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1 Abstract

I propose a theoretical model of crowded markets with private information, overconfident traders, and market power. Traders receive private signals, observe price, and make inference about the fundamental value of the asset based on the Bayes law. However, this inference may be incorrect because of the wrong perception about the total number of participants in the market. The market is supposed to be *crowded* if traders underestimate the total number of traders in economy. I compare this situation of underestimated crowding to the baseline cases of no crowding and of observed crowding. I also consider the crowding impact on liquidity, price predictability, and price efficiency.

2 Introduction

According to the statistics, starting from \$1.71 trillions in 2011 assets under management of hedge funds increase up to the level of \$2.98 trillions in 2016. This dramatic growth of hedge fund industry lead to profound changes in the structure of financial markets and in the way resources are allocated. Often considered as sophisticated investors, during the last years hedge funds are increasingly moving toward the use of quantitative strategies. Stein (2009) notes that in most cases such strategies can be treated as 'unanchored', i.e. there is no possibility to infer how many market participants are involved in a particular quantitative strategy from prices or any other public information. This condition may result in crowding, the situation when unprecedentedly high number of traders are involved in similar strategies. Crowding not only creates coordinate problem among traders (Kondor and Zawadowski, 2015), but it may also lead to large losses for hedge funds (Khandani, Lo, 2007) and to increase of overall systemic risk (Menkveld, 2014).

The empirical evidence of the increase of crowding in hedge fund industry is present in the paper of Khandani and Lo (2007). The authors reckon that the main reason for the unprecedented losses by the large number of hedge funds in August 2007 is the high share of funds involved in highly correlated leveraged Long/Short strategies. In addition, Khandani and Lo provide the evidence that on average correlation between different quantitative stratagies increases in 2000s compared to 1990s.

Stein (2009) propose a crowded-trade model with risk-averse informed trades and unobserved number of arbitrageurs. Arbitrage opportunity in this model occurs because informed traders exogenously underreact on their private signals. Stein methodology diverges from the standard market microstructure theory, since trading appear by the reason of 'artificial' underreaction. More information based approach is applied in Callahan (2004). Based on the Kyle (1985) model, Callahan examine how uncertainty about the number of participants influence total trade volume, profits and market liquidity. The main case, presented by Callahan is N-period model with 0, 1 or 2 insiders, noise traders, and a market maker, is hard to extend to more general setup (e.g. other potential number of insiders) because of the cumbersome calculations and specific choice of private signals distribution.

The key feature of the present paper is that model is based on the setup with overconfident traders (Kyle, Obizhaeva, Wang, 2016). In this context, trade occurs because each trader believe that his private signal is more informative than the signals of others. Extending this model I introduce subjective beliefs about the total number of traders. Traders strategies depend only on the subjective beliefs, while the equilibrium price also depends on the true number of the traders in the market. To investigate dynamics of prices similar to Kyle, Obizhaeva, Wang (2015) I define empirically correct beliefs. Using the aforementioned modelling devices I examine how underestimation of the total number of traders influence market liquidity, volatility of prices and price predictability. The paper examines one period model that is a starting point for further research of crowded markets in more general continuous time setup.

3 The Model

There is a risky asset with liquidation value $v \sim N(0, 1/\tau_v^{1/2})$. There are N risk-averse traders with CARA utility function with equal risk-aversion coefficients A traders on the market. Trader n privately observes his own inventory S_n. Sum of all inventory endowments is equal to zero. All traders observe public signal $i_0 := \tau_0^{1/2}(\tau_v^{1/2}v) + e_0$, where $e_0 \sim N(0,1)$ and private signal $i_n := \tau_n^{1/2}(\tau_v^{1/2}v) + e_n$, where $e_n \sim N(0,1)$. Random variables $v, e_0, e_1, .., e_n$ are independent. All traders agree about the precision of the public signal and agree to disagree about the precisions of the private signals. Trader n believe that $\tau_n = \tau_H$ and $\tau_m = \tau_L$ for all $m \neq n$, with $\tau_H > \tau_L$. All traders believe that there are only $\underline{N}$ traders on the market ($\underline{N} < N$). Linear demand function:

$$X_m(i_0, i_m, S_m, p) = \alpha i_0 + \beta i_m - \gamma p - \delta S_m$$

Trader n believes in the following market clearing condition:

$$x_n = \sum_{\substack{m \neq n \\ \underline{N}-1 \ traders}} (\alpha i_0 + \beta i_m - \gamma p - \delta S_m) \tag{1}$$

$$P(x_n) = \frac{\alpha}{\gamma} i_0 + \frac{\beta}{\gamma} i_{-n} + \frac{\delta}{(\underline{N}-1)\gamma} S_n + \frac{1}{(\underline{N}-1)\gamma} x_n \tag{2}$$

