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Chapter 1

Introduction

This dissertation deals with the development of predictive controllers within the framework of probabilistic graphical models, specifically dynamic Bayesian networks. Dynamic Bayesian networks are well suited to the study of stochastic processes i.e. processes where there is noise in both the state evolution and state measurements. By leveraging the natural formulation of stochastic processes within dynamic Bayesian networks we develop stochastic predictive controllers, focussing specifically on linear quadratic Gaussian (LQG) and chance constrained model predictive control (MPC).

The dissertation primarily deals with the two graphical models¹ shown in Figures 1.1 and 1.2. Inference is discussed in general, but we focus on filtering and prediction because they are important for control.

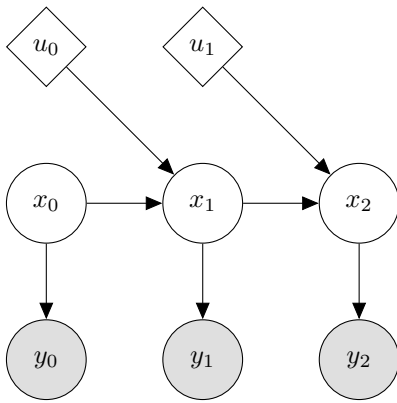


Figure 1.1: Single model probabilistic graphical model.

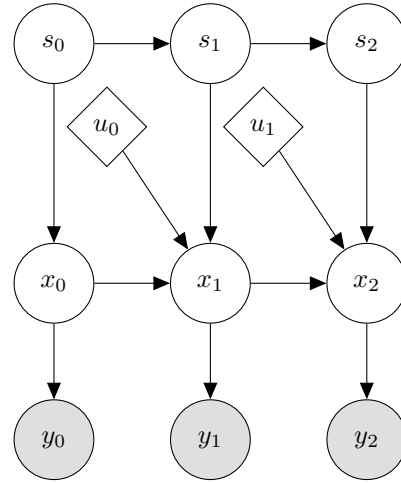


Figure 1.2: Model switching probabilistic graphical model.

The dissertation is structured in 3 parts, each composed of self contained chapters dealing

¹Clear circular nodes represent latent variables, shaded circular nodes represent observed variables and diamond nodes are deterministic variables.

with a specific problem area. Part I contains Chapters 2 to 5. The literature review, Chapter 2, deals with current papers on topics related to this dissertation. Chapter 3 mainly deals with background theory found in reference materials (e.g. books). Chapter 4 deals with hidden Markov models. The goal of this chapter is to gently introduce the uninformed reader to the power of graphical models. Finally, Chapter 5 introduces the CSTR example which is used to illustrate the techniques investigated throughout the rest of the dissertation. If the reader is familiar with graphical models, predictive control and CSTR design Part I may be safely skipped.

Parts II and III each follow the same pattern: a graphical model is introduced and studied after which a control scheme is implemented using the tenets of the preceding work. We detail the content and results of these two parts next.

In Part II the dynamic Bayesian network, shown in Figure 1.1, is investigated within the context of the Kalman filter model (linear dynamics and Gaussian noise) and the particle filter model (no assumptions about the dynamics and noise). Using the techniques endemic to the aforementioned probabilistic graphical models we show:

1. That the LQG controller reduces to the linear quadratic regulator under the assumptions of normality and linearity ².
2. That a chance constrained MPC problem can be reduced to the standard form MPC problem (a deterministic optimisation problem with linear constraints and a quadratic objective function) under the assumptions of linearity and normality. Furthermore, since the Mahalanobis distance, a statistically important measure, is used to reduce the chance constraints to linear constraints it supports the application of the aforementioned techniques to systems which are nonlinear and non-Gaussian.

In Part III the switching model filter, based on the dynamic Bayesian network shown in Figure 1.2, is investigated³. The benefit of generalising Figure 1.1 is that it allows one to infer which model, from a set of possible models, is likely to be generating the observations. This allows us to extend the stochastic MPC discussed previously to incorporate model switching. We investigate the following:

1. Using the Rao-Blackwellised particle filter as the switching model filter. In this context the resultant most likely linear model is used to move the underlying system to different regions in state space. It was found that the approach caused controller instability because the current most likely model is often not accurate enough to steer the system to the target.
2. Using a switching particle filter as the switching model filter. In this context the filter

²This result is not new but the derivation using probabilistic graphical models is both instructive and, more importantly, intuitive.

³This probabilistic graphical model uses a set of models to perform inference. The stochastic switching variables (s_0, s_1, \dots) are used to weight the likelihood of each model supporting the observations.

was used to identify when a process fault occurred and, based on this event, switch the model control is based upon. It was found that the algorithm successfully stabilised and regulated the nonlinear underlying system.

Perhaps most usefully, the dissertation illustrates the advantage of designing model predictive controllers from within the framework of probabilistic graphical models. While it may seem that the two fields are not related, most modern control systems perform filtering on system measurements which is a natural result of the application of probabilistic graphical models. Therefore, the motivation for this study is not purely esoteric but demonstrates a tacit relationship between the fields.

