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The bid-choice land-use model: an integrated economic framework†

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Abstract. Alonso's bid-rent theory and the discrete-choice random-utility theory appear in the literature as well-established alternative frameworks to model urban land use. As both approaches share the support of microeconomic theory, the main issue addressed in this paper is the theoretical comparison of the two approaches. It is demonstrated that in perfectly competitive land markets these approaches are equivalent, therefore they should be understood as complementary rather than alternative. The case of markets subject to speculative land prices is then explored for the cases of speculative supply and/or speculative demand, with the discovery that both approaches are theoretically equivalent in every case studied, thus extending the previous conclusion for the general case. These conclusions provide the base for an integrated and more comprehensive urban economic theory and for the bid-choice land-use model.

1 Introduction

Since Alonso's (1964) work on urban household location theory, the idea of the land market behaving as a bid-auction process has been accepted as the best attempt to formulate a consistent theory on land economics. His original version was formulated in a deterministic form and for a monocentric city. This deserves strong criticism, but subsequent modellers have been able to overcome such constraint and still use the basic bid-auction model to describe the market clearing process.

For instance, Herbert and Stevens (1960) introduced a discrete representation of space in a model specified in a mathematical programming format, Wilson and Bennett (1985) combined the best bidder rule with the entropy model of spatial interaction, and Ellickson (1981) proposed a disaggregate version by using the logit model. Thus, the best-bid mechanism remains a popular concept among modelling proposals, each one an improvement on Alonso's original version, to give a more flexible and realistic model.

In an alternative approach, McFadden (1978), Anas (1982), and others propose to "depart from the bid-rent approach" (Anas, 1982, page 49). Instead, they prefer the use of the probabilistic discrete-choice process based on the well-established random-utility theory (see Domencich and McFadden, 1975). This approach can generate several specifications of logit models, either in a multinomial or in a hierarchical format.

Intuitively, there seem to be some differences between these alternative approaches. First, the maximisation process takes place on different sets of alternatives. The best-bidder approach compares alternative bids for a specific land lot and assigns the lot to the best-bidder household; that is, a maximisation process takes place across alternative bidders. Thus, land suppliers' maximisation of revenue is assured in this *market* clearing process or equilibrium model. The choice approach, on the other hand, is a demand model which considers each individual and searches for his or her *individual* location with regard to his or her own specific preferences.

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In this case, the maximisation process takes place across alternative land lots on offer in the market. Therefore, the first problem is to understand the theoretical meaning behind this apparent difference in the way in which the land market is modelled, from which a priori one could expect different results.

A second important difference refers to the assumption about the formation of land prices. In the bidding theory, land rents are the result of the bid-auction process; that is, market rents must be equal to the best bid for each location, therefore land rents are endogenous variables of the location process. In contrast, in the choice approach individuals are assumed to be rent takers with apparently no direct influence on rents; that is, land rents are exogenous variables to the location model.

In this paper, a comparison of the *bidding* rule with the *choice* model will demonstrate that these approaches are indeed equivalent, in the sense that they produce the same distribution of households in space. Furthermore, they are complementary, because they provide complementary information on the land-market clearing mechanism and the individuals' behaviour, and they are also inseparable because they impose theoretical constraints on each other. As a result of this comparison I shall propose an integrated model called the bid-choice land-use model.

The key element in the bridge between the two approaches is found in the fact that land prices are the result of buyers' valuation rather than any sort of 'production costs'; thus, prices are inevitably endogenous, as the bid theory suggests. But individuals' bids respond to the process of maximisation of their utility, as is well described by the choice approach.

It is worth noting, however, that my discussion and the model will refer to the residential land market. Any extension to the housing market must be examined carefully as dwelling production and maintenance costs are present, which requires a specific supply model (for example, see Anas, 1982).

In the following section, I present the main theoretical argument used in this paper, where the consumer surplus formulation of the choice model for the deterministic and competitive case is developed. Here the argument of the quasi-unique feature of land is introduced, which leads me to the conclusion that the bid and the choice approach are equivalent; then, the bid-choice model is presented. The equivalence between stochastic models follows in section 3, with the aggregate case discussed in section 4. In section 5, I shall investigate the noncompetitive (or speculative) market, extending my conclusions to this case. A summary of the argument and some issues arising are presented in section 6. A proposal of an integrated urban economic framework is given in the last section.

2 The theoretical argument

2.1 *The consumer decision*

Let us begin with a brief revision of the maximum utility model, leading to a definition of the maximum consumer-surplus model which provides a better link with the best-bid approach.

Suppose that land lots are well described by $z = (z_1, \dots, z_n)$, with z_i measuring the ith attribute, and that consumers purchase only one lot of land. Write the utility function for consumer h as $\mathrm{U}_h(x, z)$, quasi-concave and with all other usual properties. Set the price of all other goods consumed x equal to unity and measure income Y in terms of units of x; the budget constraint is then given by

$$x + \mathrm{p}(z) = y_h ,$$

where the land price p depends on the lot attributes.

