Lecture 05

§11 Quantifiers (Continued)

- Negating Quantified Statements
  (1) $\neg\forall = \exists\neg$
  (2) $\neg\exists = \forall\neg$
  (3) $\neg(A \text{ and } B) = \neg(A) \text{ or } \neg(B)$
  (4) $\neg(A \text{ or } B) = \neg(A) \text{ and } \neg(B)$
- Examples
  (1) Disprove the following statement: $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}$, such that $x^2 - y^2 = x + y$ and $x^3 - y^3 = x - y$.

§12 Set II: Operations

- $A \cup B$: the *union* of A and B, the set of all elements that are in A or in B.
- $A \cap B$: the *intersection* of A and B, the set of all elements that are in both A and B.
- A few facts about union and intersection:
  (1) $A \cup B = B \cup A$ and $A \cap B = B \cap A$.
  (2) $A \cup (B \cup C) = (A \cup B) \cup C$, $A \cap (B \cap C) = (A \cap B) \cap C$.
  (3) $A \cup \emptyset = A$ and $A \cap \emptyset = \emptyset$.
  (4) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ and $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
  One useful tool to visualize these identities is the so-called venn diagram.
- $|A| + |B| = |A \cup B| + |A \cap B|$, or equivalently, $|A \cup B| = |A| + |B| - |A \cap B|$.
- We say A and B *disjoint* provided $A \cap B = \emptyset$. If A and B are disjoint, then $|A \cup B| = |A| + |B|$. We say sets $A_1, A_2, \cdots, A_n$ are *pairwise disjoint* provided that $A_i \cap A_j = \emptyset$ whenever $i \neq j$. If $A_1, A_2, \cdots, A_n$ are pairwise disjoint, then $|A_1 \cup A_2 \cup \cdots \cup A_n| = |A_1| + |A_2| + \cdots + |A_n|$.
- Examples
  (2) How many integers between 1 and 2000 are divisible by 3 and 5?
  (3) How many integers between 1 and 2000 are not divisible by any of $2, 3$ or 5?
- Difference and Symmetric Difference: $A - B$
  $A - B$: the *difference* of A and B, the set of all elements that are in A but not B.
  $A \Delta B = (A - B) \cup (B - A)$: the *symmetric difference* of A and B.
- Examples
  (4) $B = (A \cup B) - (A \cap B)$.
  (5) $A - (B \cup C) = (A - B) \cap (A - C)$.
  (6) $A - (B \cap C) = (A - B) \cup (A - C)$.

(7) $A\Delta(B\Delta C) = (A\Delta B)\Delta C$.

- Cartesian Product $A \times B = \{(a,b) : a \in A, b \in B\}$.
- Properties of Cartesian product: $A \times B \neq B \times A$. $|A \times B| = |A| \times |B|$.
- (Optional topic) Relation between Set Operations and Boolean Algebra
  $x \in A \cap B \iff (x \in A) \wedge (x \in B)$.
  $x \in A \cup B \iff (x \in A) \vee (x \in B)$.
  $x \in A - B \iff (x \in A) \wedge (x \notin B) \iff (x \in A) \wedge \neg(x \in B)$.
  $x \in A\Delta B \iff ((x \in A) \wedge (x \notin B)) \vee ((x \notin A) \wedge (x \in B))$.

HW3(a) (Due 2/17/2016)

- 12.3
- 12.21 (including your proof or counterexample for each statement).