Part I

Literature, theory and background material

Part II

Single model systems

Part III

Multiple model systems

Bibliography

- [1] Y. Bar-Shalom, X.R. Li, and T. Kirubarajan. *Estimation with applications to tracking and navigation*. John Wiley and Sons, 2001.
- [2] D. Barber. Expectation correction for smoothed inference in switching linear dynamical systems. *Journal of Machine Learning*, 7:2515–2540, 2006.
- [3] D. Barber. *Bayesian Reasoning and Machine Learning*. Cambridge University Press, 2012.
- [4] I. Batina, A.A. Stoorvogel, and S. Weiland. Optimal control of linear, stochastic systems with state and input constraints. In *Proceedings of the 41st IEEE Conference on Decision and Control*, 2002.
- [5] A. Bemporad and M. Morari. Control of systems integrating logic, dynamics, and constraints. *Automatics*, 35:407–427, 1999.
- [6] C.M. Bishop. *Pattern Recognition and Machine Learning*. Springer, 2006.
- [7] L. Blackmore, Hui Li, and B. Williams. A probabilistic approach to optimal robust path planning with obstacles. In *American Control Conference*, June 2006.
- [8] L. Blackmore, O. Masahiro, A. Bektassov, and B.C. Williams. A probabilistic particle-control approximation of chance-constrained stochastic predictive control. *IEEE Transactions on Robotics*, 26, 2010.
- [9] M. Cannon, B. Kouvaritakis, and X. Wu. Probabilistic constrained mpc for multiplicative and additive stochastic uncertainty. *IEEE Transactions on Automatic Control*, 54(7), 2009.
- [10] A.L. Cervantes, O.E. Agamennoni, and J.L. Figueroa. A nonlinear model predictive control system based on weiner piecewise linear models. *Journal of Process Control*, 13:655–666, 2003.
- [11] R. Chen and J.S. Liu. Mixture kalman filters. *Journal of Royal Statistical Society*, 62(3):493–508, 2000.

- [12] J.J. Dabrowski and J.P. de Villiers. A method for classification and context based behavioural modelling of dynamical systems applied to maritime piracy. *Expert Systems with Applications*, 2014.
- [13] B.N. Datta. *Numerical Methods for Linear Control Systems - Design and Analysis*. Elsevier, 2004.
- [14] F. Daum and J. Huang. Particle flow for nonlinear filters. In *Acoustics, Speech and Signal Processing (ICASSP), 2011 IEEE International Conference on*, pages 5920–5923, May 2011.
- [15] M. Davidian. Applied longitudinal data analysis. North Carolina State University, 2005.
- [16] J.P. de Villiers, S.J. Godsill, and S.S. Singh. Particle predictive control. *Journal of Statistical Planning and Inference*, 141:1753–1763, 2001.
- [17] N. Deo. *Graph Theory with Applications to Engineering and Computer Science*. Prentice-Hall, 1974.
- [18] M. Diehl, H.J. Ferreau, and N. Haverbeke. Efficient numerical methods for nonlinear mpc and moving horizon estimation. *Control and Information Sciences*, 384:391–417, 2009.
- [19] A. Doucet and A.M. Johansen. A tutorial on particle filtering and smoothing: fifteen years later. Technical report, The Institute of Statistical Mathematics, 2008.
- [20] A.D. Doucet, N.J. Gordon, and V. Krishnamurthy. Particle filters for state estimation of jump markov linear systems. *IEEE Transactions on Signal Processing*, 49(3):613–624, March 2001.
- [21] J. Du, C. Song, and P. Li. Modeling and control of a continuous stirred tank reactor based on a mixed logical dynamical model. *Chinese Journal of Chemical Engineering*, 15(4):533–538, 2007.
- [22] The Economist. In praise of bayes. Article in Magazine, September 2000.
- [23] C. Edwards, S.K. Spurgeon, and R.J. Patton. Sliding mode observers for fault detection and isolation. *Automatica*, 36:541–553, 200.
- [24] H.C. Edwards and D.E. Penny. *Elementary Differential Equations*. Pearson, 6th edition edition, 2009.
- [25] W. Forst and D. Hoffmann. *Optimisation - Theory and Practice*. Springer, 2010.
- [26] O.R. Gonzalez and A.G. Kelkar. *Electrical Engineering Handbook*. Academic Press, 2005.
- [27] N.J. Gordon, D.J. Salmond, and A.F.M. Smith. Novel approach to nonlinear/non-gaussian bayesian state estimation. *IEE Proceedings-F*, 140(2):107–113, 1993.

