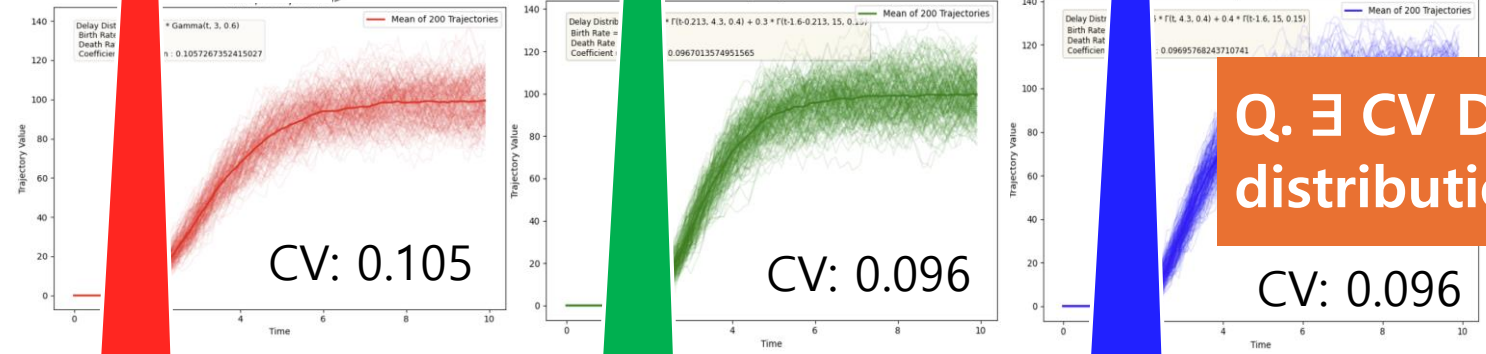
At lower birth/death ratio, there is no significant difference in the final time trace heterogeneity, (although the CV value increases with fixed distribution).

$\lambda_b = 200, \lambda_d = 1$



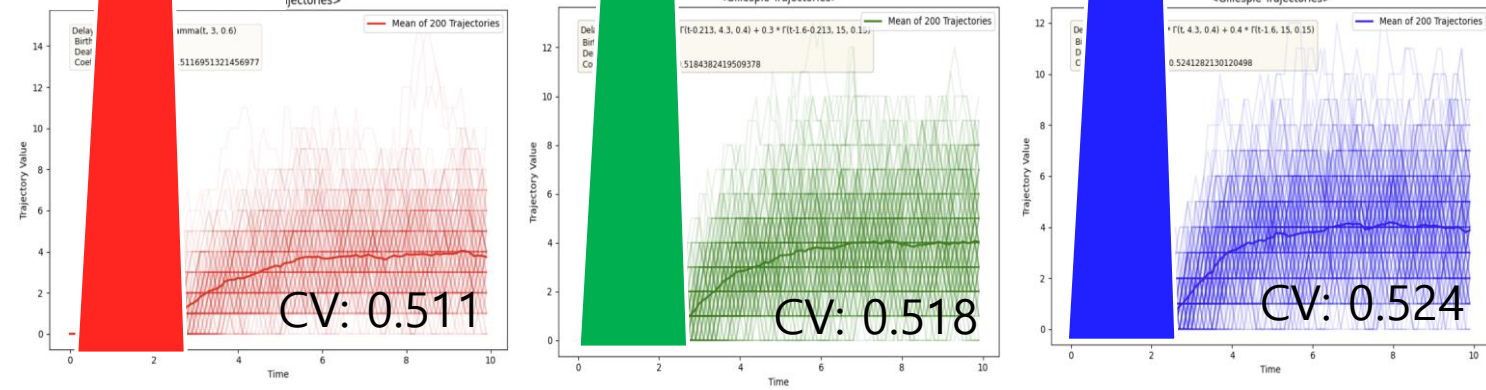
Very little CV difference (200:1)

$\lambda_b = 10, \lambda_d = 1$



Q. \exists CV Difference when changing distribution variance?

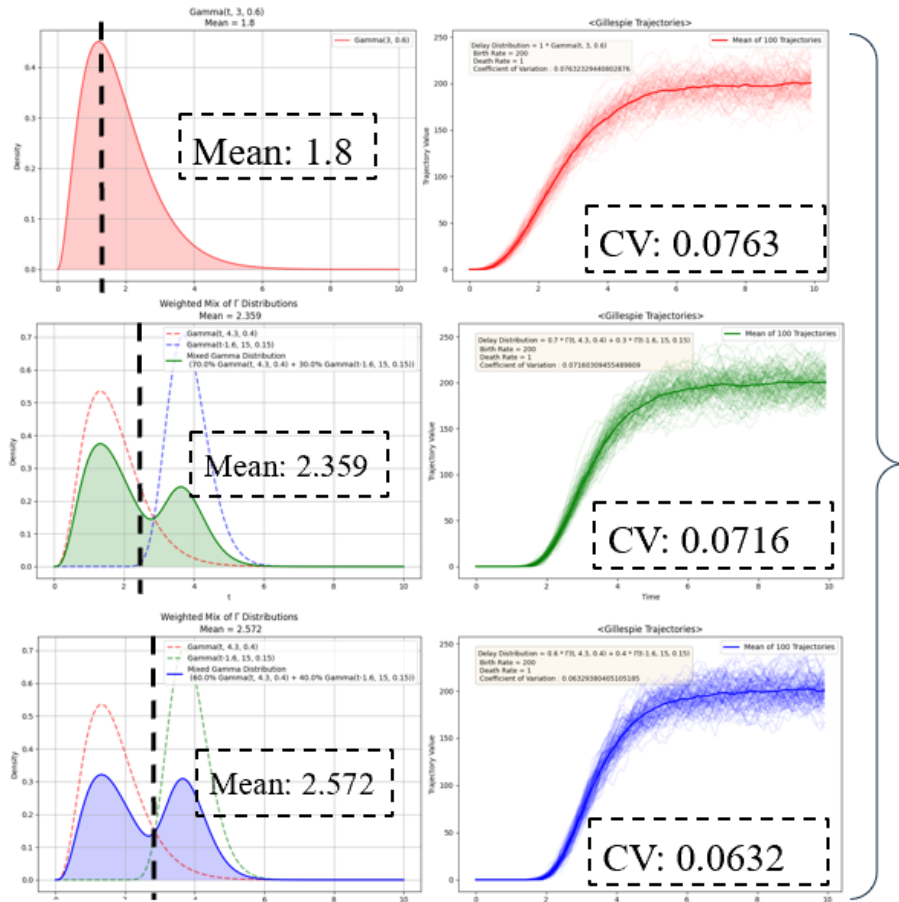
$\lambda_b = 4, \lambda_d = 1$



Very little CV difference (4:1)

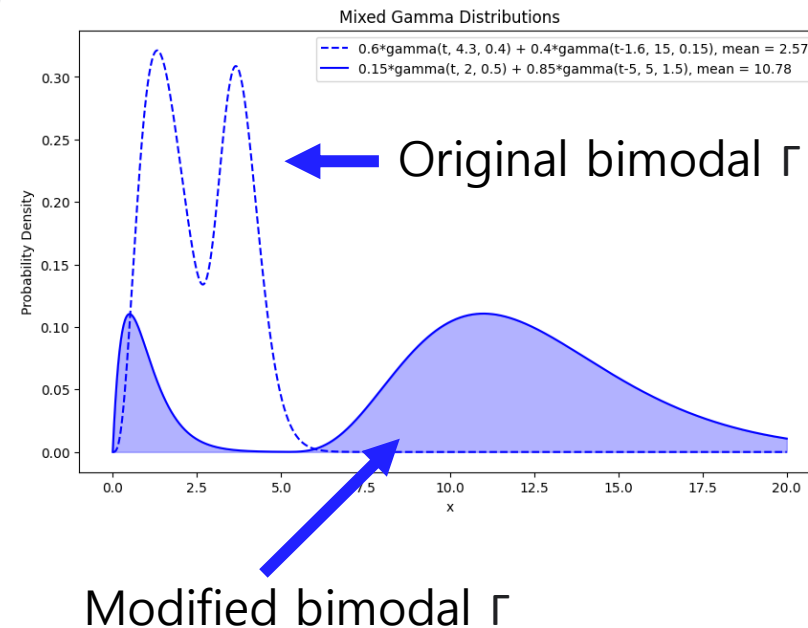
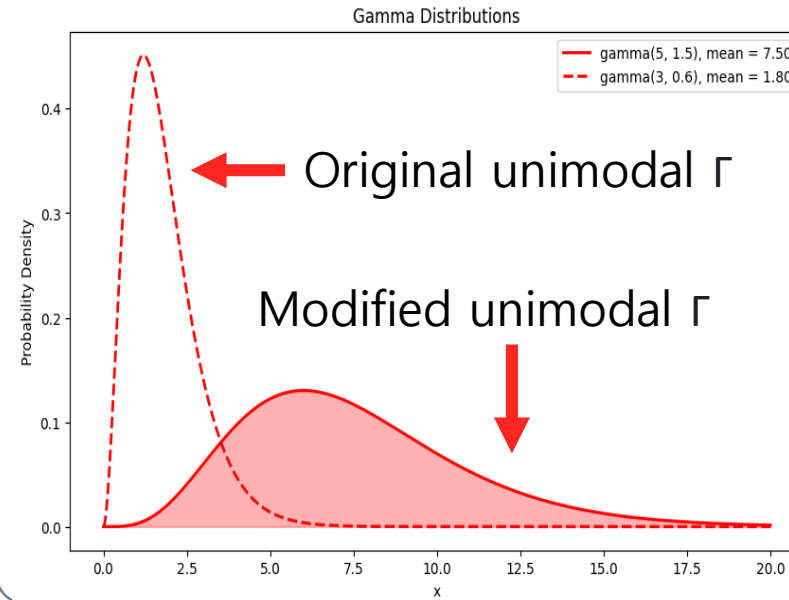
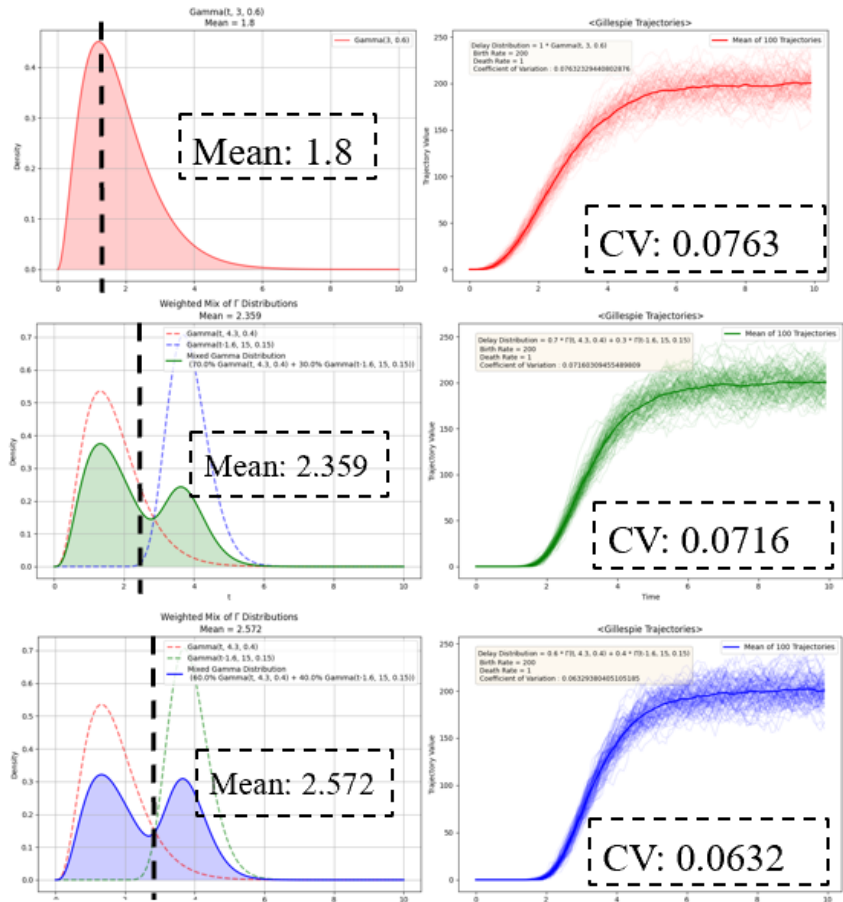
Changing distribution variance yields no significant difference in the final time trace heterogeneity.

