



# National Numeracy Learning Progression

(adapted for NSW syllabuses – May 2018)

# Table of contents

<b>What is the focus of the numeracy progression? .....</b>	<b>3</b>
<b>How is the numeracy progression structured? .....</b>	<b>3</b>
Elements and sub-elements .....	3
Levels and indicators.....	4
<b>How is the numeracy progression related to the NSW syllabuses? .....</b>	<b>5</b>
NSW Mathematics K-10 Syllabus.....	5
Other learning areas.....	6
<b>How can the numeracy progression be used? .....</b>	<b>6</b>
<b>Number sense and algebra.....</b>	<b>7</b>
Quantifying numbers .....	7
Additive strategies .....	13
Multiplicative strategies .....	15
Operating with decimals.....	17
Operating with percentages .....	19
Understanding money.....	21
Number patterns and algebraic thinking.....	23
Comparing units (ratios, rates and proportion).....	26
Interpreting fractions.....	28
<b>Measurement and geometry .....</b>	<b>30</b>
Understanding units of measurement .....	30
Understanding geometric properties .....	33
Positioning and locating .....	35
Measuring time.....	36
<b>Statistics and probability .....</b>	<b>38</b>
Understanding chance .....	38
Interpreting and representing data .....	40

# What is the focus of the numeracy progression?

Numeracy development influences student success in many areas of learning. The National Numeracy Learning Progression can be used to support students to successfully engage with the numeracy demands of the Kindergarten to Year 10 NSW syllabuses.

The progression outlines a sequence of observable indicators of increasingly sophisticated understanding of and skills in key numeracy concepts. By providing a comprehensive view of numeracy learning and how it develops over time, the progression gives teachers a conceptual tool that can assist them to develop targeted teaching and learning programs for students who are working above or below year-level expectations.

The progression does not advise on how to teach, plan, program, assess or report in schools. It recognises the importance of, but does not describe, the sequence for specific learning area content related to numeracy development such as graphing and constructing timelines.

The [Australian Core Skills Framework](#) has been used to guide decisions on the scope of the progressions. The progression is designed to assist students in reaching a level of proficiency in numeracy to at least Level 3 of the Core Skills Framework.

## How is the numeracy progression structured?

### Elements and sub-elements

The National Numeracy Learning Progression has three elements that reflect aspects of numeracy development necessary for successful learners of the *NSW Mathematics K–10 Syllabus* and in everyday life. The three elements are:

- Number sense and algebra
- Measurement and geometry
- Statistics and probability.

Each of the elements includes sub-elements that present developmental sequences for important aspects of numeracy capability. There are nine sub-elements in Number sense and algebra, four in Measurement and geometry and two in Statistics and probability.

The diagram (Figure 1) represents the elements and sub-elements in relation to the numeracy development of the student.

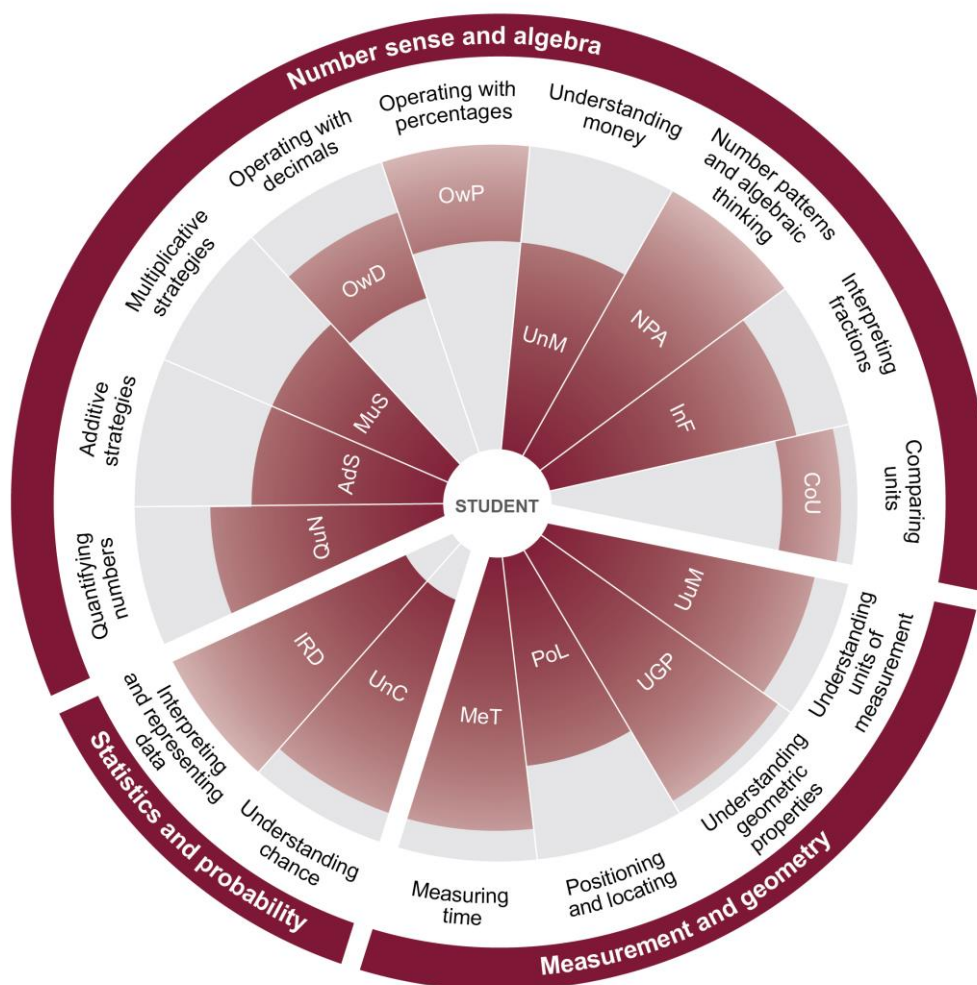


Figure 1. Elements and sub-elements of the National Numeracy Learning Progression

## Levels and indicators

Within each sub-element indicators are grouped together to form developmental levels. Each indicator describes what a student says, does or produces and begins with the implicit stem 'A student ...' as the subject of the sentence.

There are as many levels within each sub-element as can be supported by evidence. The listing of indicators within a level is non-hierarchical as the levels are collections of indicators. Each level within a sub-element has one or more indicators and is more sophisticated or complex than the preceding level.

In many of the sub-elements, subheadings have been included to assist teachers by grouping indicators into particular categories of skills that develop over a number of levels.

The amount of time it takes students to progress through each level is not specified since students progress in numeracy development at different rates.

The levels do not describe equal intervals of time in students' learning. They are designed to indicate the order in which students acquire the knowledge and skills necessary to be



numerate. As learning is very rapid in the early years of school, the initial levels tend to be more detailed than the later levels.

Moreover, the amount of detail in any level or sub-element is not an indication of importance. A single indicator at a higher level in the progression may rely on a substantial number of indicators being evident in earlier levels. The diagram (Figure 2) shows the various components included in the progression.

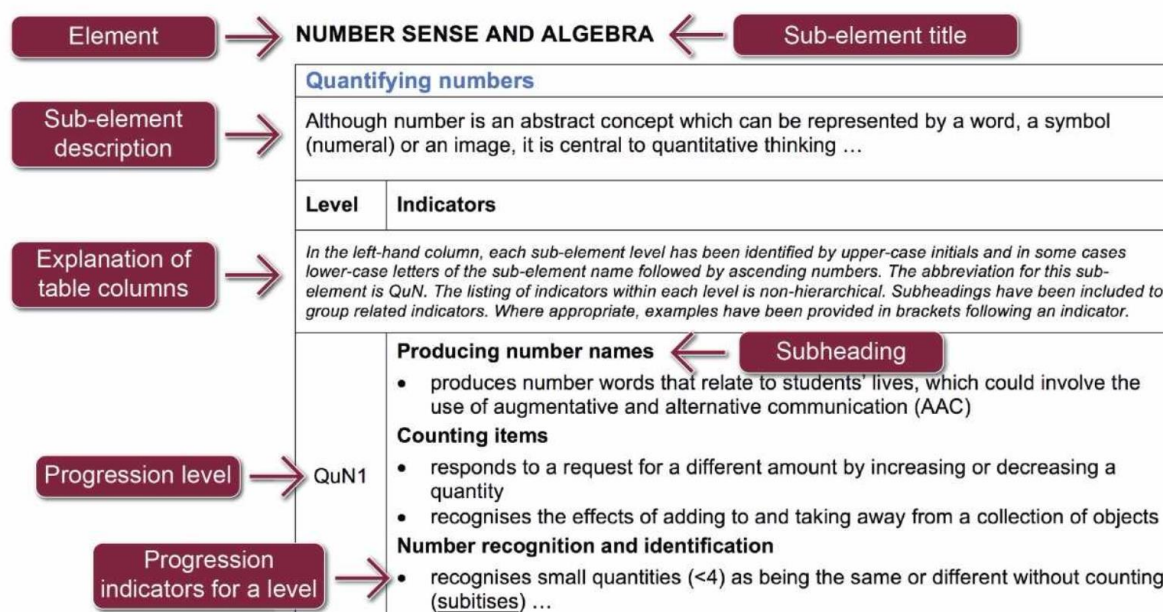


Figure 2. Annotated example of a numeracy sub-element

## How is the numeracy progression related to NSW syllabuses?

Numeracy skills are explicit in the *NSW Mathematics K–10 Syllabus*. Students need opportunities to recognise that mathematics is constantly used outside the mathematics classroom and that numerate people apply general mathematical skills in a wide range of familiar and unfamiliar situations.