The consumers' choice process can be described in two stages. First, they will maximise utility subject to budget constraint with z held fixed. The solution is the indirect utility function, expressed as

$$\mathrm{V}_h[z, y_h - \mathrm{p}(z)] , \tag{1}$$

giving the maximum utility achievable by choosing a lot described by z at price $\mathrm{p}(z)$. In the second stage, the consumer chooses $z^\star$ which maximises V_h. The first-order conditions for a maximum requires that $z^\star$ satisfies the expression

$$\frac{\delta \mathrm{V}_h}{\delta z_j} = \frac{\delta \mathrm{V}_h}{\delta y} \frac{\delta \mathrm{p}(z)}{\delta z_j} , \qquad j = 1, \dots , n ; \tag{2}$$

that is, at $z^\star$, the surface of $\mathrm{p}(z)$ is tangential to V_h.

The underlying assumption is that z is continuous, so $z^\star$ exists always and can be effectively chosen. But that condition is not appropriate in the land market where lots offer discrete bundles of characteristics. Then, the consumer may have to choose a second-best land lot defined as the one which will yield the consumer a maximum utility, that is, the z which solves

$$\underset{z}{\text{maximise}}\ \mathrm{V}_h[z, \mathrm{p}(z)] .$$

This is the discrete version of the model.

2.2 *The consumer-surplus formulation*

We shall now consider an alternative interpretation for discrete-choice models, more in line with Rosen's (1974) formulation. That is, in terms of the consumer's willingness-to-pay function for a land lot with characteristics z that will yield a given level of utility u_h for the consumer. The willingness to pay Θ_h is the solution of the equation

$$\mathrm{V}_h[z, y_h - \Theta_h] = u_h . \tag{3}$$

Assuming u_h increases monotonically with $y_h - \Theta_h$, we can invert equation (3) to obtain

$$\Theta_h[z, y_h, u_h] , \tag{4}$$

which defines a family of indifferent surfaces relating z to money at a given utility level u_h (see figure 1). Utility is maximised at $z^\star$ where

$$\Theta_h(z^\star, u_h^\star, y_h) = \mathrm{p}(z^\star) , \tag{5}$$

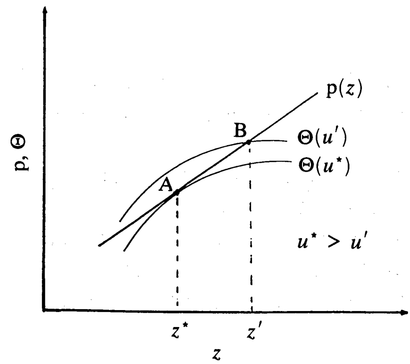

Figure 1. Consumer second-best discrete-choice model.

and

$$\frac{\delta\Theta_h(z^\star, u_h^\star, y_h)}{\delta z_j} = \frac{\delta \mathrm{p}(z^\star)}{\delta z_j}\,; \tag{6}$$

that is, where the two surfaces $\mathrm{p}(z)$ and $\Theta_h(z, u_h, y_h)$ are tangential to each other, as illustrated at point A in figure 1.

Again, if $z^\star$ is not on offer (simply because $z^\star$ may not be a feasible combination of attributes), the consumer will have to choose the second best among available land lots. He or she will consider, for example, the land lot identified by z' in figure 1, with an offer price $\mathrm{p}(z')$ yielding a utility $u_h' < u_h^\star$, as shown by point B.

The criteria to evaluate nonoptimal choices can be specified as the minimum loss in utility with respect to the optimum $u_h^\star$. In contrast to the maximum utility model, this criteria makes a relative measure of utility which, as we shall see, provides useful insights. Nevertheless, as $u_h^\star$ is at a fixed level for all alternative choices, the outcome of the choice process is exactly the same as for the maximum utility model.

The consumer loss associated with choice z' is, in terms of utility, equal to $u_h^\star - u_h'$ which can be measured at z' as

$$\mathrm{u}_h[z', y_h - \Theta_h(z', u_h^\star)] - \mathrm{u}_h[z', y_h - \mathrm{p}(z')]\,.$$

The loss in utility will effectively imply a reduction in the consumption of other goods or in the consumer savings.

The second-best choice is then found as the offered land lot which yields the consumer the minimum loss, or the maximum consumer surplus S. Such a choice is the solution of

$$\underset{z \in s}{\text{maximise}}\ \mathrm{S}_h = \underset{z \in s}{\text{maximise}}\ \{\mathrm{u}_h[z, y_h - \mathrm{p}(z)] - \mathrm{u}_h[z, y_h - \Theta(z, u_h)]\}\,, \tag{7}$$

with s the set of available choices in the land market. We shall call equation (7) the maximum consumer-surplus model or maxCS.

