Discussion Assignment #1

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Question 8: Give a detailed implementation of longitudinal IPTW to estimate parameters of an MSM without effect modifiers (Section 7-8).

Implementing the Estimator

The stabilized weights for a marginal structural model are:

$$sw_i = \frac{\prod_{k=0}^{K} g_n(A_k = a_{ki})}{\prod_{k=0}^{K} g_n(A_k = a_{ki} | \bar{A}_{k-1} = \bar{a}_{(k-1)i}, \bar{L}_k = \bar{l}_{ki})}$$

Step 1: Calculate appropriate stabilized weights

Part A:

Estimate the probability of receiving treatment, based on prior treatment and covariates using correctly specified parametric regression models (logistic regression) to get the denominator of the weights.

- **In this example** we are estimating the probability of being treated with AZT at each time point, given the covariate pattern and the prior history of AZT treatment, and the authors use this model:

$$\begin{aligned} logit(p[A_k = 1|\bar{A}_{k-1} = \bar{a}_{k-1}, \bar{L}_k = \bar{l}_k)]) &= \alpha_0 + \alpha_1 k + \alpha_2 a_{k-1} + \alpha_3 a_{k-2} \\ &+ \alpha_4 l_k + \alpha_4 l_{k-1} + \alpha_5 l_{k-1} + \alpha_6 a_{k-1} l_k \\ &+ \alpha_7 l_0 \end{aligned}$$

Where l_k is the covariate vector.

Step 1: Calculate appropriate stabilized weights

Part B:

Estimate the probability of receiving treatment, based on prior treatment (but NOT covariates) to get the numerator of the weights.

- **In this example** We simply remove those terms from the model that depended on I, our covariates, resulting in this model:

$$logitp[A_k = 1|\bar{A}_{k-1} = \bar{a}_{k-1}] = \alpha_0^* + \alpha_1^* k + \alpha_2^* a_{k-1} + \alpha_3^* a_{k-2}$$

Step 1: Calculate appropriate stabilized weights

Part C:

Predict each subject's probability of the entire exposure history, from each of the models (based on either ONLY exposure history or exposure history PLUS covariates). Those predicted values are then plugged in to this equation to get the stabilized weight for each subject, i.

- **In this example** we are estimating the probability of their entire AZT treatment hitory pattern.
- Say that ρ_{ki} is the probability of treatment for the ith subject at time k given exposure and covariates, but ρ^*_{ki} is the probability of treatment for the ith subject at time k given exposure history only. Then each subject's weight is:

$$sw_i = \frac{\prod_{k=1}^{K} (\rho^*_{ki})^{a_{ki}} (1 - \rho^*_{ki})^{1-a_{ki}}}{\prod_{k=1}^{K} (\rho_{ki})^{a_{ki}} (1 - \rho_{ki})^{1-a_{ki}}}$$

Step 2: Take the weighted average of observed outcomes across the population

Here we are trying to get our estimate of the β using the stabilized weights we calculated before. We fit a parsimonious MSM (with our stablized weights) such as:

$$logit(p[Y = 1|\bar{A} = \bar{a}]) = \beta_0 + \beta_1 \sum_{t=0}^{K} a(t)$$

β_1 from this model is then our MSM IPTW estimate.

Question 9: How would you modify the above procedure when the target causal parameter is a MSM with effect modification (Section 9)?

Including Baseline Covariates in the Model

If effect modification by baseline covariates V (a subset of $L(t)$) is of interest to the target causal parameter, inclusion of those baseline characteristics in the MSM allows for their incorporation into the counterfactual pseudopopulations.

Therefore, we want to condition on baseline covariates V in our model, where the true model is now:

$$logit(Pr[Y_{\bar{a}_k}|V]) = m(\bar{a}, V|\beta) = \beta_0 + \beta_1 \sum_{t=0}^{K} a(t) + \beta_2 V + \beta_3 \sum_{t=0}^{K} a(t) \times V$$

and the working model is now:

$$\beta(P_{U,X}|m) = \underset{\beta}{\operatorname{argmin}}\ E_{U,X}[\sum_{\bar{a} \in \mathcal{A}} (Y_{\bar{a}} - m(\bar{a}|\beta))^2]$$

Including Baseline Covariates in the Model

The choice of numerator in the MSM changes the target parameter being measured, and the stabilized weights can be improved:

$$\hat{sw}_i = \frac{g_n(\bar{A}_i(K)|V_i)}{\prod_{t=1}^{K} g_n(A_i(t)|\bar{A}(t-1), \bar{L}_i(t))}$$