Since the decision made by trader depend only on the market clearing condition that he believes and on the strategies that he believes other traders play we can proceed step-by-step as in Kyle, Obizhaeva and Wang (2016). Under the assumption that

$$\triangle_H = \frac{\tau_H^{1/2}}{\tau_L^{1/2}} - 2 - \frac{2}{\underline{N}-2}, \tag{3}$$

there is Nash equilibrium with the following strategies played by traders:

$$\alpha(\underline{N}) = \frac{\tau_0^{1/2}}{\tau_H^{1/2} + (\underline{N}-1)\tau_L^{1/2}}\beta, \tag{4}$$

$$\beta(\underline{N}) = \frac{(\underline{N}-2)\tau_H^{1/2} - 2(\underline{N}-1)\tau_L^{1/2}}{A(\underline{N}-1)}\tau_v^{1/2}, \tag{5}$$

$$\gamma(\underline{N}) = \frac{\tau(\underline{N})}{\tau_H^{1/2} + (\underline{N}-1)\tau_L^{1/2}} \frac{\beta(\underline{N})}{\tau_v^{1/2}}, \tag{6}$$

$$\delta(\underline{N}) = \frac{A}{\tau_H^{1/2} - \tau_L^{1/2}} \frac{\beta(\underline{N})}{\tau_v^{1/2}}, \tag{7}$$

where

$$\tau(\underline{N}) = \tau_v(1 + \tau_0 + \tau_H + (\underline{N}-1)\tau_L) \tag{8}$$

is a total precision (from the point of view of n-th trader) of private and public signals. To make further formulas less cumbersome I omit argument if it is equal to $\underline{N}$ (i.e. $\alpha := \alpha(\underline{N})$).

4 Equilibrium price

In spite of the fact that strategies played by the traders depend only on their beliefs, the equilibrium price is different compared to the baseline model since the true number of traders is N. The *true* market clearing condition:

$$\alpha(\underline{N})Ni_0 + \beta(\underline{N})I = \gamma(\underline{N})NP, \tag{9}$$

where $I := \sum_{k=1}^{N} i_k$. Equilibrium price:

$$P = \frac{\alpha(\underline{N})}{\gamma(\underline{N})}i_0 + \frac{\beta(\underline{N})}{N\gamma(\underline{N})}I$$

$$P = \frac{(\tau_0 \tau_v)^{1/2}}{\tau(\underline{N})} i_0 + I \frac{\tau_H^{1/2} + (\underline{N} - 1)\tau_L^{1/2}}{N\tau(\underline{N})} \tau_v$$

5 Benchmarks and empirically correct beliefs

To estimate the effect of crowding we should define the benchmark case. Definitions of this benchmark is closely related to the notion of empirically correct beliefs introduced in Kyle, Obizhaeva, Wang (2015). Since traders disagree about the precision of private signals we should define empirically correct beliefs. Further results are based on the particular choice of the empirically correct parameters of the model. Real dynamics of prices depends only on the true total precision of signal I.

Assumption 1 *Empirically correct total precision of private signals are equal to* $N\tau_e$.

The choice of the empirically correct parameters is based on the simple logic. We assume that professional traders are only able to gain some information about the fundamental value of the asset with precision τ_L while each of them believe that his information is much more informative (τ_H).

The assumption about the true number of traders were implicitly made above (we assume that the true number of traders is equal to N). Since I am mostly interested in the effect of crowding, I introduce the following configurations of the information structure and then compare equilibrium characteristics in these cases.

1. *Unobserved crowding (UC).* This is the main setup described in the previous section. The key feature of this configuration is that all traders believe that there are $\underline{N}$ traders in the market while the true number is N.

2. *Observed crowding (OC).* In this setup we assume that traders know the true number of market participants N. They still overestimate the precision of their

private signals (each trader assumes that he receives signal with precision τ_H and others receive private signals with precision τ_L).

3. *No-crowding (NC).* This configuration implies that traders know the true number of market participants and it is equal to $\underline{N}$. Similar to all previous cases traders are agree to disagree about the precisions of the signals. In fact, the comparison of this case to the cases mentioned above is out of our main interest. However, since many characteristics of the equilibrium (e.g. strategies) coincide with our main case (UC) it is convenient to consider this case as well.