Original Γ Distributions



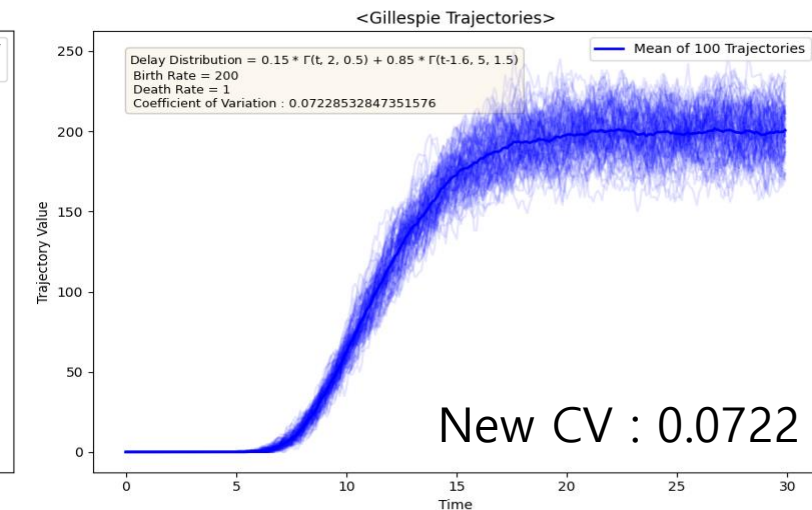
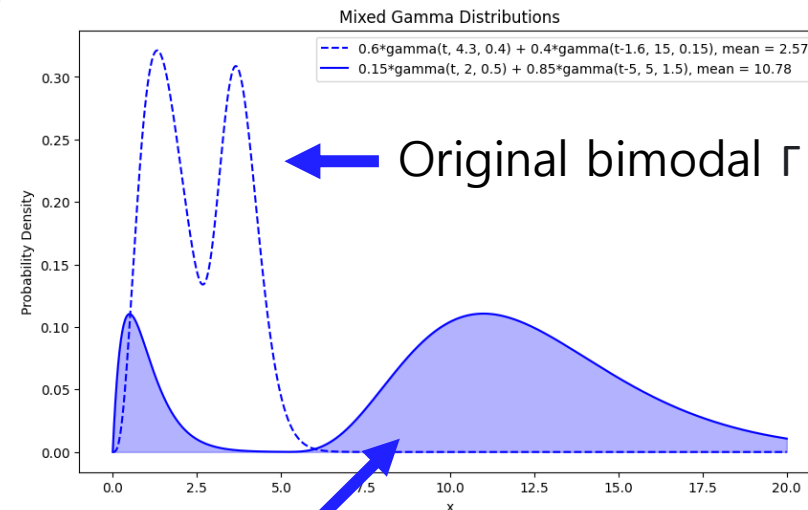
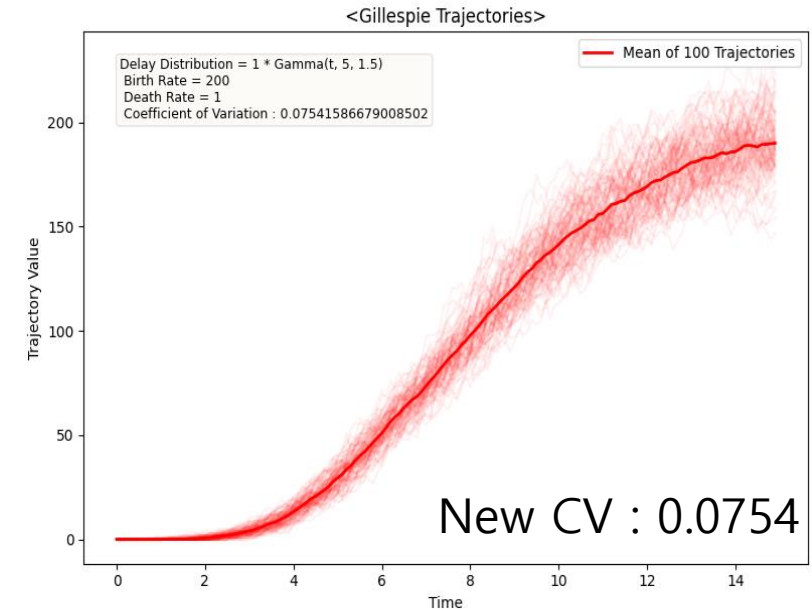
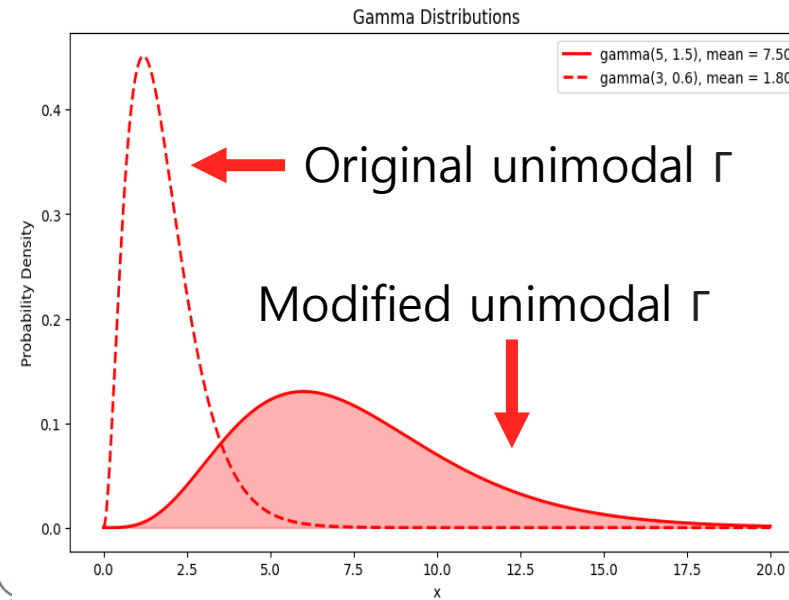
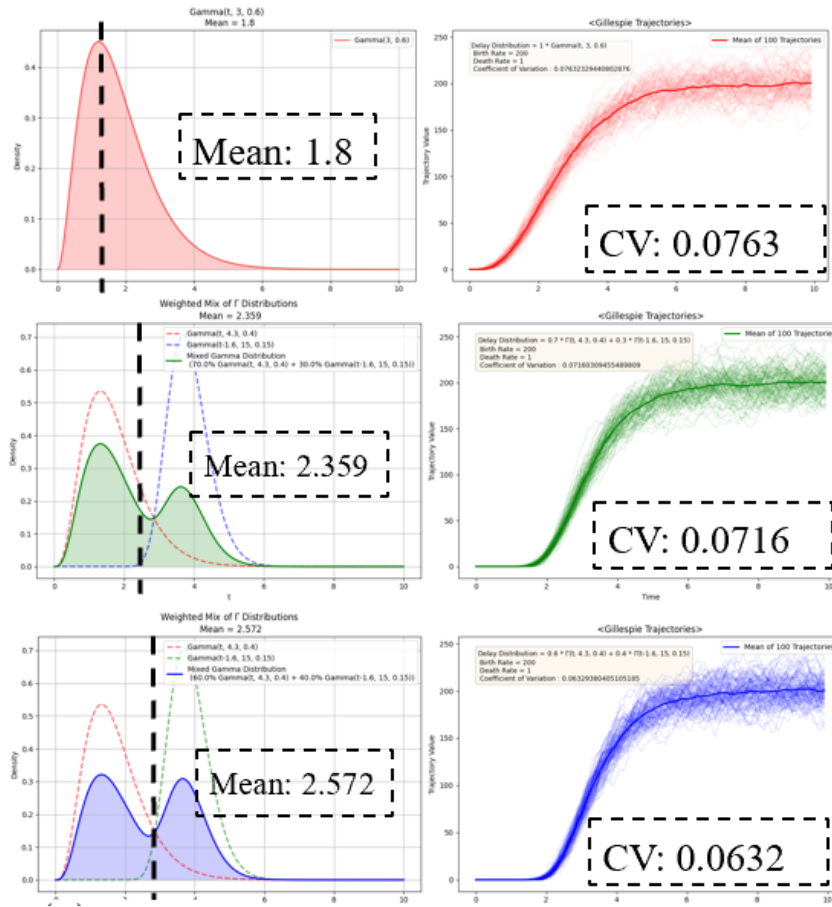
Changing distribution variance yields no significant difference in the final time trace heterogeneity.

Original Γ Distributions



Changing distribution variance yields no significant difference in the final time trace heterogeneity.

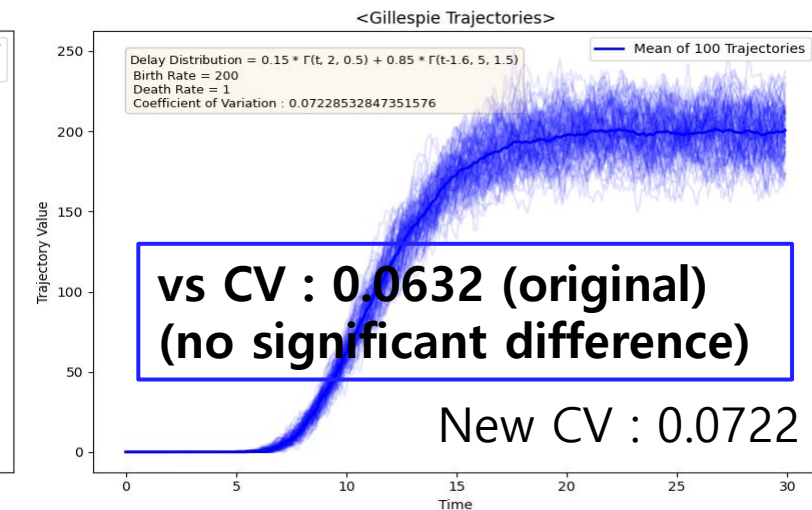
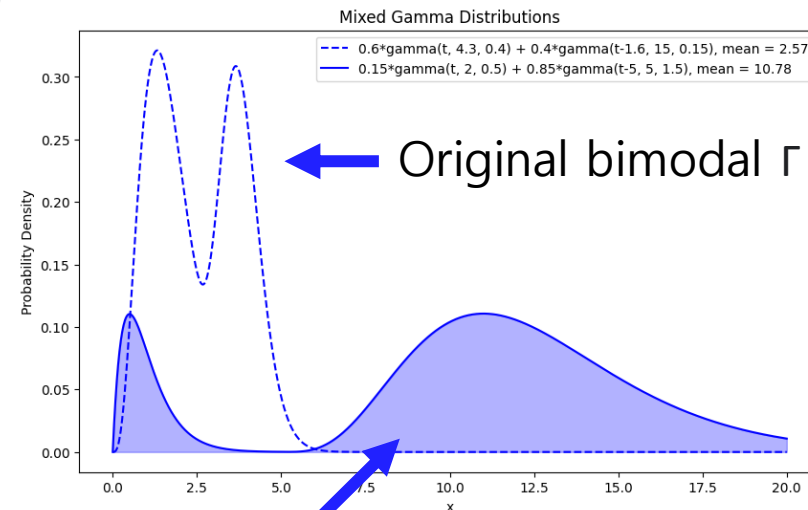
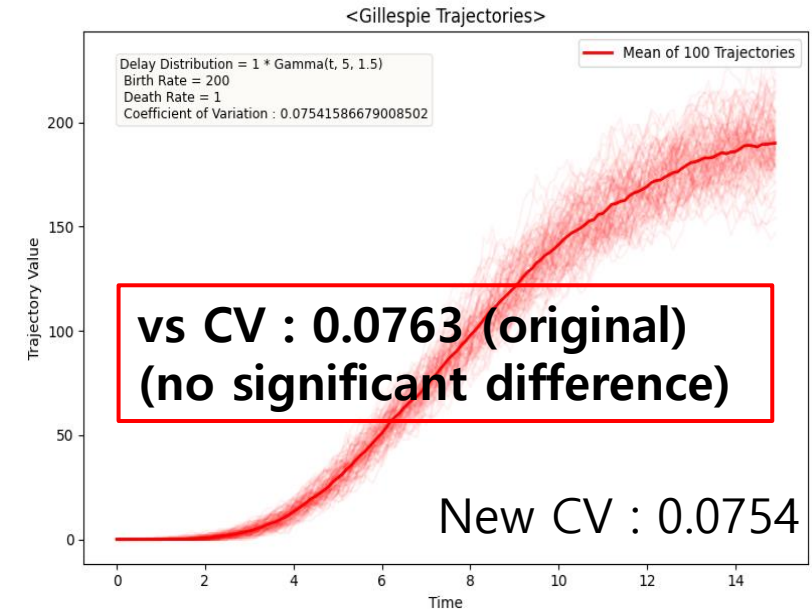
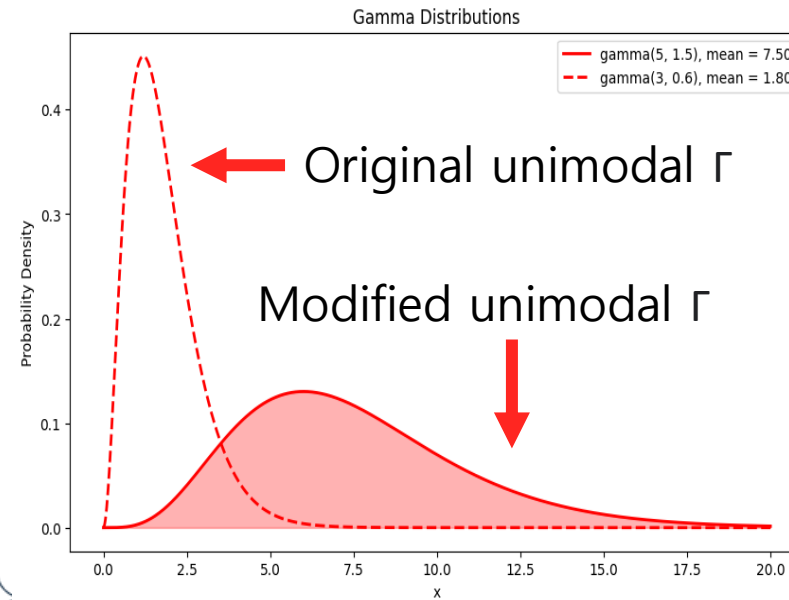
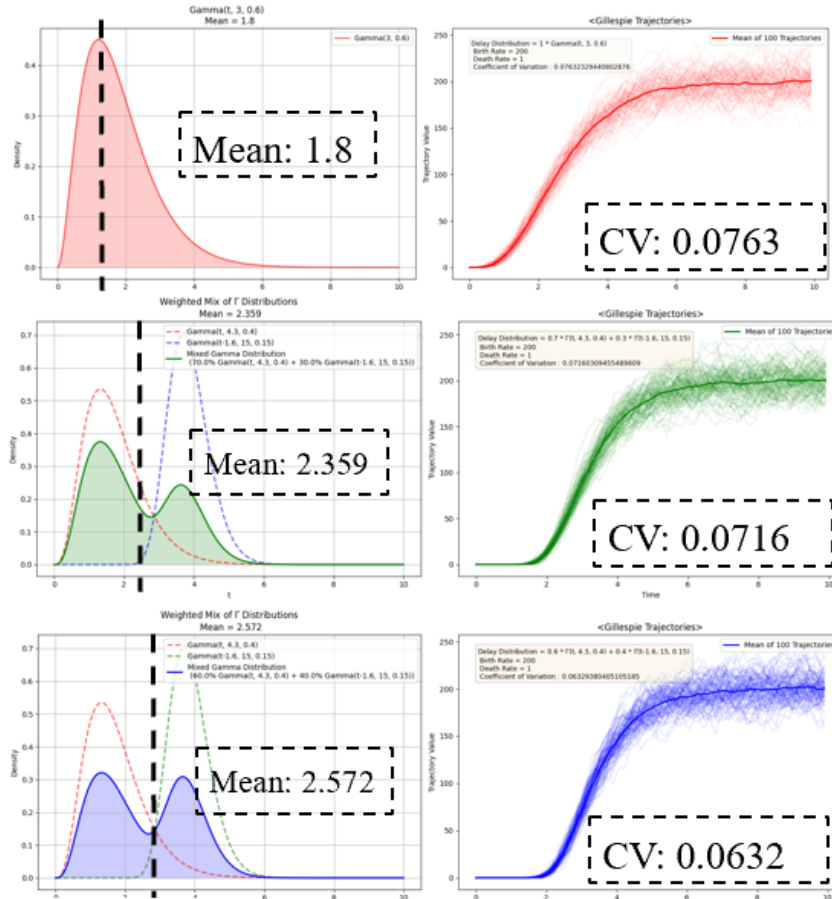
Original Γ Distributions



Modified bimodal Γ

Changing distribution variance yields no significant difference in the final time trace heterogeneity.

