# The latent factor structure of child development

**Anonymous Cogsci Submission**

**Abstract**

Hello

**Keywords:** one; two;

## Introduction

TO DO

## Data

A child's development can be thought of as the set of developmental milestones that they have reached at a particular point in time. This conceptualization results in data with the same structure as the item response data common to educational measurement. In education, item response data is most typically students responding to test items (i.e., questions) and, in the dichotomous case, getting each question either correct or incorrect. In the context of child development, the child is the "student," and each developmental milestone is the "item."

We use Kinedu, a Mexico-based child development app, as a source for this type of data. When parents first start using the Kinedu app, they are asked a series of questions about which developmental milestones their child has reached. We consider the 1946 children between 2 and 55 months of age whose parents responded to all 414 of the developmental milestones. Each developmental miletone on Kinedu is mapped to a milestone group: physical, cognitive, linguistic, or social & emotional. Table 1 shows the number of developmental milestones in each group along with an example milestone translated to English.

Table 1: Developmental milestone groups and examples

| Group | Count | Milestone |
|---|---|---|
| Physical | 180 | Stands on their toes |
| Cognitive | 100 | Finds objects on the floor |
| Linguistic | 75 | Babbles to imitate conversations |
| Social & Emotional | 59 | Complains when play is interrupted |

Figure 1 shows the age (in months) and number of developmental milestones for each child. At 12 months old, most children have reached about 200 developmental milestones. At 24 months old, most children have reached about 300 developmental milestones. Finally, at 48 months old, most children have reached about 375 of the 414 developmental milestones.

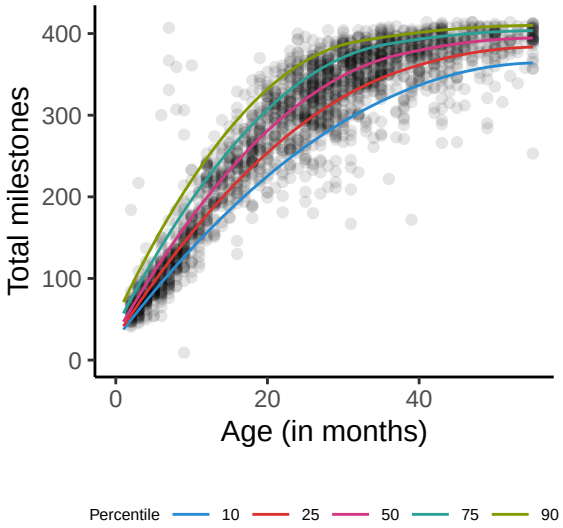


Figure 1: Number of milestones by age

## Empirical assessment of the dimensionality of child development

We frame the assessment of the dimensionality of child development as a model comparison question.

### Models

Item response theory offers a suite of models with which to model item response data. We adopt the notation used in Chalmers & others (2012). Let $i = 1, \ldots, I$ represent the distinct children and $j = 1, \ldots, J$ the developmental milestones. The Kinedu item response data is stored in a matrix, $y$, where element $y_{ij}$ denotes if the $i$th child has or has not achieved the $j$th developmental milestone as reported by their parent/guardian. Each model represents the $i$th child's development using $m$ latent factors $\boldsymbol{\theta}_i = (\theta_1, \ldots, \theta_m)$. The $j$th milestone's discriminations (i.e. slopes) $\boldsymbol{a_j} = (a_1, \ldots, a_m)$ capture the latent factor loadings onto that milestone.

We fit four two-parametric logistic (2PL) models where a child's development is represented by $m = 1$, $m = 2$, $m = 3$, and $m = 4$ latent factors. According to the 2PL model, the probability of a child having achieved a developmental milestone is

$$P(y_{ij} = 1 | \boldsymbol{\theta_i}, \boldsymbol{a_j}, b_j) = \sigma(\boldsymbol{a_j}^\top \boldsymbol{\theta_i} + b_j)$$

where $b_j$ is the milestone easiness (i.e. intercept) and $\sigma(x) = \frac{e^x}{e^x+1}$ is the standard logistic function.