### NSW Mathematics K-10 Syllabus

*Mathematics in K–10 provides students with knowledge, skills and understanding in Number and Algebra, Measurement and Geometry, and Statistics and Probability.....The Mathematics curriculum makes clear the links between the various components of mathematics, as well as the relationship between mathematics and other disciplines. Students learn to apply their mathematical knowledge, skills and understanding in a broad range of contexts beyond the mathematics classroom....* (Mathematics K–10 Syllabus, Rationale 2012)

The *Mathematics K–10 Syllabus* sets teaching expectations for mathematics learning at each stage, providing carefully paced, in-depth study of critical mathematical skills and concepts. The syllabus focuses on developing the Working Mathematically skills of communicating,

fluency, understanding, reasoning and problem solving. These skills are reflected in the National Numeracy Learning Progression rather than specifically identified.

The progression helps teachers to develop fine-grain understandings of student numeracy development in the *Mathematics K–10 Syllabus*, especially in the early years. It is particularly useful in guiding teachers to support students whose numeracy development is above or below the age-equivalent syllabus expectations of the *Mathematics K–10 Syllabus*. The progression has not been designed as a checklist and does not replace the *Mathematics K–10 Syllabus*.

The progression has been mapped to the *NSW Mathematics K–10 Syllabus* to demonstrate the range and level of numeracy skills required to access the outcomes and content.

## Other learning areas

This progression is designed to assist schools and teachers in all learning areas to support their students to successfully engage with the numeracy demands of the K-10 NSW curriculum.

## How can the numeracy progression be used?

The National Numeracy Learning Progression can be used at a whole school, team or individual teacher level. However, the progression provides maximum student learning benefits when used as part of a whole-school strategy that involves professional learning and collaboration between teachers.

The progression can be used to identify the numeracy performance of individual students within and across the 15 sub-elements. In any class there may be a wide range of student abilities. Individual students may not neatly fit within a particular level of the progressions and may straddle two or more levels within a progression. While the progression provides a logical sequence, not all students will progress through every level in a uniform manner.

When making decisions about a student's numeracy development, teachers select relevant indicators. It is important to remember indicators at a level are not a prescriptive list and the progression is not designed to be used as a checklist. Teacher judgements about student numeracy capability should be based on a range of learning experiences. Number talks, written or oral explanations, or tasks from any learning area can provide suitable evidence of a student's numeracy capability.

Teachers can use the progression to support the development of targeted teaching and learning programs and to set clearer learning goals for individual students. For example, teaching decisions can be based on judgements about student capability that relate to a single indicator rather than all indicators at a level.

# Number sense and algebra

## Quantifying numbers

Although number is an abstract concept which can be represented by a word, a symbol (numeral) or an image, it is central to quantitative thinking.

This sub-element describes how a student becomes increasingly able to count, recognise, read and interpret numbers expressed in different ways. It outlines key understandings needed to process, communicate and interpret numerical information in a variety of contexts.

Within this sub-element, place value is taken to mean more than being able to read, write and state the positional value of a digit. Place value relies on understanding the relationship between digits in a numeral, which then enables the numeral to be renamed in multiple ways. In addition to the base-ten positional value property, the place value system has both additive and multiplicative properties. That is, the quantity represented by a numeral is the sum of the values represented by its individual digits ( $326 = 300 + 20 + 6$ ) and the value of a digit is determined by multiplying its face value by the value assigned to its position in the numeral ( $326 = 3 \times 100 + 2 \times 10 + 6 \times 1$ ).

The *Quantifying numbers* sub-element underpins learning of number sense, measuring and using data.

*Some students will communicate using augmentative and alternative communication strategies to demonstrate their numeracy skills. This may include digital technologies, sign language, braille, real objects, photographs and pictographs.*

Level	Indicators
<i>Each sub-element level has been identified by upper-case initials and in some cases lower-case letters of the sub-element name followed by ascending numbers. The abbreviation for this sub-element is QuN. The listing of indicators within each level is non-hierarchical. Subheadings have been included to group related indicators. Where appropriate, examples have been provided in brackets following an indicator.</i>	
QuN1	<p><b>Producing number names</b></p> <ul style="list-style-type: none"><li>produces number words that relate to students' lives, which could involve the use of augmentative and alternative communication (AAC)</li></ul> <p><b>Counting items</b></p> <ul style="list-style-type: none"><li>responds to a request for a different amount by increasing or decreasing a quantity</li><li>recognises the effects of adding to and taking away from a collection of objects</li></ul> <p><b>Number recognition and identification</b></p> <ul style="list-style-type: none"><li>recognises small quantities (&lt;4) as being the same or different without counting (subitises)</li><li>compares two quantities and states which group has more and which less</li><li>matches one numeral with another (matches to a sample)</li><li>recognises some numerals, such as those associated with age or home address</li></ul>

## Quantifying numbers

QuN2	<p><b>Producing number names</b></p> <ul style="list-style-type: none"> <li>• produces a rote count to at least 12<sup>†</sup></li> <li>• produces a rote count down from 10</li> </ul> <p><b>Counting items</b></p> <ul style="list-style-type: none"> <li>• counts a small number of items (typically less than 4)</li> </ul> <p><b>Numeral recognition and identification</b></p> <ul style="list-style-type: none"> <li>• indicates the correct numeral from a range of different symbols for most numerals up to 10 ('which is 3?')</li> <li>• produces the matching number word for most numerals up to 10</li> </ul>
QuN3	<p><b>Producing number names</b></p> <ul style="list-style-type: none"> <li>• produces the number word just after a given number word in the range 1–10 (but drops back to 1 when doing so)</li> <li>• produces the number word just before a given number word in the range 1–10 (but drops back to 1 when doing so)</li> </ul> <p><b>Counting items</b></p> <ul style="list-style-type: none"> <li>• recognises that the last number word said in a count answers 'How many?'</li> <li>• matches the count (up to 10) to objects, using the one-to-one principle</li> </ul> <p><b>Numeral recognition and identification</b></p> <ul style="list-style-type: none"> <li>• recognises and identifies all numerals in the range 1–10</li> <li>• selects the largest numeral from an unordered group of 3 or more, in the range 1–10</li> </ul>
QuN4	<p><b>Producing number names</b></p> <ul style="list-style-type: none"> <li>• produces the number word just after a given number word in the range 1–10 (without dropping back to count from 1)</li> <li>• produces the number word just before a given number word in the range 1–10 (without dropping back)</li> </ul> <p><b>Counting items</b></p> <ul style="list-style-type: none"> <li>• matches number words within the current known counting range to quantities of items</li> <li>• correctly indicates the larger or smaller of two numerals in the range from 1 to 10</li> </ul> <p><b>Numeral recognition and identification</b></p> <ul style="list-style-type: none"> <li>• recognises and identifies all numerals in the range 1–10 as well as 20, 30, 40, 50, 60, 70, 80, 90 and 100</li> <li>• orders numerals to at least 10</li> </ul>

<sup>†</sup> Reference to a rote count to at least 12 rather than 10 is because approximately 87% of children start schooling being able to produce an oral count at least to 10. For most children, the first major hurdle in learning to count occurs at 12, with the start of the 'teen' sequence.



## Quantifying numbers

QuN5	<p><b>Producing number names</b></p> <ul style="list-style-type: none"> <li>counts to at least 20</li> <li>continues a count from a number other than 1</li> <li>counts forwards by tens to 100</li> </ul> <p><b>Counting items</b></p> <ul style="list-style-type: none"> <li>counts groups of up to 20 items</li> </ul> <p><b>Numeral recognition and identification</b></p> <ul style="list-style-type: none"> <li>points to the correct numeral in response to a verbal request, for numerals up to 20 as well as 30, 40, 50, 60, 70, 80, 90 and 100</li> </ul>
QuN6	<p><b>Producing number names</b></p> <ul style="list-style-type: none"> <li>counts to at least 30</li> <li>produces the number word just after a given number in the range 1–30 (without dropping back)</li> <li>produces the number word just before a given number word in the range 1–30 (without dropping back)</li> <li>counts forwards and backwards by tens to and from 100</li> </ul> <p><b>Counting items</b></p> <ul style="list-style-type: none"> <li>matches known numerals (to 20) to quantities</li> </ul> <p><b>Numeral recognition and identification</b></p> <ul style="list-style-type: none"> <li>identifies all numerals up to 30 as well as 40, 50, 60, 70, 80, 90 and 100 (is shown the numeral 17 and produces its name)</li> <li>orders numbers to at least 20 (determines the largest number in a group of numbers selected from 1 to 20)</li> </ul>
QuN7	<p><b>Producing number names to at least 120<sup>†</sup></b></p> <ul style="list-style-type: none"> <li>counts forwards and backwards to and from 120 and beyond</li> <li>continues counting from any number up to 120 and beyond</li> <li>counts forwards and backwards by fives</li> </ul> <p><b>Grouping and counting items by tens</b></p> <ul style="list-style-type: none"> <li>counts items in groups of twos, fives and tens</li> <li>recognises that a count of one ten is the same as ten counts of one</li> </ul> <p><b>Numeral recognition and identification</b></p> <ul style="list-style-type: none"> <li>identifies numerals from 0 to at least 100 (is shown the numeral 45 and produces its name)</li> <li>recognises a numeral from a given range up to 100 (is shown the numerals 70, 38, 56 and 26 and when asked which is 38, indicates the numeral)</li> </ul>

<sup>†</sup>Reference to producing number names to at least 120 rather than 100 is because of the higher proportion of students in the early years who encounter a hurdle at 109 compared to 100.