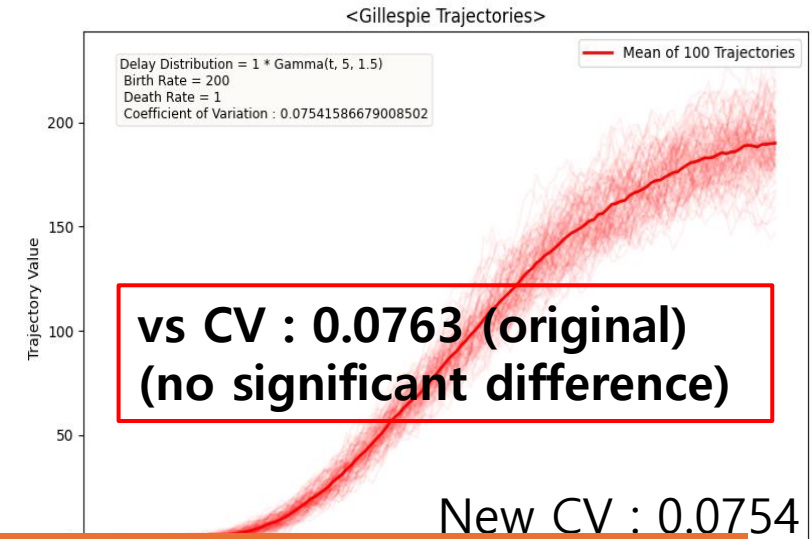
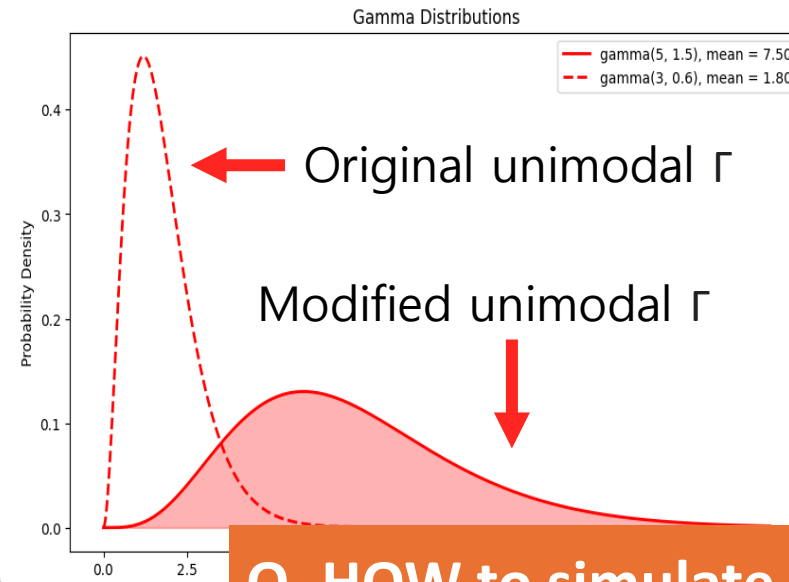
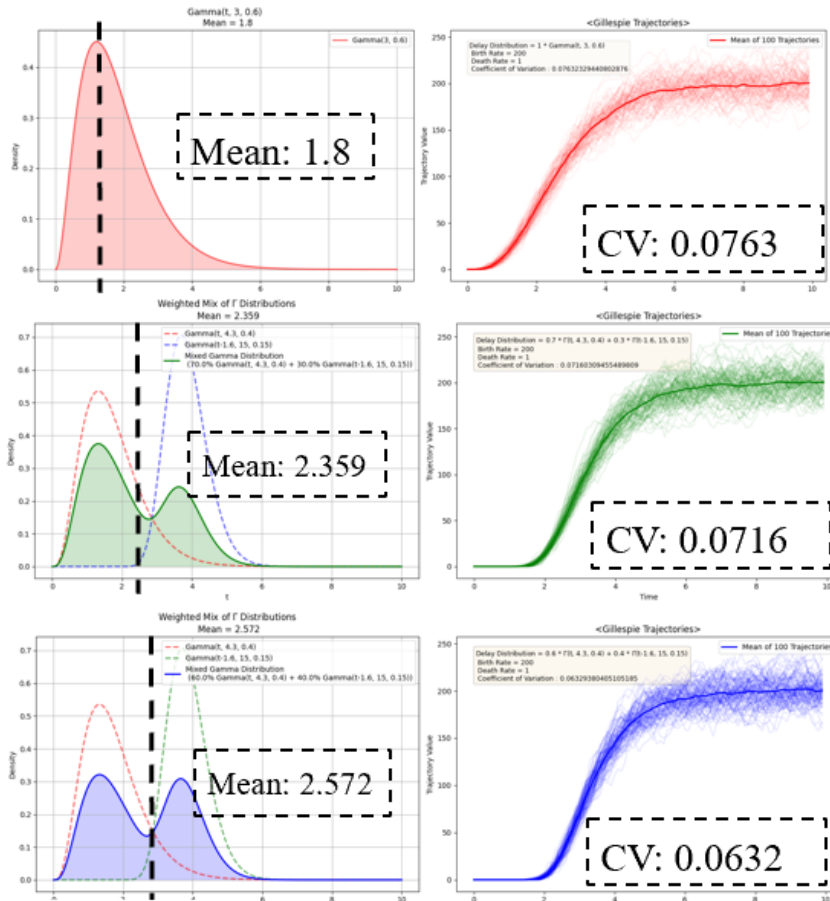
Original Γ Distributions



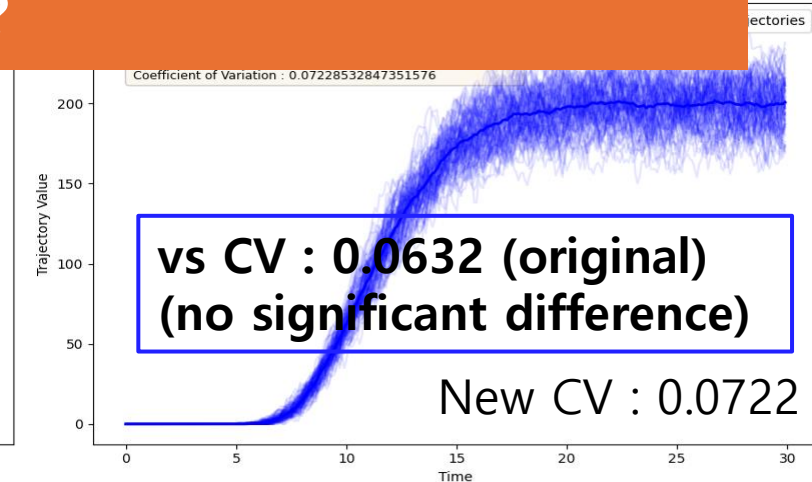
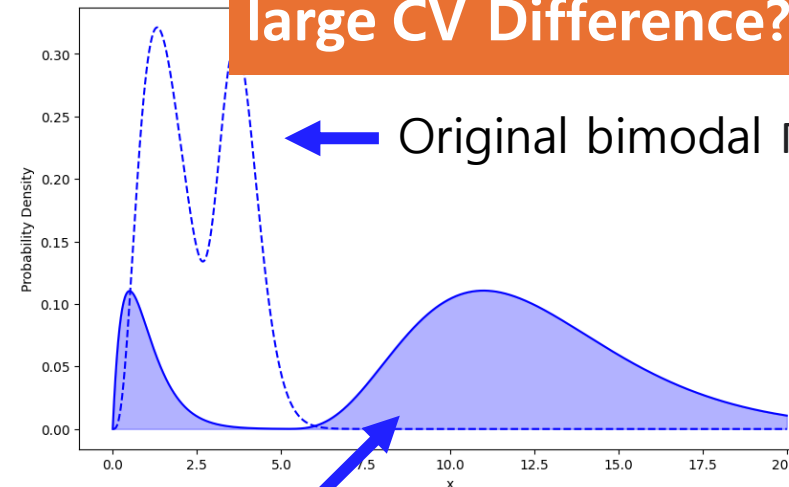
Modified bimodal Γ

Changing distribution variance yields no significant difference in the final time trace heterogeneity.

Original Γ Distributions



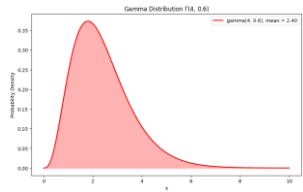
Q. HOW to simulate with Gillespie such that \exists large CV Difference??



Modified bimodal Γ

Lowering 'volume' in Gillespie SSA for **single timescale pathways** raises heterogeneity and produces closer CV estimates to real data, across different birth/death ratios.

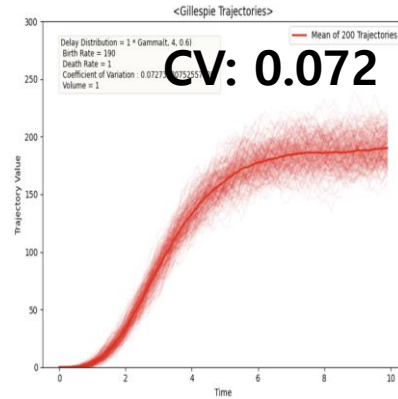
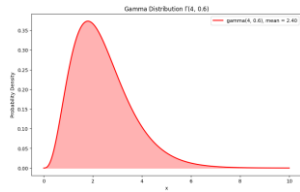
$$\lambda_b/\lambda_d = 190$$



Lowering 'volume' in Gillespie SSA for **single timescale pathways** raises heterogeneity and produces closer CV estimates to real data, across different birth/death ratios.

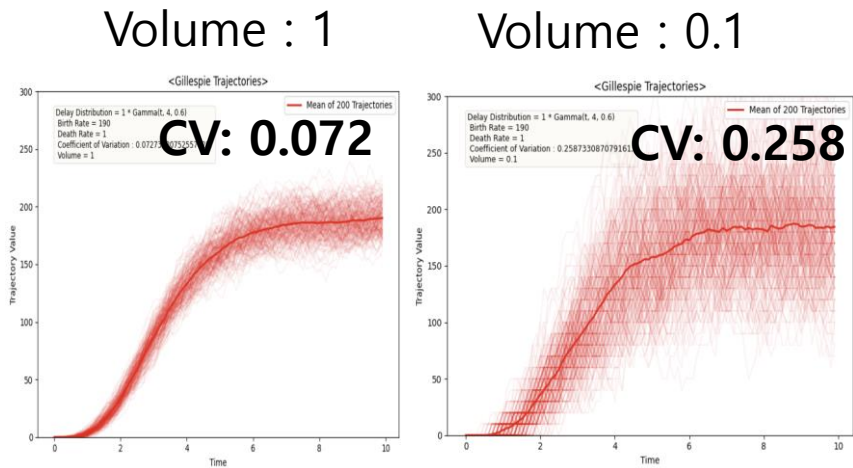
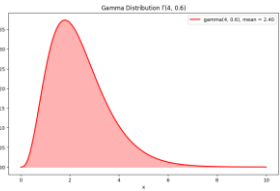
Volume : 1

$$\lambda_b / \lambda_d = 190$$



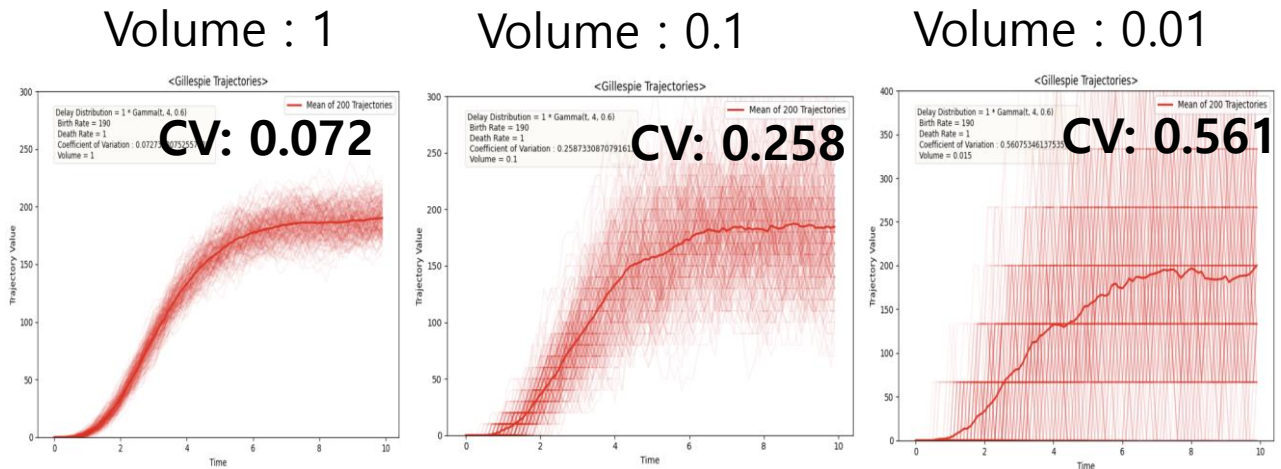
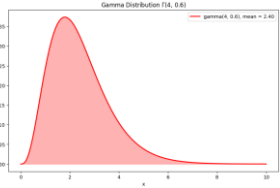
Lowering 'volume' in Gillespie SSA for **single timescale pathways** raises heterogeneity and produces closer CV estimates to real data, across different birth/death ratios.

$\lambda_b/\lambda_d = 190$



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$\lambda_b/\lambda_d = 190$

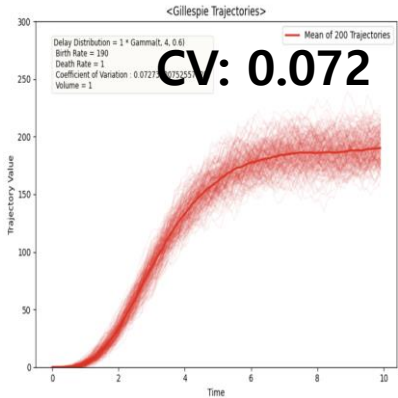


Lowering 'volume' in Gillespie SSA for **single timescale pathways** raises heterogeneity and produces closer CV estimates to real data, across different birth/death ratios.