The 2PL models learn the latent factor structure entirely from the data, making them exploratory. The bifactor model offers an alternative specification where each milestone loads onto a general factor $\theta_0$ and a specific factor $\theta_s$ (Cai, Yang, & Hansen, 2011). The assignment of each developmental milestone to its specific factor is an opportunity to specify the latent factor structure, making the model confirmatory as opposed to exploratory. We map each milestone to its specific factor according to the four developmental milestone groups shown in Table X. For the bifactor model, the probability of a child having achieved a developmental milestone is

$$P(y_{ij} = 1|\theta_0, \theta_s, a_0, a_s) = \sigma(a_0\theta_0 + a_s\theta_s + b_j).$$

## Model comparison

Model comparison in IRT typically uses information criterion such as AIC and BIC (Maydeu-Olivares, 2013). However, these methods are not guaranteed to work with modest sample sizes (McDonald & Mok, 1995). Instead, we prefer a marginalized version of cross-validation. In essence, we partition the data into folds based on the children (i.e. the rows of the item response matrix). Then for each fold, we estimate the item parameters using all but that fold, and calculate the likelihood of that fold by integrating over $g(\theta)$.

Mathematically and following notation similar to Vehtari, Gelman, & Gabry (2017), we partition the data into $K$ subsets $y^{(k)}$ for $k = 1, \ldots, K$. Each model is fit separately to each training set $y^{(-k)}$ yielding item parameter estimates which we compactly denote $\Psi_j^{(-k)}$. The predictive (i.e. out-of-sample) likelihood of $y^{(k)}$ is

$$p(y^{(k)}|y^{(-k)}) = \prod_{i \in i^{(k)}}^{I} \int_\theta \prod_{j=1}^{J} \hat{\Pr}(y_{ij}^{(k)}|\Psi_j^{(-k)}, \theta) g(\theta) d\theta.$$

The ultimate quantity of interest for each model is the log predictive likelihood for the entire item response matrix, which is defined as

$$\text{lpl } y = \sum_{k=1}^{K} \log p(y^{(k)}|y^{(-k)}).$$

## Results

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## Understanding the latent factor structure

To understand each of the three factors in the best performing model, we fit the model to the the full dataset. We then estimate the factor loadings (i.e. discriminations or slopes) using a varimax rotation. The varimax rotation results in orthogonal and, therefore, more interpretable factors (Kaiser, 1959). Figure 2 shows the distribution of factor loadings for each group on each of the three factors. The first factor is mainly cognitive and linguistic. The second factor is a combination of each of the groups with the strongest loadings on the physical and social & emotional milestones. The third mainly loads positively on linguistic milestones and negatively on physical milestones.

\begin{CodeChunk} \begin{CodeOutput}
Rotation: varimax
Rotated factor loadings:

