Задача 3

Исходные данные:

$m := 9$ $\quad m_1 := m \quad m_2 := m$

$E := 2\cdot 10^{11}$ $\quad h := 0.012 \quad L := 4.3$

$D := 0.21$ $\quad l_{01} := 1.2$

$d := 0.046$ $\quad l_{12} := 0.8$

$\rho := 8000$ $\quad l_{23} := 1.2$

Задание : Определить собственные частоты поперечных колебаний системы (несимметричной), состоящей из 3-х дисков равной массы

Решение :

$$Jxx := \pi\cdot\frac{d^4}{64} = 2.198\times 10^{-7}$$

Определим податливости:

$$\delta_{11} := \left(l_{01}\right)^2\cdot\frac{\left(L-l_{01}\right)^2}{3\cdot E\cdot Jxx\cdot L} = 2.44\times 10^{-5}$$

$$\delta_{12} := \left(l_{01}\right)^2\cdot\frac{\left(L-l_{01}\right)^2}{6\cdot E\cdot Jxx\cdot L}\cdot\left[2\cdot\frac{\left(L-l_{01}-l_{12}\right)}{\left(L-l_{01}\right)}+\frac{\left(L-l_{01}-l_{12}\right)}{l_{01}}-\frac{\left(L-l_{01}-l_{12}\right)^3}{l_{01}\cdot\left(L-l_{01}\right)^2}\right] = 2.862\times 10^{-5}$$

$$\delta_{13} := \left(l_{01}\right)^2\cdot\frac{\left(L-l_{01}\right)^2}{6\cdot E\cdot Jxx\cdot L}\cdot\left[2\cdot\frac{\left(L-l_{01}-l_{12}-l_{23}\right)}{\left(L-l_{01}\right)}+\frac{\left(L-l_{01}-l_{12}-l_{23}\right)}{l_{01}}-\frac{\left(L-l_{01}-l_{12}-l_{23}\right)^3}{l_{01}\cdot\left(L-l_{01}\right)^2}\right] = 1.844\times 10^{-5}$$

$$\delta_{22} := \left(l_{01}+l_{12}\right)^2\cdot\frac{\left(L-l_{01}-l_{12}\right)^2}{\left(3\cdot E\cdot Jxx\cdot L\right)} = 3.732\times 10^{-5}$$

$$\delta_{21} := \left(l_{01}+l_{12}\right)^2\cdot\frac{\left(L-l_{01}-l_{12}\right)^2}{\left(6\cdot E\cdot Jxx\cdot L\right)}\cdot\left[2\cdot\frac{l_{01}}{l_{01}+l_{12}}+\frac{l_{01}}{L-l_{01}-l_{12}}-\frac{\left(l_{01}\right)^3}{\left(l_{01}+l_{12}\right)^2\cdot\left(L-l_{01}-l_{12}\right)}\right] = 2.862\times 10^{-5}$$

$$\delta_{23} := \left(l_{01}+l_{12}\right)^2\cdot\frac{\left(L-l_{01}-l_{12}\right)^2}{\left(6\cdot E\cdot Jxx\cdot L\right)}\cdot\left[2\cdot\frac{L-l_{01}-l_{12}-l_{23}}{L-l_{01}-l_{12}}+\frac{L-l_{01}-l_{12}-l_{23}}{l_{01}+l_{12}}-\frac{\left(L-l_{01}-l_{12}-l_{23}\right)^3}{\left(l_{01}+l_{12}\right)\cdot\left(L-l_{01}-l_{12}\right)^2}\right] = 2.576\times 10^{-5}$$

$$\delta_{33} := \left(l_{01}+l_{12}+l_{23}\right)^2\cdot\frac{\left(L-l_{01}-l_{12}-l_{23}\right)^2}{\left(3\cdot E\cdot Jxx\cdot L\right)} = 2.185\times 10^{-5}$$

$$\delta_{31} := \left(l_{01}+l_{12}+l_{23}\right)^2\cdot\frac{\left(L-l_{01}-l_{12}-l_{23}\right)^2}{\left(6\cdot E\cdot Jxx\cdot L\right)}\cdot\left[2\cdot\frac{l_{01}}{l_{01}+l_{12}+l_{23}}+\frac{l_{01}}{L-l_{01}-l_{12}-l_{23}}-\frac{\left(l_{01}\right)^3}{\left(l_{01}+l_{12}+l_{23}\right)^2\cdot\left(L-l_{01}-l_{12}-l_{23}\right)}\right]$$

