Lecture 06

§14 Relations

- A *relation* is a set of ordered pairs. Notation: $x \; R \; y \iff (x, y) \in R$. $x \not R \; y \iff (x, y) \notin R$.
- We say R is a *relation on* a set A provided $R \subseteq A \times A$, and R is a *relation from* set A *to* set B provided $R \subseteq A \times B$.
- *Inverse relation*: $R^{-1} = \{(x, y) : (y, x) \in R\}$. $(R^{-1})^{-1} = R$.
- Let R be a relation on a set A.
  - If for all $x \in A$ we have $x \; R \; x$, we call R *reflexive*.
  - If for all $x \in A$ we have $x \not R \; x$, we call R *irreflexive*.
  - If for all $x, y \in A$ we have $x \; R \; y \implies y \; R \; x$, we call R *symmetric*.
  - If for all $x, y \in A$ we have $(x \; R \; y) \wedge (y \; R \; x) \implies x = y$, we call R *antisymmetric*.
  - If for all $x, y, z \in A$ we have $(x \; R \; y) \wedge (y \; R \; z) \implies x \; R \; z$, we call R *transitive*.
- Examples
  (1) Relations on $\mathbb{Z}$: $\leq$, $<$,$\geq$, $>$, $=$.
  (2) Define a relation R on the set $\{(x, y) : x, y \in \mathbb{R}\}$: $(x, y) \; R \; (x', y')$ provided $(x - x')(y - y') \geq 0$. Is this relation reflexive? symmetric? transitive?

§15 Equivalence Relations

- Let R be a relation on a set A. We say R is an *equivalence relation* provided it is reflexive, symmetric, and transitive.
- *Equivalence Class*: Let R be an equivalence relation on a set A and $a \in A$. The *equivalent class of* a, denoted $[a]$, is the set of all elements of A related (by R) to a; that is, $[a] = \{x \in A : x \; R \; a\}$.
- If R is an equivalence relation on a set A. Then the equivalence classes of R are nonempty, pairwise disjoint subsets of A whose union is A.
- **Congruence modulo** n: Let n be a positive integer. We say that integers x and y are *congruent modulo* n provided $n|(x - y)$. The notation is $x \equiv y (\mod n)$. This congruence-mod-n relation is an equivalence relation on $\mathbb{Z}$.
- Examples
  (3) Let $x, y \in \mathbb{Z}$. $x \equiv y (\mod 2)$ if and only if x, y are both odd or both even.
  (4) There are n equivalence classes of the congruence-mod-n relation.
  (5) Let X be the set of ordered pairs of integers (a, b) with $b \neq 0$, and define a relation R on X according to which $(a, b) \; R \; (c, d)$ if and only if $ad = bc$. Then R is an equivalence relation. Moreover, the equivalence class of the pair (a, b) can be identified with the rational number a/b.

HW3(b) (Due 2/17/2016)

- 14.10
- 15.3 (b), (c), (e)
- 15.7 (b), (c), (f)