To illustrate the maxCS model, consider the linear indirect utility function

$$\mathrm{V}_h = \sum_k \beta_k^h z_k + \beta^h[y_h - \mathrm{p}(z)]\,, \tag{8}$$

with β^h the unknown set of taste coefficients associated with consumer h. This functional form is typical of land-use models in which the logit expression is used. The willingness-to-pay function is the solution of

$$u_h = \sum_k \beta_k^h z_k + \beta^h(y_h - \Theta_h)\,, \tag{9}$$

which is also linear and given by

$$\Theta_h = y_h - \frac{1}{\beta^h}\left(\sum_k \beta_k^h z_k + u_h\right). \tag{10}$$

Therefore, the loss associated with any land lot z' is obtained from the difference between equations (9) and (8). After some simplifications, the consumer surplus can be expressed as

$$\mathrm{S}_h(z) = \frac{1}{\beta^h}(u_h' - u_h^\star) = \Theta_h(z, u_h^\star) - \mathrm{p}(z)\,, \tag{11}$$

and the second-best choice is found by solving

$$\underset{z \in s}{\text{maximise}}\ \mathrm{S}_h = \underset{z \in s}{\text{maximise}}\ [\Theta_h(z, u_h^\star) - \mathrm{p}(z)]\,. \tag{12}$$

Note that in equation (12) the consumer surplus is measured in monetary terms and represents the maxCS model for linear utility functions which is an alternative specification of the usual linear maximum utility model. Although this specification is valid for a linear specification of utility, with the general form of the CS model given by equation (7), equation (12) provides a useful interpretation of the maximum utility model, which allows us to build the bridge between the bid and the choice approaches.

2.3 *The quasi-unique feature of land*

I now introduce Alonso's market clearing condition that available land lots are assigned to the best bidder. This condition is justified here as the natural consequence of the fact that land is a scarce resource which cannot be produced. This is not only because space is limited, but also because land attributes (accessibility, neighbourhood quality, etc) are the result of nonpecuniary urban external economies, costless for landowners (except through taxes), which cannot be voluntarily modified by the owner in the short term. All these characteristics make each land lot highly peculiar, or *quasi-unique*.

Additionally, we assume that suppliers are profit maximisers so they will always sell to the best bidder. This does not include the case of governments following social policies instead of maximum-profit objectives. Therefore, land lots are sold in the market to the best bidder through a process similar to an auction.

Then, under competitive conditions and given individuals bid according to their willingness to pay, land prices are given by the best bid; that is

$$\mathrm{p}(z) = \underset{g \in H}{\text{maximise}}\ \Theta_g(z, u_g^\star)\,, \tag{13}$$

where H is the group of potential buyers of land. Potential buyers include those buying not only households but also land for every other activity, for example, business, services, shops, schools, etc.

2.4 *The bid-choice model*

Then, by replacing $\mathrm{p}(z)$ in equation (12) by equation (13) we obtain

$$\underset{z \in s}{\text{maximise}}\ \mathrm{S}_h = \underset{z \in s}{\text{maximise}}\ \{\Theta_h(z, u_h^\star) - [\underset{g \in H}{\text{maximise}}\ \Theta_g(z, u_g^\star)]\}\,, \tag{14}$$

which is the discrete and deterministic formulation of the bid-choice model.

Note that equation (14) represents an equilibrium model. Indeed, equation (12) is a demand model with exogenous prices, whereas equation (13) represents a supply model, or the price which maximises suppliers profit. Then, equilibrium is found by solving the system of equations (12) and (13), which in this case is achieved by replacing equation (13) in equation (12) generating the equilibrium model of equation (14).

The bid-choice model not only states the second-best choice for consumer h, but also whether he or she is able to outbid other potential buyers. Assume, for example, that the willingness of consumer h to pay for land lot z' is higher than other bidders, then the price of z' is settled at $\mathrm{p}(z') = \Theta_h(z')$ and, from equation (11), the consumer surplus $\mathrm{S}_h(z') = 0$. But, if consumer h is not the best bidder for z', then $\Theta_h(z') < \mathrm{p}(z')$ and $\mathrm{S}_h(z') < 0$. Hence, in this deterministic model, the upper limit of the consumer surplus is equal to 0, which is achieved only at a land lot where the consumer is able to outbid other buyers.

What is remarkable about equation (14) is its ability to embrace both the bidding approach, describing the competition among potential bidders (group H), and the choice approach which provides the spatial scope of the location problem through available sites (group s). Second, one should recognise that the best-bidder condition is both necessary and sufficient to allocate land, as it always assures zero consumer surplus. On the other hand, in the maximum utility model with exogenous prices our argument of the quasi-unique feature of land, which implies that prices are necessarily explained by the demand for land (that is, prices are endogenous) is not recognised. Nevertheless, if the quasi-unique feature and the maximum profit condition are accepted, the demand-choice model should be interpreted as an equilibrium model.

Now, we are in a position to make the main conclusion of this paper. If the quasi-unique assumption is accepted, the second-best location choice identified by the maximum utility model coincides with the outbidding location given by the bid model. Hence, the bidding approach is *equivalent* to the maximum utility (choice) approach; both produce the same land-use distribution.