$\delta_{31} = 1.844 \times 10^{-5}$

$$\delta_{32} := \left(l_{01} + l_{12} + l_{23}\right)^2 \cdot \frac{\left(L - l_{01} - l_{12} - l_{23}\right)^2}{(6 \cdot E \cdot Jxx \cdot L)} \cdot \left[2 \cdot \frac{l_{01} + l_{12}}{l_{01} + l_{12} + l_{23}} + \frac{l_{01} + l_{12}}{L - l_{01} - l_{12} - l_{23}} - \frac{\left(l_{01} + l_{12}\right)^3}{\left(l_{01} + l_{12} + l_{23}\right)^2 \cdot \left(L - l_{01} - l_{12} - l_{23}\right)}\right]$$

$\delta_{32} = 2.576 \times 10^{-5}$

Закон парности выполняется:

$\delta_{12} = \delta_{21} = 2.818 \times 10^{-5}$

$\delta_{13} = \delta_{31} = 2.183 \times 10^{-5}$

$\delta_{23} = \delta_{32} = 3.184 \times 10^{-5}$

$m := 8.81$

Given

$$\left|\begin{pmatrix} \delta_{11} \cdot m \cdot p^2 - 1 & \delta_{12} \cdot m \cdot p^2 & \delta_{13} \cdot m \cdot p^2 \\ \delta_{21} \cdot m \cdot p^2 & \delta_{22} \cdot m \cdot p^2 - 1 & \delta_{23} \cdot m \cdot p^2 \\ \delta_{31} \cdot m \cdot p^2 & \delta_{32} \cdot m \cdot p^2 & \delta_{33} \cdot m \cdot p^2 - 1 \end{pmatrix}\right| = 0$$

$$\mathrm{Find}(p) \rightarrow \begin{pmatrix} -\frac{126629}{3320} & \frac{345257}{2224} & -\frac{417495}{1133} & -\frac{345257}{2224} & \frac{126629}{3320} & \frac{417495}{1133} \end{pmatrix}$$

$p_1 := 36.2$

$p_2 := 151.4$

$p_3 := 392.7$

$$0 = \left(\delta_{11} \cdot m \cdot p^2 - 1\right) \cdot Y_1 + \delta_{12} \cdot m \cdot Y_2 p^2 + \delta_{13} \cdot m \cdot p^2 \cdot Y_3$$

$$0 = \left(\delta_{21} \cdot m \cdot p^2 - 1\right) \cdot Y_1 + \delta_{22} \cdot m \cdot Y_2 p^2 + \delta_{23} \cdot m \cdot p^2 \cdot Y_3$$

$$0 = \left(\delta_{31} \cdot m \cdot p^2 - 1\right) \cdot Y_1 + \delta_{32} \cdot m \cdot Y_2 p^2 + \delta_{33} \cdot m \cdot p^2 \cdot Y_3$$

$Y_{11} := 1 \qquad p := p_1$

Given

$$0 = \left(\delta_{22} \cdot m \cdot p^2 - 1\right) \cdot Y21 + \delta_{21} \cdot m \cdot Y_{11} \cdot p^2 + \delta_{23} \cdot m \cdot Y31 \cdot p^2$$

$$0 = \left(\delta_{33} \cdot m \cdot p^2 - 1\right) \cdot Y31 + \delta_{32} \cdot m \cdot Y21 \cdot p^2 + \delta_{31} \cdot m \cdot Y_{11} \cdot p^2$$

$$\mathrm{Find}(Y21, Y31) \rightarrow \begin{pmatrix} 0.92057013966469303261 \\ 0.65081928099583512227 \end{pmatrix} \qquad \begin{matrix} Y21 := 1.37 \\ Y31 := 1.18 \end{matrix}$$

$Y_{12} := 1 \qquad p := 151.4$

Given

$$0 = \left(\delta_{11} \cdot m \cdot p^2 - 1\right) \cdot Y_{12} + \delta_{12} \cdot m \cdot Y22 p^2 + \delta_{13} \cdot m \cdot Y32$$

$$0 = \left(\delta_{33} \cdot m \cdot p^2 - 1\right) \cdot Y32 + \delta_{32} \cdot m \cdot Y22 \cdot p^2 + \delta_{31} \cdot m \cdot Y_{12} \cdot p^2$$

$$\mathrm{Find}(Y22, Y32) \rightarrow \begin{pmatrix} -0.67968055251903889878 \\ -0.05485318189278975844 5 \end{pmatrix} \qquad Y22 := -0.63 \qquad Y32 := -0.067$$

$Y_1 := 1 \qquad p := 392.7$

Given

$$0 = \left(\delta_{11}\cdot m\cdot p^2 - 1\right)\cdot Y_1 + \delta_{12}\cdot m\cdot Y23\,p^2 + \delta_{13}\cdot m\cdot Y33$$

$$0 = \left(\delta_{33}\cdot m\cdot p^2 - 1\right)\cdot Y33 + \delta_{31}\cdot m\cdot Y_1\,p^2 + \delta_{32}\cdot m\cdot Y23$$

