Fast AHRS Filter for Accelerometer, Magnetometer, and Gyroscope Combination with Separated Sensor Corrections

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$$\dot{q}_x = \sin\frac{\omega_x}{2}, \dot{q}_y = \sin\frac{\omega_y}{2}, \dot{q}_z = \sin\frac{\omega_z}{2}, \dot{q}_s = \sqrt{1 - \left(\dot{q}_x^2 + \dot{q}_y^2 + \dot{q}_z^2\right)}$$

$$\text{FSCF: } \dot{q} \cong \left(1, \frac{\omega_x}{2}, \frac{\omega_y}{2}, \frac{\omega_z}{2}\right)$$

$$q_{pre} = q \otimes \dot{q}$$

$$a_{ref} = (0, 0, 1) \rightarrow a_{pre} = q \otimes a_{ref} \otimes q^* = M_q \cdot a_{ref}$$

 $e_a = |a_{msr} \times a_{pre}|, \alpha_a = \cos^{-1}(a_{msr} \cdot a_{pre})$

$$\begin{split} m_z &= a_{pre,x} m_{msr,x} + a_{pre,y} m_{msr,y} + a_{pre,z} m_{msr,z}, \ m_y = \sqrt{1 - m_z^2} \\ m_{ref} &= (0, m_y, m_z) \ \rightarrow \ m_{pre} = q \otimes m_{ref} \otimes \dot{q} = M_q \cdot m_{ref} \\ e_m &= \left| m_{mse} \times m_{pre} \right|, \alpha_m = \cos^{-1}(m_{msr} \cdot m_{pre}) \end{split}$$

$$e = f_{\lambda_a}(\alpha_a) e_a + f_{\lambda_m}(\alpha_m) e_m \text{ with } f_{\lambda}(\alpha) = \min(\alpha \lambda_1, \lambda_2)$$

$$q_{corr} = \left(s = \sqrt{1 - \|\gamma e\|}, v = \gamma e\right), \gamma = \frac{\sin\|e\|}{\|e\|}$$

$$\text{FSCF: } q_{corr} \cong (s = 1, v = e)$$

$$q = q_{pre} \otimes q_{corr}$$