## Quantifying numbers

QuN8	<p><b>Producing number names to at least 1000</b></p> <ul style="list-style-type: none"> <li>counts forwards and backwards by 100s to 1000 (100, 200 ... 1000)</li> <li>counts forwards and backwards by tens off the decade to 100 (2, 12, 22, ...)</li> </ul> <p><b>Numeral recognition and identification of place value</b></p> <ul style="list-style-type: none"> <li>recognises and describes teen numbers as 1 ten and some more (16 is 1 ten and 6 more)</li> <li>represents and renames two-digit numbers as separate tens and ones (68 is 6 tens and 8 ones, 68 ones, or 60 + 8)</li> <li>applies an understanding of zero in place value notation when reading numerals that include internal zeros (correctly recognises 101 as one hundred and one, not 1001)</li> </ul>
QuN9	<p><b>Producing number names of any size</b></p> <ul style="list-style-type: none"> <li>counts forwards and backwards from any number</li> <li>produces and reads numbers to at least 1000</li> </ul> <p><b>Numeral recognition and identification of place value</b></p> <ul style="list-style-type: none"> <li>recognises and identifies numerals from a given range up to 1000 (is shown the numerals 170, 318, 576 and 276 and when asked which is 276, points to the 276)</li> </ul> <p><b>Understanding place value</b></p> <ul style="list-style-type: none"> <li>represents and flexibly renames three-digit numbers as counts of hundreds, tens and ones (247 is 2 hundreds, 4 tens and 7 ones, or 2 hundreds and 47 ones, or 24 tens and 7 ones)</li> </ul> <p><b>Understanding decimal place value</b></p> <ul style="list-style-type: none"> <li>recognises that the place value system can be extended to tenths and hundredths</li> <li>uses an understanding of the magnitude of decimals to compare them to two decimal places (0.20 is smaller than 0.4)</li> <li>orders decimals to one decimal place by placing them on an interval between 0 and 1</li> </ul>

## Quantifying numbers

QuN10	<p><b>Numerals recognition and identification of place value</b></p> <ul style="list-style-type: none"> <li>identifies numerals in the range 0–10 000 (is shown the numeral 2001 and responds <i>two thousand and one</i>)</li> <li>recognises a numeral from a given range of numerals up to 10 000 (when presented with the numerals 1701, 9318, 2050 and 2500 and when asked which is 2050, indicates the correct numeral)</li> </ul> <p><b>Understanding place value</b></p> <ul style="list-style-type: none"> <li>reads and writes numbers beyond 1000 applying knowledge of the place value periods of ones and thousands</li> <li>partitions numbers by their place value into thousands, hundreds, tens and ones</li> </ul> <p><b>Understanding decimal place value</b></p> <ul style="list-style-type: none"> <li>locates and orders decimals between 0 and 1 up to two decimal places</li> <li>recognises that the place value system can be extended to thousandths</li> <li>compares the size of decimals (including ragged decimals such as 0.5, 0.25, 0.125)</li> <li>reads, compares and renames decimal numbers (0.4 is greater than 0.355 because 0.4 has 4 tenths and 0.355 only has 3 tenths)</li> </ul>
QuN11	<p><b>Understanding place value</b></p> <ul style="list-style-type: none"> <li>reads and writes numbers applying knowledge of the place value periods of ones, thousands, millions (how numbers are written with the digits organised in groups of three – 10 000 is read as ten thousand, where thousand is the place value period)</li> <li>partitions numbers by their place value into tens of thousands, thousands, hundreds, tens and ones and beyond</li> <li>recognises the relationship between adjacent positions in place value (200 is 10 times as large as 20, which is 10 times as large as 2)</li> <li>estimates whole numbers to the nearest hundred thousand, ten thousand, etc. (crowd numbers at a football match)</li> </ul> <p><b>Understanding decimal place value</b></p> <ul style="list-style-type: none"> <li>compares and orders decimals beyond 1 including ragged decimals (those expressed with unequal numbers of places)</li> <li>recognises the relationship between adjacent positions in place value for decimals (0.20 is 10 times larger than 0.02)</li> </ul>

## Quantifying numbers

<b>QuN12</b>	<p><b>Understanding place value (directed numbers)</b></p> <ul style="list-style-type: none"><li>• orders negative numbers (recognises that <math>-10^{\circ}\text{C}</math> is colder than <math>-2.5^{\circ}\text{C}</math>)</li></ul> <p><b>Representing place value</b></p> <ul style="list-style-type: none"><li>• recognises, reads and interprets very large and very small numbers</li><li>• expresses numbers as powers of 10 in scientific notation and determines the order of magnitude of quantities (a nanometre has an order of magnitude of <math>-9</math>)</li><li>• relates place value parts to indices (<math>1000</math> is 100 times larger than 10, and that is why <math>10^1 \times 10^2 = 10^3</math> and why <math>10^3</math> divided by <math>10^1</math> is equal to <math>10^2</math>)</li></ul>
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## Additive strategies

This sub-element describes how a student becomes increasingly able to choose and use additive computational strategies for different purposes. The transition from counting by one to more flexible methods of dealing with quantity, where numbers are treated as the sums of their parts, is a critical hurdle to be addressed in students becoming fluent users of number. Rather than only focusing on the speed of producing correct answers, an emphasis on attending to the relation of given numbers to sums and differences is needed for flexibility. This supports the development of additive strategies such as adding the same to both numbers to reach an easier calculation ( $47 - 38 = 49 - 40$ ).

The capacity to make reasonable adjustments to numbers is essential in estimating. Estimating is not a basic skill as it requires students to be able to conceptualise and mentally manipulate numbers. The estimation process involves selecting numbers to simplify mental manipulation.

Additive strategies apply equally to subtraction, as can be seen in 'Giving change' in the *Understanding money* sub-element.

*Some students will communicate using augmentative and alternative communication strategies to demonstrate their numeracy skills. This may include digital technologies, sign language, braille, real objects, photographs and pictographs.*

Level	Indicators
<p><i>Each sub-element level has been identified by upper-case initials and in some cases lower-case letters of the sub-element name followed by ascending numbers. The abbreviation for this sub-element is AdS. The listing of indicators within each level is non-hierarchical. Subheadings have been included to group related indicators. Where appropriate, examples have been provided in brackets following an indicator.</i></p>	
AdS1	<b>Emergent strategies</b> <ul style="list-style-type: none"> <li>combines two groups of objects and attempts to find the total</li> <li>compares two quantities of up to 10 and states which group has more</li> </ul>
AdS2	<b>Perceptual strategies</b> <ul style="list-style-type: none"> <li>counts items that can be perceived by ones to find the total of two groups with one-to-one matching of number words and objects</li> <li>builds and subtracts numbers by using objects or fingers</li> <li>makes combinations to form numbers up to 10</li> </ul>
AdS3	<b>Figurative (imagined units)</b> <ul style="list-style-type: none"> <li>solves additive tasks involving two concealed collections of items by visualising, counting from one to determine the total</li> </ul>