We can easily calculate equilibrium prices from $(4)-(7)$ and market clearing condition in each case.

$$P_{UC} = \frac{(\tau_0\tau_v)^{1/2}}{\tau(\underline{N})} i_0 + I \frac{\tau_H^{1/2} + (\underline{N}-1)\tau_L^{1/2}}{N\tau(\underline{N})} \tau_v^{1/2} \tag{10}$$

$$P_{OC} = \frac{(\tau_0\tau_v)^{1/2}}{\tau(N)} i_0 + I \frac{\tau_H^{1/2} + (N-1)\tau_L^{1/2}}{N\tau(N)} \tau_v^{1/2} \tag{11}$$

$$P_{NC} = \frac{(\tau_0\tau_v)^{1/2}}{\tau(\underline{N})} i_0 + I \frac{\tau_H^{1/2} + (\underline{N}-1)\tau_L^{1/2}}{\underline{N}\tau(\underline{N})} \tau_v^{1/2} \tag{12}$$

6 Price impact

Similar to the paper of Kyle (1989) I calculate temporary (κ) and permanent (λ) price impact as a derivative of price with respect to x_n and S_n respectively. In the model of Kyle (1989) the market clearing condition traders believe coincide with the true market clearing condition. In contrast, in our model we have two different market clearing conditions. For further analysis I introduce notions of *subjective* and *objective* market clearing conditions, temporary and permanent price impacts.

Subjective: $P_S(x_n) = \frac{\alpha}{\gamma} i_0 + \frac{\beta}{\gamma} i_{-n} + \frac{\delta}{(\underline{N}-1)\gamma} S_n + \frac{1}{(\underline{N}-1)\gamma} x_n$, i.e.

$$\lambda_S = \frac{\delta}{(\underline{N}-1)\gamma} \qquad (13)$$

$$\kappa_S = \frac{1}{(\underline{N}-1)\gamma} \qquad (14)$$

Objective: $P_O(x_n) = \frac{\alpha}{\gamma} i_0 + \frac{\beta}{\gamma} i_{-n} + \frac{\delta}{(N-1)\gamma} S_n + \frac{1}{(N-1)\gamma} x_n$, i.e.

$$\lambda_O = \frac{\delta}{(N-1)\gamma} \qquad (15)$$

$$\kappa_O = \frac{1}{(N-1)\gamma} \qquad (16)$$

It is easy to see that subjective price impact (both temporary and permanent component) is (i) equal to the price impact of no-crowding case and (ii) is higher than objective price impact. The result (i) is just the consequence of The result (ii) is quite reasonable since when traders underestimate the number of traders present in the market they underestimate available liquidity, and, as a result, overestimate price impact.

It is interesting to compare objective price price impact to the case of observed crowding.

$$\lambda_{OC} = \frac{\delta(N)}{(N-1)\gamma(N)} \qquad (17)$$

$$\kappa_{OC} = \frac{1}{(N-1)\gamma(N)} \qquad (18)$$

To compare κ_O and κ_{OC} I calculate derivative of $\gamma(\cdot)$ with respect to number of traders. From (6) and (5):

$$\gamma'(n) = \frac{-\tau_L^{1/2}\left((1-(n-1)^{-1})\tau_H^{1/2} - 2\tau_L^{1/2}\right) + (n-1)^{-1}\tau_H^{1/2}(\tau_H^{1/2} + (n-1)\tau_L^{1/2})}{A\left(\tau_H^{1/2} + (n-1)\tau_L^{1/2}\right)^2} =$$

$$\frac{2(n-1)\tau_L + \tau_H + \tau_H^{1/2}\tau_L^{1/2}(n-1)}{A(n-1)\left(\tau_H^{1/2} + (n-1)\tau_L^{1/2}\right)^2} > 0 \qquad (19)$$

Hence, from (13), (15), (19), (18), the following proposition hold:

Proposition 1 *Subjective temporary price impact is greater than objective temporary price impact that is greater than temporary price impact in the case of observed crowding.*

$$\kappa_S > \kappa_O > \kappa_{OC} \qquad (20)$$

The intuition behind right inequality in proposition 1 is straightforward. For trader n, to implement bet x_n he should 'provide' the price that makes it reasonable for other traders to take the other side of the trade. Since in case of unobserved crowding traders believe that the equilibrium price is the result of interaction of only $\underline{N}$ traders, they believe that such price incorporates less information (compared to the case of observed crowding). Hence, greater price movement is needed to reach market clearing condition.