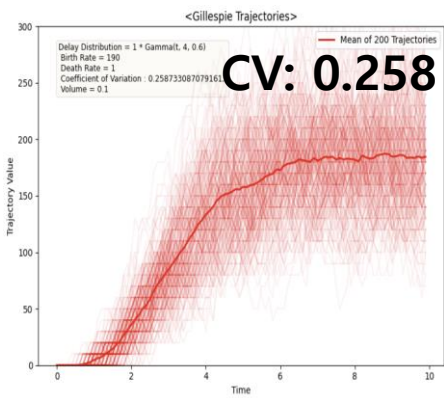
$\lambda_b/\lambda_d = 190$



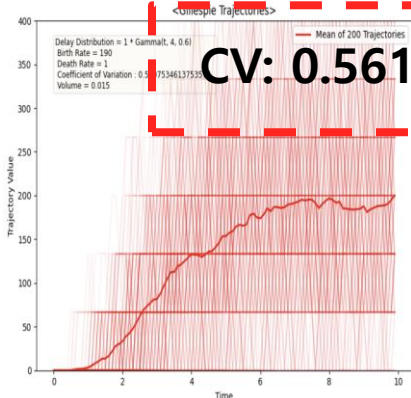
Volume : 1



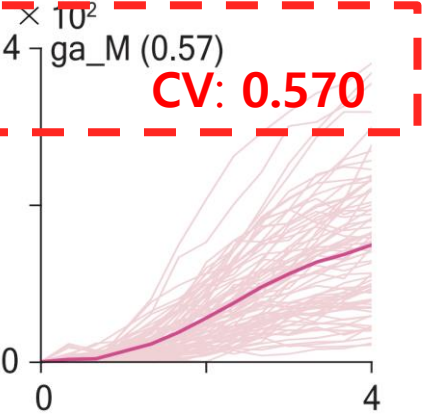
Volume : 0.1



Volume : 0.01

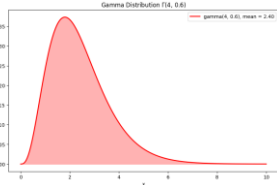


$\lambda_b/\lambda_d = 190$

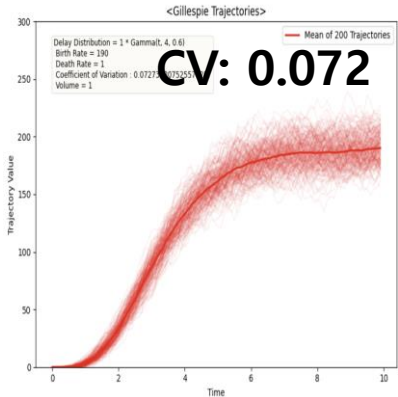


Lowering 'volume' in Gillespie SSA for **single timescale pathways** raises heterogeneity and produces closer CV estimates to real data, across different birth/death ratios.

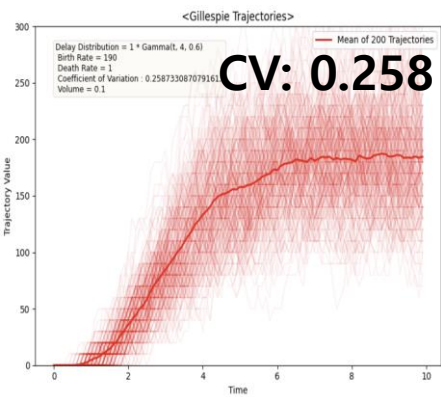
$\lambda_b/\lambda_d = 190$



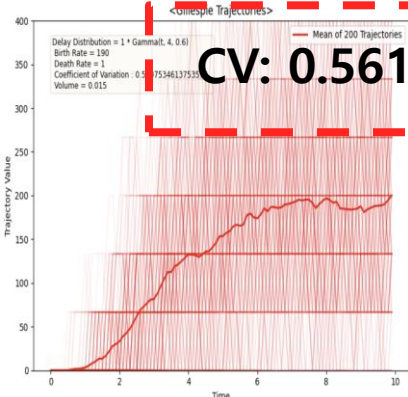
Volume : 1



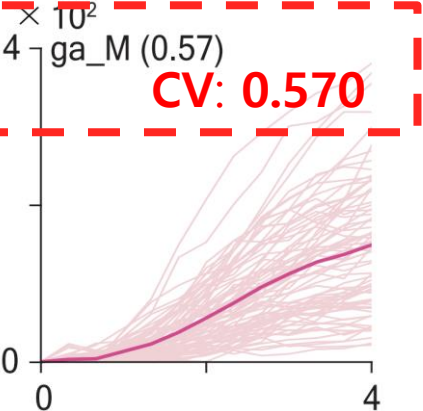
Volume : 0.1



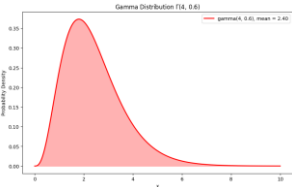
Volume : 0.01



$\lambda_b/\lambda_d = 190$

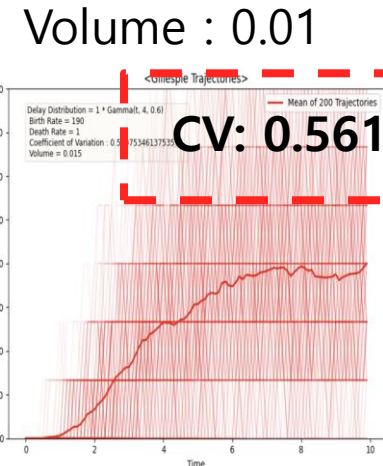
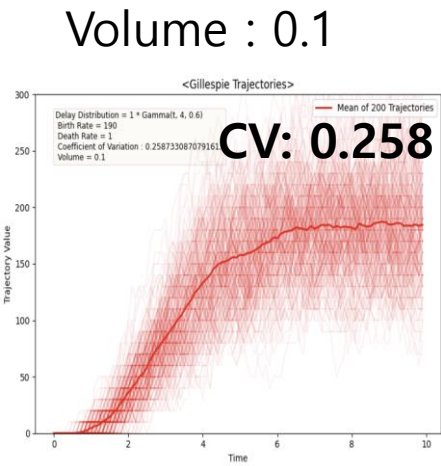
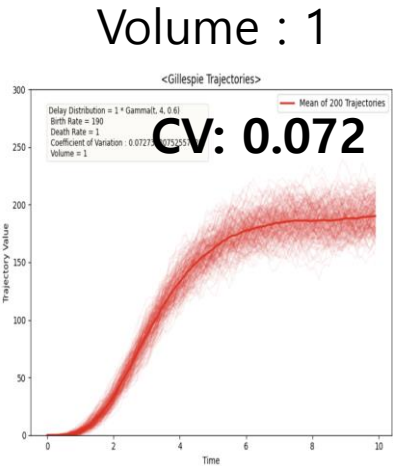
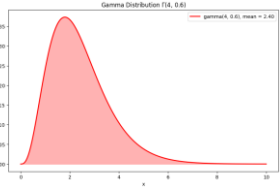


$\lambda_b/\lambda_d = 350$

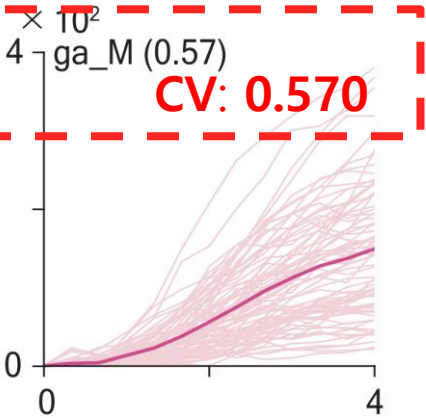


Lowering 'volume' in Gillespie SSA for **single timescale pathways** raises heterogeneity and produces closer CV estimates to real data, across different birth/death ratios.

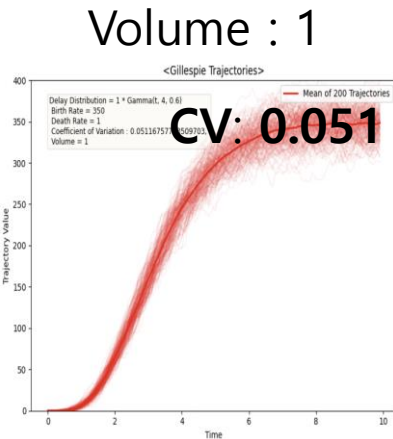
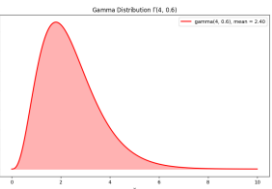
$$\lambda_b/\lambda_d = 190$$



$$\lambda_b/\lambda_d = 190$$

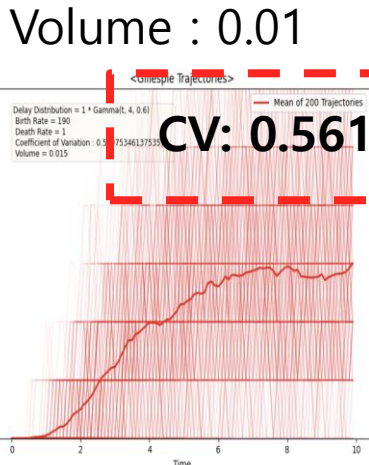
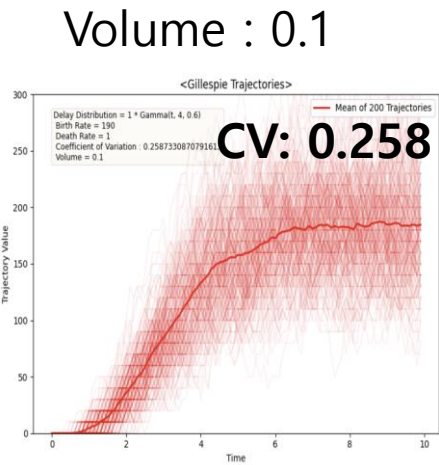
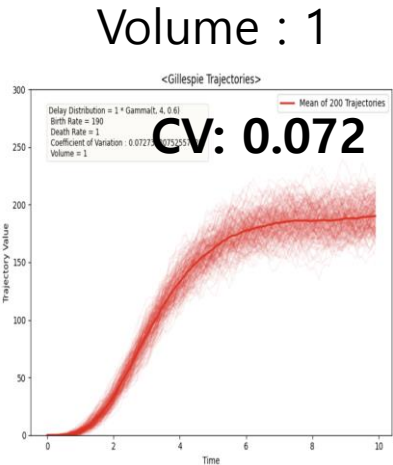
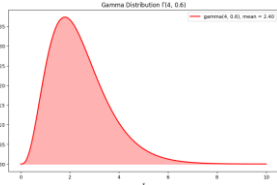


$$\lambda_b/\lambda_d = 350$$

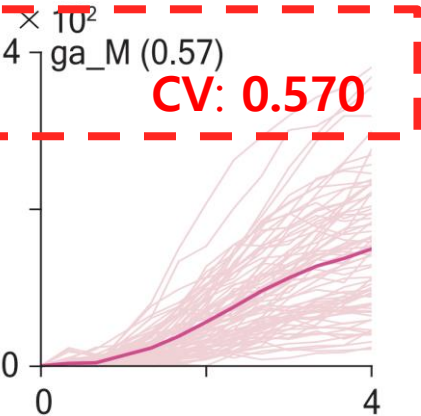


Lowering 'volume' in Gillespie SSA for **single timescale pathways** raises heterogeneity and produces closer CV estimates to real data, across different birth/death ratios.

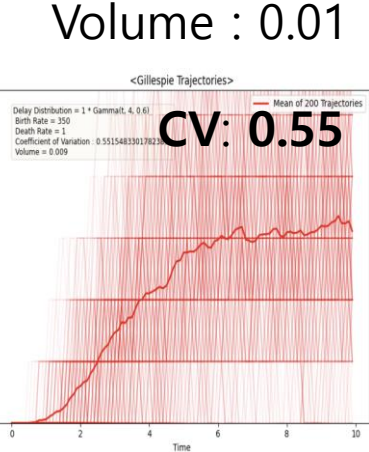
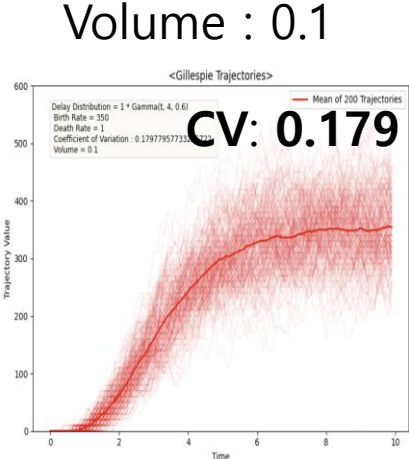
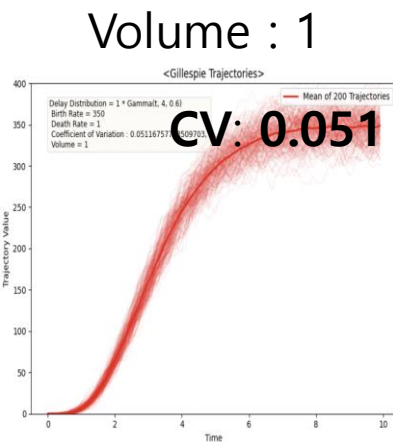
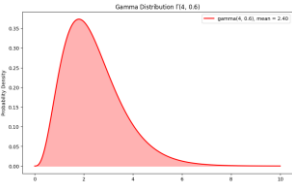
$$\lambda_b/\lambda_d = 190$$



$$\lambda_b/\lambda_d = 190$$

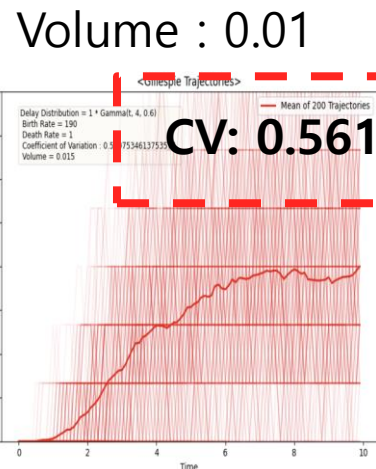
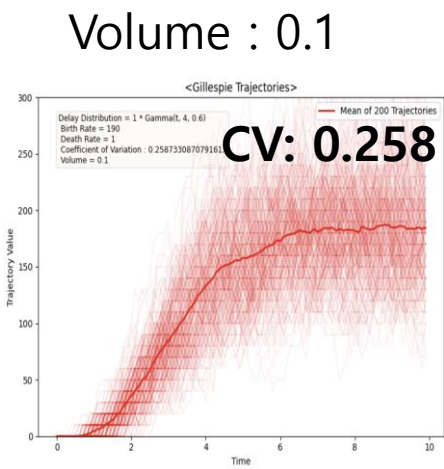
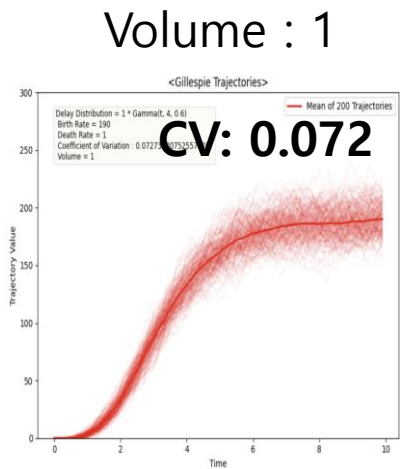


$$\lambda_b/\lambda_d = 350$$

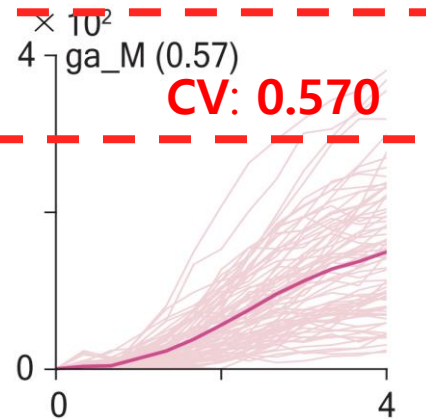


Lowering 'volume' in Gillespie SSA for **single timescale pathways** raises heterogeneity and produces closer CV estimates to real data, across different birth/death ratios.