## Additive strategies

AdS4	<p><b>Counting on (by ones)</b></p> <ul style="list-style-type: none"> <li>treats a number word as a completed count when solving problems ('I have 7 apples. I want 10. How many more do I need?' Treats the 7 as a completed count)</li> <li>uses a strategy of count-up-from to calculate addition (to find <math>6 + 3</math>, responds 6, 7, 8, 9. <i>It's 9</i>)</li> <li>uses a strategy of count-up-to to solve missing addends tasks (to solve <math>6 + ? = 9</math>, responds 6 ... 7, 8, 9. <i>It's 3</i>)</li> </ul>
AdS5	<p><b>Counting back (by ones)</b></p> <ul style="list-style-type: none"> <li>uses count-down-from for subtraction tasks (<math>9 - 3 = ?</math>, 9 ... 8, 7, 6. <i>It equals 6</i>)</li> <li>uses count-down-to to calculate (9 take away something equals 6, responds 9 ... 8, 7, 6 ... <i>It's 3</i>)</li> <li>finds the difference between two numbers less than 20</li> <li>counts back to find the difference between two quantities where the difference is no greater than 4</li> </ul>
AdS6	<p><b>Flexible strategies with combinations to 10</b></p> <ul style="list-style-type: none"> <li>uses a range of non-count-by-one strategies when adding or subtracting two or more numbers (bridging to 10, near doubles)</li> <li>uses part-whole construction of number to partition a whole number into parts (partitions 7 into 5 and 2, 6 and 1, 4 and 3)</li> <li>applies inverse relationship of addition and subtraction</li> </ul>
AdS7	<p><b>Flexible strategies with two-digit numbers</b></p> <ul style="list-style-type: none"> <li>applies knowledge of 10 as a unit to add and subtract 2 two-digit numbers (jump strategy, split strategy or compensation)</li> <li>manipulates tens and ones flexibly for addition and subtraction (to add 45 and 37, adds the tens on 45 ... 55, 65, 75, then partitions the 7 into 5 and 2, adds the 5 to make 80 and 2 more to make 82)</li> <li>uses part-whole knowledge of numbers to 20 to calculate two-digit addition and subtraction (when finding <math>53 - 27</math>, recognises the subtraction within as <math>13 - 7</math> and regroupes the 53 into 40 and 13 and the 27 into 20 and 7)</li> </ul>
AdS8	<p><b>Flexible strategies with three-digit numbers and beyond</b></p> <ul style="list-style-type: none"> <li>manipulates hundreds, tens and ones flexibly to add and subtract 2 three-digit numbers (to add 250 and 457, ungroups the 250 into 2 hundreds and 5 tens, responds <i>457 plus 2 hundred is 657, plus 50 is 707</i>)</li> <li>manipulates place value of numbers flexibly in regrouping for addition of three-digit numbers and beyond (when adding 650 and 550, regroupes 650 as 600 and 50, adds 50 to 550, then doubles 600)</li> <li>manipulates place value of numbers flexibly in regrouping for subtraction (when solving <math>3000 - 260</math>, treats the 3000 as 2700 and 300 to aid in mental calculation)</li> <li>regroups for subtraction involving trading or exchange of units with different place values</li> <li>chooses and uses multiple strategies for solving everyday problems involving addition and subtraction</li> </ul>

## Multiplicative strategies

This sub-element describes how a student becomes increasingly able to use multiplicative strategies in computation. The coordination of units multiplicatively involves using the values of one unit applied to each of the units of the other, the multiplier. This process of coordinating units is equally relevant to problems of division.

Although multiplication of whole numbers can be achieved by repeated addition, this isn't necessarily the best way to think of multiplication. To determine how many shoes are in 100 pairs of shoes it is possible (but not practical) to add 100 lots of 2. Coordinating '100' as one unit as well as '2' as a unit leads to appreciating a multiplicative relationship between the quantities. Recognising that 100 lots of 2 is the same as 2 lots of 100 is an important multiplicative strategy. This same understanding relates to seeing the two forms of division as being equivalent.

In the *sharing* model of division, the divisor indicates a whole number of equal groups and the quotient, the result of division, is the size of each part. In  $12 \div 3 = 4$ , twelve is *shared* into 3 equal groups and there are 4 in each group. An over-reliance on the sharing model of division can contribute to misconceptions about division with decimals.<sup>†</sup> This model is inadequate when the division has a divisor that is less than one.

In the *measurement* division model, the divisor indicates the size of the subset (number in each group) and the quotient is the number of equal-sized subsets. For  $12 \div 3 = 4$ , 12 is divided into groups of 3, and 4 is the number of groups of 3. The measurement division model is sometimes described as quotitive division.

Multiplicative strategies are used in the sub-elements *Operating with decimals*, *Operating with percentages* and *Interpreting fractions*.

*Some students will communicate using augmentative and alternative communication strategies to demonstrate their numeracy skills. This may include digital technologies, sign language, braille, real objects, photographs and pictographs.*

Level	Indicators
<p><i>Each sub-element level has been identified by upper-case initials and in some cases lower-case letters of the sub-element name followed by ascending numbers. The abbreviation for this sub-element is MuS. The listing of indicators within each level is non-hierarchical. Subheadings have been included to group related indicators. Where appropriate, examples have been provided in brackets following an indicator.</i></p>	
<b>MuS1</b>	<p><b>Forming equal groups</b></p> <ul style="list-style-type: none"> <li>• shares collections equally by dealing (that is, distributing items one to one until they are exhausted)</li> <li>• makes equal groups and counts by ones to find the total</li> </ul>
<b>MuS2</b>	<p><b>Perceptual multiples</b></p> <ul style="list-style-type: none"> <li>• uses groups or multiples in perceptual counting and sharing (rhythmic or skip counting with all items visible)</li> </ul>

<sup>†</sup> Fischbein, E, Deri, M, Nello, MS & Merino, MS 1985, 'The role of implicit models in solving verbal problems in multiplication and division', *Journal for Research in Mathematics Education*, 16, pp. 3–17.

## Multiplicative strategies

<b>MuS3</b>	<b>Figurative (imagined units)</b> <ul style="list-style-type: none"> <li>relies on perceptual markers to represent each group</li> <li>uses equal grouping and counting without individual items visible but needs to represent the groups before determining the total</li> <li>counts by twos, fives and tens, matching the count to groups of the corresponding size</li> </ul>
<b>MuS4</b>	<b>Repeated abstract composite units</b> <ul style="list-style-type: none"> <li>uses composite units in repeated addition and subtraction using the unit a specified number of times</li> <li>uses skip counting and may use fingers to keep track of the number of groups as the counting occurs</li> <li>determines the total or number of equal groups where the individual items cannot be seen</li> </ul>
<b>MuS5</b>	<b>Coordinating composite units</b> <ul style="list-style-type: none"> <li>coordinates two composite units (mentally) as an operation (that is, both the number of groups and the number in each group are treated as composite units)</li> <li>represents multiplication in various ways (arrays, factors, 'for each')</li> <li>represents division as sharing division and measurement or grouping division</li> </ul>
<b>MuS6</b>	<b>Flexible strategies for multiplication</b> <ul style="list-style-type: none"> <li>draws on the structure of multiplication to use known multiples in calculating related multiples (uses multiples of 4 to calculate multiples of 8)</li> <li>uses known single-digit multiplication facts (7 boxes of 6 donuts is 42 donuts altogether because <math>7 \times 6 = 42</math>)</li> <li>applies known facts and strategies for multiplication to mentally calculate (3 sixes is 'double 6' plus 1 more row of 6, <math>5 \times 19</math> is half of <math>10 \times 19</math> or <math>5 \times 19</math> is <math>5 \times 20</math> take away 5)</li> <li>uses commutative properties of numbers (<math>5 \times 6</math> is the same as <math>6 \times 5</math>)</li> </ul> <b>Flexible strategies for division</b> <ul style="list-style-type: none"> <li>applies known multiples and strategies for division to mentally calculate (to find 64 divided by 4, halves 64 then halves 32)</li> <li>explains the idea of a remainder as an incomplete next row or multiple, and determines what is 'left over' from the division</li> </ul>
<b>MuS7</b>	<b>Flexible number properties</b> <ul style="list-style-type: none"> <li>uses multiplication and division as inverse operations</li> <li>uses factors of a number to carry out multiplication and division (to multiply a number by 72, first multiply by 12 and then multiply the result by 6)</li> <li>uses knowledge of distributive property of multiplication over addition (<math>7 \times 83</math> equals <math>7 \times 80</math> plus <math>7 \times 3</math>)</li> <li>uses decomposition into hundreds, tens and ones to calculate using partial products with numbers of any size (<math>327 \times 14</math> is equal to <math>4 \times 327</math> plus <math>10 \times 327</math>)</li> <li>uses estimation and rounding to check the reasonableness of products and quotients</li> </ul>

## Operating with decimals

This sub-element focuses on understanding the use of place value in operating with decimals. Decimals are better suited to estimating magnitude than fractions because decimals use the base-ten system to record quantity and fractions do not. However, the base-ten system used with whole numbers can also contribute to misconceptions with decimals. For example, recognising that whole numbers with more digits are always larger and applying this to decimals may lead to incorrectly believing 0.75 is larger than 0.8 and 0.320 is larger than 0.32. Understanding that fractions with larger denominators result in smaller magnitudes and longer decimals contain smaller parts can lead to believing longer decimals must be smaller than shorter decimals.

Decimals are commonly used to record metric quantities and have applications in areas that range from nutritional advice to expressing tolerances in precision engineering.

(NB: The notation for fractions is distinct from the place value notation used with decimals. This progression treats the development of decimal notation separately from the development of common fractions).