$Y_{13} := Y_1 = 1$

$$\mathrm{Find}(Y23, Y33) \rightarrow \begin{pmatrix} -0.82698467500034510894 \\ -0.87315602933350793871 \end{pmatrix}$$

$Y23 := -0.7798557710097087784$

$Y33 := -0.71925079948187783717$

Проверим условие ортогональности :

$$\sqrt{m}\cdot Y_{11}\cdot\left(\sqrt{m}\cdot Y_{12}\right) + m\cdot Y21\cdot Y22 + m\cdot(Y31\cdot Y32) = 0.51$$ выполняется

Задача 4

Исходные данные: (задача3) $\theta d := 53400 \cdot 10^{-6}$

Задание : определить частоту крутильных колебаний

Решение :

модуль сдвига

$G := 79.3 \cdot 10^{9}$

полярный момент инерции вала

$$Ip := \pi \cdot \frac{d^4}{32}$$

жесткость вала

$$c_{12} := G \cdot \frac{Ip}{l_{12}} = 4.357 \times 10^{4}$$

$$c_{23} := G \cdot \frac{Ip}{l_{23}} = 2.905 \times 10^{4}$$

Запишем систему уравнений:

$$0 + \theta d \cdot \frac{d^2 \cdot \varphi_1}{2} + c_{12} \cdot (\varphi_1 - \varphi_2) = 0$$

$$-c_{12}(\varphi_1 - \varphi_2) + \theta d \cdot \frac{d^2 \cdot \varphi_2}{2} + c_{23} \cdot (\varphi_2 - \varphi_3) = 0$$

$$-c_{23} \cdot (\varphi_2 - \varphi_3) + \theta d \cdot \frac{d^2 \cdot \varphi_3}{dt^2} + 0 = 0$$

Система имеет следующие решения:

$$\varphi_1 = \Phi_1 \cdot e^{ip\tau}$$

$$\varphi_2 = \Phi_2 \cdot e^{ip\tau}$$

$$\varphi_3 = \Phi_3 \cdot e^{ip\tau}$$

Подставим решения в систему и получим:

$$0 - p^2 \cdot \theta d \cdot \Phi_1 + c_{12} \cdot (\Phi_1 - \Phi_2) = 0$$

$$-c_{12} \cdot (\Phi_1 - \Phi_2) - p^2 \cdot \theta d \cdot \Phi_2 + c_{23} \cdot (\Phi_2 - \Phi_3) = 0$$

$$-c_{23} \cdot (\Phi_2 - \Phi_3) - p^2 \cdot \theta d \cdot \Phi_3 + 0 = 0$$

1 способ (через подбор частот)

1. Пусть $\Phi 1 := 1$ и $p_1 := 5$ тогда

Given

$$0 - (p_1)^2 \cdot \theta d \cdot \Phi 1 + c_{12} \cdot (\Phi 1 - \Phi 2) = 0$$

$$-c_{12} \cdot (\Phi 1 - \Phi 2) - (p_1)^2 \cdot \theta d \cdot \Phi 2 + c_{23} \cdot (\Phi 2 - \Phi 3) = 0$$

$\text{Find}(\Phi 2, \Phi 3) \rightarrow \begin{pmatrix} 0.99996936154620965216 \\ 0.99987744759291088465 \end{pmatrix}$

$\Phi_2 := 0.99997056571203160895$

$\Phi_3 := 0.99988646898408950338$

$\Delta_1 := -c_{23} \cdot (\Phi_2 - \Phi_3) - (p_1)^2 \cdot \theta d \cdot \Phi_3 + 0 = -3.778$

2. Пусть $\Phi_1 := 1$ и $p_2 := 25$ тогда

Given

$0 - (p_2)^2 \cdot \theta d \cdot \Phi 1 + c_{12} \cdot (\Phi 1 - \Phi 2) = 0$

$-c_{12} \cdot (\Phi 1 - \Phi 2) - (p_2)^2 \cdot \theta d \cdot \Phi 2 + c_{23} \cdot (\Phi 2 - \Phi 3) = 0$

$\text{Find}(\Phi 2, \Phi 3) \rightarrow \begin{pmatrix} 0.99923403865524130411 \\ 0.99693703466613771325 \end{pmatrix}$

$\Phi 2 := 0.99926414280079022369$

$\Phi 3 := 0.99716246721135890409$

$\Delta_2 := -c_{23} \cdot (\Phi_2 - \Phi_3) - (p_2)^2 \cdot \theta d \cdot \Phi_3 + 0 = -35.814$