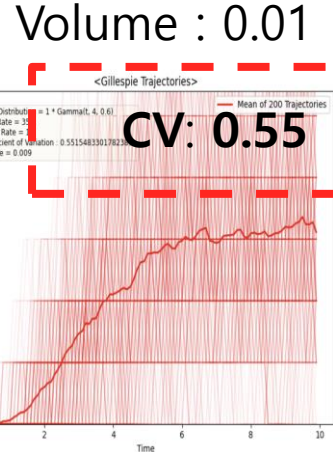
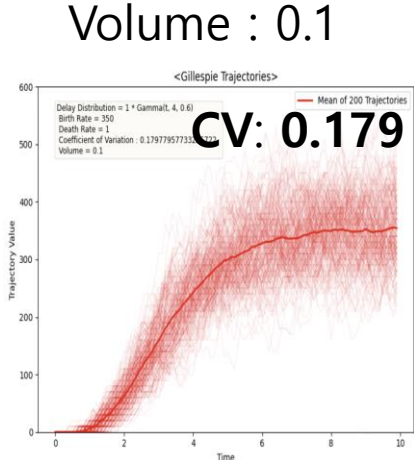
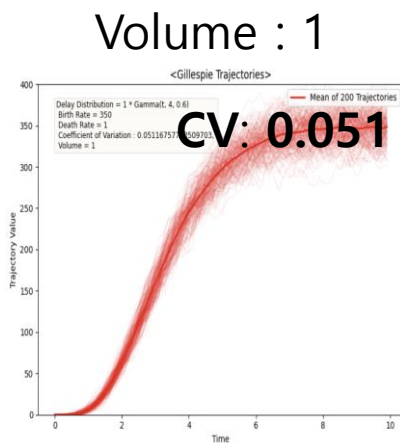
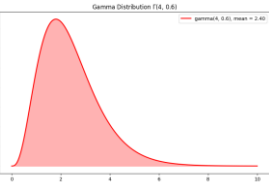
$\lambda_b/\lambda_d = 190$



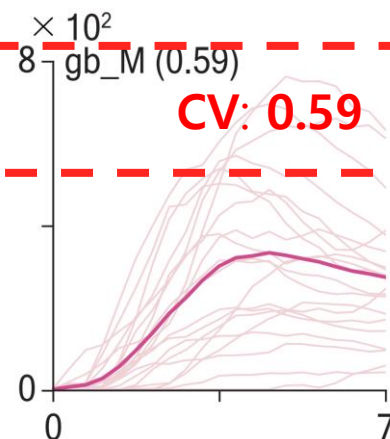
$\lambda_b/\lambda_d = 190$



$\lambda_b/\lambda_d = 350$

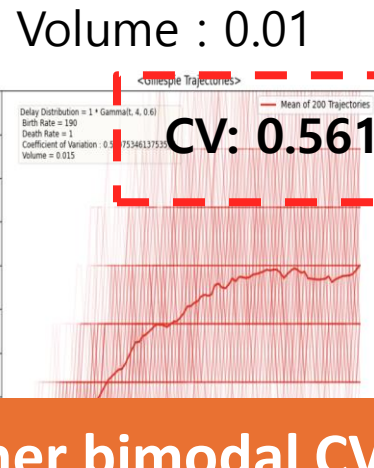
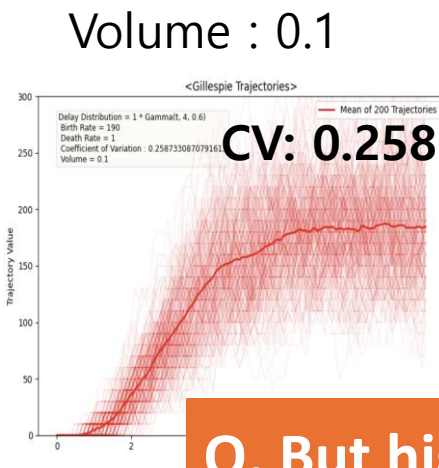
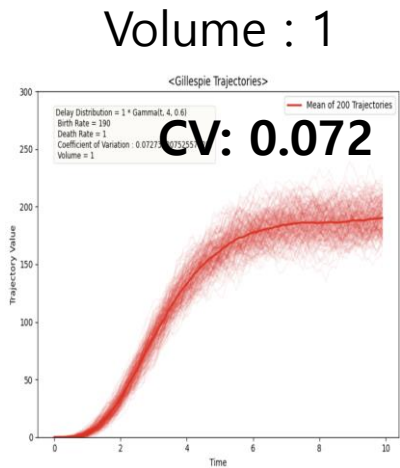
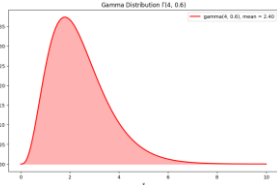


$\lambda_b/\lambda_d = 350$

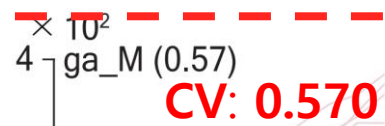


Lowering 'volume' in Gillespie SSA for **single timescale pathways** raises heterogeneity and produces closer CV estimates to real data, across different birth/death ratios.

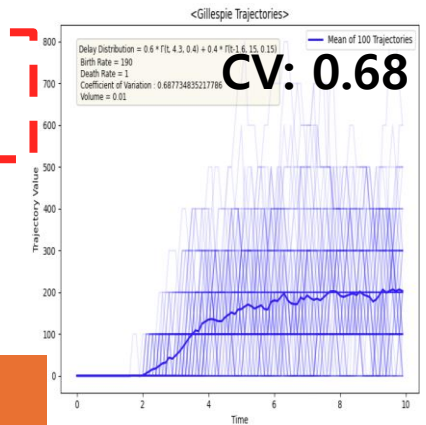
$\lambda_b/\lambda_d = 190$



$\lambda_b/\lambda_d = 190$

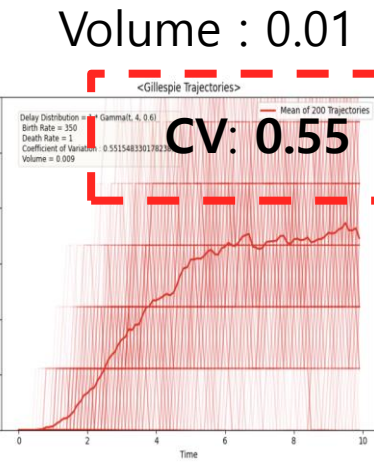
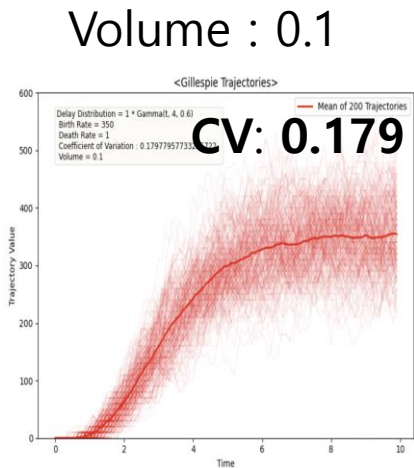
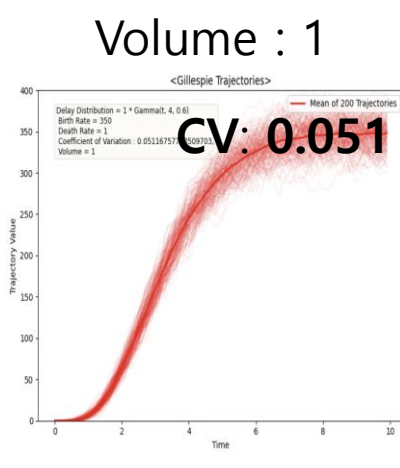
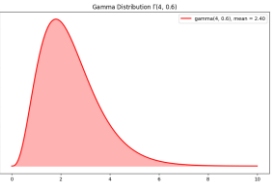


Volume : 0.01

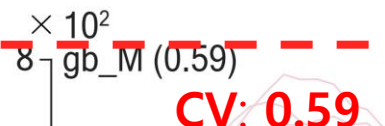


Q. But higher bimodal CV too...

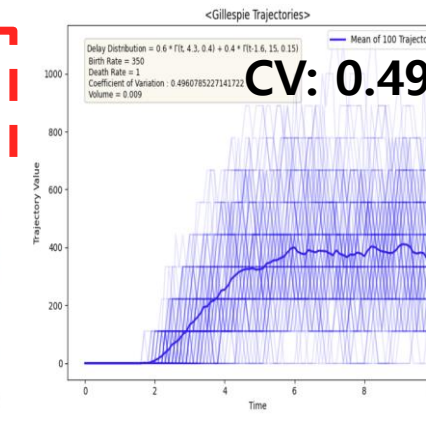
$\lambda_b/\lambda_d = 350$



$\lambda_b/\lambda_d = 350$

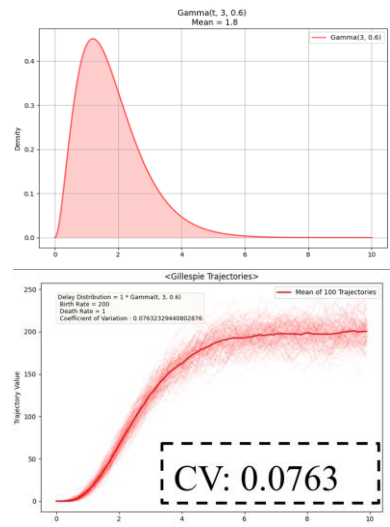


Volume : 0.01



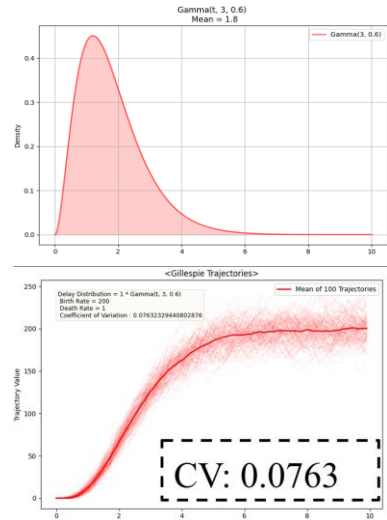
Changing the delay distribution type does not significantly impact heterogeneity.

Gamma

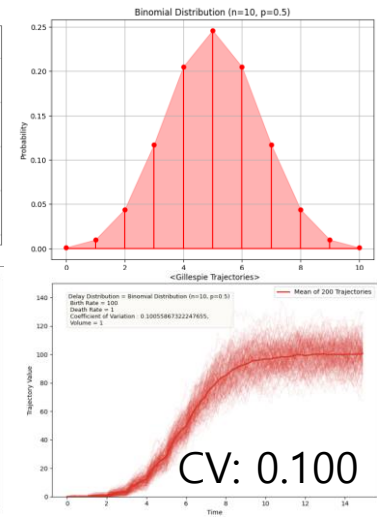


Changing the delay distribution type does not significantly impact heterogeneity.