- [28] R. Isermann and P. Balle. Trends in the application of model based fault detection and diagnosis of technical processes. *Control Engineering Practice*, 5(5):709–719, 1997.
- [29] K. Ito and K. Xiong. Gaussian filters for nonlinear filtering problems. *IEEE Transactions on Automatic Control*, 45(5):910–928, 2000.
- [30] R. J. Jang and C.T. Sun. *Neuro-fuzzy and soft computing: a computational approach to learning and machine intelligence*. Prentice-Hall, 1996.
- [31] D. Koller and N. Friedman. *Probabilistic Graphical Models*. MIT Press, 2009.
- [32] K. B. Korb and A. E. Nicholson. *Bayesian Artificial Intelligence*. Series in Computer Science and Data Analysis. Chapman & Hall, first edition edition, 2004.
- [33] M. Kvasnica, M. Herceg, L. Cirka, and M. Fikar. Model predictive control of a cstr: a hybrid modeling approach. *Chemical Papers*, 64(3):301–309, 2010.
- [34] J.H. Lee, M. Morari, and C.E. Garcia. *Model Predictive Control*. Prentice Hall, 2004.
- [35] U.N. Lerner. *Hybrid Bayesian Networks for Reasoning about Complex Systems*. PhD thesis, Stanford Univesity, 2002.
- [36] P. Li, M. Wendt, H. Arellano-Garcia, and G. Wozny. Optimal operation of distrillation processes under uncertain inflows accumulated in a feed tank. *American Institute of Chemical Engineers*, 2002.
- [37] P. Li, M. Wendt, and G. Wozny. A probabilistically constrained model predictive controller. *Automatica*, 38:1171–1176, 2002.
- [38] W.L. Luyben. *Process Modeling, Simulation and Control for Chemical Engineers*. McGraw-Hill, 2nd edition edition, 1990.
- [39] J.M. Maciejowski. *Predictive Control with constraints*. Prentice-Hall, 2002.
- [40] O. Masahiro. Joint chance-constrained model predictive control with probabilistic resolvability. *American Control Conference*, 2012.
- [41] P. Mhaskar, N.H. El-Farra, and P.D. Christofides. Stabilization of nonlinear systems with state and control constraints using lyapunov-based predictive control. *Systems and Control Letters*, 55:650–659, 2006.
- [42] K.P. Murphy. Switching kalman filters. Technical report, Compaq Cambridge Research Lab, 1998.
- [43] K.P. Murphy. *Dynamic Bayesian Networks: Representation, Inference and Learning*. PhD thesis, University of California, Berkeley, 2002.
- [44] K.P. Murphy. *Machine Learning: A Probabilistic Perspective*. MIT Press, 2012.

- [45] N. Nandola and S. Bhartiya. A multiple model approach for predictive control of non-linear hybrid systems. *Journal of Process Control*, 18(2):131–148, 2008.
- [46] L. Ozkan, M. V. Kothare, and C. Georgakis. Model predictive control of nonlinear systems using piecewise linear models. *Computers and Chemical Engineering*, 24:793–799, 2000.
- [47] T. Pan, S. Li, and W.J. Cai. Lazy learning based online identification and adaptive pid control: a case study for cstr process. *Industrial Engineering Chemical Research*, 46:472–480, 2007.
- [48] J.B. Rawlings and D.Q. Mayne. *Model Predictive Control*. Nob Hill Publishing, 2009.
- [49] B. Reiser. Confidence intervals for the mahalanobis distance. *Communications in Statistics: Simulation and Computation*, 30(1):37–45, 2001.
- [50] Y. Sakakura, M. Noda, H. Nishitani, Y. Yamashita, M. Yoshida, and S. Matsumoto. Application of a hybrid control approach to highly nonlinear chemical processes. *Computer Aided Chemical Engineering*, 21:1515–1520, 2006.
- [51] A.T. Schwarm and Nikolaou. Chance constrained model predictive control. Technical report, University of Houston and Texas A&M University, 1999.
- [52] C. Snyder, T. Bengtsson, P. Bickel, and J. Anderson. Obstacles to high-dimensional particle filtering. *Mathematical Advances in Data Assimilation*, 2008.
- [53] S.J. Streicher, S.E. Wilken, and C. Sandrock. Eigenvector analysis for the ranking of control loop importance. *Computer Aided Chemical Engineering*, 33:835–840, 2014.
- [54] D.H. van Hessem and O.H. Bosgra. Closed-loop stochastic dynamic process optimisation under input and state constraints. In *Proceedings of the American Control Conference*, 2002.
- [55] D.H. van Hessem, C.W. Scherer, and O.H. Bosgra. Lmi-based closed-loop economic optimisation of stochastic process operation under state and input constraints. In *Proceedings of the 40th IEEE Conference on Decision and Control*, 2001.
- [56] H. Veeraraghavan, P. Schrater, and N. Papanikolopoulos. Switching kalman filter based approach for tracking and event detection at traffic intersections. *Intelligent Control*, 2005.
- [57] D. Wang, W. Wang, and P. Shi. Robust fault detection for switched linear systems with state delays. *Systems, Man and Cybernetics*, 39(3):800–805, 2009.
- [58] R.S. Wills. Google’s pagerank: the math behind the search engine. Technical report, North Carolina State University, 2006.

- [59] J. Yan and R.R. Bitmead. Model predictive control and state estimation: a network example. In *15th Triennial World Conference of IFAC*, 2002.
- [60] J. Yan and R.R. Bitmead. Incorporating state estimation into model predictive control and its application to network traffic control. *Automatica*, 41:595–604, 2005.
- [61] M.B. Yazdi and M.R. Jahed-Motlagh. Stabilization of a cstr with two arbitrarily switching modes using model state feedback linearisation. *Chemical Engineering Journal*, 155(3):838–843, 2009.