3 The empirical comparison

Because of the difficulties in measuring individuals' behaviour, it is more realistic to depart from the deterministic analysis and develop stochastic models which allow a more flexible handling of available data sources. In this section, the equivalency between bidding and choice stochastic models is explored, leading to the stochastic formulation of the bid-choice model.

To begin, it is worth noting that most of the criticism of the best-bidder approach is overcome simply by allowing a stochastic and discrete formulation, as in Ellickson's model. In order to avoid confusing criticism of model form and underlying theory, a fair comparison is achieved by considering two multinomial logit models, one based on the bidding rule, the other on the individual's utility maximising condition.

Let us introduce some notation:

i is the index for a specific land lot from a total of s available;
h is the index for a specific individual from the population H;
z_i is the vector of the lot attributes which best describes i, including, among others, the size of the lot, environmental characteristics, and accessibility advantages;
p_i is the observed market price for lot i;
$\mathit{\Theta}_{hi}$ is the willingness to pay of individual h for land lot i. [In the notation of the previous section, $\mathit{\Theta}_{hi} = \Theta_h(z_i, u_h^\star)$.]

In this section the analysis is concentrated on the individual or disaggregate case.

According to the bidder approach, a given lot i will be assigned to h if he or she is willing to pay the highest price compared with all other alternative bidders. In the stochastic bidding approach, willingness to pay functions are assumed to be random variables, then

$$\mathit{\Theta} = \Theta(z) + \varepsilon_{\Theta}\,, \tag{15}$$

where the random error ε_{Θ} is an independent and identically distributed (IID) Gumbel across different individuals. Then, the probability of individual h making a successful bid for a given location i is

$$P_{h/i} = P[\mathit{\Theta}_{hi} \geqslant \max \mathit{\Theta}_{gi}, \forall g \in H]\,, \tag{16}$$

where the assumption that individuals will bid according to their willingness to pay is implicit. Then,

$$P_{h/i} = \frac{\exp \mu \mathit{\Theta}_{hi}}{\sum\limits_{g} \exp \mu \mathit{\Theta}_{gi}}\,, \tag{17}$$

which is Ellickson's multinomial logit expression, with μ the dispersion parameter.

In this formulation, the expected market price b_i is, by definition, equal to the expected maximum bid, that is:

$$b_i = \frac{1}{\mu}\ln\sum_g \exp\mu\Theta_{gi} \ . \qquad (18)$$

By replacing equation (18) in equation (17), I reduced the logit expression to

$$P_{h/i} = \exp\mu(\Theta_{hi} - b_i) \ . \qquad (19)$$

Now, by using equation (19) it is possible to estimate the probability that an individual will be assigned a *given location i*, and this calculation can be done for all individuals and available locations. After the auction of all available locations is simulated, one can observe the resulting spatial distribution of individuals. From that distribution, the probability of a *given individual h* being located at i, $P_{i/h}$, compared with his or her own possibilities of being finally located at any alternative location $j \in s$, can be worked out by dividing $P_{h/i}$ by the conditional probability

$$P_h = \sum_j P_{h/j} ;$$

that is,

$$P_{i/h} = \frac{P_{h/i}}{\sum_j P_{h/j}} , \qquad (20)$$

which describes the distribution of location probabilities across the space.(1) Note that location probabilities $P_{i/h}$ and $P_{h/i}$ are not meant to be identical but consistent; what should be identical is the spatial distribution of households, which is assured by equation (20).

Replacing equation (19) in equation (20) we obtain

$$P_{i/h} = \frac{\exp\mu(\Theta_{hi} - b_i)}{\sum_j \exp\mu(\Theta_{hj} - b_j)} , \qquad (21)$$

an expression of the normal logit format. Note that the difference $\Theta_{hi} - b_i$ is interpreted as the individual surplus of locating at i and paying the price b_i. Therefore, equation (21) is the logit expression for the individual's maximisation of his or her own (consumer) surplus; that is

$$P_{i/h} = P[(\Theta_{hi} - b_i) > \max(\Theta_{hj} - b_j), \forall j \in s] \ . \qquad (22)$$

Hence, the result of equation (21) shows that the spatial probability distribution obtained from the bidding rule is identical to the probability distribution obtained by the maximisation of individuals' (consumer) surplus, which resembles the previous conclusion to the deterministic case. Moreover, it proves the equivalence between the bid and the choice approaches for the stochastic formulation of a competitive land market.

It is worth mentioning that, in fact, previous utility maximising models, which include land rents or prices as an attribute in their indirect utility function, have no formal difference from the consumer-surplus expression of equation (21), and the estimated coefficients have a similar interpretation as 'marginal values at equilibrium'. For example, see Anas (1982). If one divides the linear utility function by the rent coefficient, the new expression can be interpreted as the consumer surplus being equal to willingness to pay minus rent.

(1) Note that the denominator in equation (20) is, in general, not equal to unity as summation is on lots not on households.