3. Пусть $\Phi 1 := 1$ и $p_3 := 45$ тогда

Given

$0 - (p_3)^2 \cdot \theta d \cdot \Phi 1 + c_{12} \cdot (\Phi 1 - \Phi 2) = 0$

$-c_{12} \cdot (\Phi 1 - \Phi 2) - (p_3)^2 \cdot \theta d \cdot \Phi 2 + c_{23} \cdot (\Phi 2 - \Phi 3) = 0$

$\text{Find}(\Phi 2, \Phi 3) \rightarrow \begin{pmatrix} 0.99751828524298182531 \\ 0.99008237933413010389 \end{pmatrix}$

$\Phi 2 := 0.99761582267456032474$

$\Phi 3 := 0.99081200788976002522$

$\Delta_3 := -c_{23} \cdot (\Phi_2 - \Phi_3) - (p_3)^2 \cdot \theta d \cdot \Phi_3 + 0 = -110.566$

4. Пусть $\Phi 1 := 1$ и $p_4 := 55$ тогда

Given

$0 - (p_4)^2 \cdot \theta d \cdot \Phi 1 + c_{12} \cdot (\Phi 1 - \Phi 2) = 0$

$-c_{12} \cdot (\Phi 1 - \Phi 2) - (p_4)^2 \cdot \theta d \cdot \Phi 2 + c_{23} \cdot (\Phi 2 - \Phi 3) = 0$

$\text{Find}(\Phi 2, \Phi 3) \rightarrow \begin{pmatrix} 0.99629274709136791188 \\ 0.98519160395166448912 \end{pmatrix}$

$\Phi 2 := 0.99643845115582468264$

$\Phi 3 := 0.98628071821556584274$

$\Delta_4 := -c_{23} \cdot (\Phi_2 - \Phi_3) - (p_4)^2 \cdot \theta d \cdot \Phi_3 + 0 = -163.96$

5. Пусть $\Phi 1 := 1$ и $p_5 := 60$ тогда

Given

$0 - (p_5)^2 \cdot \theta d \cdot \Phi 1 + c_{12} \cdot (\Phi 1 - \Phi 2) = 0$

$-c_{12} \cdot (\Phi 1 - \Phi 2) - (p_5)^2 \cdot \theta d \cdot \Phi 2 + c_{23} \cdot (\Phi 2 - \Phi 3) = 0$

$\text{Find}(\Phi 2, \Phi 3) \rightarrow \begin{pmatrix} 0.99558806265418991165 \\ 0.98238144840347467727 \end{pmatrix}$

$\Phi 2 := 0.99576146253255168843$

$\Phi 3 := 0.98367702005393217458$

$\Delta_5 := -c_{23} \cdot (\Phi_2 - \Phi_3) - (p_5)^2 \cdot \theta d \cdot \Phi_3 + 0 = -194.661$

6. Пусть $\Phi 1 := 1$ и $p_6 := 58$ тогда

Given

$0 - (p_6)^2 \cdot \theta d \cdot \Phi 1 + c_{12} \cdot (\Phi 1 - \Phi 2) = 0$

$-c_{12} \cdot (\Phi 1 - \Phi 2) - (p_6)^2 \cdot \theta d \cdot \Phi 2 + c_{23} \cdot (\Phi 2 - \Phi 3) = 0$

$\text{Find}(\Phi 2, \Phi 3) \rightarrow \begin{pmatrix} 0.99587728965797079522 \\ 0.98353465374272959274 \end{pmatrix}$

$\Phi 2 := 0.99603932221097329997$

$\Phi 3 := 0.98474550991168057088$

$\Delta_6 := -c_{23} \cdot (\Phi_2 - \Phi_3) - (p_6)^2 \cdot \theta d \cdot \Phi_3 + 0 = -182.06$

7. Пусть $\Phi 1 := 1$ и $p_7 := 54$ тогда

Given

$0 - (p_7)^2 \cdot \theta d \cdot \Phi 1 + c_{12} \cdot (\Phi 1 - \Phi 2) = 0$

$-c_{12} \cdot (\Phi 1 - \Phi 2) - (p_7)^2 \cdot \theta d \cdot \Phi 2 + c_{23} \cdot (\Phi 2 - \Phi 3) = 0$

$\text{Find}(\Phi 2, \Phi 3) \rightarrow \begin{pmatrix} 0.99642633074989382844 \\ 0.98572447966743904537 \end{pmatrix}$

$\Phi 2 := 0.99656678465136686763$

$\Phi 3 := 0.98677443644661723328$

$\Delta_7 := -c_{23} \cdot (\Phi_2 - \Phi_3) - (p_7)^2 \cdot \theta d \cdot \Phi_3 + 0 = -158.14$

