An efficient orientation filter for inertial and inertial/magnetic sensor arrays

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$$a_{ref} = (0,0,1) \;\rightarrow\; a_{pre} = q \otimes a_{ref} \otimes q^* = M_q \cdot a_{ref}$$

$$\mathrm{h} = q^* \otimes |\mathrm{m_{msr}}| \otimes q = M_{\mathrm{q}}^{-1} \cdot |\mathrm{m_{msr}}|$$

$$m_{ref} = (0, \sqrt{\mathrm{h_x^2} + \mathrm{h_y^2}}, \mathrm{h}_z) \;\rightarrow\; m_{pre} = q \otimes m_{ref} \otimes \dot{q} = M_q \cdot m_{ref}$$

$$\mathrm{F(q,a)} = \begin{cases} \|\mathrm{f_a}\,(\mathrm{q,a})\,\| \\ \|\mathrm{f_m}(\mathrm{q,m})\| \end{cases} \text{ with } \mathrm{f(q,a)} = \begin{cases} M_q \cdot \mathrm{a}_{ref} \;- a_{\mathrm{msr}} \\ M_q \cdot \mathrm{m}_{ref} - \mathrm{m_{msr}} \end{cases}$$

$$\nabla\mathrm{F} \;=\; \mathrm{J_f} \cdot \mathrm{f}$$

$$\omega_{\Delta\mathrm{t}} = 2\mathrm{q} \otimes \frac{\nabla\mathrm{F}}{\|\nabla\mathrm{F}\|} \qquad\qquad \omega_{\mathrm{bias}} = \gamma \sum \omega_{\Delta\mathrm{t}} \Delta\mathrm{t}$$

$$\dot{q}_\omega = \frac{1}{2}\,\mathrm{q} \otimes (\omega - \omega_{\mathrm{bias}})$$

$$\dot{q}_{\mathrm{est}} = \dot{q}_\omega - \beta \frac{\nabla\mathrm{F}}{\|\nabla\mathrm{F}\|}$$

$$q_\omega = \mathrm{q} + \dot{q}_{\mathrm{est}} \Delta\mathrm{t}$$

$$\beta = \sqrt{0.75}\,\omega_{\mathrm{err}} \qquad\qquad \gamma = \sqrt{0.75}\,\dot{\omega}_{\mathrm{drift}}$$

$$\underset{\textit{mean gyroscope measurement error}}{} \qquad\qquad \underset{\textit{rate of bias drift}}{}$$