MATICOVÁ METODA SPH

$$W_{ij} = \frac{1}{\pi h^2} \cdot e^{-\frac{(x_j - x_i)^2 + (y_j - y_i)^2}{h^2}} =$$

$$= \frac{1}{\pi h^2} \cdot e^{-\frac{dx^2 + dy^2}{h^2}}$$

$$\boxed{\begin{aligned} dx_{ij} &= x_j - x_i \\ dy_{ij} &= y_j - y_i \end{aligned}}$$

$$\boxed{\begin{aligned} \nabla W_{SIZE} &= \frac{2}{\pi h^4} \cdot e^{-\frac{dx^2 + dy^2}{h^2}} \\ \nabla W_x &= dx \cdot \nabla W_{SIZE} \\ \nabla W_y &= dy \cdot \nabla W_{SIZE} \end{aligned}}$$

MATICE INTERAKCÍ I

$$I = \begin{array}{c|ccc} i \downarrow \; \overset{j}{\rightarrow} & 1 & 2 & 3 \\ \hline 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 1 \\ 3 & 0 & 1 & 0 \end{array}$$

interakce částice 1

- SYMETRICKÁ, ŘÍDKÁ

MATICE DX, DY, DU, DV

$\vec{x} = [x_1 \quad x_2 \quad x_3]$ $\quad \vec{y} = [y_1 \quad y_2 \quad y_3]$

$\vec{u} = [u_1 \quad u_2 \quad u_3]$ $\quad \vec{v} = [v_1 \quad v_2 \quad v_3]$

$$MX = \vec{x} \cdot I = \begin{array}{ccc} x_1 & x_2 & x_3 \\ \downarrow & \downarrow & \downarrow \end{array} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & x_2 & 0 \\ x_1 & 0 & x_3 \\ 0 & x_2 & 0 \end{bmatrix}$$

součin přes sloupce

antisymetrická

rozdíl přes prvky

$$DX = MX - MX^T =$$

$$= \begin{bmatrix} 0 & x_2 & 0 \\ x_1 & 0 & x_3 \\ 0 & x_2 & 0 \end{bmatrix} - \begin{bmatrix} 0 & x_1 & 0 \\ x_2 & 0 & x_2 \\ 0 & x_3 & 0 \end{bmatrix} =$$

$$= \begin{bmatrix} 0 & x_2 - x_1 & 0 \\ x_1 - x_2 & 0 & x_3 - x_2 \\ 0 & x_2 - x_3 & 0 \end{bmatrix} = \begin{bmatrix} 0 & dx_{12} & 0 \\ dx_{21} & 0 & dx_{23} \\ 0 & dx_{32} & 0 \end{bmatrix}$$

$DY = MY - MY^T$

$DU = MU - MU^T$ $\quad DV = MV - MV^T$

MATICE ∇W_x A ∇W_y

$$MR2 = DX^2 + DY^2 = \begin{bmatrix} 0 & dx_{12}^2 + dy_{12}^2 & 0 \\ \cdots & 0 & \cdots \\ 0 & \cdots & 0 \end{bmatrix}$$

$$MEXP = -MR2 \cdot \frac{1}{h^2}$$

$$\nabla W_{SIZE} = \exp(M_{EXP}) \cdot \frac{1}{\pi h^4}$$

umocní pouze nenulové prvky M_{EXP}

$$\nabla W_x = DX \cdot \nabla W_{SIZE} = \quad ; \quad \nabla W_y = DY \cdot \nabla W_{SIZE}$$

přes prvky

$$= \begin{bmatrix} 0 & \frac{2dx_{12}}{\pi h^4} \cdot e^{-\frac{dx_{12}^2 + dy_{12}^2}{h^2}} & 0 \\ \cdots & 0 & \cdots \\ 0 & \cdots & 0 \end{bmatrix}$$

