Lecture 02

§6 Counterexample

- To disprove "if A then B", we just need to find an example where A is true but B is false.
- Examples:

  (1) Disprove: If a, b, c are positive integers such that $a \mid bc$, then $a \mid b$ or $a \mid c$.

  (2) Disprove: if p is prime, then $2^p - 1$ is also prime.

§7 Boolean algebra

- Boolean algebra includes expressions containing letters and operations, where each letter stands for the value TRUE or FALSE. The basic operations are

  $\wedge$ *and*: $x \wedge y$ is true if and only if both x and y are true.

  $\vee$ *or*: $x \vee y$ is true if and only if at least one of x and y is true.

  $\neg$ *not*: $\neg x$ is true if and only if x is false.

  $\rightarrow$ *if$\cdots$ then$\cdots$*:$x \rightarrow y$ is true if and only if the statement "if x then y" is true. In other words, $x \rightarrow y$ is always true unless x is true but y is false.

  $\leftrightarrow$ *if and only if*: $x \leftrightarrow y$ is true if and only if x and y are both true or both false.

- A technique to prove the equivalence of Boolean expressions: use 1 and 0 to represent true and false, then each Boolean expression has a value 0 or 1. In other words, define a function ϕ from Boolean expressions to 1 and 0: ϕ(true)=1 and ϕ(false)=0. The operations of Boolean expressions becomes the following calculations of 1and 0:

  (1) $\phi(x \wedge y) = \phi(x)\phi(y)$.

  (2) $\phi(x \vee y) = \phi(x) + \phi(y) - \phi(x)\phi(y)$.

  (3) $\phi(\neg x) = 1 - \phi(x)$.

  (4) $\phi(x \rightarrow y) = 1 - \phi(x) + \phi(x)\phi(y)$.

  (5) $\phi(x \leftrightarrow y) = \phi(x)\phi(y) + (1 - \phi(x))(1 - \phi(y))$.

- Examples:

  (3) Prove that $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$ and that $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$.

  (4) Prove that $\neg(x \vee y) = (\neg x) \wedge (\neg y)$ and that $\neg(x \wedge y) = (\neg x) \vee (\neg y)$.

  (5) Prove that $(x \wedge y) \vee (x \wedge \neg y)$ is equivalent to x.

  (6) Prove that $(x \wedge (x \rightarrow y)) \rightarrow y$ is always true.

HW1(b) (Due 2/1/2016)

- 6.9
- 7.8
- 7.13 (b),(c)