8. Пусть $\Phi 1 := 1$ и $p_8 := 54.2$ то

Given

$0 - (p_8)^2 \cdot \theta d \cdot \Phi 1 + c_{12} \cdot (\Phi 1 - \Phi 2) = 0$

$-c_{12} \cdot (\Phi 1 - \Phi 2) - (p_8)^2 \cdot \theta d \cdot \Phi 2 + c_{23} \cdot (\Phi 2 - \Phi 3) = 0$

$\mathrm{Find}(\Phi 2, \Phi 3) \rightarrow \begin{pmatrix} 0.99639981010429290335 \\ 0.98561868246809934062 \end{pmatrix}$

$\Phi 2 := 0.99654130633170142834$

$\Phi 3 := 0.98667641379640712196$

$\Delta_8 := -c_{23} \cdot (\Phi_2 - \Phi_3) - (p_8)^2 \cdot \theta d \cdot \Phi_3 + 0 = -159.295$

9. Пусть $\Phi 1 := 1$ и $p_9 := 90$ тогда

Given

$0 - (p_9)^2 \cdot \theta d \cdot \Phi 1 + c_{12} \cdot (\Phi 1 - \Phi 2) = 0$

$-c_{12} \cdot (\Phi 1 - \Phi 2) - (p_9)^2 \cdot \theta d \cdot \Phi 2 + c_{23} \cdot (\Phi 2 - \Phi 3) = 0$

$\mathrm{Find}(\Phi 2, \Phi 3) \rightarrow \begin{pmatrix} 0.99007314097192730122 \\ 0.96044037768295404755 \end{pmatrix}$

$\Phi 2 := 0.99046329069824129897$

$\Phi 3 := 0.96334547672793965545$

$\Delta_9 := -c_{23} \cdot (\Phi_2 - \Phi_3) - (p_9)^2 \cdot \theta d \cdot \Phi_3 + 0 = -434.934$

10. Пусть $\Phi 1 := 1$ и $p_{10} := 120$ тогда

Given

$0 - (p_{10})^2 \cdot \theta d \cdot \Phi 1 + c_{12} \cdot (\Phi 1 - \Phi 2) = 0$

$-c_{12} \cdot (\Phi 1 - \Phi 2) - (p_{10})^2 \cdot \theta d \cdot \Phi 2 + c_{23} \cdot (\Phi 2 - \Phi 3) = 0$

$\mathrm{Find}(\Phi 2, \Phi 3) \rightarrow \begin{pmatrix} 0.98235225061675964661 \\ 0.92987616705447907687 \end{pmatrix}$

$\Phi 2 := 0.98304585013020675373$

$\Phi 3 := 0.93501605507052235227$

$\Delta_{10} := -c_{23} \cdot (\Phi_2 - \Phi_3) - (p_{10})^2 \cdot \theta d \cdot \Phi_3 + 0 = -771.316$

11. Пусть $\Phi 1 := 1$ и $p_{11} := 95$ тогда

Given

$0 - (p_{11})^2 \cdot \theta d \cdot \Phi 1 + c_{12} \cdot (\Phi 1 - \Phi 2) = 0$

$-c_{12} \cdot (\Phi 1 - \Phi 2) - (p_{11})^2 \cdot \theta d \cdot \Phi 2 + c_{23} \cdot (\Phi 2 - \Phi 3) = 0$

$\mathrm{Find}(\Phi 2, \Phi 3) \rightarrow \begin{pmatrix} 0.98893951818168443 13 \\ 0.95594157411381765 91 \end{pmatrix}$

$\Phi 2 := 0.98937422204341083003$

$\Phi 3 := 0.95917615239198854018$

$$\Delta_{11} := -c_{23} \cdot \left(\Phi_2 - \Phi_3\right) - \left(p_{11}\right)^2 \cdot \theta d \cdot \Phi_3 + 0 = -484.323$$

12. Пусть $\Phi 1 := 1$ и $p_{12} := 96.5$ тогда

Given

$$0 - \left(p_{12}\right)^2 \cdot \theta d \cdot \Phi 1 + c_{12} \cdot (\Phi 1 - \Phi 2) = 0$$

$$-c_{12} \cdot (\Phi 1 - \Phi 2) - \left(p_{12}\right)^2 \cdot \theta d \cdot \Phi 2 + c_{23} \cdot (\Phi 2 - \Phi 3) = 0$$

$$\mathrm{Find}(\Phi 2, \Phi 3) \rightarrow \begin{pmatrix} 0.98858748234763333467 \\ 0.95454529772928170978 \end{pmatrix}$$