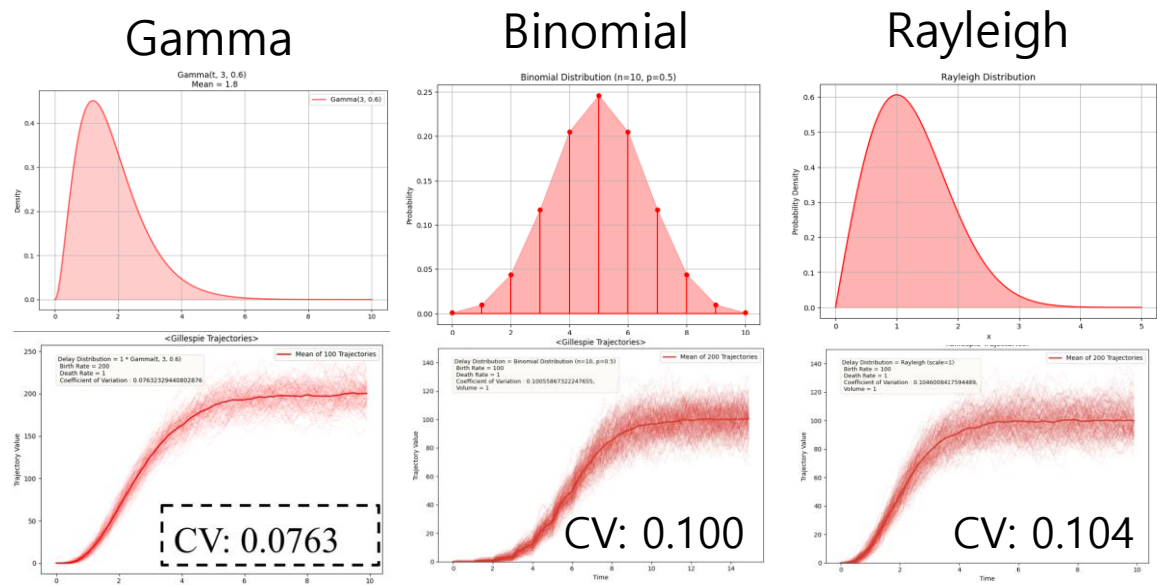
Gamma



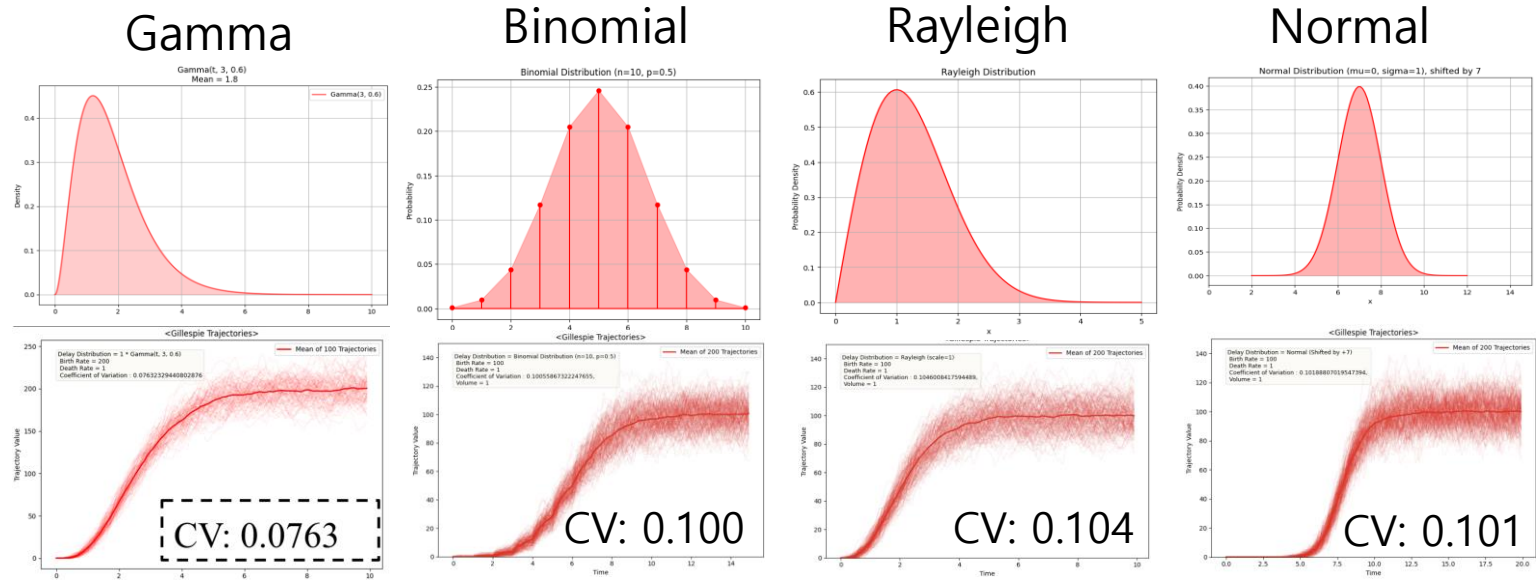
Binomial



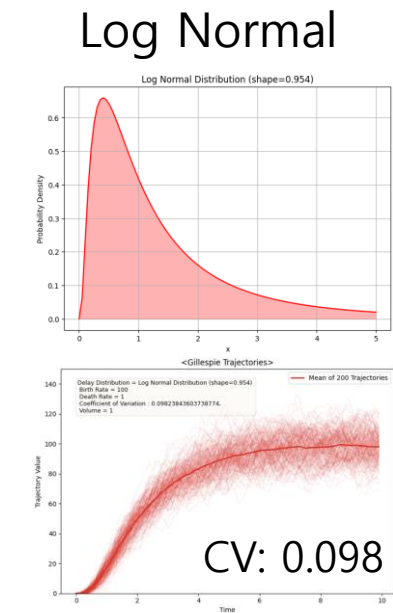
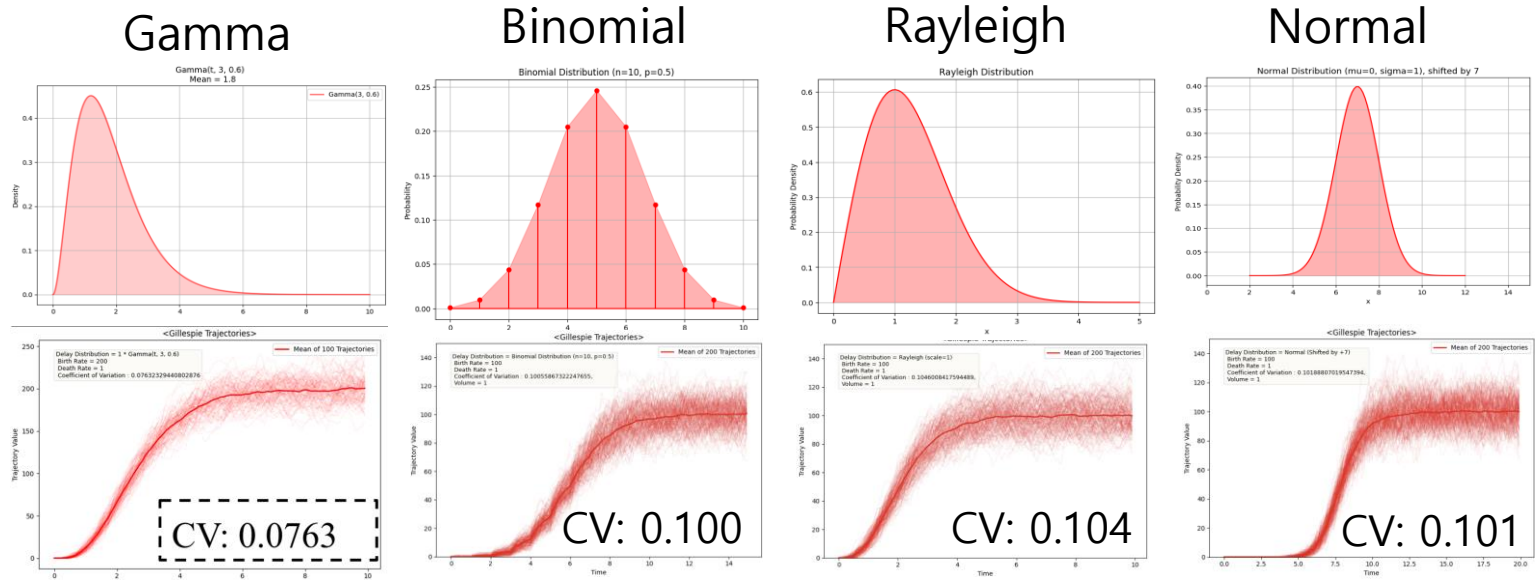
Changing the delay distribution type does not significantly impact heterogeneity.



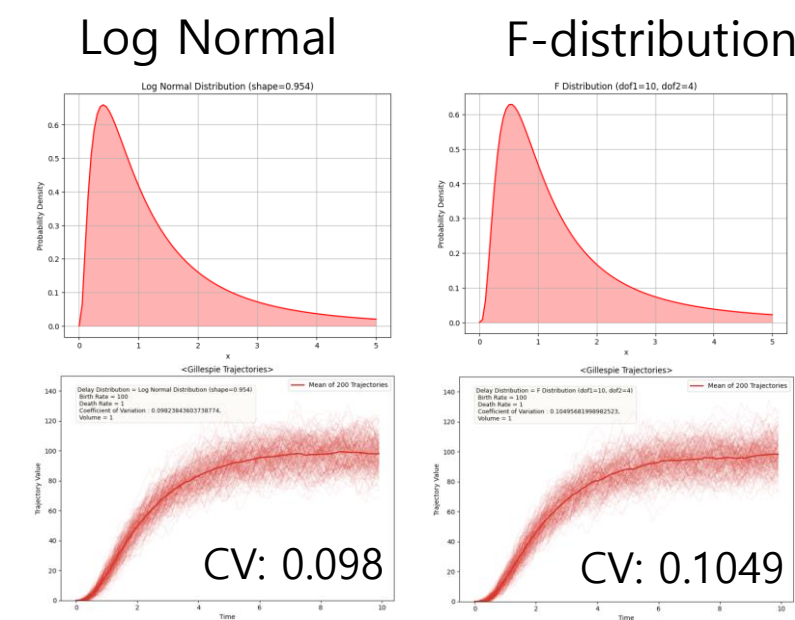
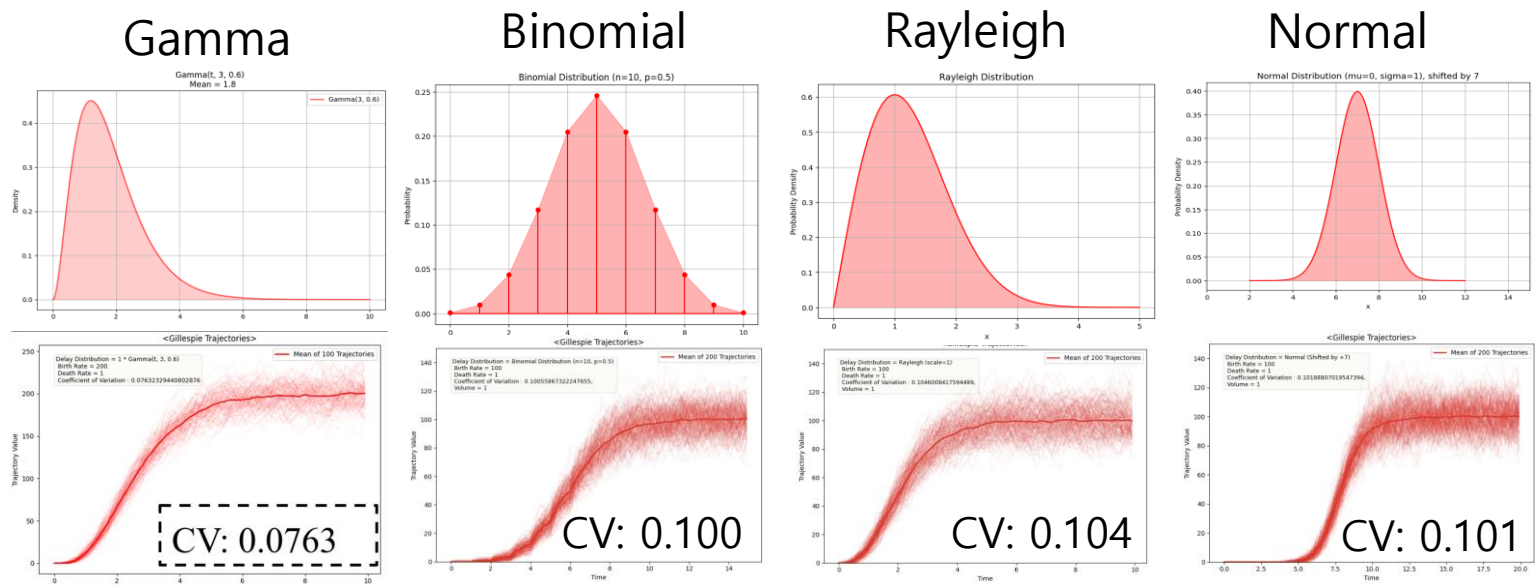
Changing the delay distribution type does not significantly impact heterogeneity.



Changing the delay distribution type does not significantly impact heterogeneity.

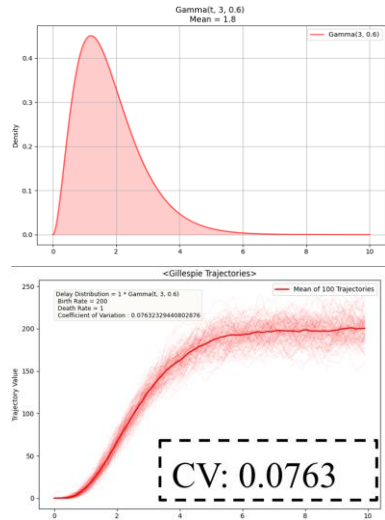


Changing the delay distribution type does not significantly impact heterogeneity.

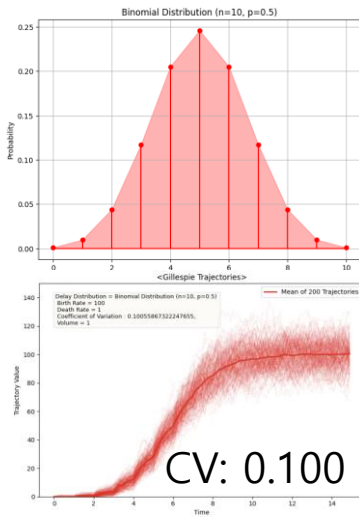


Changing the delay distribution type does not significantly impact heterogeneity.