To compare λ_O and λ_{OC} I calculate derivative of $\frac{\delta(\cdot)}{\gamma(\cdot)}$ with respect to number of traders. From (6), (7) and (8):

$$\left(\frac{\delta(n)}{\gamma(n)}\right)' = \left(\frac{A}{\tau_H^{1/2}-\tau_L^{1/2}} \frac{\tau_H^{1/2}+(n-1)\tau_L^{1/2}}{\tau_v(1+\tau_0+\tau_H+(n-1)\tau_L)}\right)' =$$

$$= \frac{\tau_L^{1/2}\tau_v(1+\tau_0+\tau_H+(n-1)\tau_L)-\tau_L\tau_v(\tau_H^{1/2}+(n-1)\tau_L^{1/2})}{A^{-1}(\tau_H^{1/2}-\tau_L^{1/2})\tau_v^2(1+\tau_0+\tau_H+(n-1)\tau_L)^2} =$$

$$= \frac{\tau_L^{1/2}\tau_v(1+\tau_0+\tau_H-\tau_L^{1/2}\tau_H^{1/2})}{A^{-1}(\tau_H^{1/2}-\tau_L^{1/2})\tau_v^2(1+\tau_0+\tau_H+(n-1)\tau_L)^2} > 0 \quad (21)$$

To compare subjective permanent price impact to the permanent price impact in the case of observed crowding one can evaluate the function $\frac{\delta(n)}{n\gamma(n)}$.

$$\frac{\delta(n)}{(n-1)\gamma(n)} = \frac{A}{\tau_H^{1/2}-\tau_L^{1/2}} \cdot \frac{\tau_H^{1/2}+(n-1)\tau_L^{1/2}}{n-1} \cdot \frac{1}{\tau(n)} \quad (22)$$

In (22), the first multiplier does not depend on the number of traders (n), the second and the third ones decrease in n. Hence, function $\frac{\delta(n)}{n\gamma(n)}$ is decreasing. Consequently, from (13), (15), (17) and (21) we can derive the following proposition.

Proposition 2 *Subjective permanent price impact is greater than the permanent price impact in case of observed crowding that is greater than the objective permanent price impact.*

$$\lambda_S > \lambda_{OC} > \lambda_O \quad (23)$$

The dominant term that determines the left inequality in proposition (2) is the number of traders in the denominator in left side of (22). I.e. permanent price impact is decreasing function of number of traders mainly because the aggregate risk aversion of the market is increasing function (in n). Or similarly, if the risk that traders bear is shared between higher number of traders, the permanent price impact is lower.

The main effect that explains the right inequality proposition (2) is the increasingness of function $\delta(\cdot)$. Following the interpretation provided in Kyle, Obizhaeva, Wang (2016) we can introduce the notion of target inventories as inventories trader wants to hold in perfect competition case. Then, δ coefficient reflects the fraction of inventory adjustment to the target level in imperfect competition case. Function δ is increasing in number of traders, since higher liquidity allows traders to implement greater share of desired inventory adjustment. As a result, when the traders believe that there are not many participants in the market ($\underline{N}$), for each particular trader price is less dependent on the initial inventory level, since others implement only small fraction of desired adjustments, because of the wrong perception about the total number of participants.

7 Volatility of prices

We have three different prices in each of the models described in the section 5. $P_0 = 0$ is the price before any information about the fundamental value received. P_1 is the price in trading period, i.e. for each model it is the price from (10) – (12). $P_2 = v$ is the price at the moment when fundamental value of the asset is realized. It is easy to see that independently of the model the variance of $(P_2 - P_0)$ is equal to τ_v^{-1}.

$$\sigma_{02}^2 = \tau_v^{-1} \tag{24}$$

We can easily calculate σ_{01} and σ_{02} using (10) – (12) and assumption 1.

$$P_{UC} = \frac{(\tau_0 \tau_v)^{1/2}}{\tau(\underline{N})} i_0 + I \frac{\tau_H^{1/2} + (\underline{N} - 1)\tau_L^{1/2}}{N\tau(\underline{N})} \tau_v^{1/2} =$$

$$= \frac{\tau_0 \tau_v v + \tau_v^{1/2} \tau_0^{1/2} e_0 + \left(\tau_e^{1/2} \tau_v^{1/2} v + \overline{e_N}\right)(\tau_H^{1/2} + (\underline{N} - 1)\tau_L^{1/2})\tau_v^{1/2}}{\tau(\underline{N})} =$$

