Fast AHRS Filter for Accelerometer, Magnetometer, and Gyroscope Combination with Separated Sensor Corrections

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$$\mathrm{v} = \omega\,\Delta\mathrm{t}$$

$$\tilde{\mathrm{q}}_x = \sin\frac{\mathrm{v}_x}{2}, \tilde{\mathrm{q}}_y = \sin\frac{\mathrm{v}_y}{2}, \tilde{\mathrm{q}}_z = \sin\frac{\mathrm{v}_z}{2}, \tilde{\mathrm{q}}_s = \sqrt{1 - \left(\tilde{\mathrm{q}}_x^2 + \tilde{\mathrm{q}}_y^2 + \tilde{\mathrm{q}}_z^2\right)}$$

$$FSCF\colon \tilde{\mathrm{q}} \cong \left(1, \frac{\omega_x}{2}, \frac{\omega_y}{2}, \frac{\omega_z}{2}\right)$$

$$q_{pre} = q \otimes \tilde{\mathrm{q}}$$

$$a_{ref} = (0, 0, 1) \;\rightarrow\; a_{pre} = q \otimes a_{ref} \otimes q^{*} = M_q \cdot a_{ref}$$

$$e_a = \left|a_{msr} \times a_{pre}\right|, \alpha_a = \cos^{-1}(\,a_{msr} \cdot a_{pre})$$

$$m_z = a_{pre,x} m_{msr,x} + a_{pre,y} m_{msr,y} + a_{pre,z} m_{msr,z},\; m_y = \sqrt{1 - m_z^2}$$

$$m_{ref} = (0, m_y, m_z) \;\rightarrow\; m_{pre} = q \otimes m_{ref} \otimes \dot{q} = M_q \cdot m_{ref}$$

$$e_m = \left|m_{mse} \times m_{pre}\right|, \alpha_m = \cos^{-1}(\,m_{msr} \cdot m_{pre})$$

$$e = f_{\lambda_a}(\alpha_a)\, e_a + f_{\lambda_m}(\alpha_m)\, e_m \; with \; f_\lambda(\alpha) = min(\alpha\lambda_1, \lambda_2)$$

$$q_{corr} = \left(s = \sqrt{1 - \|\gamma e\|}, v = \gamma e\right), \gamma = \frac{sin\|e\|}{\|e\|}$$

$$FSCF\colon q_{corr} \cong (s = 1, v = e)$$

$$q = q_{pre} \otimes q_{corr}$$