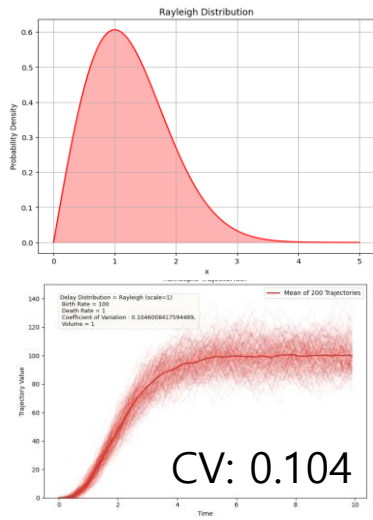
Gamma



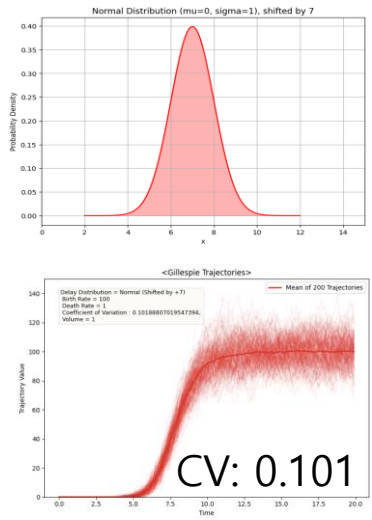
Binomial



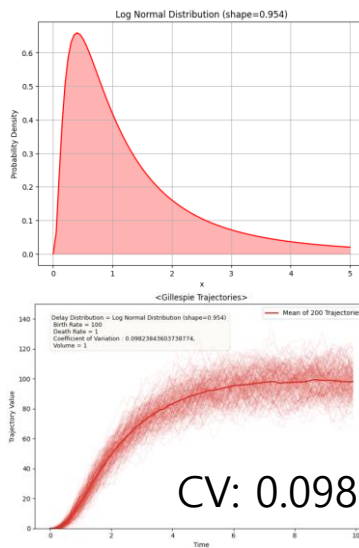
Rayleigh



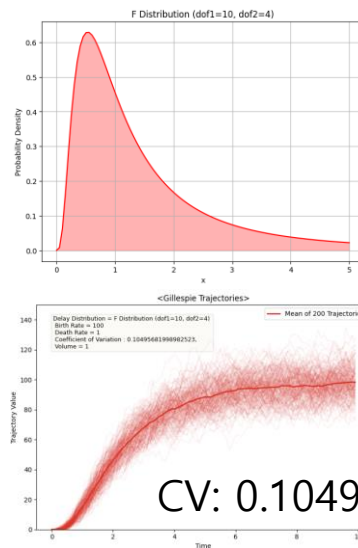
Normal



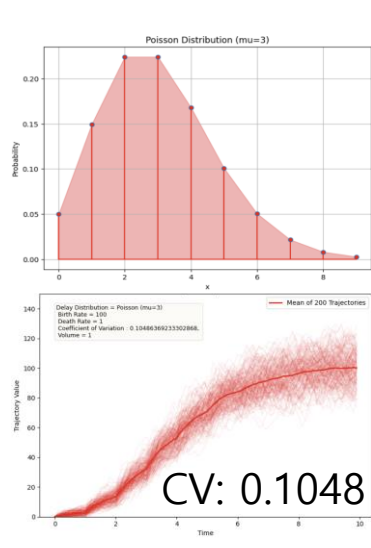
Log Normal



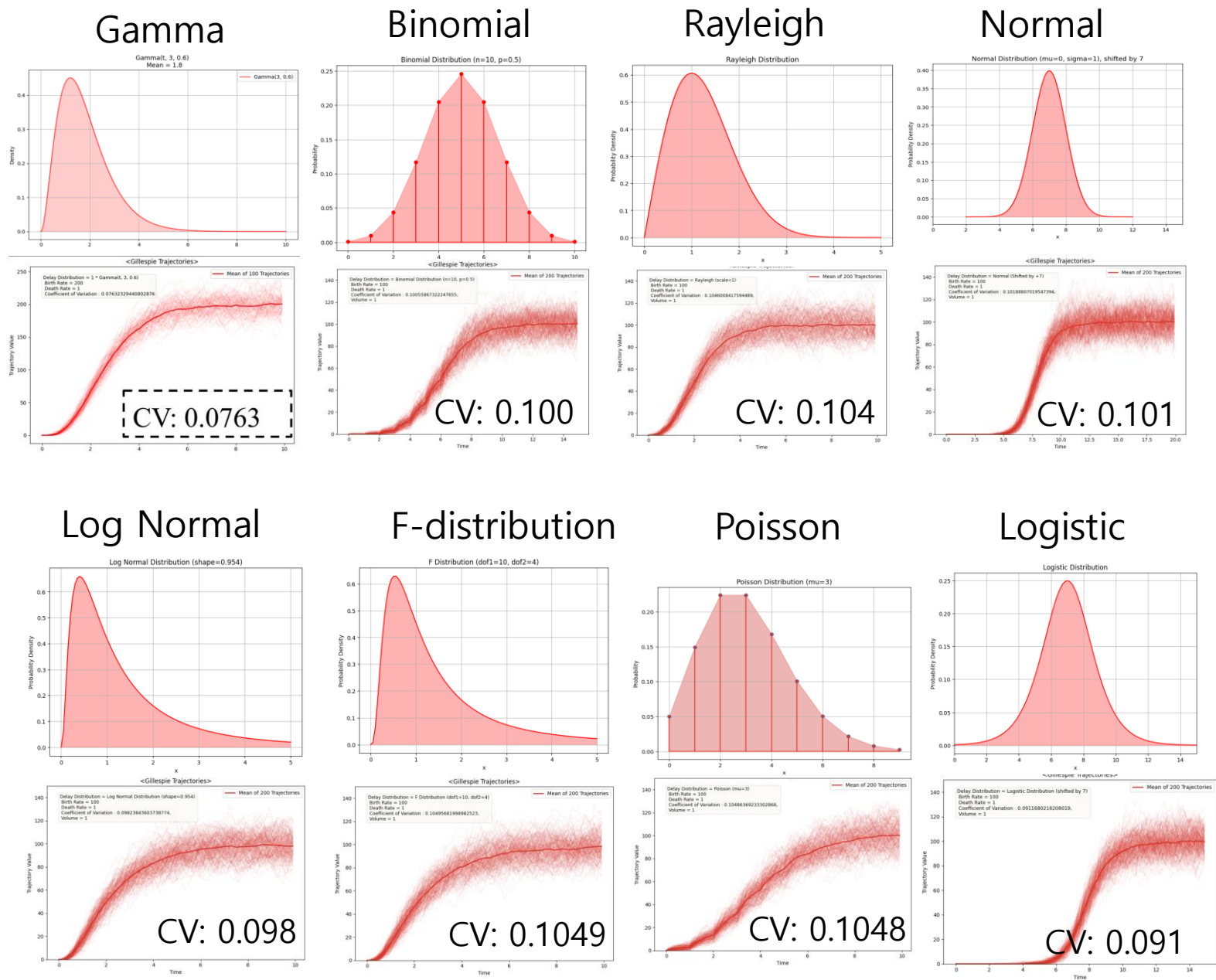
F-distribution



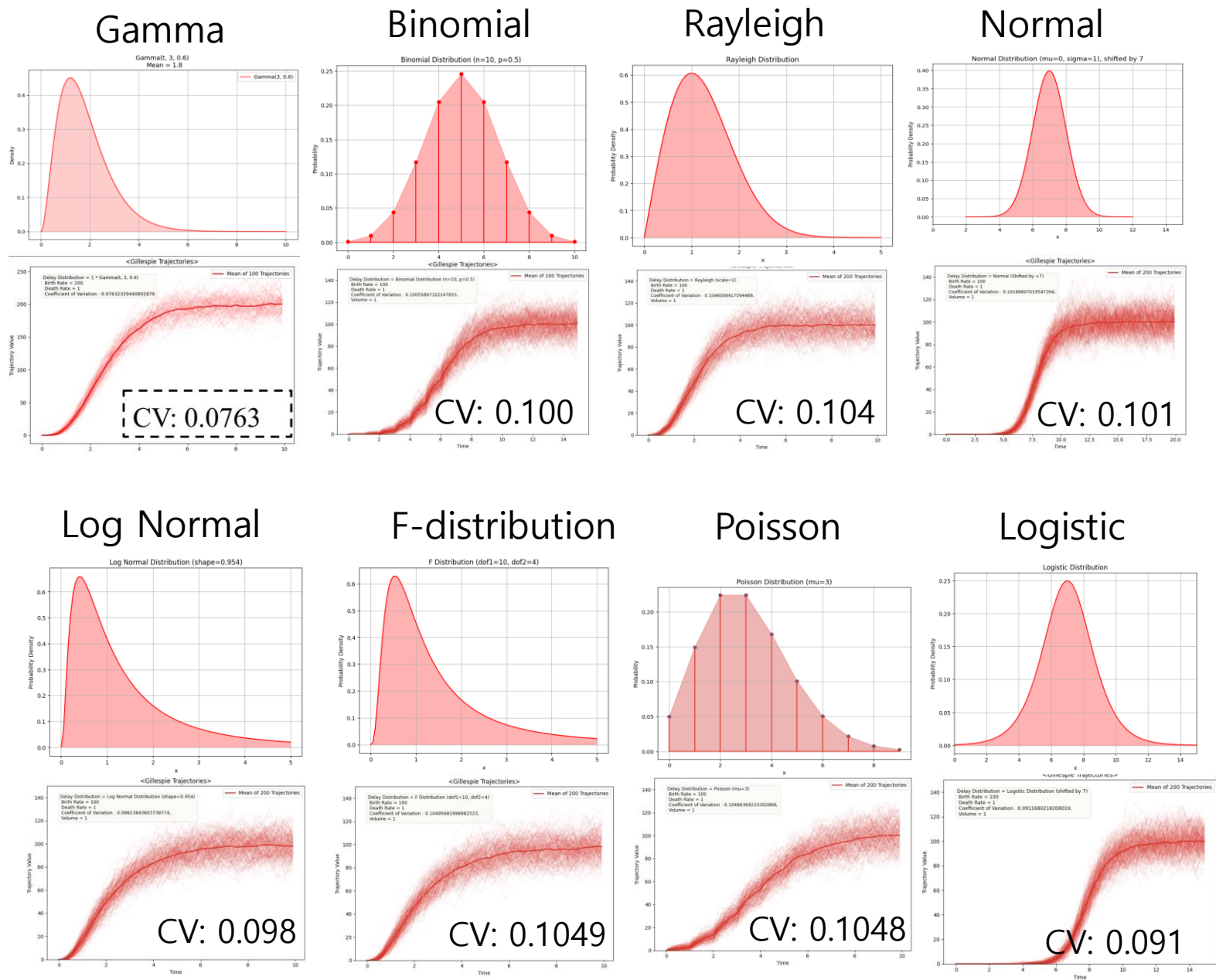
Poisson



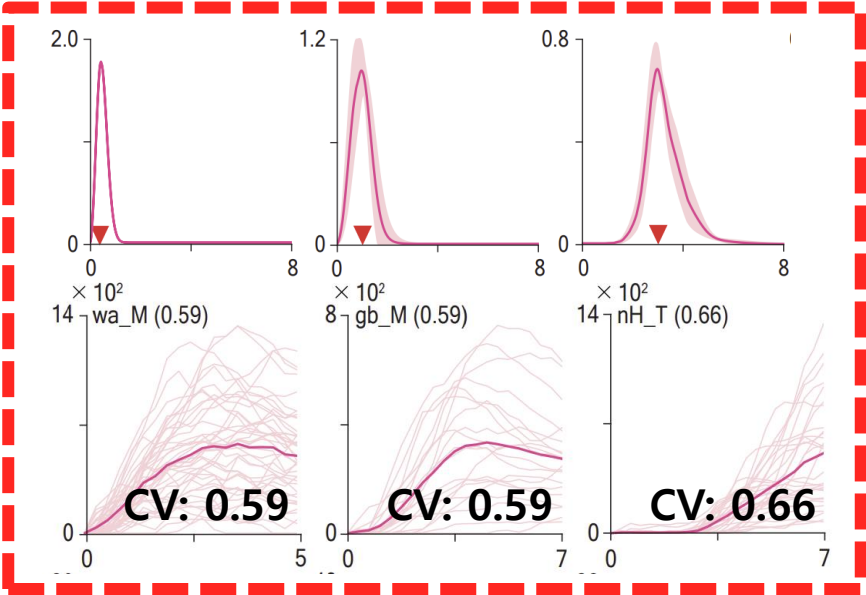
Changing the delay distribution type does not significantly impact heterogeneity.



Changing the delay distribution type does not significantly impact heterogeneity.

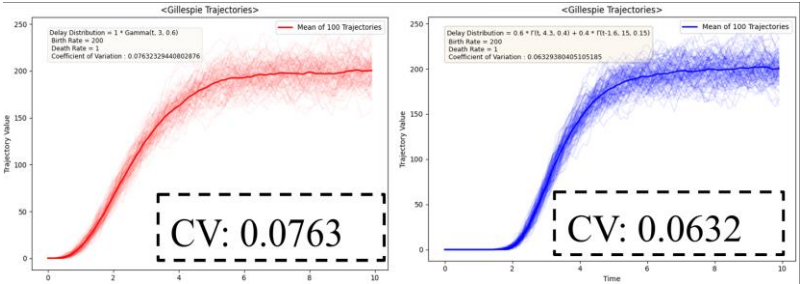


How to generate high CV values??

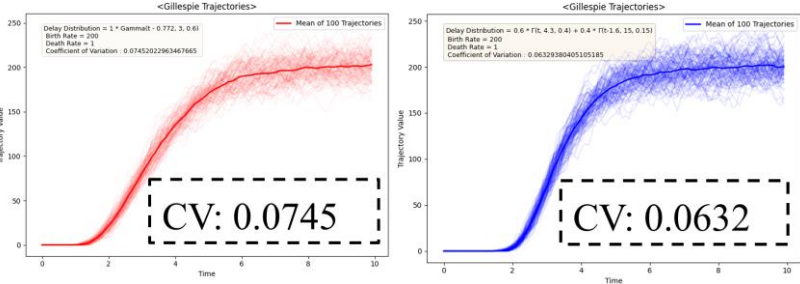


Summary: Modifying delay distribution (mean, variance, type) / volume / birth-death ratio do not work.

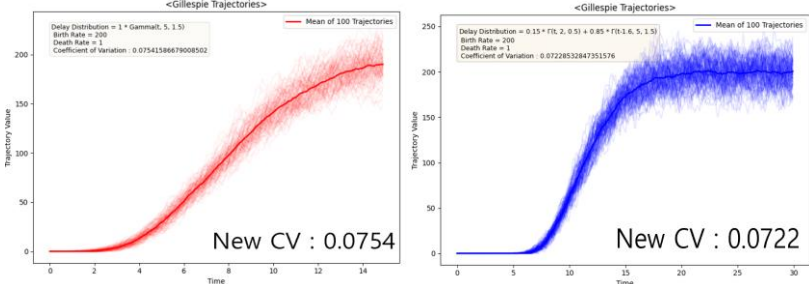
Original [Paper]



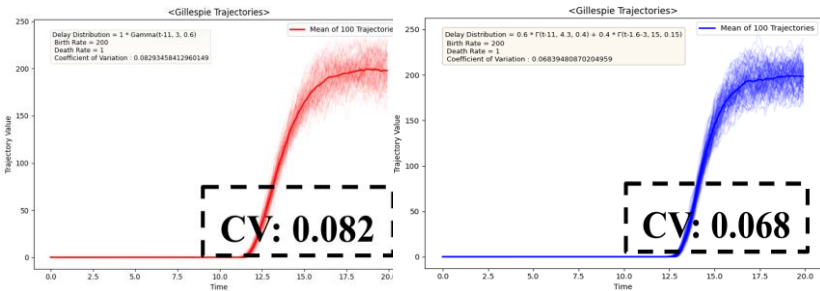
Same Mean



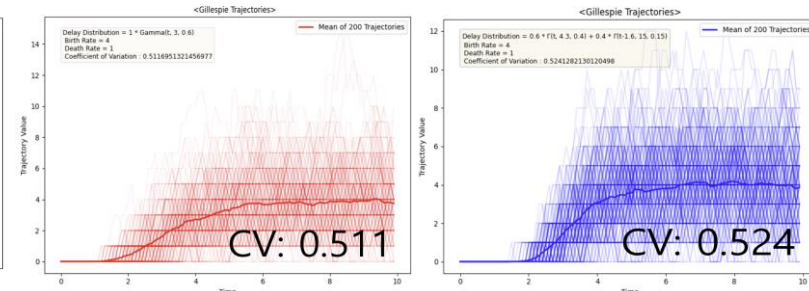
Increased Variance



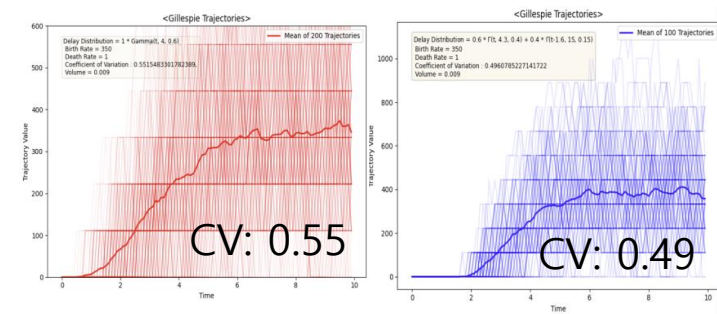
Horizontal Shift



Lower Birth/Death Ratio (4:1)