$\Phi 2 := 0.98903602207465401684$

$\Phi 3 := 0.95788209773359045746$

$$\Delta_{12} := -c_{23} \cdot \left(\Phi_2 - \Phi_3\right) - \left(p_{12}\right)^2 \cdot \theta d \cdot \Phi_3 + 0 = -499.661$$

13. Пусть $\Phi 1 := 1$ и $p_{13} := 96.6$ тогда

Given

$$0 - \left(p_{13}\right)^2 \cdot \theta d \cdot \Phi 1 + c_{12} \cdot (\Phi 1 - \Phi 2) = 0$$

$$-c_{12} \cdot (\Phi 1 - \Phi 2) - \left(p_{13}\right)^2 \cdot \theta d \cdot \Phi 2 + c_{23} \cdot (\Phi 2 - \Phi 3) = 0$$

$$\mathrm{Find}(\Phi 2, \Phi 3) \rightarrow \begin{pmatrix} 0.98856381720592567 \\ 0.95445144823905190261 \end{pmatrix}$$

$\Phi 2 := 0.98901328703062723159$

$\Phi 3 := 0.95779511834937844262$

$$\Delta_{13} := -c_{23} \cdot \left(\Phi_2 - \Phi_3\right) - \left(p_{13}\right)^2 \cdot \theta d \cdot \Phi_3 + 0 = -500.692$$

14. Пусть $\Phi 1 := 1$ и $p_{14} := 100$ тогда

Given

$$0 - \left(p_{14}\right)^2 \cdot \theta d \cdot \Phi 1 + c_{12} \cdot (\Phi 1 - \Phi 2) = 0$$

$$-c_{12} \cdot (\Phi 1 - \Phi 2) - \left(p_{14}\right)^2 \cdot \theta d \cdot \Phi 2 + c_{23} \cdot (\Phi 2 - \Phi 3) = 0$$

$$\mathrm{Find}(\Phi 2, \Phi 3) \rightarrow \begin{pmatrix} 0.9877446184838608657 \\ 0.95120376549960264994 \end{pmatrix}$$

$\Phi 2 := 0.98822628481264357898$

$\Phi 3 := 0.954785127662072359$

$$\Delta_{14} := -c_{23} \cdot \left(\Phi_2 - \Phi_3\right) - \left(p_{14}\right)^2 \cdot \theta d \cdot \Phi_3 + 0 = -536.382$$

2 способ решения (через определитель)

$\Phi 1 = 1$

Given

$$0 - p^2 \cdot \theta d \cdot \Phi 1 + c_{12} \cdot (\Phi 1 - \Phi 2) = 0$$

$$-c_{12} \cdot (\Phi 1 - \Phi 2) - p^2 \cdot \theta d \cdot \Phi 2 + c_{23} \cdot (\Phi 2 - \Phi 3) = 0$$

$$-c_{23} \cdot (\Phi 2 - \Phi 3) - p^2 \cdot \theta d \cdot \Phi 3 + 0 = 0$$

$$\mathrm{Find}(\Phi 2, \Phi 3, p) \rightarrow \begin{pmatrix} 1.0 & -1.5485837703548635302 & 0.21525043702153019683 & 0.21525043702153019683 & -1.5485837703548635302 \\ 1.0 & 0.54858377035486353017 & -1.2152504370215301968 & -1.2152504370215301968 & 0.54858377035486353017 \\ 0 & 1442.0689927113954784 & -800.20658967242787686 & 800.20658967242787686 & -1442.0689927113954784 \end{pmatrix}$$

$$P1n := 1482.84 \quad \frac{рад}{с}$$

$$P2n := 830.04 \quad \frac{рад}{с}$$

Задача 5

Исходные данные: см. предыдущие задачи

$\omega := 62.8 \ \frac{\text{рад}}{\text{c}}$ (10Гц) $P_3 := 1$ Н величина вынуждающей силы

Собственные частоты:

$p_1 = 5 \quad \frac{\text{рад}}{\text{c}}$

$p_2 = 25 \quad \frac{\text{рад}}{\text{c}}$

$p_3 = 45 \quad \frac{\text{рад}}{\text{c}}$

Задание:

Определить частоту вынужденных колебаний
(вынуждающая сила приложена к третьему диску)

Решение:

1 способ решения

Найдем коэффициент динамичности

$$\lambda_1 := \frac{1}{1 - \left(\frac{\omega}{p_1}\right)^2} = -6.379 \times 10^{-3}$$

$$\lambda_2 := \frac{1}{1 - \left(\frac{\omega}{p_2}\right)^2} = -0.188$$

$$\lambda_3 := \frac{1}{1 - \left(\frac{\omega}{p_3}\right)^2} = -1.055$$

Амплитуды (из задачи по определению частот собственных колебаний):