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                F1        F2         F3      h2
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abs_12 -0.51680 -0.58278 0.044006 0.6087 abs_148 -0.77483 -0.34897 0.119377 0.7364 abs_183 -0.60944 -0.57465 0.001097 0.7016 abs_199 -0.62202 -0.36144 0.086368 0.5250 abs_203 -0.54057 -0.57353 0.063470 0.6252 abs_206 -0.56340 -0.50044 -0.046780 0.5700 abs_317 -0.53209 -0.49228 0.151364 0.5484 abs_385 -0.79606 -0.26714 0.025069 0.7057 abs_387 -0.78031 -0.34180 0.085323 0.7330 abs_69 -0.86880 -0.23527 0.002409 0.8102 attach_111 -0.23760 -0.35900 -0.059810 0.1889 attach_122 -0.23534 -0.32500 -0.207071 0.2039 attach_129 -0.01819 -0.17826 -0.197429 0.0711 attach_186 -0.19208 -0.18009 0.060800 0.0730 attach_20 -0.20588 -0.63238 0.100786 0.4524 attach_252 -0.07626 -0.42677 -0.344363 0.3065 attach_283 0.15579 -0.19063 -0.208809 0.1042 attach_308 -0.27002 -0.35564 -0.103310 0.2101 attach_36 -0.24869 -0.29504 -0.116073 0.1624 attach_441 -0.76651 -0.35221 -0.003165 0.7116 attach_97 -0.38659 -0.57228 -0.254446 0.5417 babbling_126 -0.01834 -0.61770 -0.093935 0.3907 babbling_143 -0.22360 -0.22116 -0.255054 0.1640 babbling_196 -0.04657 -0.57924 -0.129663 0.3545 babbling_197 0.02970 -0.54101 -0.243293 0.3528 babbling_23 0.19703 -0.54914 -0.210168 0.3846 babbling_247 0.16482 -0.37034 -0.293189 0.2503 babbling_301 0.35609 -0.58954 -0.235290 0.5297 babbling_31 -0.02729 -0.67950 -0.154188 0.4862 babbling_589 0.12125 -0.62995 -0.090468 0.4197 balance_635 -0.68247 -0.39343 0.274479 0.6959 balance_638 -0.73792 -0.24225 0.267574 0.6748 balance_640 -0.66218 -0.45047 0.277316 0.7183 balance_641 -0.68935 -0.23409 0.188265 0.5654 balance_671 -0.63827 -0.26058 0.215091 0.5216 balance_683 -0.67314 -0.26341 0.163717 0.5493 balance_708 -0.52089 -0.30648 0.175395 0.3960 balance_712 -0.57953 -0.29436 0.179695 0.4548 care_411 -0.71984 -0.33239 0.180054 0.6611 care_565 -0.76149 -0.44909 0.170122 0.8105 care_606 -0.64523 -0.36614 0.159087 0.5757 care_607 -0.59098 -0.25592 0.216885 0.4618 care_608 -0.75596 -0.27365 0.230677 0.6996 care_609 -0.64086 -0.30787 0.171594 0.5349 care_610 -0.79137 -0.18894 0.221224 0.7109 care_611 -0.72682 -0.19154 0.271319 0.6386 care_612 -0.78151 -0.17374 0.241659 0.6993 care_85 -0.74392 -0.29226 0.086208 0.6463 color_629 -0.63097 -0.25557 0.214753 0.5096 color_630 -0.84911 -0.22171 0.179524 0.8024 color_666 -0.84270 -0.14231 0.140191 0.7500 color_677 -0.84025 -0.15780 0.161535 0.7570 color_678 -0.79747 -0.16492 0.195257 0.7013 color_679 -0.82426 -0.15502 0.192348 0.7404 compreh_109 -0.83074 -0.33811 0.031930 0.8055 compreh_258 -0.75256 -0.50313 0.034655 0.8207 com-

