Discussion Assignment #1

Sabrina Boyce, Shelley Facente, and Steph Holm

10/7/2019

Question 8: Give a detailed implementation of longitudinal IPTW to estimate parameters of an MSM without effect modifiers (Section 7-8).

Implementing the Horvitz-Thompson Estimator

We are interested in the expected Y if everyone got treatment regime $\bar{A}(t) = \bar{a}$, for t= 0,1.

$$\Psi^F(P_{U,X}) = E_{U,X}[Y_{\bar{A}(t)=\bar{a}}]$$

The Horvitz-Thompson IPTW estimator for $Psi^F\ (P_{U,X})$ is:

$$\hat{\Psi}(P_n) = \frac{1}{n}\sum_{i=1}^{n} \frac{\mathbb{I}[\bar{A}_i(t) = \bar{a}]}{g_n(A_i(0)|L_i(0)) \times g_n(A_i(1)|A_i(0), L_i(0), L_i(1))} Y_i$$

Step 1: Calculate appropriate stabilized weights using the modified Horvitz-Thomas estimator

$$Weights = \frac{1}{g_n(A_i(0)|L_i(0)) \times g_n(A_i(1)|A_i(0), L_i(0), L_i(1))}$$

Part a:

Estimate the probability of receiving treatment using correctly specified parametric regression models (logistic regression)

$$g_0(A(0) = a(0)|L(0)) = expit[\beta_0 + \beta_1 L(0)]$$
$$g_0(A(0) = a(1)|\bar{L}(1), A(0)) = expit[\beta_0 + \beta_1 L(0) + \beta_2 L(1)]$$

In this example we are estimating the probability of being treated with AZT at each time point, given the covariate pattern and the prior history of AZT treatment.

Step 1: Calculate appropriate stabilized weights using the modified Horvitz-Thomas estimator

Part b:

Predict each subject's probability of the exposure at each time t, given his or her observed exposure and covariate history.

$$g_n(A_i(t) = a_i(t)|\bar{A}_i(t-1), L_i(t))$$

In this example:

- for time points where AZT treatment is NOT occuring it is the predicted probability of NOT being treated, given the observed past.
- for time points where AZT treatment IS occuring it is the predicted probability of being treated, given the observed past.

Step 1: Calculate appropriate stabilized weights using the modified Horvitz-Thomas estimator

Part c:

Predict each subject's probability of the entire exposure history, which is the product of the time point specific probabilities.

$\prod_{t=1}^{k}(A_i(t)|\bar{A}_i(t-1), \bar{L}_i(t))$

In this example we are estimating the probability of their entire AZT treatment hitory pattern.

The weights, as given earlier, are thus the inverse of these products.

Step 2: Take the weighted average of observed outcomes across the population

The Horvitz-Thompson IPTW estimator for $Psi^F\left(P_{U,X}\right)$ is:

$$\hat{\Psi}(P_n) = \frac{1}{n}\sum_{i=1}^{n} \frac{\mathbb{I}[\bar{A}_i(t) = \bar{a}]}{g_n(A_i(0)|L_i(0)) \times g_n(A_i(1)|A_i(0), L_i(0), L_i(1))} Y_i$$

The Modified or Stabilized Horvitz-Thompson IPTW estimator for $Psi^F\left(P_{U,X}\right)$ is:

$$\hat{\Psi}(P_n) = \frac{\frac{1}{n}\sum_{i=1}^{n} \frac{\mathbb{I}[\bar{A}_i(t)=\bar{a}]}{g_n(A_i(0)|L_i(0))\times g_n(A_i(1)|A_i(0),L_i(0),L_i(1))} Y_i}{\frac{1}{n}\sum_{i=1}^{n} \frac{\mathbb{I}[\bar{A}_i(t)=\bar{a}]}{g_n(A_i(0)|L_i(0))\times g_n(A_i(1)|A_i(0),L_i(0),L_i(1))}}$$

Question 9: How would you modify the above procedure when the target causal parameter is a MSM with effect modification (Section 9)?