$$= \frac{\left(\tau_0 + \tau_e^{1/2}\tau_H^{1/2} + \tau_e^{1/2}\tau_L^{1/2}(\underline{N}-1)\right) v + \tau_v^{-1/2}\tau_0^{1/2} e_0 + \tau_v^{-1/2}(\tau_H^{1/2} + (\underline{N}-1)\tau_L^{1/2})\overline{e_N}}{1 + \tau_0 + \tau_H + (\underline{N}-1)\tau_L}$$

$$(\sigma_{01}^{UC})^2 = \frac{\left(\tau_0 + \tau_e^{1/2}\tau_H^{1/2} + \tau_e^{1/2}\tau_L^{1/2}(\underline{N}-1)\right)^2 + \tau_0 + N^{-1}(\tau_H^{1/2} + (\underline{N}-1)\tau_L^{1/2})^2}{\tau_v(1 + \tau_0 + \tau_H + (\underline{N}-1)\tau_L)^2} \tag{25}$$

We can consider $(\sigma_{01}^{UC})^2$ as a function of $\underline{N}$. Then, $(\sigma_{01}^{UC})^2(N)$ is equal to $(\sigma_{01}^{OC})^2$. Straightforward calculations of the derivative of $(\sigma_{01}^{UC})^2(\cdot)$ imply the following result.

Proposition 3 *Unobserved crowding 01-volatility is lower than the observed crowding 01-volatility.*

$$\sigma_{01}^{UC} < \sigma_{01}^{OC}$$

Similar inferences can be applied to 12-period. Then we obtain the following proposition.

Proposition 4 *Unobserved crowding 12-volatility is greater than the observed crowding 12-volatility.*

$$\sigma_{12}^{UC} > \sigma_{12}^{OC}$$

Propositions 3 and 4 can be interpreted in the following way. In unobserved crowding case traders believe that market receives relatively (to the observed crowding case) small amount of information. As a result, prices incorporate less amount of information. In contrast, observed-crowding prices are sensitive to information arrived to the market. Thus, 01-volatility is higher for OC case. At the same time, when the fundamental value of the asset is realized (in period 2) UC prices adjusts by more than OC prices, since OC prices in period 1 already contains large fraction of information available from signals.

8 Predictability of returns

Economist with empirically correct returns can easily calculate the expected 12-return conditional on the price in period 1. He can derive the value of I from observed market clearing price and i_0. The same calculations can be done by each trader (say n-th). To distinguish between traders and economists expectations and variances I use superscripts n and e respectively.

Each trader makes wrong inference about the total information arrived to the market. Hence, for trader n, his subjective perception differs from the real value of i_{-n}. To avoid ambiguity I denote i^n_{-n} as a subjective result of inference from price. From (10), it is easy to see that each trade makes correct inference about the average of all private signals received by traders (i.e. $I/N = I^n/\underline{N}$). Hence,

$$i^n_{-n} = I^n - i_n = I\frac{\underline{N}}{N} - i_n$$

$$\begin{aligned}
E^n(v - P_{UC}|P_{UC}, i_0, i_n) &= E^n(v|P_{UC}, i_0, i_n) - P_{UC} = \\
&= i_n \frac{\tau_H^{1/2}}{\tau_v^{-1/2}\tau(\underline{N})} + i^n_{-n}\frac{\tau_L^{1/2}}{\tau_v^{-1/2}\tau(\underline{N})} - I\frac{\tau_H^{1/2} + (\underline{N}-1)\tau_L^{1/2}}{N\tau(\underline{N})}\tau_v^{1/2} = \\
&= i_n \frac{\tau_H^{1/2} - \tau_L^{1/2}}{\tau_v^{-1/2}\tau(\underline{N})} + I\frac{\tau_L^{1/2}\underline{N}/N - N^{-1}(\tau_H^{1/2} + (\underline{N}-1)\tau_L^{1/2})}{\tau_v^{-1/2}\tau(\underline{N})} = \\
&= i_n \frac{\tau_H^{1/2} - \tau_L^{1/2}}{\tau_v^{-1/2}\tau(\underline{N})} - I\frac{N^{-1}(\tau_H^{1/2} - \tau_L^{1/2})}{\tau_v^{-1/2}\tau(\underline{N})} = (i_n - I/N)\,\frac{\tau_H^{1/2} - \tau_L^{1/2}}{\tau_v^{-1/2}\tau(\underline{N})} \qquad (26)
\end{aligned}$$