Incidentally, the dispersion parameter μ appears to be unique for both approaches. Note that it is a known property that the inclusive value of a Gumbel distributed variable Θ, namely b, follows a Gumbel distribution with the same dispersion parameter μ. Then equation (21) may be interpreted as a nested logit model with a scale parameter μ', $\mu' \geqslant \mu$. Hence, a unique value for μ represents a special case of a family of nested models. For example, Hayashi and Doi (1989) found values for a bid MNL (multinomial logit) scale parameter of about one third of the choice MNL scale parameter. This issue might be a source of interesting contributions from a more statistical analysis, which is beyond our present concern.

Now, we extend our conclusion to cover the empirical model. That is, the difference between the two approaches is merely formal, and reveals two alternative ways to formulate the same problem. The location distribution of individuals in space is identical in both approaches. Both approaches are indeed equivalent and equations (17), (18), and (21) represent the stochastic bid-choice model.

What happens is that, as the supply model is completely describd by consumers' bids, the equilibrium is solely described by consumers' behaviour with suppliers remaining as passive agents in the market. The underpinning assumption leading to the equivalence between bid and choice logit models is, again, the quasi-unique feature of land implicit in equation (18).

4 The aggregate model

It is worth noting that the result obtained above can be extended to the case of aggregate alternatives, as will be shown in this section. This extension is important, as spatial aggregation of location alternatives into zones is a normal practice in the choice approach, whereas aggregation of individuals into homogeneous groups is common in the bidding approach. In both cases, the aim is to reduce the available competitive alternatives to a manageable number.

To compare the bid rule with the choice approach in aggregate models, consider for example the classification method for aggregated alternatives, plus a given aggregation of individuals into groups and of land lots into zones. The bidding logit expression of equation (17) has to be amended in order to obtain the aggregated version. That is

$$P_{h/i} = \frac{H_h B_{hi} \exp \mu \Theta_{hi}}{\sum\limits_g H_g B_{gi} \exp \mu \Theta_{gi}}, \tag{23}$$

where

$P_{h/i}$ is the proportion of lots at zone i occupied by group h,

Θ_{hi} is the average willingness to pay of group h for land lots in zone i,[2]

H_h is the size of group h,

B_{hi} is the measure of variability of the willingness to pay of the elemental alternatives in the aggregate alternatives.

It is worth mentioning that a special case is the spatial aggregation of land lots of different size into one zone. It is widely accepted in urban economics that the size of the lot, q, is a key variable in location decisions. Indeed, the trade-off between q and accessibility advantages has been the basic argument in most of the classical urban economic theory. Moreover, if lot sizes are not homogeneous within a zone, aggregation will implicitly mean an a priori average location density for that zone. Nevertheless, B_{hi} allows us to aggregate land lots of some difference

(2) From Ben-Akiva and Lerman's (1987, pages 258-259) discussion pages, Θ functions are assumed to be independent of the definition of aggregated alternatives.

in size, still assuming an average density, but without introducing aggregation bias in the estimation of parameters.

From equation (23), the expected rent for land located at i is

$$b_i = \frac{1}{\mu} \ln \sum_g H_g B_{gi} \exp \mu \Theta_{gi} \,, \tag{24}$$

which can be replaced in equation (23) to obtain

$$P_{h/i} = H_h B_{hi} \exp \mu (\Theta_{hi} - b_i) \,. \tag{25}$$

Now, the expected number of households of type h being located at i, H_i, will be proportional to the number of land lots of type i available, s_i. Then,

$$H_{hi} = s_i P_{h/i} \,, \tag{26}$$

and

$$H_h = \sum_j s_j P_{h/j} \,. \tag{27}$$

By dividing equation (26) by equation (27) we obtain the proportion of the group h located at i compared with the total population in the corresponding group; that is,

$$P_{i/h} = \frac{H_{hi}}{H_h} = \frac{s_i P_{h/i}}{\sum_j s_j P_{h/j}} \,, \tag{28}$$

where equation (25) is used to obtain

$$P_{i/h} = \frac{s_i B_{hi} \exp \mu (\Theta_{hi} - b_i)}{\sum_j s_j B_{hj} \exp \mu (\Theta_{hj} - b_j)} \,. \tag{29}$$

Equation (29) is the expression of the aggregate choice model, and the proof that the market bidding rule is identical to the choice approach if b, as defined in equation (24), is able to reproduce the observed land-market prices (that is, the quasi-unique feature of land applies).

Moreover, equations (23), (24), and (29) represent the aggregate bid-choice model, with the assumption of simultaneous IID distribution of residuals. The model can be estimated by using either equation (23) or (29) but, in any case, imposing the additional equation (24).

5 From competitive to speculative markets

The conclusion that the bid approach is equivalent to the choice approach holds only under the stated condition on the formation of land rents; that is, actual rents should be equal to the expected bid-auction outcome [equations (18) and (24)]. If perfect conditions prevail in the land market (that is, if prices are not distorted by any sort of monopoly power) it is theoretically acceptable to assume that this condition holds.