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-0.288733 0.3294 jump_670 -0.74580 -0.29754 0.286775 0.7270 jump_680 -0.82880 -0.18018 0.277116 0.7962 jump_681 -0.77504 -0.12656 0.227203 0.6683 jump_682 -0.75346 -0.27676 0.260403 0.7121 jump_706 -0.79468 -0.20487 0.289809 0.7575 jump_713 -0.74917 -0.29081 0.314067 0.7445 jump_715 -0.80917 -0.27147 0.277880 0.8057 kick_168 -0.58525 -0.47296 0.192321 0.6032 kick_528 -0.74386 -0.37167 0.256035 0.7570 kick_642 -0.64909 -0.33087 0.253715 0.5952 kick_643 -0.73177 -0.38029 0.264351 0.7500 kick_714 -0.61106 -0.34817 0.265703 0.5652 memory_19 -0.80137 -0.38548 0.069265 0.7956 memory_421 -0.82079 -0.35578 0.030471 0.8012 memory_435 -0.82658 -0.17295 -0.083011 0.7200 memory_436 -0.51936 -0.23053 -0.054047 0.3258 memory_472 -0.76943 -0.31820 0.014306 0.6935 memory_478 -0.84481 -0.17671 -0.034799 0.7461 memory_524 -0.85426 -0.16151 -0.048335 0.7582 memory_613 -0.78525 -0.33762 0.102208 0.7410 memory_627 -0.81539 -0.23578 -0.004477 0.7205 memory_664 -0.83838 -0.20419 0.013822 0.7448 move_1 -0.25649 -0.77474 -0.010334 0.6661 move_114 -0.10789 -0.71923 -0.003777 0.5289 move_130 -0.30672 -0.51057 0.085440 0.3621 move_133 -0.40593 -0.55777 0.006601 0.4759 move_146 -0.16985 -0.44974 -0.105350 0.2422 move_149 -0.11143 -0.73172 0.004735 0.5479 move_153 -0.21642 -0.86206 0.197434 0.8290 move_159 -0.10137 -0.51849 -0.176788 0.3104 move_165 -0.34304 -0.77071 0.087298 0.7193 move_207 -0.04477 -0.40226 -0.024879 0.1644 move_288 -0.29735 -0.64061 0.018620 0.4991 move_294 -0.11344 -0.70484 0.037914 0.5111 move_299 -0.22245 -0.69838 -0.017745 0.5375 move_86 0.16859 -0.37996 -0.213382 0.2183 music_352 -0.78063 -0.23336 0.039766 0.6654 music_468 -0.66147 -0.21850 0.037773 0.4867 music_537 -0.85428 -0.05421 -0.066689 0.7372 music_655 -0.86011 -0.01980 -0.054284 0.7431 newborn_131 0.15394 -0.23865 -0.261474 0.1490 newborn_136 0.13158 -0.31932 -0.308159 0.2142 newborn_150 0.03536 -0.18037 -0.220922 0.0826 newborn_193 0.08219 -0.25083 -0.201681 0.1103 newborn_201 -0.03314 -0.13060 -0.162080 0.0444 newborn_21 -0.13688 -0.28194 -0.162996 0.1248 newborn_236 0.09735 -0.17050 -0.237761 0.0951 newborn_285 0.02998 -0.15195 -0.233612 0.0786 newborn_289 0.08493 -0.22928 -0.300269 0.1499 newborn_43 0.06656 -0.36740 -0.266369 0.2104 problem_409 -0.71081 -0.27185 0.160110 0.6048 problem_624 -0.65174 -0.35570 0.133339 0.5691 problem_626 -0.77449 -0.25373 0.170742 0.6934 problem_662 -0.59130 -0.25913 0.054810 0.4198 problem_669 -0.72710 -0.21314 0.182396 0.6074 prod_419 -0.91900 -0.03064 -0.143428 0.8661 prod_465 -0.91184 -0.13097 -0.124980 0.8642 prod_483 -0.93286 -0.01455 -0.122331 0.8854 prod_527 -0.90502 -0.11902 -0.041584 0.8350 prod_540 -0.76236 -0.17400 -0.141680 0.6315 prod_544 -0.91625 -0.05138 -0.117634 0.8560 prod_564 -0.91308 -0.11127 -0.083298 0.8530 prod_572 -0.90990 -0.05347 -0.136354 0.8494 pronoun_391 -0.82980 -0.11075 -0.203992 0.7424 pronoun_394 -0.87368 -0.10156 -0.185804 0.8081 pronoun_490 -0.68382 -0.16827 -0.121986 0.5108 pronoun_587 -0.86451 -0.01878 -0.191532 0.7844 pronoun_688 -0.86463 -0.05043 -0.153052 0.7735 pronoun_689 -0.86222 -0.04727 -0.147430 0.7674 pronoun_690 -0.79422 -0.04742 -0.092606 0.6416 pronoun_691 -0.83273 -0.03756 -0.134896 0.7131 pronoun_692 -0.85976 -0.02751 -0.167505 0.7680 pronoun_693 -0.84794 -0.05947 -0.138355 0.7417 relation_619 -0.80794 -0.12863 -0.062645 0.6732 relation_648 -0.43180 -0.30943 0.049950 0.2847 relation_649 -0.45181 -0.39761 0.055438 0.3653 relation_694 -0.60133 -0.34735 0.003749 0.4823 relation_84 -0.37381 -0.40225 0.087247 0.3091 run_637 -0.74455 -0.43411 0.281636 0.8221 run_711 -0.82411 -0.32618 0.328455 0.8934 run_721 -0.62137 -0.34187 0.228998 0.5554 run_723 -0.67167 -0.35261 0.243492 0.6348 run_725 -0.65397 -0.35414 0.291894 0.6383 run_726 -0.75445 -0.33216 0.345903 0.7992 run_727 -0.78941 -0.36483 0.267860 0.8280 scrib_330 -0.50138 -0.46482 0.172502 0.4972 scrib_377 -0.48003 -0.56073

