Lecture 04

§10 Sets I: Introduction, Subsets

- A *set* is a repetition-free, unordered collection of objects. An object that belongs to a set is called an *element* of the set. The notation for membership in a set is $\in$. The notation $x \in A$ means that the object x is a member of the set A. It reads "x is a member of A", or "x is an element of A", or simply "x is in A". The notation $x \notin A$ means x is not a member of the set A.
- *Cardinality*: the number of elements of a set is called the *cardinality* (or *size*) of the set. Notation $|A|$ denotes the cardinality of a set A.
- Ways to specify a set: (1) small size set: list all the elements between curly braces; (2) *set-builder notation*: {dummy variable: conditions}; (3){dummy variable $\in$ set: conditions}.
- A few common sets: $\mathbb{Z}$, $\mathbb{N}$, $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$, and $\emptyset$.
- Relation between two sets:

  Two sets A and B are equal if and only if they have exactly the same elements. Notation: $A = B$.

  If all the elements of a set A are in a set B, then we call A is a *subset* of B. Notation: $A \subseteq B$.

  Note that $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.
- Examples:
  (1) Let P be the set of Pythagorean triples: $P = \{(a, b, c) : a, b, c \in \mathbb{Z} \text{ and } a^2 + b^2 = c^2\}$, and T be the set $T = \{(x^2 - y^2, 2xy, x^2 + y^2) : x, y \in \mathbb{Z}\}$. Prove $T \subseteq P$. Furthermore, show $T \neq Q$.
- If A is a finite set, then the number of subsets of A is $2^{|A|}$. All the subsets of A form a new set which is called the *power set* of A. Notation: 2^A.
- Examples:
  (2) Find the cardinality of the set $\{x \in 2^{\{1,2,3,4\}} : |x| = 2\}$.
  (3) Put $\in$ or $\subseteq$ in $\bigcirc$:
    (a) $\emptyset \bigcirc \{1, 2, 3\}$
    (b) $\{2\} \bigcirc \{\{1\}, \{2\}, \{3\}\}$
    (c) $\{2\} \bigcirc \{1, 2, 3\}$
    (d) $\emptyset \bigcirc 2^{\mathbb{N}}$
    (e) $2^{\mathbb{N}} \bigcirc 2^{2^{\mathbb{N}}}$

§11 Quantifiers

- Existential quantifier *there is* $\cdots$.
- Existential statement: $\exists x \in A$, such that assertions about x hold.
- Universal quantifier *for all* $\cdots$.
- Universal statement: $\forall x \in A$, assertions about x hold.
- Combining Quantifiers:

$\exists x \in A$ such that $\forall y \in B$, assertions about x and y hold.

$\forall y \in B$, $\exists x \in A$, such that assertions about x and y hold.

- Examples:

  (4) The two statements are different: (1) $\forall x, \exists y, x + y = 0$. (2) $\exists y, \forall x, x + y = 0$.

HW2(b) (Due 2/8/2016)

- 10.3
- 10.12
- 11.1 (c),(d),(h),(i)