We can see that (subjective) expected return is decreasing function of $\underline{N}$. I.e, for a given signals received by the traders, each trader believe in higher returns (in absolute value), when he believe in lower $\underline{N}$. We can treat coefficient $\frac{\tau_H^{1/2} - \tau_L^{1/2}}{\tau_v^{-1/2}\tau(\underline{N})}$ as *overconfidence relative to market*. Thus, with low $\underline{N}$ trader believe that the informativeness of the aggregate market signal is low enough and rely more on his private signal. In contrary, with high $\underline{N}$ trader expect less return with the same divergence between private and aggregate signal. In particular:

$$E^n(v - P_{UC}|P_{UC}, i_0, i_n) > E^n(v - P_{OC}|P_{OC}, i_0, i_n)$$

$$E^e(v - P_{UC}|P_{UC}, i_0) = E^e(v|P_{UC}, i_0) - P_{UC} = \frac{(\tau_0\tau_v)^{1/2}}{\tau_v(1+\tau_0+N\tau_e)} i_0 + I\frac{\tau_e^{1/2}\tau_v^{1/2}}{\tau_v(1+\tau_0+N\tau_e)} - P_{UC} =$$
$$= \left[\frac{1}{\tau_v(1+\tau_0+N\tau_e)} - \frac{1}{\tau(\underline{N})}\right](\tau_0\tau_v)^{1/2} i_0 +$$
$$+ I\frac{\tau_e^{1/2}(1+\tau_0+\tau_H+(\underline{N}-1)\tau_L) - N^{-1}(\tau_H^{1/2}+(\underline{N}-1)\tau_L^{1/2})(1+\tau_0+N\tau_e)}{\tau_v^{1/2}(1+\tau_0+N\tau_e)(1+\tau_0+\tau_H+(\underline{N}-1)\tau_L)} \quad (27)$$

For market with unobserved crowing to be efficient two following conditions must hold:

1. $\tau_v(1+\tau_0+N\tau_e) = \tau(\underline{N})$

2. $\tau_e^{1/2}(1+\tau_0+\tau_H+(\underline{N}-1)\tau_L) - N^{-1}(\tau_H^{1/2}+(\underline{N}-1)\tau_L^{1/2})(1+\tau_0+N\tau_e) = 0$

The first condition implies that traders should be 'correct on average', i.e.

$$\tau_e = N^{-1}(\tau_H + (\underline{N}-1)\tau_L) \quad (28)$$

Identity (28) together with the second condition gives

$$\tau_e^{1/2} = N^{-1}(\tau_H^{1/2} + (\underline{N}-1)\tau_L^{1/2}) \quad (29)$$

But (29) and (28) can not hold simultaneously by Jensen inequality and ($\underline{N} < N$) condition. Indeed, let (28) holds. Then,

$$N/\underline{N}\tau_e = \underline{N}^{-1}(\tau_H + (\underline{N}-1)\tau_L)$$

By Jensen inequality,

$$(N/\underline{N}\tau_e)^{1/2} \geq \underline{N}^{-1}(\tau_H^{1/2} + (\underline{N}-1)\tau_L^{1/2})$$

Hence, for $N > \underline{N}$,

$$\tau_e^{1/2}(1+\tau_0+\tau_H+(\underline{N}-1)\tau_L) - N^{-1}(\tau_H^{1/2}+(\underline{N}-1)\tau_L^{1/2})(1+\tau_0+N\tau_e) > 0$$

This implies that the coefficient of I in (27) is positive, i.e. there is potential for profitable momentum strategy in the market.

Since (29) and (28) can not hold simultaneously, we obtain that similar to the baseline case considered in Kyle, Obizhaeva, Wang (2015) market is inefficient for any possible parameters of the model because of the price dampening effect of beliefs averaging.

9 Discussion and extensions

The results obtained in the paper reveals the key properties of unobserved crowding effect. I define unobserved crowding as a situation when there is a large number of traders in the market and each of them underestimate the total number of participants. In propositions 1 and 2 I establish the effect of crowding on the price impact. The interesting fact is that temporary price impact increases when traders believe in lower (compared to the true level) number of traders in the market, while permanent price impact decreases. Another result discussed in section 7 implies that crowding lead to the lower price efficiency in the sense that prices incorporates lower fraction of information available at the market compared to the situation with observed number of traders.

One-period model provides important insight about the effect of crowding on various characteristics of the market. However, to obtain empirically testable results one should elaborate dynamic model. The current paper is a stepping-stone for the the continuous-time crowding model. Dynamic model will allow to establish how crowding affects price dynamics and efficiency of the market.

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