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Level	Indicators
<i>Each sub-element level has been identified by upper-case initials and in some cases lower-case letters of the sub-element name followed by ascending numbers. The abbreviation for this sub-element is OwD. The listing of indicators within each level is non-hierarchical. Subheadings have been included to group related indicators. Where appropriate, examples have been provided in brackets following an indicator.</i>	
OwD1	<b>Understanding positional value of decimals</b> <ul style="list-style-type: none"><li>uses knowledge of positional value of numbers to add and subtract decimals of up to three decimal places</li></ul>
OwD2	<b>Understanding and estimating relative size of decimals</b> <ul style="list-style-type: none"><li>interprets the relative size of decimals, and rounds to estimate answers</li><li>estimates the size of answers without doing the exact calculations (1.23 + 3.4 cannot be 1.57 because the sum must be greater than 4)</li></ul>

## Operating with decimals

<b>OwD3</b>	<p><b>Understanding the effects of multiplication and division with decimals</b></p> <ul style="list-style-type: none"> <li>• understands that multiplying and dividing decimals by 10, 100, 1000 changes the positional value of the numerals</li> <li>• explains that multiplication does not always make the answer larger (when multiplying whole numbers by a decimal less than 1, <math>15 \times 0.5 = 7.5</math>)</li> <li>• connects and converts decimals to fractions to assist in mental computation involving multiplication (to find <math>16 \times 0.25</math>, recognises 0.25 as a quarter, and finds a quarter of 16)</li> <li>• connects and converts decimals to fractions to assist in mental computation involving division (to determine <math>0.5 \div 0.25</math>, recognises the answer is 2 as there are two quarters in one-half)</li> <li>• recognises the equivalence of decimals to benchmark fractions (<math>\frac{1}{4} = 0.25</math>, <math>\frac{1}{2} = 0.5</math>, <math>\frac{3}{4} = 0.75</math>, <math>\frac{1}{10} = 0.1</math>, <math>\frac{1}{100} = 0.01</math>)</li> </ul>
<b>OwD4</b>	<p><b>Flexible strategies for multiplication and division of decimals</b></p> <ul style="list-style-type: none"> <li>• uses knowledge of positional value of numbers to multiply and divide decimals</li> <li>• uses knowledge of approximate answers to check accuracy of solutions when using a variety of strategies</li> </ul>



## Operating with percentages

This sub-element focuses on understanding the use of percentages in representing quantities. It begins with understanding the concept of a percentage, progresses through calculations of a percentage, finding a percentage change first through two steps (calculating the percentage and adding or subtracting) before progressing to one-step methods (10% increase in cost is achieved by multiplying by 1.1) and then to calculating multiple changes.

The multiplicative nature of percentages coupled with the practice of frequently only implying the quantity the percentage refers to can lead to an incomplete understanding of percentages.

Addition and subtraction are inverse operations, yet increasing a price by 10%, followed by decreasing by 10%, does not return to the original price. Operating with percentages relates to comparing units in ratios and proportions.

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Level	Indicators
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OwP1	<p><b>Understanding percentages and relative size</b></p> <ul style="list-style-type: none"> <li>interprets per cent as meaning 'out of 100'</li> <li>recognises that 100% is a complete whole</li> <li>interprets a percentage as an operator (percentage is of an amount, 17% of \$80, 17% does not exist alone without its referent)</li> <li>uses percentages to describe and compare relative size (select which beaker is 75% full, describes an object as 50% larger)</li> <li>represents relative size of percentages of an amount</li> </ul>
OwP2	<p><b>Find percentage as a part of a whole</b></p> <ul style="list-style-type: none"> <li>uses fraction benchmarks to find percentages of quantities (to find 75% of 160, I know that 50% (half) of 160 is 80, and 25% (quarter) is 40 so 75% is 120)</li> <li>finds a percentage of a quantity (10%, 20%, 25%, 50%, 75% and multiples of these)</li> <li>multiplies to calculate a percentage of any amount (finds 13% of 160)</li> <li>finds percentages of quantities and expresses one quantity as a percentage of another (finds 20% of \$13 and determines what percentage \$13 is of \$20)</li> </ul>
OwP3	<p><b>Find a part of a whole as a percentage</b></p> <ul style="list-style-type: none"> <li>uses a strategy to find a percentage that represents part of a whole (what per cent is 7 of 28, may use benchmark fractions, or what per cent is 7 of 29, may multiply to calculate)</li> </ul>

## Operating with percentages

<b>OwP4</b>	<b>Find the whole from a percentage and a part</b> <ul style="list-style-type: none"> <li>determines the whole given a percentage (given 20% is 13 mL, determines the whole is 65 mL)</li> <li>identifies the whole for a range of multiplicative situations (percentages for calculating discounts and rates for best buys)</li> </ul>
<b>OwP5</b>	<b>Adding a percentage as multiplying</b> <ul style="list-style-type: none"> <li>increases and decreases quantities by a percentage (to determine discounts and mark-ups)</li> <li>uses percentages to calculate simple interest on loans and investments</li> <li>recognises that adding a percentage is equivalent to multiplication (adding 3% is multiplying by 1.03)</li> </ul>
<b>OwP6</b>	<b>Repeatedly adding a percentage</b> <ul style="list-style-type: none"> <li>uses percentage increases or decreases as an operator (a 3% increase is achieved by multiplying 1.03, and 4 successive increases is multiplying by 1.034)</li> <li>chooses appropriate strategies for problems in a range of multiplicative situations (percentage of a percentage for calculating successive discounts)</li> <li>uses percentages to calculate compound interest on loans and investments</li> <li>evaluates, critically, claims based on numerical multiplicative operations (Why is a 10% increase followed by a 10% discount different from the original price?)</li> </ul>

## Understanding money

The descriptive term *decimal* applied to money refers to the basic conversion units being multiples of ten. However, money is not a true decimal system. Amounts such as \$2.99 are spoken as two whole numbers (2 dollars 99). Due to the withdrawal of 1-cent and 2-cent coins, can no longer be represented with coins. Money is more of a system based on the face value of coins and notes than a place value system. Understanding how to use currency draws on both additive and multiplicative strategies. Giving change requires being able to round values and work with multiples of 5, 20 or 50 as well as 10. Interest and compound interest involve operating with percentages and are outlined in that sub-element. Similarly, unitary pricing or determining 'best buys' is an application of rates found in the *Comparing units* sub-element.

This sub-element focuses on understanding the use of Australian coins in operating with money.

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UnM1	<b>Matching</b> <ul style="list-style-type: none"> <li>matches like coins and notes (matches 10-cent coins as being alike)</li> </ul>
UnM2	<b>Face value</b> <ul style="list-style-type: none"> <li>recognises 5c, 10c, 20c and 50c coins based on face value</li> <li>recognises \$1 and \$2 coins based on face value</li> </ul>
UnM3	<b>Sorts</b> <ul style="list-style-type: none"> <li>sorts and counts the number of coins with the same face value</li> <li>identifies situations that involve the use of money</li> </ul>
UnM4	<b>Counting value of coins</b> <ul style="list-style-type: none"> <li>determines the equivalent value of coins to a maximum of 10 coins of one denomination</li> </ul>
UnM5	<b>Coins of one value to \$5<sup>†</sup></b> <ul style="list-style-type: none"> <li>determines the equivalent value of coins to \$5 using one denomination of 5c, 10c, 20c or 50c coins (Sam has \$1.20 in 5-cent coins. How many 5-cent coins does Sam have?)</li> </ul>

<sup>†</sup> \$5 is the limit of legal tender in combinations of 5c, 10c, 20c and 50c coins according to the *Currency Act 1965* (section 16)..

## Understanding money

<b>UnM6</b>	<b>Coins of mixed values</b> <ul style="list-style-type: none"><li>determines the equivalent value of coins to \$5 using any combination of 5c, 10c, 20c or 50c coins</li></ul>
<b>UnM7</b>	<b>Giving change</b> <ul style="list-style-type: none"><li>uses complementary addition (the shopkeeper's method of adding change to obtain the amount tendered) to determine the difference between two amounts, rounding as necessary</li></ul>

## Number patterns and algebraic thinking

Figuring out how a pattern works brings predictability and allows the making of generalisations. This sub-element describes how a student becomes increasingly able to identify a pattern as something that is a discernible regularity in a group of numbers or shapes. As students become increasingly able to connect patterns with the structure of numbers, they create a foundation for algebraic thinking (that is, thinking about generalised quantities). Number patterns are evident in house numbers on opposite sides of streets. Algebra enables the 'generalisation' of patterns from one situation to another.

Algebraic thinking is also used to capture the relationship between quantities such as  $F=ma$  or force equals mass multiplied by acceleration.

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<b>NPA1</b>	<b>Identifying patterns</b> <ul style="list-style-type: none"> <li>recognises simple patterns in everyday contexts</li> <li>copies simple patterns</li> </ul>
<b>NPA2</b>	<b>Identifying and creating patterns</b> <ul style="list-style-type: none"> <li>identifies standard patterns (dice or domino) without counting individual items</li> <li>creates repeating patterns with numbers and shapes (circle, square, circle, square or 1,2,3 1,2,3 1,2,3)</li> </ul>
<b>NPA3</b>	<b>Identifying repeating patterns</b> <ul style="list-style-type: none"> <li>identifies the pattern unit within a simple repeating pattern (continues a simple pattern)</li> <li>identifies standard patterns up to 10 (patterns in ten frames, finger patterns, playing cards)</li> <li>finds the missing element in a pattern involving shapes or objects</li> </ul>



