Error Variance

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$$\text{cov}(A, B) = E[AB] - E[A]E[B]$$

Recognize that $Y = X + e$:

$$Y - X = e$$

$$Y^2 - X^2 = (Y - X)(Y + X) = e(X + e + X) = 2Xe + e^2$$

$$\text{cov}(Y - X, Y^2 - X^2) = \text{cov}(e, (2Xe + e^2)) = E[2Xe^2 + e^3] - E[e]E[2Xe - e^2]$$

Assuming e is normally distributed with $\mu_e = 0$, we know $E[e] = 0$:

$$\text{cov}(Y - X, Y^2 - X^2) = E[2Xe^2 + e^3] - 0$$

$$\text{cov}(Y - X, Y^2 - X^2) = E[2Xe^2 + e^3]$$

$$\text{cov}(Y - X, Y^2 - X^2) = 2E[Xe^2] + E[e^3]$$

Note that X and e (and by extension, e^2) are independent, so:

$$\text{cov}(Y - X, Y^2 - X^2) = 2E[X]E[e^2] + E[e^3]$$

Assuming e is normally distributed with $\mu_e = 0$, we know:

$$E[e^2] = \sigma_e^2$$

$$E[e^3] = 0$$

Therefore:

$$\text{cov}(Y - X, Y^2 - X^2) = 2\sigma_e^2 E[X]$$