CPSC 540 — Assignment 1

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1 Fundmanentals

Stephanie

2 Convex Functions

2.1 Minimizing Stricly Convex Quadratic Functions

1.
$$f(w) = \frac{1}{2} \parallel w - v \parallel^2 \tag{1}$$

Taking the gradient of $f(w)$, we get

$$\begin{aligned} \nabla f(w) &= \nabla \left(\frac{1}{2}(w-v)^T(w-v) \right) \\ &= \frac{1}{2} \nabla \left(w^T w - 2w^T v + v^T v \right) \\ &= w^T - v^T. \end{aligned} \tag{2}$$

We now set the gradient to 0 to find the critical points

$$\nabla f(w) = 0 = w^T - v^T, \tag{3}$$

implies $w = v$.

2.
$$f(w) = \frac{1}{2} \parallel Xw - y \parallel^2 + \frac{1}{2} w^T \Lambda w \tag{4}$$

Taking the gradient of $f(w)$, we get

$$\begin{aligned} \nabla f(w) &= \nabla \left(\frac{1}{2} \parallel Xw - y \parallel^2 + \frac{1}{2} w^T \Lambda w \right) \\ &= \frac{1}{2} \nabla \left[w^T \left(X^T X + \Lambda \right) w - 2w^T X^T y + y^T y \right] \\ &= \frac{1}{2} \left(2X^T X + \Lambda + \Lambda^T \right) w - y^T X. \end{aligned} \tag{5}$$

Setting the gradient to 0, we find the critical point

$$0 = \frac{1}{2} \left(2X^T X + \Lambda + \Lambda^T \right) w - y^T X \tag{6}$$

which gives,

$$\left(2X^T X + \Lambda + \Lambda^T \right) w = 2y^T X \tag{7}$$

3 Numerical Optimization

Alex