## Number patterns and algebraic thinking

NPA4	<p><b>Continuing number patterns</b></p> <ul style="list-style-type: none"> <li>continues patterns where the difference between each term is the same number (2, 4, 6, 8, 10 ...)</li> <li>describes rules for continuing patterns where the difference between each term is the same number (to find the next number in the pattern 3, 6, 9, 12 ... you add 3)</li> <li>sequences numbers to identify a pattern or rule</li> </ul> <p><b>Introducing number sentences</b></p> <ul style="list-style-type: none"> <li>recognises the equals sign as meaning 'is equivalent to' or 'is the same as' not just 'makes' (recognises that <math>5 + 3 = 6 + 2</math>)</li> <li>finds missing values in a number sentence (<math>5 + ? = 6 + 2</math>)</li> </ul>
NPA5	<p><b>Generalising patterns</b></p> <ul style="list-style-type: none"> <li>identifies elements, including missing elements, in a one-operation number pattern</li> </ul> <p><b>Number sentences</b></p> <ul style="list-style-type: none"> <li>uses equivalent number sentences involving addition or subtraction to find an unknown (<math>527 + 96 = ?</math> is the same as <math>527 + 100 - 4 = ?</math>)</li> <li>applies knowledge of factors associated with the row and column structure of arrays to explain the commutative property of multiplication (<math>3 \times 4 = 4 \times 3</math>)</li> </ul>
NPA6	<p><b>Generalising patterns</b></p> <ul style="list-style-type: none"> <li>identifies a single operation rule in numerical patterns and records it as a numerical expression (2, 4, 6, 8, 10 ... is <math>n + 2</math>, or 2, 6, 18, 54 ... is <math>3n</math>)</li> <li>predicts a higher term of a pattern using the pattern's rule</li> </ul> <p><b>Number properties</b></p> <ul style="list-style-type: none"> <li>creates and interprets number sentences demonstrating the inverse relationship between multiplication and division</li> <li>balances number sentences involving one or more operations following conventions of order of operations (<math>5 \times 2 + 4 = 4 \times 2 + ?</math>, <math>5 + 2 \times 3 = 11</math>)</li> <li>recognises that any number multiplied by 0 equals 0 which means that one of the factors is 0 (<math>3 \times ? = 0</math>)</li> </ul>
NPA7	<p><b>Representing unknowns</b></p> <ul style="list-style-type: none"> <li>uses words or symbols (including letters) to express relationships involving unknown values</li> <li>finds the value of formulae or algebraic expressions by substituting</li> <li>creates algebraic expressions from word problems involving one operation</li> </ul>
NPA8	<p><b>Algebraic expressions</b></p> <ul style="list-style-type: none"> <li>creates and identifies algebraic expressions from word problems involving two operations and one unknown</li> <li>recognises equivalent algebraic expressions</li> </ul>

## Number patterns and algebraic thinking

**NPA9**

### **Algebraic relationships**

- interprets and uses formulae and algebraic representations that describe relationships in various contexts (Body Mass Index – BMI)
- creates an algebraic expression in two unknowns to represent a formula or relationship (Anna has 6 times as many stickers as Carol)

## Comparing units (ratios, rates and proportion)

This sub-element addresses comparing units in ratios, rates and proportions. A ratio describes a situation in comparative terms, and a proportion is taken to mean when this comparison is used to describe a related situation in the same comparative terms. For example, if the ratio of boys to girls in a class is 2 to 3, the comparison is the number of boys to the number of girls. Knowing that there are 30 children in the class, proportionally, the number of boys is 12 and the number of girls is 18. Applying the base comparison to the whole situation uses proportional reasoning. Proportional reasoning is knowing the multiplicative relationship between the base ratio and the proportional situation to which it is applied.

Learning to reason using proportion is a complex process that develops over an extended time. Proportional reasoning also includes numerical comparison tasks involving a comparison of different rates or ratios.<sup>†</sup> For example, if one dog grows from 5 kilograms to 8 kilograms and another dog grows from 3 kilograms to 6 kilograms, which dog grew more?

The sub-element of comparing units applies to Measurement, Interpreting fractions and Representing data.

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CoU1	<b>Building ratios</b> <ul style="list-style-type: none"><li>uses knowledge of fractions as part-whole relationships to divide and compare quantities</li><li>represents and models ratios using diagrams or objects (in a ratio 1:4 of red to blue counters, for each red counter there are four blue counters)</li></ul>

<sup>†</sup> Cramer, K & Post, T 1993, 'Connecting research to teaching proportional reasoning', *Mathematics Teacher*, 86(5), May, pp. 404–407.

## Comparing units (ratios, rates and proportion)

CoU2	<p><b>Ratios</b></p> <ul style="list-style-type: none"> <li>interprets ratios as a comparison between the same units of measure (students to teachers in a school is 20:1)</li> <li>expresses a ratio as equivalent fractions or percentages (ratio 1:1, each part represents <math>\frac{1}{2}</math> or 50% of the whole)</li> <li>uses a ratio to increase or decrease quantities to maintain a given consistency (doubling a recipe)</li> </ul> <p><b>Rates</b></p> <ul style="list-style-type: none"> <li>interprets rates as a relationship between two different types of quantities (money per unit of fuel)</li> <li>uses rates to determine how quantities change</li> </ul>
CoU3	<p><b>Applying proportion</b></p> <ul style="list-style-type: none"> <li>interprets proportion as the equality of two ratios or rates</li> <li>uses common fractions and decimals for proportional division</li> <li>demonstrates how increasing one quantity in a ratio will affect the total proportion</li> <li>performs operations with negative integers involving rates (rates of descent or cooling)</li> <li>explains and applies the difference between direct and indirect proportion (direct – working more hours will result in earning more money; indirect – travelling at a greater speed will mean the journey takes less time)</li> </ul>

## Interpreting fractions

This sub-element emphasises the development of the fraction concept and the size of fractions rather than the development of procedures or algorithmic skills. Understanding the size of a fraction is an indicator of the depth of a student's understanding of the fraction concept.<sup>†</sup>

This sub-element describes how a student becomes increasingly able to use fractions as numbers that describe a relationship between two abstract measures of quantity. Rather than representing two numbers, the fraction  $\frac{a}{b}$  represents the result of dividing one by the other.

That is,  $\frac{2}{3}$  is the *number* that results from dividing 2 by 3. Although the notation used with fractions is very powerful, its meaning can often remain opaque. A common misconception is thinking of a fraction as two whole numbers and not as a single number.

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InF1	<p><b>Creating halves</b></p> <ul style="list-style-type: none"> <li>identifies the part and the whole</li> <li>recognises dividing a whole into 2 parts can create equal or unequal parts</li> <li>creates equal halves by attending to the linear aspect of a model (folds a paper strip in half to make equal pieces by aligning the edges or makes 2 groups of 3 when halving a collection of 6 counters in a linear arrangement)</li> <li>distinguishes between halfway and half</li> </ul>
InF2	<p><b>Repeated halving</b></p> <ul style="list-style-type: none"> <li>recognises quarters and eighths formed by repeated halving of a length (finds halfway then halves each half, or repeatedly halves using a linear arrangement of discrete items— 8 counters halved and then halved again into 4 groups of 2)</li> </ul>
InF3	<p><b>Repeating fractional parts</b></p> <ul style="list-style-type: none"> <li>accumulates fractional parts of a length (knows that two-quarters is inclusive of one-quarter and twice one-quarter, not just the second quarter)</li> <li>checks the equality of parts by iterating one part to form the whole (when given a representation of one-quarter of a length and asked, 'what fraction is this of the whole length?', compares the size of the unit to the whole)</li> </ul>

<sup>†</sup> Post, TP, Behr, MJ, Lesh, R & Wachsmuth, I 1986, Selected results from the Rational Number Project, <http://education.umn.edu/rationalnumberproject>



## Interpreting fractions

InF4	<p><b>Re-imagining the whole</b></p> <ul style="list-style-type: none"> <li>calculates thirds by visualising or approximating and adjusting (imagines a paper strip in 3 parts, then adjusts and folds)</li> <li>identifies examples and non-examples of partitioned representations of thirds and fifths</li> <li>recognises the whole can be redivided into different fractional parts for different purposes (a strip of paper divided into quarters can be redivided to show fifths)</li> <li>demonstrates that the more parts into which a whole is divided, the smaller the parts become</li> </ul>
InF5	<p><b>Equivalence of fractions</b></p> <ul style="list-style-type: none"> <li>identifies the need to have equal wholes to compare fractional parts (explains why one-third as a number is larger than one-quarter)</li> <li>creates fractions larger than 1 by recreating the whole (when creating four-thirds, recognises that three-thirds corresponds to the whole and the fourth third is part of an additional whole)</li> <li>creates equivalent fractions by dividing the same-sized whole into different parts (shows two-sixths is the same as one-third of the same whole)</li> <li>links partitioning to establish relationships between fractions (creates one-sixth as one-third of one-half)</li> </ul>
InF6	<p><b>Fractions as numbers</b></p> <ul style="list-style-type: none"> <li>connects the concepts of fractions and division: a fraction is a quotient, or a division statement (two-sixths is the same as <math>2 \div 6</math> or 2 partitioned into 6 equal parts)</li> <li>justifies where to place fractions on a number line (to place two-thirds on a number line, divides the space between 0 and 1 into 3 equal parts)</li> <li>understands the relationship between a fraction, decimal and percentage as different representations of the same quantity (<math>\frac{1}{2} = 0.5 = 50\%</math>)</li> <li>shows an understanding that a fraction represents a single number, not two separate whole numbers (explains why <math>\frac{2}{4}</math> is not halfway between <math>\frac{1}{3}</math> and <math>\frac{3}{5}</math>, although 2 is midway between 1 and 3 and 4 is midway between 3 and 5)</li> </ul>
InF7	<p><b>Using fractions</b></p> <ul style="list-style-type: none"> <li>uses knowledge of equivalence to compare fractions (when comparing two-thirds and three-quarters, subdivides the whole into twelfths)</li> <li>justifies the need for the same denominators to add or subtract fractions</li> <li>uses strategies to find a fraction of a quantity (to find two-thirds of 27, finds one-third then doubles)</li> <li>demonstrates why dividing by a fraction can result in a larger number</li> <li>understands the difference between multiplying and dividing fractions (recognises <math>\frac{1}{2} \times \frac{1}{4}</math> as one-half of a quarter and <math>\frac{1}{2} \div \frac{1}{4}</math> as how many quarters are in one half)</li> </ul>

# Measurement and geometry

## Understanding units of measurement

This sub-element describes how a student becomes increasingly able to recognise attributes that can be measured and how units of measure are used and calculated. In making the transition from informal to formal units, a student attends to the structure of units used to measure, how they are assembled end-to-end, side-by-side or in layers without gaps or overlapping. The structure of the units gives rise to ways of calculating length, area and volume.