ZMĚNA HUSTOTY

$\vec{\rho} = [\rho_1\ \rho_2\ \rho_3]$

$\vec{V} = [V_1\ V_2\ V_3]$

BEZ DIFUZNÍHO ČLENU:

$$\frac{D\rho_i}{Dt} = -\rho_i \sum_j (\vec{u}_j - \vec{u}_i) \cdot \overrightarrow{\nabla W_{ij}} \cdot V_j =$$

$$\frac{D\rho_i}{Dt} = -\rho_i \sum_j \left[(u_j - u_i) \nabla W_{x_{ij}} + (v_j - v_i) \nabla W_{y_{ij}} \right] V_j$$

$$\frac{D\rho_i}{Dt} = -\rho_i \sum_j \left[du_{ij} \nabla W_{x_{ij}} + dv_{ij} \nabla W_{y\,ij} \right] V_j$$

přes prvky

$$\frac{D\vec{\rho}}{Dt} = -\vec{\rho} \cdot \underbrace{\left[\underbrace{(DU \cdot \nabla W_x + DV \cdot \nabla W_y)}_{\text{matice } A} \cdot \vec{V} \right]}_{\text{vektor } \vec{b}}$$

klasický součin $A \cdot \vec{V}$

přes prvky

ROZEPSÁNÍ ZMĚNY HUSTOTY

$$A = DU \cdot \nabla W_x + DV \cdot \nabla W_y =$$

$$= \begin{bmatrix} 0 & du_{12} \nabla W_{x12} + dv_{12} \nabla W_{y12} & 0 \\ du_{21} \nabla W_{x21} + dv_{21} \nabla W_{y21} & 0 & \cdots \\ 0 & \cdots & 0 \end{bmatrix}$$

$$\vec{b} = A \cdot \vec{V} = \begin{bmatrix} 0 \cdot V_1 + (du_{12} \nabla W_{x12} + dv_{12} \nabla W_{y12}) V_2 + 0 V_3 \\ \cdots \\ \cdots \end{bmatrix}$$

$$\frac{D\vec{\rho}}{Dt} = -\vec{\rho} \cdot \vec{b} = \begin{bmatrix} -\rho_1 b_1 \\ -\rho_2 b_2 \\ -\rho_3 b_3 \end{bmatrix}$$

přes prvky

ZMĚNA RYCHLOSTI – POUZE TLAKOVÝ ČLEN

$\vec{p} = [p_1 \ p_2 \ p_3]$

$$\frac{D\vec{u}_i}{Dt} = -\frac{1}{\rho_i} \sum_j (p_j + p_i) \nabla W_{ij} V_j$$

$$\frac{Du_i}{Dt} = -\frac{1}{\rho_i} \sum_j (p_j + p_i) \nabla W_{x\,ij} V_j$$

$$\begin{array}{cc} & \begin{matrix} p_1 & p_2 & p_3 \end{matrix} \\ M_P = \vec{p} \cdot \mathbb{1} = & \begin{bmatrix} 0 & p_2 & 0 \\ p_1 & 0 & p_3 \\ 0 & p_2 & 0 \end{bmatrix} \end{array}$$

$$D_P = M_P + M_P^T = \begin{bmatrix} 0 & p_2 + p_1 & 0 \\ p_1 + p_2 & 0 & p_3 + p_2 \\ 0 & p_2 + p_3 & 0 \end{bmatrix}$$

↳ symetrická!

$$\frac{\vec{Du}}{Dt} = -\frac{1}{\vec{\rho}} \overset{\text{přes prvky}}{\cdot} \underbrace{\left[\underbrace{(D_P \cdot \nabla W_x)}_{\text{matice}} \overset{\text{klasický}}{\cdot} \vec{V} \right]}_{\text{vektor}}$$

$$\frac{\vec{Dv}}{Dt} = -\frac{1}{\vec{\rho}} \cdot \left[(D_P \cdot \nabla W_y) \cdot \vec{V} \right]$$

VISKÓZNÍ ČLEN

$$\Pi_{ij} = 8 \, \frac{(\vec{u}_j - \vec{u}_i)\cdot(\vec{r}_j - \vec{r}_i)}{\| r_j - r_i \|^2}$$

$$\Pi = 8 \cdot \left[(DU \cdot DX) + (DV \cdot DY) \right] \cdot \frac{1}{MR2}$$

všechny operace přes prvky

$$\frac{D\vec{u}_i}{Dt} = \nu \, \frac{\rho_0}{\rho_i} \sum_j \Pi_{ij} \, \nabla W_{ij} \, V_j$$

přes prvky klasický

$$\frac{\vec{Du}}{Dt} = \nu \cdot \rho_0 \cdot \frac{1}{\vec{\rho}} \cdot \underbrace{\left[\underbrace{(\Pi \cdot \nabla W_x)}_{\text{matice}} \cdot \vec{V} \right]}_{\text{vektor}}$$

$$\frac{\vec{Dv}}{Dt} = \nu \rho_0 \, \frac{1}{\vec{\rho}} \cdot \left[(\Pi \cdot \nabla W_y) \cdot \vec{V} \right]$$

