

Lecture 01

§3 Definition

- Definition (in mathematics) gives a precise meaning of a term.
- Some typical definition forms:
An object X is called *the term being defined* provided it satisfies *specific conditions*.
We say X is *the term being defined* provided it satisfies *specific conditions*.
- Examples:
 - (1) Define the following terms: divisible, odd, even, prime, composite. You could use the basic concept of integers (including additions and multiplications).
 - (2) Suppose the distance between two points in the plane is defined already. Give a definition of *collinear* of three points.

§4 Theorem

- A *theorem* is a declarative statement (about mathematics) for which there is a proof.
- Differences between theorems, conjectures, and mistakes.
- The table of “If A then B ” statement

A	B	
True	True	Possible
True	False	Impossible
False	True	Possible
False	False	Possible

- Equivalent expressions to “If A then B ”:
 - (1) A implies B .
 - (2) Whenever A , we have B .
 - (3) A is sufficient for B .
 - (4) B is necessary for A .
 - (5) $A \implies B$.
- (Vacuous truth) Sometimes the condition A is always impossible. In this case the statement “If A then B ” is always true.
- The table of “ A if and only if B ” statement

A	B	
True	True	Possible
True	False	Impossible
False	True	Impossible
False	False	Possible

- Equivalent expressions to “ A if and only if B ”:
 - (1) A iff B .
 - (2) A is sufficient and necessary for B .

- (3) A is true exactly when B is true.
- (4) $A \iff B$.
- And, Or, and Not statements.

§5 Proof

- Direct proof of “if A then B ”
 - (1) Invent suitable notations and restate A .
 - (2) Write down B .
 - (3) Figure out a bridge between A and B .
- Direct proof of “ A if and only if B ”: prove “if A then B ” and “if B then A ”.
- Examples:
 - (3) Let x be an integer. If $x > 1$, then $x^3 + 1$ is composite.
 - (4) Suppose x, y, z are positive real numbers. Prove that $x^3 + y^3 + z^3 \geq 3xyz$.

HW1(a) (Due 2/1/2016)

- 3.4
- 4.2(a),(g),(h)
- 5.21