In dealing with mass and capacity, experience helps develop estimates associated with commonly available reference objects (a cupful in cooking or the mass of an egg). Developing standard and agreed units of measurement is critically vital in areas as diverse as medicine and trade.

The relationship between units of measurement is applied in ratios, rates and proportions as well as decimals and percentages.

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UuM1	<b>Describing length</b> <ul style="list-style-type: none"><li>identifies the attribute of length (using gestures)</li><li>identifies the longest object using direct comparison</li><li>compares the length of two objects by aligning the ends</li><li>uses everyday language to describe attributes that can be measured</li></ul>
UuM2	<b>Comparing and ordering objects</b> <ul style="list-style-type: none"><li>compares objects and explains how they have been ordered using comparative language (shorter, longer, lighter, heavier)</li><li>orders three or more objects by comparing the size of each of the objects</li><li>makes a copy of the length of one object (with fingers) to then make a comparison with another object</li></ul>
UuM3	<b>Using informal units of measurement</b> <ul style="list-style-type: none"><li>estimates the total number of units needed to measure</li><li>uses multiple informal units to measure length, mass or capacity (uses paper clips to measure the length of a line)</li><li>chooses and uses a selection of the same size and type of units to measure length, area and volume (without gaps or overlaps)</li><li>counts the individual units used by ones to find a total to then make comparisons</li></ul>

## Understanding units of measurement

<b>UuM4</b>	<b>Using equal units for indirect comparison</b> <ul style="list-style-type: none"> <li>describes the qualitative relationship between the size and number of units (with bigger units you need fewer of them)</li> <li>chooses and uses a selection of the same size and type of units to make indirect comparisons of mass and capacity</li> </ul>
<b>UuM5</b>	<b>Repeating a single informal unit to measure</b> <ul style="list-style-type: none"> <li>measures the length and area of a shape using a single informal unit repeatedly (iteration) (uses one paper clip when measuring the length of a line, making the first unit, marking its place, then moving the unit along the line and repeating this process)</li> <li>estimates length or area by visualising how many of the units will fit into the space to be measured</li> <li>explains that the distance measured is the space between the marks or 'ends' of each unit, not the marks themselves</li> <li>uses appropriate uniform units when measuring mass and capacity</li> </ul>
<b>UuM6</b>	<b>Identifying the structure of units</b> <ul style="list-style-type: none"> <li>draws and describes the column and row structure to represent area as an array, moving beyond counting of squares by ones</li> <li>calculates the total area using rows and/or columns as composite units</li> <li>uses familiar household items as benchmarks when estimating mass and capacity (compares capacities based on knowing the size of a bottle of water)</li> <li>estimates lengths that lie between full units by visualising subdivisions of the unit</li> </ul>
<b>UuM7</b>	<b>Using the structure of units</b> <ul style="list-style-type: none"> <li>explains the difference between different measures of the same shape or object (area and perimeter, volume and mass, volume and capacity)</li> <li>uses rows, columns and layers to find the number of units needed to measure volume</li> <li>creates and uses the structure of repeated layers in determining the volume of a rectangular prism</li> <li>uses dissection and rearrangement to calculate composite areas of unfamiliar shapes</li> </ul> <b>Using formal units</b> <ul style="list-style-type: none"> <li>measures, compares and estimates length, area, mass, volume and capacity using standard formal units</li> <li>calculates perimeter using properties of two-dimensional shapes to determine unknown lengths</li> </ul>
<b>UuM8</b>	<b>Converting units</b> <ul style="list-style-type: none"> <li>converts between formal units of measurement</li> <li>recognises the relationship between metric units of measurement and the base-ten place value system</li> <li>explains why having 100 cm in a metre results in 10 000 cm<sup>2</sup> in a square metre (using a diagram)</li> </ul>

## Understanding units of measurement

<b>UuM9</b>	<p><b>Calculating measurements</b></p> <ul style="list-style-type: none"><li>• uses dissection and rearrangement to calculate volumes of objects</li><li>• identifies appropriate levels of precision with measurement (significant figures)</li></ul> <p><b>Circle measurements</b></p> <ul style="list-style-type: none"><li>• understands that the relationship between the circumference and the diameter of a circle is constant (<math>\pi</math>)</li><li>• uses the constant <math>\pi</math> to determine the circumference and the area of a circle</li></ul>
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## Understanding geometric properties

This sub-element describes how a student becomes increasingly able to identify the attributes of shapes and objects and how they can be combined or transformed. Being able to use spatial reasoning and geometric properties to solve problems is important for a range of tasks. For example, dissection and rearrangement combined with basic geometric properties underpins surveying and building design, as well as interpreting plans.

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Level	Indicators
<p><i>Each sub-element level has been identified by upper-case initials of the sub-element name followed by ascending numbers. The abbreviation for this sub-element is UGP. The listing of indicators within each level is non-hierarchical. Subheadings have been included to group related indicators. Where appropriate, examples have been provided in brackets following an indicator.</i></p>	
UGP1	<b>Familiar shapes and objects</b> <ul style="list-style-type: none"> <li>uses everyday language to describe and compare shapes and objects</li> <li>finds similar shapes or objects in the environment</li> </ul>
UGP2	<b>Features of shapes and objects</b> <ul style="list-style-type: none"> <li>identifies and describes features of shapes and objects</li> <li>describes what an object may look like from a different perspective</li> <li>recognises features of shapes of different sizes and in different orientations following flips, slides and turns</li> <li>sorts objects based on their features</li> </ul> <b>Angles</b> <ul style="list-style-type: none"> <li>identifies angles as greater than, less than or equal to a right angle</li> </ul>
UGP3	<b>Properties of shapes and objects</b> <ul style="list-style-type: none"> <li>relates the faces of a three-dimensional object to two-dimensional shapes</li> <li>aligns the corresponding faces of an object and its net</li> <li>identifies the relationship between the number of edges of a shape and the number of corners (if the shape has 4 edges, it has 4 corners)</li> <li>represents shapes and objects (sketching, model building, digital drawing packages)</li> </ul>
UGP4	<b>Symmetry</b> <ul style="list-style-type: none"> <li>recognises that shapes can have lines of symmetry (by folding shapes or using mirrors)</li> <li>identifies the different shapes that enable the creation of symmetrical designs</li> </ul>
UGP5	<b>Angles and lines</b> <ul style="list-style-type: none"> <li>recognises the angles at a point add to <math>360^\circ</math></li> <li>estimates and identifies measures of angles in degrees up to one revolution</li> <li>uses angle properties to identify perpendicular and parallel lines</li> </ul>

## Understanding geometric properties

<b>UGP6</b>	<b>Geometric properties</b> <ul style="list-style-type: none"><li>• uses relevant properties of geometrical figures to find unknown lengths and angles</li></ul>
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## Positioning and locating

This sub-element describes how a student becomes increasingly able to recognise the attributes of position and location, and to use positional language to describe himself/herself and objects in various locations. A student learns to reason with representations of shapes and objects regarding position and location, and to visualise and orientate objects to solve problems in spatial contexts, such as navigating. The use of scales on maps is an application of proportional reasoning.