$u_{11} := Y_{11} = 1$ $u_{21} := Y21 = 1.37$ $u_{31} := Y31 = 1.18$

$u_{12} := Y_{12} = 1$ $u_{22} := Y22 = -0.63$ $u_{32} := Y32 = -0.067$

$u_{13} := Y_{13} = 1$ $u_{23} := Y23 = -0.78$ $u_{33} := Y33 = -0.719$

Обобщенные массы:

$$M_1 := m\cdot\left[\left(u_{11}\right)^2 + \left(u_{21}\right)^2 + \left(u_{31}\right)^2\right] = 37.613 \quad \text{кг}$$

$$M_2 := m\cdot\left[\left(u_{12}\right)^2 + \left(u_{22}\right)^2 + \left(u_{32}\right)^2\right] = 12.346 \quad \text{кг}$$

$$M_3 := m\cdot\left[(u_{13})^2 + (u_{23})^2 + (u_{33})^2\right] = 18.726$$ кг

Определим главные координаты:

$$q_1 := \frac{\lambda_1\cdot P_3\cdot u_{31}}{M_1\cdot(p_1)^2} = -8.006\times 10^{-6}$$

$$q_2 := \frac{\lambda_2\cdot P_3\cdot u_{32}}{M_2\cdot(p_2)^2} = 1.635\times 10^{-6}$$

$$q_3 := \frac{\lambda_3\cdot P_3\cdot u_{33}}{M_3\cdot(p_3)^2} = 2.002\times 10^{-5}$$

Находим формы вынужденных колебаний:

$$y_1 := q_1\cdot u_{11} + q_2\cdot u_{12} + q_3\cdot u_{13} = 1.365\times 10^{-5}$$ м

$$y_2 := q_1\cdot u_{21} + q_2\cdot u_{22} + q_3\cdot u_{23} = -2.761\times 10^{-5}$$ м

$$y_3 := q_1\cdot u_{31} + q_2\cdot u_{32} + q_3\cdot u_{33} = -2.395\times 10^{-5}$$ м

2 способ решения

Податливости системы:

$$\delta_{11} := (l_{01})^2\cdot\frac{(L-l_{01})^2\cdot P_3}{3\cdot E\cdot Jxx\cdot L} = 2.44\times 10^{-5}$$

$$\delta_{12} := (l_{01})^2\cdot\frac{(L-l_{01})^2\cdot P_3}{6\cdot E\cdot Jxx\cdot L}\cdot\left[2\cdot\frac{(L-l_{01}-l_{12})}{(L-l_{01})} + \frac{(L-l_{01}-l_{12})}{l_{01}} - \frac{(L-l_{01}-l_{12})^3}{l_{01}\cdot(L-l_{01})^2}\right] = 2.862\times 10^{-5}$$

$$\delta_{13} := (l_{01})^2\cdot\frac{(L-l_{01})^2\cdot P_3}{6\cdot E\cdot Jxx\cdot L}\cdot\left[2\cdot\frac{(L-l_{01}-l_{12}-l_{23})}{(L-l_{01})} + \frac{(L-l_{01}-l_{12}-l_{23})}{l_{01}} - \frac{(L-l_{01}-l_{12}-l_{23})^3}{l_{01}\cdot(L-l_{01})^2}\right] = 1.844\times 10^{-5}$$

$$\delta_{22} := (l_{01}+l_{12})^2\cdot\frac{(L-l_{01}-l_{12})^2\cdot P_3}{(3\cdot E\cdot Jxx\cdot L)} = 3.732\times 10^{-5}$$

$$\delta_{21} := (l_{01}+l_{12})^2\cdot\frac{(L-l_{01}-l_{12})^2\cdot P_3}{(6\cdot E\cdot Jxx\cdot L)}\cdot\left[2\cdot\frac{l_{01}}{l_{01}+l_{12}} + \frac{l_{01}}{L-l_{01}-l_{12}} - \frac{(l_{01})^3}{(l_{01}+l_{12})^2\cdot(L-l_{01}-l_{12})}\right] = 2.862\times 10^{-5}$$

$$\delta_{23} := (l_{01}+l_{12})^2\cdot\frac{(L-l_{01}-l_{12})^2\cdot P_3}{(6\cdot E\cdot Jxx\cdot L)}\cdot\left[2\cdot\frac{L-l_{01}-l_{12}-l_{23}}{L-l_{01}-l_{12}} + \frac{L-l_{01}-l_{12}-l_{23}}{l_{01}+l_{12}} - \frac{(L-l_{01}-l_{12}-l_{23})^3}{(l_{01}+l_{12})\cdot(L-l_{01}-l_{12})^2}\right] = 2.576\times 10^{-5}$$

