Assignment 8
Probability

1. Two faces of a fair six-sided die are painted red, two are painted green, and two are painted blue. The die is rolled three times, and the colors that are rolled are recorded. Let RRG be the outcome where red is rolled the first two times, and green is rolled on the third roll.

    (a) List all 27 possible outcomes.

$$
\begin{array}{ccccccccc}
\text{BBB} & \text{BGB} & \text{BRB} & \text{BBG} & \text{BGG} & \text{BRG} & \text{BBR} & \text{BGR} & \text{BRR} \\
\text{GBB} & \text{GGB} & \text{GRB} & \text{GBG} & \text{GGG} & \text{GRG} & \text{GBR} & \text{GGR} & \text{GRR} \\
\text{RBB} & \text{RGB} & \text{RRB} & \text{RBG} & \text{RGG} & \text{RRG} & \text{RBR} & \text{RGR} & \text{RRR}
\end{array}
$$

    (b) Consider the event where all three rolls produce a different color. One outcome in this event is RGB. List all outcomes in the event. What is the probability of the event?

        Let event in which all three rolls produce a different color be denoted by A.

$$A = \{\text{BGB}, \text{BRG}, \text{GRB}, \text{GBR}, \text{RGB}, \text{RBG}\}$$

$$Pr(A) = \frac{6}{27} \approx 0.22$$

    (c) Consider the event where two of the colors rolled are the same. One outcome in this event is RRB. List all outcomes in this event. What is the probability of the event?

        Let event in which two of the colors rolled are the same be denoted by B.

$$
B = \left\{
\begin{array}{cccccc}
\text{BGB}, & \text{BRB}, & \text{BBG}, & \text{BGG}, & \text{BBR}, & \text{BRR}, \\
\text{GBB}, & \text{GGB}, & \text{GBG}, & \text{GRG}, & \text{GGR}, & \text{GRR}, \\
\text{RBB}, & \text{RRB}, & \text{RGG}, & \text{RRG}, & \text{RBR}, & \text{RGR}
\end{array}
\right\}
$$

$$Pr(B) = \frac{18}{27} \approx 0.67$$

2. You have two hats. The first hat contains two orange marbles and seven red marbles; the second hat contains four orange marbles and three red marbles. Bob selects a marble by first choosing one of the two hats at random. He then selects one of the marbles in this hat at random. If Bob has selected a red marble, what is the probability that he selected a marble from the first hat?

3. Suppose that the words "Act now" occurs in 250 of 2000 messages known to be spam and in 5 of 1000 messages known not to be spam. Estimate the probability that an incoming message containing the words "Act now" is spam, assuming that it is initially equally likely that an incoming message is spam or not spam. If our threshold for rejecting a message as spam is 0.9, will we reject this incoming message?