0.173518 0.5750 scrib_481 -0.61539 -0.33095 0.208608 0.5317 scrib_516 -0.70865 -0.34658 0.185030 0.6565 scrib_518 -0.70703 -0.33653 0.172910 0.6430 scrib_519 -0.75983 -0.26339 0.212147 0.6917 scrib_559 -0.68848 -0.36513 0.197957 0.6465 self_603 -0.73496 -0.28367 0.091723 0.6291 self_618 -0.91067 -0.04858 -0.050417 0.8342 self_653 -0.90629 -0.08695 -0.107573 0.8405 self_658 -0.91740 0.00107 -0.091571 0.8500 senses_138 -0.53163 -0.25761 -0.371878 0.4873 senses_144 -0.11080 -0.28714 -0.344842 0.2136 senses_145 -0.27109 -0.55199 -0.296297 0.4660 senses_212 -0.38546 -0.32213 -0.133449 0.2702 senses_214 -0.23047 -0.32015 -0.183881 0.1894 senses_243 -0.48732 -0.24346 -0.115004 0.3100 senses_303 -0.10882 -0.28717 -0.416676 0.2679 senses_304 -0.43548 -0.13717 -0.100307 0.2185 senses_38 0.08274 -0.14217 -0.199517 0.0669 senses_577 -0.39413 -0.50233 0.039061 0.4092 senses_6 -0.20941 -0.40904 -0.219205 0.2592 senses_93 -0.27825 -0.21354 -0.069720 0.1279 shapes_396 -0.86533 -0.11358 0.073181 0.7671 shapes_397 -0.87163 -0.12943 0.073097 0.7818 shapes_398 -0.87673 -0.14244 0.086914 0.7965 shapes_399 -0.88285 -0.12244 0.088169 0.8022 shapes_427 -0.88796 -0.03677 0.019242 0.7902 shapes_429 -0.87530 -0.15893 0.123530 0.8067 shapes_430 -0.81969 -0.12291 0.092841 0.6956 shapes_431 -0.82982 -0.14239 0.110102 0.7210 shapes_502 -0.86349 -0.23439 0.168516 0.8289 shapes_503 -0.82046 -0.18519 0.146524 0.7289 shapes_521 -0.88792 -0.17512 0.100310 0.8291 shapes_660 -0.89847 -0.12306 0.066866 0.8269 sitting_118 -0.41699 -0.77118 0.108220 0.7803 sitting_151 -0.35771 -0.65760 0.068490 0.5651 sitting_160 -0.30939 -0.57811 -0.037519 0.4313 sitting_162 0.00407 -0.60536 -0.177203 0.3979 sitting_227 -0.01430 -0.60573 -0.076956 0.3730 sitting_270 -0.51735 -0.72748 0.188113 0.8323 sitting_272 -0.19567 -0.57296 -0.072943 0.3719 sitting_33 -0.01823 -0.46609 -0.150684 0.2403 sitting_92 -0.43001 -0.64712 0.083659 0.6107 standing_113 -0.41478 -0.76007 0.169243 0.7784 standing_208 -0.59154 -0.64677 0.317316 0.8689 standing_209 -0.54853 -0.64767 0.167364 0.7484 standing_245 -0.47631 -0.69602 0.206391 0.7539 standing_271 -0.17725 -0.58876 -0.016846 0.3783 standing_325 -0.62938 -0.55131 0.297733 0.7887 standing_350 -0.69098 -0.51985 0.323909 0.8526 standing_438 -0.65127 -0.54868 0.351565 0.8488 standing_459 -0.69313 -0.50986 0.329312 0.8488 standing_470 -0.67139 -0.52038 0.351033 0.8448 steps_124 -0.33700 -0.68834 0.107535 0.5989 steps_15 -0.12503 -0.64711 -0.070198 0.4393 steps_16 -0.66264 -0.61743 0.267613 0.8919 steps_309 -0.73323 -0.47663 0.332452 0.8753 steps_355 -0.59069 -0.64577 0.263696 0.8355 steps_4 -0.63048 -0.54870 0.393996 0.8538 steps_415 -0.53416 -0.61373 0.206056 0.7044 throw_456 -0.63879 -0.35951 0.240175 0.5950 throw_457 -0.71088 -0.48364 0.291987 0.8245 throw_458 -0.63159 -0.38259 0.222453 0.5948 throw_529 -0.60432 -0.33654 0.176118 0.5095 throw_530 -0.50243 -0.32162 0.235224 0.4112 throw_644 -0.62198 -0.32936 0.214171 0.5412 throw_672 -0.59106