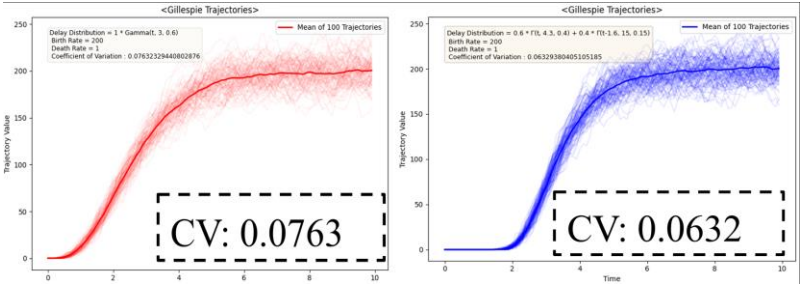


Lower Volume (1 → 0.009)



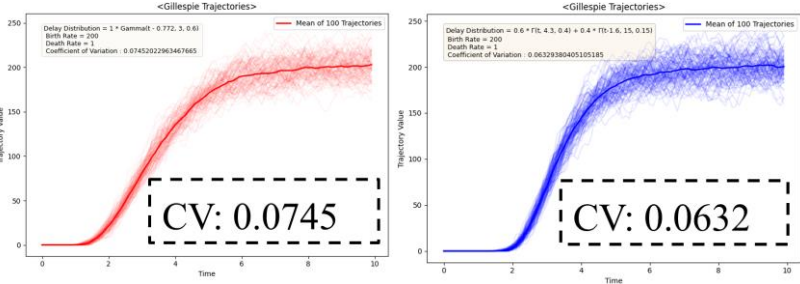
Summary: Modifying delay distribution (mean, variance) / volume / birth-death ratio do not work.

Original [Paper]

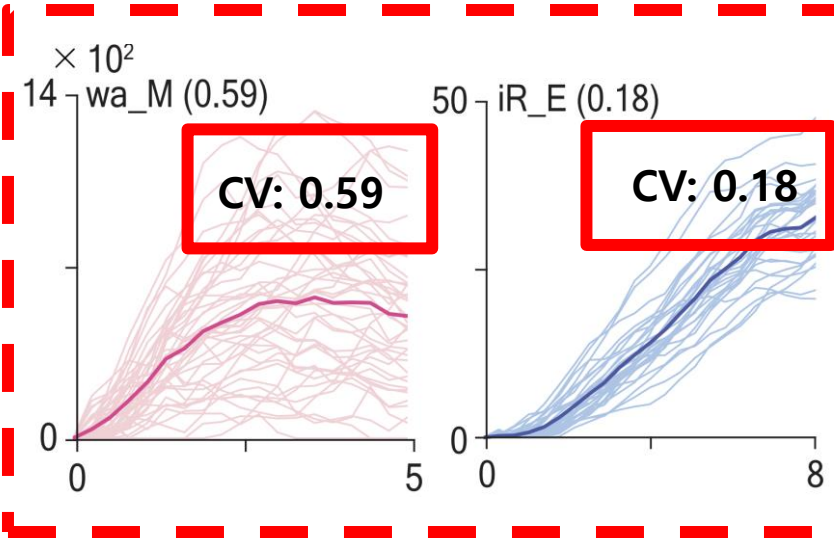
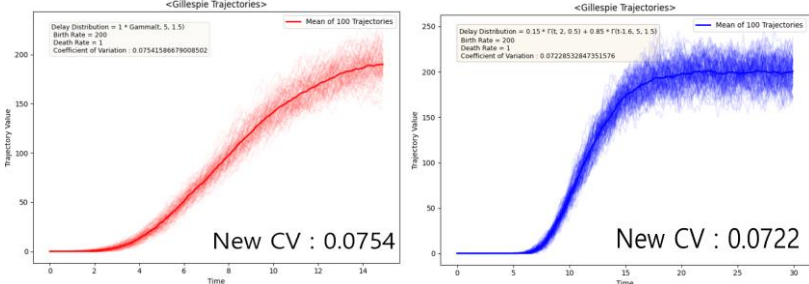


Q. How to get Unimodal CV >> Bimodal CV??

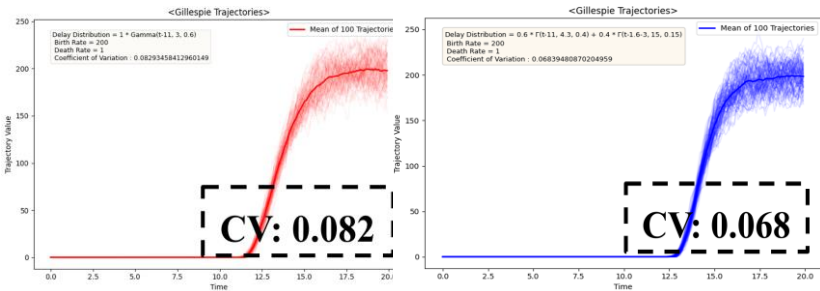
Same Mean



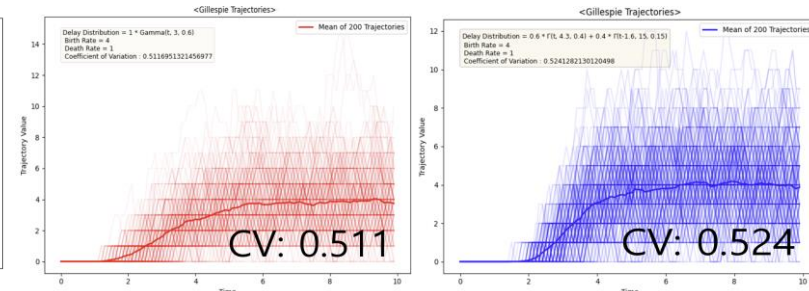
Increased Variance



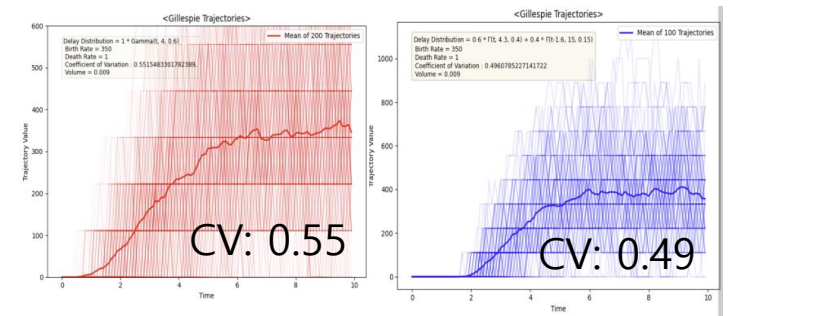
Horizontal Shift



Lower Birth/Death Ratio (4:1)

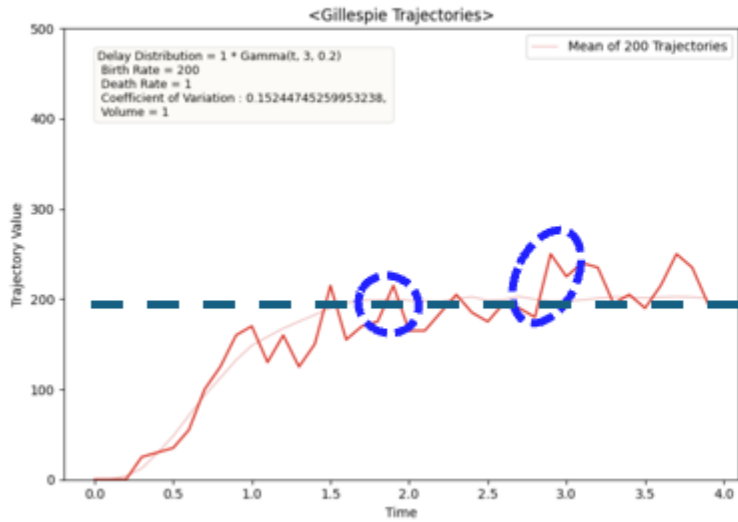


Lower Volume (1 → 0.009)

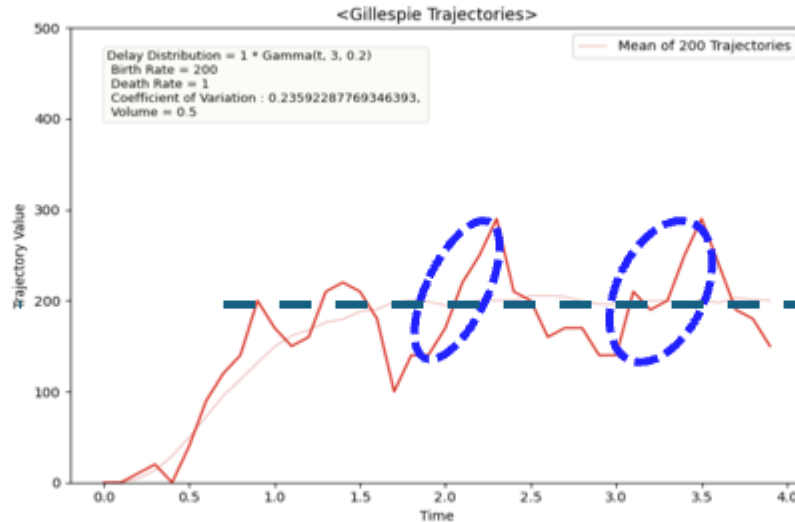


Future Directions : Identify why lowering volume increases CV, find how low volume is related to single timescale pathways.

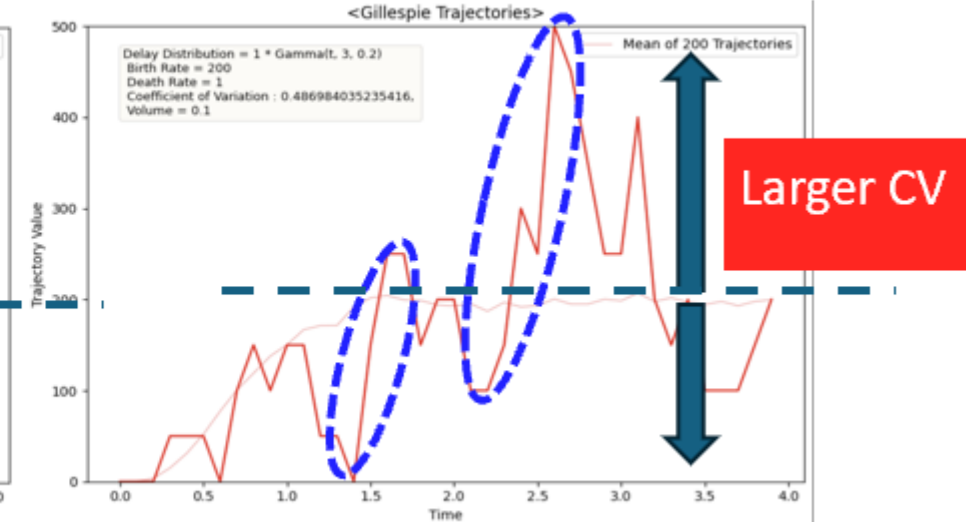
Volume : 1



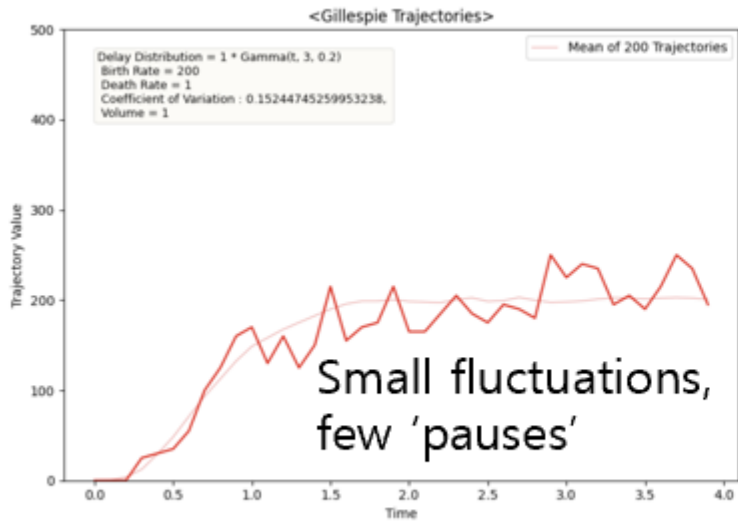
Volume : 0.5



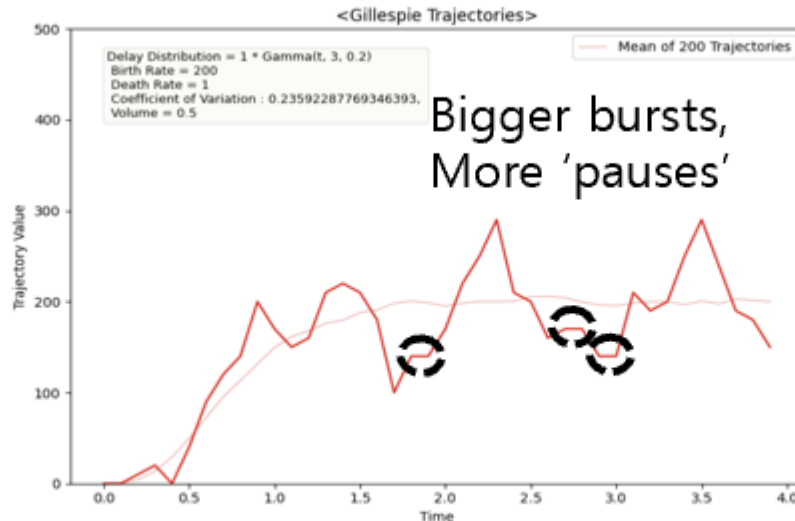
Volume : 0.1



Volume : 1



Volume : 0.5



Volume : 0.1