However, this might not be the case under conditions where prices are subject to speculative forces. The question I intend to address in this section is to what extent the best-bid and the choice approaches differ in the case of speculative markets. For our purpose, speculation is understood as a form of monopoly power, for example, information on future land prices. In the hands of landowners (or developers) it can be used to generate expectations about the future price of the land, and in the hands of consumers, this power might be used to speculate with their bids for land.

The main objective here is to extend our previous analysis so as to cover the case where consumers behave as price takers. Therefore we shall consider the case of supply speculation, where prices are assumed to be exogenously fixed by landowners. However, the analysis is complemented by two other cases: in the second case individuals make bids which include a speculative element, and in the third case speculation in both sides, supply and demand, is assumed simultaneously.

For our purpose, it will be useful to assume that the speculative market was developed from an initially competitive equilibrium arising from a sudden change in market conditions. Analytically, we will assume that speculative values (prices and bids) are well described by the addition of a distortion element to the theoretical competitive values, though these elements should not be confused with a parallel vertical move of price or bid surfaces.

5.1 *Case 1: pure supply speculation*

Now, consider the case of exogenous land prices, and assume that supply speculation distorts competitive rents by an amount equal to δ_i (positive, negative, or zero). Then, observed new prices are given by

$$p_i = b_i + \delta_i, \qquad \forall i \in s\,, \tag{30}$$

with δ_i different for every lot i, so $\mathrm{p}(z)$ and $\mathrm{b}(z)$ are not parallel surfaces.

The effect of speculative prices is that consumers will no longer be able to pay the price and still achieve the utility level $u^\star$ enjoyed at competitive prices. If the consumer persists in satisfying his or her wish to buy land, some adjustments in expenditure on other goods will be necessary. Then the consumer's new equilibrium is achieved at a lower utility level, say $u^{\star\star}$, which may imply a different second-best choice z'' as illustrated in figure 2.(3) The original competitive equilibrium for the consumer h is indicated by point A, with prices $\mathrm{b}(z^\star)$ and optimum choice $(z^\star, u_h^\star)$. Point B shows the associated second-best choice. Under exogenous prices $\mathrm{p}(z)$, the new equilibrium is at $(z^{\star\star}, u_h^{\star\star})$, shown as point C, and the second-best choice is shown by point D.

As land prices are distorted, the choice approach is appropriate because individuals are assumed to be price takers without any assumption about the formation of prices. As in the competitive case, maximum utility is equivalent to maxCS for those who persist in buying land. Then, for the linear utility case,

$$\mathrm{S}_h = \Theta_h(z, u_h^{\star\star}) - \mathrm{p}(z) = \Theta'_h(z) - \mathrm{p}(z)\,.$$

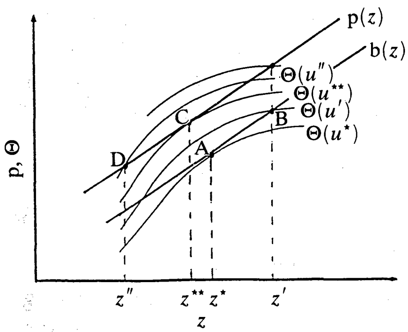

Figure 2. Consumer choice under a speculative market.

(3) Note that the rationale for consumers to accept the loss $u^\star - u^{\star\star}$ is found in their expectations to recover this loss by future increase in land prices, which is also a speculative behaviour.

Note that in the choice approach it is implicitly assumed that consumer h has made the adjustments in expenditure and in the achievable utility level in order to meet the exogenous price $p(z)$.

Now, in this case prices are not formed as the outcome of the bid-auction process. But, consumers who buy land in the speculative market must have adjusted their willingness to pay in order to match the price $p(z)$.[(4)] Additionally, $p(z)$ must still reflect the best bid for land, otherwise landowners would not behave as profit maximisers, which is an assumption in the model. Therefore, from the modeller's point of view, prices can still be explained by the modified consumers' (revealed) willingness to pay for land.

Indeed, once the new equilibrium is established it is difficult to determine whether prices are exogenous or competitive; one can only observe transactions of land lots. Hence, the market can be modelled as if it were competitive, and the equivalence between the two approaches is evident. For the analytical proof, simply replace u_h^* by u_h^{**} in section 2, or $\Theta_h(z, u_h^*)$ by $\Theta_h(z, u_h^{**})$ for its example of linear utility.

Let us now concentrate on the case of the stochastic linear utility model to illustrate how the market moves from the competitive to the speculative equilibrium, or indeed between two feasible equilibrium points. We express the new Θ values as:

$$\Theta'_{hi} = \Theta_{hi} + \delta_{hi} + \varepsilon_\Theta \ , \tag{31}$$

where δ_{hi} represents the adjustment of consumer h's willingness to pay to the new market conditions, which is assumed to be different for each lot i; that is, surfaces Θ_h and Θ'_h are not parallel across the urban space.

