Lecture 11

§23 Recurrence

- Linear homogeneous recurrence
  A *linear homogeneous recurrence* of degree k is a relation of the form
  $$a_n = s_1 a_{n-1} + s_2 a_{n-2} + \cdots + s_k a_{n-k},$$
  where $s_1, \cdots, s_k$ are fixed numbers and $s_1 \neq 0, s_k \neq 0$.
- *Linear inhomogeneous recurrence* of degree k is a relation of the form
  $$a_n = s_1 a_{n-1} + s_2 a_{n-2} + \cdots + s_k a_{n-k} + c,$$
  where $s_1, \cdots, s_k, c$ are fixed numbers and $s_1 \neq 0, s_k \neq 0, c \neq 0$.
- In this course, we will learn how to solve linear homogeneous/inhomogeneous recurrences of degree 1 or 2.
- For linear homogeneous recurrence relation:
  (1) If degree is 1, $a_n = s_1 a_{n-1}$. Then a_n is a geometric series given by $a_n = a_0 s_1^n$.
  (2) If degree is 2, $a_n = s_1 a_{n-1} + s_2 a_{n-2}$. Usually we can find two geometric sequences satisfying this relation: $a_n = r_1^n$ and $a_n = r_2^n$, where r_1, r_2 are two solutions to the following *characteristic equation*:
  $$r^2 = s_1 r + s_2.$$
  Then a general solution is given by $a_n = c_1 r_1^n + c_2 r_2^n$, where c_1, c_2 are both constants which are determined by a_0, a_1.
  (3) In some special cases, the characteristic equation has a double root r. Then the general solution is given byu $a_n = c_1 r^n + c_2 n r^n$, where c_1, c_2 are both constants which are determined by a_0, a_1.
- For inhomogeneous recurrence relation: first try to find a special solution by guess (constant, or polynomials of n, or power functions of n, or exponential function, etc.). Then the special solution plus that of the homogeneous solution gives the general solution.
- Examples: solve the following recurrence relations.
  (1) $a_n = 5a_{n-1} - 6a_{n-2}$, $a_0 = 2$, $a_1 = 5$.
  (2) $b_n = 3b_{n-1} + 4b_{n-2}$, $b_0 = 3, b_1 = 2$.
  (3) $F_n = F_{n-1} + F_{n-2}$, $F_0 = F_1 = 1$.
  (4) $c_n = 2c_{n-1} - 2c_{n-2}$, $c_0 = 1$, $c_1 = 3$.
  (5) $d_n = 4d_{n-1} - 4d_{n-2}$, $d_0 = 1$, $d_1 = 3$.
  (6) $e_n = 2e_{n-1} + 2$, $e_0 = 1$.
  (7) $g_n = g_{n-1} + n$, $g_0 = 1$.
  (8) $h_n = 2h_{n-1} - h_{n-2} + 2$, $h_0 = 4$, $h_1 = 2$.
  (9*) $p_n = 3p_{n-1} - 2q_{n-1}, q_n = 2p_{n-1} - q_{n-1}$, $p_1 = 1$, $q_1 = 2$.

HW6(a) (Due 3/28/2016)

- 23.2 (f), (g), (i), (j), (k), (m)
  Note: Do not require to calculate a_9 for this problem.