Lecture 03

§8 Lists

- A *list* is an ordered sequence of objects. The number of elements in a list is called its *length*. A list of length 2 is also called an *ordered pair*. A list of length 0 is called the *empty list*.
- Note that in a list an element might repeat. And two lists are equal provided they have the same length and elements in the corresponding positions on the two lists are equal.
- One basic method to count the number of lists satisfying certain conditions: *multiplication principle*.
- Examples:

  (1) (a) How many 2-digit numbers are there including 10 and 99?

  (b) How many 2-digit numbers are even?

  (c) How many 2-digit numbers are formed by two different digits?

  (d) How many 2-digit numbers are even and are formed by two different digits?

  (2) Airports have names, but they also have three-letter codes. How many different airport codes are possible?

  (3) How many ways can we arrange 5 people in 7 seats?
- The number of lists of length k whose elements are chosen from a pool of n possible elements is n^k if repetitions are permitted, or $n(n-1)\cdots(n-k+1)$ if repetitions are forbidden.
- *Falling factorial*: $(n)_k = n(n-1)\cdots(n-k+1)$.
- Sometimes we also need the *addition principle* or other techniques. Examples:

  (4) How many 8-digit numbers have exact one 8?

  (5) A class contains ten boys and five girls. In how many different ways can they stand in a line if no two girls are standing next to one another?

§9 Factorial

- $(n)_n$ is also called n *factorial* and is written $n!$.
- Production notation: $\Pi_{i=a}^{b} f(i) = f(a)f(a+1)\cdots f(b)$ for all integers a and b satisfying $a \leq b$, where f is an function of a.
- For convenience we define $\Pi_{k=a}^{b} g(k) = 1$ if $a > b$. Compare it with the summation $\sum_{k=a}^{b} h(k) = 0$ if $a > b$.
- Examples:

  (6) Prove the following inequality: $1 - \frac{1}{\sqrt{n}} < \left(\frac{1}{n}\right)^{\frac{1}{n}} < 1 + \frac{1}{\sqrt{n}}$. This fact suggests $0^0 = 1$.

  (7) Prove the following identity:

$$\Pi_{k=1}^{n} k! = \Pi_{k=1}^{n} k^{n+1-k}. \tag{1}$$

HW2(a) (Due 2/8/2016)

- 8.12 (b), (d), (g)
- 8.15
- 9.10