$$\delta_{33} := \left(l_{01}+l_{12}+l_{23}\right)^2\cdot\frac{\left(L-l_{01}-l_{12}-l_{23}\right)^2\cdot P_3}{(3\cdot E\cdot Jxx\cdot L)} = 2.185\times10^{-5}$$

$$\delta_{31} := \left(l_{01}+l_{12}+l_{23}\right)^2\cdot\frac{\left(L-l_{01}-l_{12}-l_{23}\right)^2\cdot P_3}{(6\cdot E\cdot Jxx\cdot L)}\cdot\left[2\cdot\frac{l_{01}}{l_{01}+l_{12}+l_{23}}+\frac{l_{01}}{L-l_{01}-l_{12}-l_{23}}-\frac{\left(l_{01}\right)^3}{\left(l_{01}+l_{12}+l_{23}\right)^2\cdot\left(L-l_{01}-l_{12}-l_{23}\right)}\right]$$

$$\delta_{31} = 1.844\times10^{-5}$$

$$\delta_{32} := \left(l_{01}+l_{12}+l_{23}\right)^2\cdot\frac{\left(L-l_{01}-l_{12}-l_{23}\right)^2\cdot P_3}{(6\cdot E\cdot Jxx\cdot L)}\cdot\left[2\cdot\frac{l_{01}+l_{12}}{l_{01}+l_{12}+l_{23}}+\frac{l_{01}+l_{12}}{L-l_{01}-l_{12}-l_{23}}-\frac{\left(l_{01}+l_{12}\right)^3}{\left(l_{01}+l_{12}+l_{23}\right)^2\cdot\left(L-l_{01}-l_{12}-l_{23}\right)}\right]$$

$$\delta_{32} = 2.576\times10^{-5}$$

Given

$$\left|\begin{pmatrix}\delta_{11}\cdot m\cdot\omega^2-1 & \delta_{12}\cdot m\cdot\omega^2 & \delta_{13}\cdot m\cdot\omega^2\\ \delta_{21}\cdot m\cdot\omega^2 & \delta_{22}\cdot m\cdot\omega^2-1 & \delta_{23}\cdot m\cdot\omega^2\\ \delta_{31}\cdot m\cdot\omega^2 & \delta_{32}\cdot m\cdot\omega^2 & \delta_{33}\cdot m\cdot\omega^2-1\end{pmatrix}\right| \to 1.389436602971309478539035$$

$$\Delta := 1.622898349009062671861$$

$$\left|\begin{pmatrix}\delta_{13}\cdot P_3 & \delta_{12}\cdot m\cdot\omega^2 & \delta_{13}\cdot m\cdot\omega^2\\ \delta_{23}\cdot P_3 & \delta_{22}\cdot m\cdot\omega^2-1 & \delta_{23}\cdot m\cdot\omega^2\\ \delta_{33}\cdot P_3 & \delta_{32}\cdot m\cdot\omega^2 & \delta_{33}\cdot m\cdot\omega^2-1\end{pmatrix}\right| \to 0.0000201497551429987530377145$$

$$\Delta1 := 0.00002374288144751264423694$$

$$\left|\begin{pmatrix}\delta_{11}\cdot m\cdot\omega^2-1 & \delta_{13}\cdot P_3 & \delta_{13}\cdot m\cdot\omega^2\\ \delta_{21}\cdot m\cdot\omega^2 & \delta_{23}\cdot P_3 & \delta_{23}\cdot m\cdot\omega^2\\ \delta_{31}\cdot m\cdot\omega^2 & \delta_{33}\cdot P_3 & \delta_{33}\cdot m\cdot\omega^2-1\end{pmatrix}\right| \to 0.00002225075659995797859007645$$

$$\Delta2 := 0.00002808853521305329870793$$

$$\left|\begin{pmatrix}\delta_{11}\cdot m\cdot\omega^2-1 & \delta_{12}\cdot m\cdot\omega^2 & \delta_{13}\cdot P_3\\ \delta_{21}\cdot m\cdot\omega^2 & \delta_{22}\cdot m\cdot\omega^2-1 & \delta_{23}\cdot P_3\\ \delta_{31}\cdot m\cdot\omega^2 & \delta_{32}\cdot m\cdot\omega^2 & \delta_{33}\cdot P_3\end{pmatrix}\right| \to 0.00001023192771407450657962163$$

$$\Delta3 := 0.00001705798612864918171368$$

Получим:

$$Y1 := \frac{\Delta1}{\Delta} = 1.463\times10^{-5}$$

$$Y2 := \frac{\Delta 2}{\Delta} = 1.731 \times 10^{-5}$$

$$Y3 := \frac{\Delta 3}{\Delta} = 1.051 \times 10^{-5}$$