Then, the exogenous price is matched by the expected value of the set of adjusted bids, b_i^s, that is

$$p_i = b_i^s = \frac{1}{\mu} \ln \sum_g \exp \mu \Theta'_{gi} \ , \tag{32}$$

which can also be expressed in terms of the changes with respect to the competitive case as follows:

$$\delta_i = \frac{1}{\mu} \ln \sum_g \exp \mu \delta_{gi} \ . \tag{33}$$

These last two equations illustrate the formation of bids in the case of exogenous prices, or how demand reacts to exogenous prices in order to adjust bids to new trading conditions. Equation (33) indicates how consumers' adjustments finally match the exogenously imposed distortion in prices. In a more general interpretation, these equations describe the path between two feasible market conditions through equilibrium stages, which can be seen as an interesting dynamic model of the land market.

The changes in location distribution can be expressed as:

$$P_{h/i}^s = P_{h/i} \exp \mu (\delta_{hi} - \delta_i) \ , \tag{34}$$

with P^s indicating the final distribution under the speculative market.

The necessary condition for these equations to hold is that observations belong to transactions under the new market conditions; that is, the market is in 'trading

(4) Other consumers may not be willing (or be able) to persist in buying land. As they will not be observed trading in the speculative land market, they are simply ignored in the analysis.

equilibrium' where necessary adjustments are already made.[5] Note that equations (32) and (33) are also valid for the case of the limit when exogenous prices are equal to the competitive ones, for all or some locations. Then, the competitive case can be seen as a particular case of equilibrium—that without speculation.

We can now extend our previous conclusions formulated for the competitive case. That is, allowing for the required adjustment of willingness to pay functions, as the natural expected response to externally imposed prices, the bid-auction and the choice approach remain equivalent for those transactions that still occur; both predict the same new distribution of individuals in space. Moreover, the allowance for adjustments in willingness-to-pay functions is implicit in the choice approach where prices are seen as exogenous; we have only made the point explicit in order to prove the equivalence of these approaches.

This conclusion is important as it has been argued that the advantage of the choice approach is that exogenous prices are assumed, thus the model does not require any assumption on land-price formation which could be particularly risky under noncompetitive conditions. One should now reconsider such interpretation as both approaches appear to be equivalent even under the speculative condition.

5.2 *Case 2: pure demand speculation*

In the case of demand speculation, individuals speculate with their bids in order to get benefits from some sort of monopsony power; the supply side can only adjust prices to the new market conditions (endogenous prices). This type of market behaviour is assumed by Hayashi and Doi's (1989) model.

Although this case is different from the previous one, we can anticipate that the expressions involved are identical in form, but they need some careful interpretation. Now, δ_{hi} is interpreted as the households' own assessment of their monopsony power while still being able to make successful bids, thus it is an assessment of a 'likely successful distortion in willingness to pay'. Some consumers may raise their bids in order to outbid for a land lot whose price they expect will be increased in the future. On the other hand, organised consumer groups may reduce their bids, forcing suppliers to sell at prices lower than the competitive level. Of course the consumers' objective is still to maximise their consumer surplus, given by $\Theta_{hi} - p_i$, so they make the best use of their power if they make a successful bid; that is, $\Theta'_{hi} = \Theta_{hi} - \delta_{hi}$.

Consequently, δ_i [given in equation (33)] is the result of various distortions δ_{gi}, made by different consumers according to their own assessment of their monopsony power, weighted by the odds of making successful bids.

Note that, compared with the household spatial distribution in competitive markets, a differential (assessment) of monopsony power across households will change the location distribution. As an example, consider the existence of monopsony power in only one area i while the market in other areas behaves competitively. Then the price is, say, lowered at i by δ_i affecting the consumer surplus by a differential amount $\delta_{hi} - \delta_i$ compared with the competitive case, as are the associated probabilities.

It can be demonstrated that, again, the bid-auction and the choice approaches are equivalent if observations belong to a trading equilibrium under new conditions. As in case 1, the proof follows the competitive case with the appropriate interpretation of bids and prices. Again, equations (32) and (33) represent the

[5] 'Trading equilibrium' indicates a market state of stable trading of land under prevailing (external and internal) market conditions (that is, moving households reach their optimum location). By 'general equilibrium' we imply that every household in the urban area is at their optimum (perfect mobility).

transition between two feasible equilibrium stages, and bids represent the true willingness to pay under prevailing conditions.

5.3 *Case 3: supply-and-demand speculation*
Last, consider the third hypothetical case in which simultaneous speculation both by supply and by demands sides is assumed; that is, both sides have some sort of monopoly power to impose on their transactions. As we have seen in the previous two cases, both will attempt to dominate the price formation through an arbitrary distortion of prices and bids, respectively.

The initial result would be that prices and bids will only match by chance, leading to very few and unpredictable transactions. However, after a bargaining process the market will tend to stabilise in a new trading equilibrium, where transactions must satisfy the compromise between demand and supply stated in equations (32) and (33). Therefore, households will be located according to the speculative formulae of case 1, but in this case, both δ_{hi} and δ_i are interpreted as endogenous elements. The equivalence between the bid-auction and the choice approach is then proved to be valid for a stable market; that is, after an initial bargaining process between speculative powers.