VÝSLEDNÉ SCHÉMA

$M_x = \text{colmult}(\vec{x}, I)$; M_y M_u M_v M_p

$DX = M_x - M_x^T$; DY DU DV, $D\rho$

$D\rho = M_\rho + M_\rho^T$

$MR2 = D_x^2 + D_y^2$

$\nabla W_{size} = \frac{1}{\pi h^4} \text{nonzero}\exp\left(\frac{-MR2}{h^2}\right)$

$\nabla W_x = DX \, \nabla W_{SIZE}$; ∇W_Y

$$\pi = \frac{8\left[(DU\, DX) + (DV\, DY)\right]}{MR2}$$

$$\frac{\overrightarrow{D\rho}}{Dt} = -\vec{\rho}\left[(DU\, \nabla W_x + DV\, \nabla W_y) \cdot \vec{V}\right]$$

$$\frac{\overrightarrow{Du}}{Dt} = \frac{-1}{\vec{\rho}}\left[(D\rho\, \nabla W_x) \cdot \vec{V}\right] + \frac{\nu\rho_0}{\vec{\rho}}\left[(\pi\, \nabla W_x) \cdot \vec{V}\right] + \vec{g_x}$$

$$\frac{\overrightarrow{Dv}}{Dt} = \frac{-1}{\vec{\rho}}\left[(D_\rho\, \nabla W_y) \cdot \vec{V}\right] + \frac{\nu\rho_0}{\vec{\rho}}\left[(\pi\, \nabla W_y) \cdot \vec{V}\right] + \vec{g_y}$$

$$\vec{p} = c_0^2(\vec{\rho} - \vec{\rho_0})$$

OPTIMALIZACE

protože matice

$Dx, Dy, Du, Dv, Dp, MR2, \nabla W_{size}, \nabla W_x, \nabla W_y, \Pi$

mají všechny stejné uspořádání a pozice prvků, lze pro všechny operace mezi nimi použít pouze vektor jejich dat. zápis:

$$\vec{Dx} = Dx.data()$$

→ zjednodušení většiny výpočtů na operace s prvky vektorů

$$\vec{MR2} = \vec{Dx}^2 + \vec{Dy}^2 \; ; \; \vec{\nabla W_{size}} = \frac{1}{\pi h^4} \exp\left(\frac{-\vec{MR2}}{h^2}\right)$$

$$\vec{\nabla W_x} = \vec{Dx}\,\vec{\nabla W_{size}}, \quad \vec{\nabla W_y} = \vec{Dy}\,\vec{\nabla W_{size}}$$

$$\frac{D\vec{\rho}}{Dt} = -\vec{\rho}\left[\text{matrix}\left(\vec{Du}\,\vec{\nabla W_x} + \vec{Dv}\,\vec{\nabla W_y}\right) \cdot \vec{V}\right]$$

$$\cancel{\frac{D\vec{u}}{Dt} = \frac{1}{\vec{\rho}}\left\{-\text{matrix}\left(\vec{Dp}\,\vec{\nabla W_x}\right)\right.}$$

$$\frac{D\vec{u}}{Dt} = \frac{1}{\vec{\rho}}\left[\text{matrix}\left(-\vec{Dp}\,\vec{\nabla W_x} + \nu\rho_0\vec{\Pi}\,\vec{\nabla W_x}\right) \cdot \vec{V}\right] + \vec{g_x}$$

$$\frac{D\vec{v}}{Dt} = \frac{1}{\vec{\rho}}\left[\text{matrix}\left(-\vec{Dp}\,\vec{\nabla W_y} + \nu\rho_0\vec{\Pi}\,\vec{\nabla W_y}\right) \cdot \vec{V}\right] + \vec{g_y}$$

DIFUZNÍ ČLEN

$$\frac{\overrightarrow{D\rho}}{Dt} = C_0 \left[\frac{D\rho \left(D_x \nabla W_x + D_y \nabla W_y \right)}{\sqrt{MR2}} \right] \cdot \vec{V}$$