data (k centered samples from normal distribution with unknown expected value a_k and variance σ):

$$x_k := \{x_{1,k}, x_{2,k}, \dots, x_{n,k}\}, \quad k = 1, 2, 3;$$

a_k may vary from sample to sample, variance is assumed to be the same

posterior pdfs:

$$p_k(\sigma) \sim \int_{-\infty}^{+\infty} p_{k-1}(\sigma) \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left\{-\frac{(x_{i,k}-a_k)^2}{2\sigma^2}\right\} da_k, \quad p_0(\sigma) := \frac{1}{\sigma}$$

$$p_k(\sigma) \sim \frac{p_{k-1}(\sigma)}{\sigma^n} \int_{-\infty}^{+\infty} \exp\left\{-\frac{\sum_{i=1}^{n}(x_{i,k}-a_k)^2}{2\sigma^2}\right\} da_k$$

$$\sum_{i=1}^{n} x_{i,k} = 0 \quad \Rightarrow \quad p_k(\sigma) \sim \frac{p_{k-1}(\sigma)}{\sigma^n} \int_{-\infty}^{+\infty} \exp\left\{-\frac{\sum_{i=1}^{n} x_{i,k}^2 + n a_k^2}{2\sigma^2}\right\} da_k$$

$$\int_{-\infty}^{+\infty} \exp\left\{-\frac{\sum_{i=1}^{n} x_{i,k}^2 + n a_k^2}{2\sigma^2}\right\} da_k = \sqrt{\frac{2\pi}{n}}\,\sigma \exp\left\{-\frac{\sum_{i=1}^{n} x_{i,k}^2}{2\sigma^2}\right\} \quad \Rightarrow$$

$$p_k(\sigma) \sim \frac{p_{k-1}(\sigma)}{\sigma^{n-1}} \exp\left\{-\frac{s_k}{\sigma^2}\right\}, \quad s_k := \frac{1}{2}\sum_{i=1}^{n} x_{i,k}^2$$

$$p_1(\sigma) = \frac{c_1}{\sigma^n} \exp\left\{-\frac{s_1}{\sigma^2}\right\}, \quad p_2(\sigma) = \frac{c_2}{\sigma^{2n-1}} \exp\left\{-\frac{s_1+s_2}{\sigma^2}\right\}, \quad p_3(\sigma) = \frac{c_3}{\sigma^{3n-2}} \exp\left\{-\frac{s_1+s_2+s_3}{\sigma^2}\right\},$$

or

$$p_k(\sigma) = \frac{c_k}{\sigma^{k(n-1)+1}} \exp\left\{-\frac{S_k}{\sigma^2}\right\}, \quad S_k := \sum_{j=1}^{k} s_j,$$

where c_k is norming factor

$$c\int_0^{+\infty} \frac{1}{\sigma^m} \exp\left\{-\frac{s}{\sigma^2}\right\} d\sigma = 1 \quad \Rightarrow \quad c = \frac{2s^{\frac{m-1}{2}}}{\Gamma\left(\frac{m-1}{2}, 0\right)} \quad \Rightarrow \quad c_k = \frac{2S_k^{\frac{k(n-1)}{2}}}{\Gamma\left(\frac{k(n-1)}{2}, 0\right)}$$

expected value for σ on k-th step:

$$E^{(k)} := E\sigma\Big|_k = \int_0^{+\infty} \sigma\, p_k(\sigma)\, d\sigma = \frac{c_k}{2S_k^{\frac{k(n-1)-1}{2}}} \Gamma\left(\frac{k(n-1)-1}{2}, 0\right) = \frac{\Gamma\left(\frac{k(n-1)-1}{2}, 0\right)}{\Gamma\left(\frac{k(n-1)}{2}, 0\right)} \sqrt{S_k}$$