6 Summary
The main conclusion in this paper is the coincidence between the individual discrete choice approach and the market bid-auction approach in the urban land market, valid either under competitive or under speculative conditions. Moreover, it is shown that the underpinning assumptions of these approaches are identical.

The main assumption to give these findings is that suppliers are profit maximisers in a land market described by its quasi-unique feature. Under such assumptions, the maximum utility or maximum consumer surplus model represents an equilibrium model rather than the typical demand model.

I have described a mechanism of natural adjustments in a land market facing speculative distortions which is summarised as follows: suppliers and consumers are, respectively, forced to adjust their profit and utility via price and bids so as to meet the exogenous constraints, otherwise they will not participate in land transactions. Under no external constraints, suppliers and consumers will follow a bargaining process leading to a trading equilibrium.

As exogenous prices will be necessarily matched at the equilibrium following a process of endogenous adjustments of bids, then they can be explained through modified bids. In that sense one can assume that prices are always endogenously explained.

These conclusions are valid in the applied field under the necessary condition that observations are drawn from real transactions at trading equilibrium. A sufficient condition is the assumption of general equilibrium in the land market, where not only observed transactions satisfy equations (32) and (33), but also every household already located in the urban area is itself in equilibrium. This assumption is, of course, more difficult to accept; nonetheless, it is normally necessary in practical work in order to use available household surveys.

Under these conditions, the relevant willingness-to-pay functions are revealed and can be estimated directly. The importance of this conclusion can be found in the area of policy evaluation, because only the willingness-to-pay function (and its coefficients) provide useful information in the derivation of benefit measures.

In any case, equation (33) is a necessary condition for a trading equilibrium to exist; hence, it represents the locus of feasible equilibrium stages of the land market.

It assures that the sum of competitive forces, speculative forces, and their induced reactions 'meet' each other making the trading possible. In this model, the competitive equilibrium can be seen as the particular case of equilibrium where speculation is eliminated.

Moreover, the change from competitive to speculative equilibrium, used in this paper as an illustrative example of a market movement from one equilibrium to another, can easily be generalised for any pair of feasible states of equilibrium. A natural extension is to interpret the analysis as a formulation of a dynamic land-use model, where equation (32) indicates the path following successive equilibrium states, and equation (33) provides an incremental formulation of the dynamic model. Nevertheless, future research is necessary for this extension of the theory.

We can also conclude that the usual key issue in modelling land markets, namely whether land prices are assumed to be exogenously given or endogenously derived, needs to be reconsidered. These conditions seem to say nothing about the appropriateness of the two approaches compared here; as both approaches are equivalent in every case discussed they lead to the same result by making the same assumptions of the market—that observations are drawn from observed transactions at trading equilibrium or that the market is in general equilibrium.

A recent empirical attempt to compare the outcome of a bidding with a utility maximising model was performed by Gross (1988) on the housing data of Bogotá, Colombia. His results show important differences in the estimated parameters which are attributed by the author to 'shortcomings of both models', namely econometric difficulties. However, according to our analysis of the land market, the simultaneous assumption of IID Gumbel distribution of disturbances might not be appropriate in the housing market of Bogotá. Nevertheless, additional considerations should be made for housing models.

7 Towards an integrated framework

Let us now discuss some important further implications following our previous conclusions. It is well known that, in the absence of externalities, the Paretian equilibrium of the land market requires two simultaneous conditions: consumers maximise their utility (or surplus) and suppliers maximise their profit (take the best bid). We note that these conditions are in fact the basic assumptions of the choice and the bid approach, respectively, and, second, that they are satisfied even under exogenous prices. Therefore, one would expect that both approaches should be satisfied simultaneously as a condition for equilibrium. However, the conclusion of equivalency between them ensures this condition is fulfilled automatically under the quasi-unique assumption; that is, whichever approach is preferred and whether it is explicitly established or not, the above requirement is satisfied.

A consequence of this nice characteristic is that, in equilibrium, land prices are *necessarily* formed following the rules of the expected best bid. This condition is unavoidable without violating the conditions for Paretian equilibrium (assuming, again, no external economies). In other words, market prices ought to be consistent with the underlying bid functions, which leads us to the further conclusion that the bid and choice approaches are not only equivalent and consistent but inseparable.

Nevertheless, it is fair to recognise that the bid-auction theory simultaneously maximises utility (or consumer surplus) and suppliers profit, leading to a Paretian equilibrium. I have only extended the validity of this approach to noncompetitive markets.

The clear link of the marriage between consumers and suppliers (at equilibrium) is found in the set of consumers' bids. Indeed, bids appear to be able to explain the land-market equilibrium on their own. They contain the elements of the law of consumers' behaviour and also of the law of price formation; that is, utility and profit are solely a function of bids, whether bids are formed under competitive or monopsony conditions.

Then, the final conclusion to this paper is that the bid and the choice approaches are *equivalent*, *consistent*, *and inseparable*. Effectively, the bid and the choice approaches collapse into a more comprehensive urban economic theory, which I call the bid-choice model.

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