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Level	Indicators
<p><i>Each sub-element level has been identified by upper-case initials and in some cases lower-case letters of the sub-element name followed by ascending numbers. The abbreviation for this sub-element is PoL. The listing of indicators within each level is non-hierarchical. Subheadings have been included to group related indicators. Where appropriate, examples have been provided in brackets following an indicator.</i></p>	
PoL1	<p><b>Position to self</b></p> <ul style="list-style-type: none"> <li>locates positions in the classroom relevant to self (hangs bag on own hook, puts materials in own tray)</li> <li>orients self to obtain a desired object</li> <li>follows simple instructions using positional language (stand up, sit down, put your lunch box in your bag)</li> </ul>
PoL2	<p><b>Position to other</b></p> <ul style="list-style-type: none"> <li>uses positional terms with reference to themselves (left and right)</li> <li>interprets a simple diagram or picture to describe the position of an object (the house is between the river and the school)</li> <li>gives and follows directions from one place to another</li> </ul>
PoL3	<p><b>Using an informal map</b></p> <ul style="list-style-type: none"> <li>draws an informal map or sketch to provide directions</li> <li>locates positions on an informal map</li> <li>orients an informal map using recognisable landmarks and current location</li> <li>locates self on an informal map to select an appropriate path to a given location</li> </ul>
PoL4	<p><b>Using formal maps and plans</b></p> <ul style="list-style-type: none"> <li>locates position on maps using grid references</li> <li>identifies features on maps and plans</li> <li>describes routes using landmarks and directional language</li> </ul>
PoL5	<p><b>Interpreting maps and plans</b></p> <ul style="list-style-type: none"> <li>interprets the scale as a ratio used to create plans, drawings or maps</li> <li>interprets plans involving scale</li> <li>uses compass directions, latitude and longitude to locate position</li> </ul>

## Measuring time

This sub-element describes how a student becomes increasingly aware of the passage of time. A student appreciates units of time are associated with regularly occurring events, such as the rotation of Earth or the swing of a pendulum. They apply units and conventions associated with measuring and recording the succession and duration of time. Analogue clocks are used in telling time as they better reflect the non-decimal units of hours, minutes and seconds. The hour locations of the clock face can also be used as a way of providing bearings (at '3 o'clock').

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Level	Indicators
<p><i>Each sub-element level has been identified by upper-case initials and in some cases lower-case letters of the sub-element name followed by ascending numbers. The abbreviation for this sub-element is MeT. The listing of indicators within each level is non-hierarchical. Subheadings have been included to group related indicators. Where appropriate, examples have been provided in brackets following an indicator.</i></p>	
MeT1	<b>Sequencing time</b> <ul style="list-style-type: none"> <li>uses the language of time to describe events in relation to past, present and future (yesterday, today, tomorrow, next week)</li> <li>applies understanding of passage of time to sequence daily events</li> </ul>
MeT2	<b>Telling time</b> <ul style="list-style-type: none"> <li>uses the appropriate time unit to describe the duration of events (uses minutes to describe time taken to clean teeth whereas uses hours to describe the duration of a long-distance car trip)</li> <li>reads time on analogue clocks to the hour, half-hour and quarter-hour</li> </ul> <b>Calendars</b> <ul style="list-style-type: none"> <li>names and orders months of the year</li> <li>recognises a sequence of seasons on a calendar</li> <li>uses a calendar to identify the date</li> </ul>
MeT3	<b>Units of time</b> <ul style="list-style-type: none"> <li>uses standard instruments and units to describe and measure time to minutes</li> <li>reads and interprets different representations of time on an analogue clock, digital clock or timer</li> </ul>
MeT4	<b>Relating units of time</b> <ul style="list-style-type: none"> <li>explains the relationship between different units of time (months and years; seconds, minutes and hours)</li> <li>uses am and pm notation to distinguish between 12-hour time and 24-hour notation</li> <li>determines elapsed time using different units (hours and minutes, days and weeks)</li> </ul>

## Measuring time

<b>MeT5</b>	<b>Time zones</b> <ul style="list-style-type: none"><li>• uses appropriate units for measuring both large and small durations of time (millenniums, nanoseconds)</li><li>• interprets 12- and 24-hour time within a single time zone</li><li>• identifies issues associated with different time zones</li><li>• identifies the relationship between longitude and time zones (investigates the location of the international date line)</li></ul>
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# Statistics and probability

## Understanding chance

Our modern understandings of probability date from the second half of the 17th century with the analysis of games of chance. Many of the basic ideas of probability can run contrary to common beliefs. Recognising how what has happened in the past does not influence what will happen in the future with independent events often needs people to overcome strongly held beliefs. This sub-element describes how a student becomes increasingly able to use the language of chance and the numerical values of probabilities when determining the likelihood of an event. Understanding chance is often essential to interpret data.

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Level	Indicators
<p><i>Each sub-element level has been identified by upper-case initials and in some cases lower-case letters of the sub-element name followed by ascending numbers. The abbreviation for this sub-element is UnC. The listing of indicators within each level is non-hierarchical. Subheadings have been included to group related indicators. Where appropriate, examples have been provided in brackets following an indicator.</i></p>	
UnC1	<b>Describing chance</b> <ul style="list-style-type: none"> <li>describes everyday occurrences that involve chance</li> <li>recognises that some events might or might not happen</li> <li>makes predictions on the likelihood of simple, everyday occurrences</li> </ul>
UnC2	<b>Comparing chance</b> <ul style="list-style-type: none"> <li>explains why one result is more likely than another (if there are more blue than red marbles in a bag, blue is more likely to be selected)</li> <li>explains why outcomes of chance experiments may differ from expected results</li> </ul>
UnC3	<b>Fairness</b> <ul style="list-style-type: none"> <li>identifies all possible outcomes from simple experiments</li> <li>explains that 'fairness' of chance outcomes is related to the equal likelihood of all possible outcomes</li> <li>identifies unfair elements in games that affect the chances of winning (having an unequal number of turns)</li> <li>recognises that all probabilities must lie between impossible (no chance) and certain</li> </ul>
UnC4	<b>Probabilities</b> <ul style="list-style-type: none"> <li>expresses probability as the number of ways an event can happen out of the total number of possibilities</li> <li>describes probabilities as fractions of one (the probability of an even number when rolling a dice is <math>\frac{1}{2}</math>)</li> </ul>

## Understanding chance

UnC5

### Calculating probabilities

- describes the likelihood of events using a fraction or percentage
- interprets the odds of an event (odds of 5:1, the odds against rolling a 6, means a wager of \$1 stands to win \$5)
- explains how probability is not affected by previous results (If a coin is tossed and heads have come up 7 times in a row, it is still equally likely that the next toss will be heads or tails)
- recognises that the chance of something occurring or not occurring has a total probability of 1 (the probability of rolling a 3 is  $\frac{1}{6}$  and the probability of not rolling a 3 is  $\frac{5}{6}$ )
- determines the probability of compound events (tossing 2 coins)
- compares expected and actual results of a chance event

## Interpreting and representing data

This sub-element describes how a student becomes increasingly able to recognise and use visual and numerical displays to describe data associated with statistical investigations, and to critically evaluate investigations by others. Making sense of data draws on knowing the concepts and tools that are being used to describe the global features of data. A student understands how these concepts and tools make meaning of data in context, and develops the ability to think critically about any claims, either questioning or confirming them.

Arguments presented in the media often need to be considered in terms of the supporting data.

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Level	Indicators
<p><i>Each sub-element level has been identified by upper-case initials of the sub-element name followed by ascending numbers. The abbreviation for this sub-element is IRD. The listing of indicators within each level is non-hierarchical. Subheadings have been included to group related indicators. Where appropriate, examples have been provided in brackets following an indicator.</i></p>	
IRD1	<p><b>One-to-one data displays</b></p> <ul style="list-style-type: none"> <li>displays information using real objects or photographs</li> <li>responds to questions about the information in one-to-one data displays</li> <li>interprets general observations made about data represented in one-to-one data displays</li> <li>makes comparisons from categorical data displays using relative heights from a common baseline</li> <li>draws reasonable conclusions from one-to-one data displays</li> </ul>
IRD2	<p><b>Collecting and displaying data</b></p> <ul style="list-style-type: none"> <li>justifies data collection methods to fit the context</li> <li>interprets and uses structural elements in data displays (labels, symbols)</li> </ul>
IRD3	<p><b>Interpreting data scales</b></p> <ul style="list-style-type: none"> <li>interprets categorical data using a many-to-one graphical display, as well as simple histograms and stacked dot plots</li> <li>explains how data displays can be misleading (whether a scale should start at zero)</li> <li>interprets data displayed using a multi-unit scale, reading values between the marked units</li> </ul>
IRD4	<p><b>Shape of data displays</b></p> <ul style="list-style-type: none"> <li>determines and calculates the most appropriate statistic to describe the data</li> <li>uses simple descriptive statistics (arithmetic mean or median) as measures to represent typical values of a distribution</li> <li>compares the usefulness of different representations of the same data</li> </ul>



## Interpreting and representing data

IRD5	<p><b>Graphical representations of data</b></p> <ul style="list-style-type: none"> <li>• uses graphical representations relevant to the purpose of the collection of the data</li> <li>• uses features of graphical representations to make predictions</li> <li>• recognises that continuous variables depicting growth or change often vary over time (growth charts, temperature charts)</li> <li>• interprets graphs depicting motion such as distance–time graphs</li> <li>• interprets and describes patterns in graphical representations in real-life situations (roller-coasters, flight trajectory)</li> <li>• interprets the impact of outliers in data</li> <li>• determines whether to use data from a sample or a population</li> <li>• determines what type of sample to use from a population</li> <li>• makes reasonable statements about a population based on evidence from samples</li> </ul>
IRD6	<p><b>Recognising bias</b></p> <ul style="list-style-type: none"> <li>• applies an understanding of distributions to evaluate claims based on data (the larger the sample taken, the more accurate the prediction of the population value is likely to be)</li> <li>• recognises and explains bias as a possible source of error in media reports of survey data</li> <li>• justifies criticisms of data sources that include biased statistical elements (inappropriate sampling from populations)</li> </ul>