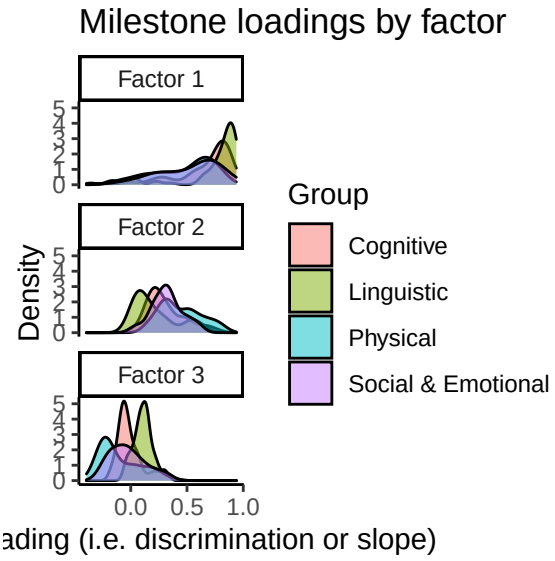


Figure 2: Factor loadings by group

-0.31941 0.245875 0.5118 throw_716 -0.58665 -0.33365 0.226027 0.5066 throw_717 -0.63433 -0.28103 0.251128 0.5444 throw_718 -0.47945 -0.29647 0.245500 0.3780 walk_182 -0.67615 -0.46837 0.325353 0.7824 walk_382 -0.53664 -0.60750 0.276979 0.7338 walk_471 -0.58521 -0.54243 0.280812 0.7156 walk_538 -0.72923 -0.45487 0.278307 0.8161 walk_639 -0.69834 -0.35391 0.290175 0.6971 walk_676 -0.73628 -0.26597 0.257621 0.6792 walk_684 -0.63499 -0.44943 0.217972 0.6527 walk_685 -0.68929 -0.38416 0.226307 0.6739 walk_709 -0.69429 -0.31448 0.296527 0.6689 walk_710 -0.72521 -0.27748 0.268015 0.6748 words_24 -0.79808 -0.23025 -0.150630 0.7126 words_249 -0.78007 -0.32015 -0.083887 0.7180 words_353 -0.88525 -0.09676 -0.218593 0.8408 words_373 -0.89474 -0.06526 -0.164544 0.8319 words_375 -0.81848 -0.16940 -0.136419 0.7172 words_437 -0.92674 -0.07124 -0.161292 0.8899 words_439 -0.85292 -0.15842 -0.108072 0.7643 words_445 -0.91155 -0.12918 -0.144866 0.8686 words_469 -0.92259 0.00185 -0.143191 0.8717 words_534 -0.89222 -0.07983 -0.067816 0.8070 words_54 -0.75873 -0.19707 -0.211569 0.6593 words_55 -0.85537 -0.24372 -0.047173 0.7933 words_573 -0.93519 -0.01089 -0.134006 0.8926 words_614 -0.91353 -0.09142 -0.193700 0.8804 words_7 -0.67352 -0.26548 -0.303189 0.6160 words_702 -0.63849 -0.10040 -0.334253 0.5295

Rotated SS loadings: 168.336 67.452 12.188

Factor correlations:

F1 F2 F3 F1 1 0 0 F2 0 1 0 F3 0 0 1 \end{CodeOutput}

\end{CodeChunk}

We also estimate the factor scores for each child using expected a posteriori (EAP) with a three dimensional standard normal distribution (Embretson & Reise, 2013). Figure 3 shows the relationship between age and factor score for each factor. The first factor, perhaps unsurprisingly, has a high correlation (r = 0.90) with age. The second factor has a strong

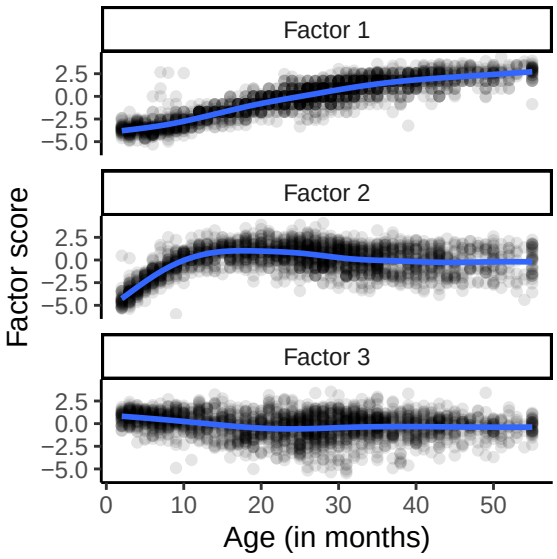


Figure 3: The first factor is highly associated with age

association with age from 2 to 16 months but thereafter is unrelated to age. By and large, the third factor does not have any association with age.

## Acknowledgements

We'd like to thank Kinedu for providing the data that made this research possible.

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