Modelling Predator and Prey Populations After the Reintroduction of Wolves to a Wildlife Refuge

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I. Abstract

A predator-prey model was developed to analyze the relationship between a hypothetical deer and wolf population. The model was run under different scenarios to test its stability and sensitivity to varying time steps, initial parameters, and initial deer and wolf population sizes. Without a carrying capacity, the model is sensitive to differing parameter and initial conditions. For reasonable time limits and small times steps it is cyclically stable, but with large variations in initial parameters and populations sizes it is often unstable and the two populations approach zero. The introduction of a carrying capacity makes the model ultimately stable for all reasonably large changes in variables. Additionally, if the deer population levels are to become endangered, it is recommended to modestly hunt wolves, as this will increase the deer population steady-state level, whilst having a minimal affect on the long-term steady-state of the wolf population.

II. Background

Anthropogenic climate change has the potential to significantly affect the viability of many species in the decades to come. For the deer population size, food availability and climatic conditions have the greatest effect on long-term population levels, while human hunting pressures are a major determinant for short-term deer abundance fluctuations. Predation interactions between deer and wolves will affect both populations at a regional level. Although unregulated and high levels of hunting may have accelerated the decline of deer populations, appropriate hunting may encourage a stable or stable state deer population. The life history of a wolf population is also decidedly shaped by its availability of food—deer population levels. Thus, monitoring the levels of predation can determine the inflow and outflow of both the deer and wolf population levels; this will allow us to derive mathematical equations to model the levels of each population over time.

Henceforth, our paper seeks to 1) conceptually model the deer-wolf interactions; 2) mathematically model how the deer and wolf populations changes over time, with varying time steps, initial parameters, and initial population sizes; and 3) whether we should permit the hunting of wolves in order to sustain a healthy deer population.

The model is useful for studying the fluctuations of the abundance of wolves and their prey. Although government management seeks to maintain steady—i.e. non-zero—population levels, not understanding the self-regulating and cyclical nature of these oscillations or steady-states may pose as an impediment to the very regulations that they may put in place. Therefore, our work seeks to understand how the reintroduction of wolves into a wildlife refuge will affect deer population levels and whether this will result in a steady or steady-state system. Additionally, we will be modelling these two populations under varying initial parameters, including the birth and death rates of both populations, the conversion of deer consumed into newborn wolves, the deer carrying capacity, and the initial population levels of both deer and wolves.

III. Methods

A conceptual model was developed to visually depict a hypothetical deer and wolf dynamical system (Figure 1).

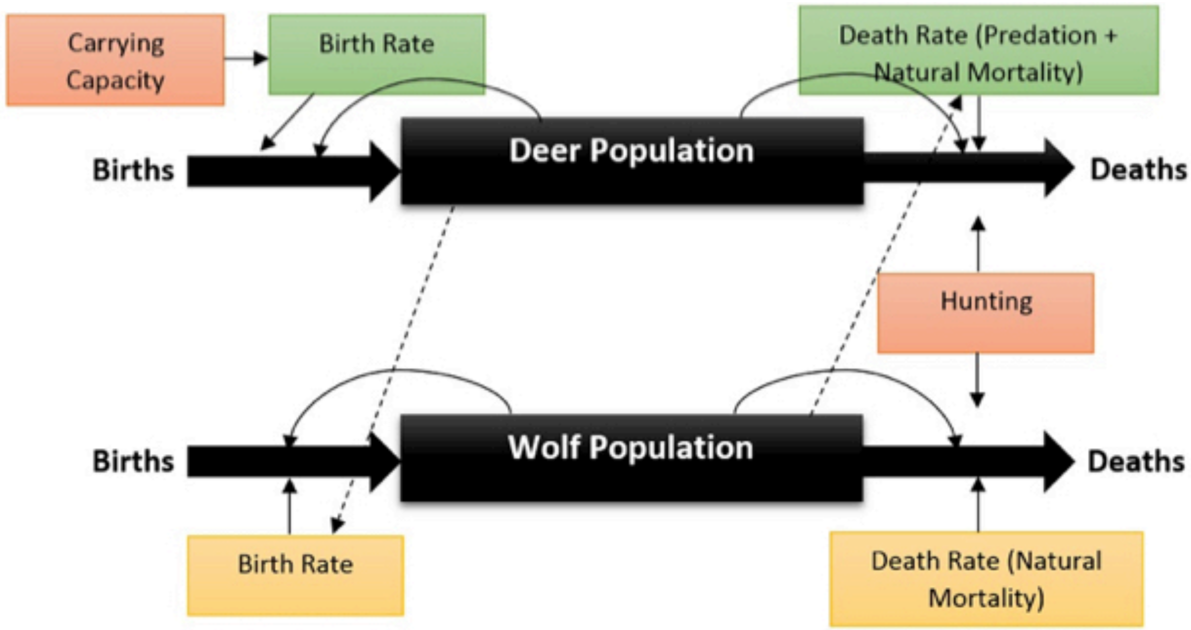

Figure 1. *Conceptual model of a hypothetical wolf and deer dynamical system. The black boxes represent the reservoirs, the large black arrows represent the inflows/outflows to the reservoirs, and the coloured boxes represent converters. The small black arrows indicate how the converters and reservoirs affect the various converters.*

Our two reservoirs are the deer and wolf populations, with birth and death rates being the inflows and outflows respectively. Although the birth and death rates are assumed to remain constant, the number of births and deaths per annum is dependent on the population size[1]. In addition, the size of the wolf population affects the deer death rate due to predation; the size of the deer population affects the growth of the wolf population due to the conversion of prey into predator growth.

To translate this conceptual model into a mathematical model, we investigated how the converters impacted the wolf and deer populations. We first considered the case where there was no carrying capacity for the deer. We defined the number of births for deer and wolves without carrying capacity (Figure 2a) such that

$$\text{number of deer births} = a * D$$

$$\text{number of wolf births} = b * c * D * W$$

[1]Assuming constant birth and death rates, a larger population will have a *larger* number of births and a *larger* number of deaths; conversely, a smaller population will have a *smaller* number of births and a *smaller* number of deaths.

where a is the deer birth rate, b the deer death rate, c the conversion of deer consumed into new wolves, D the deer population size, and W the wolf population size. The number of deaths for deer and wolves without carrying capacity (Figure 2b) is given by

$$\text{number of deer deaths} = b * D * W$$

$$\text{number of wolf births} = d * W$$

where d is the death rate of wolves. Combining the above equations results in two differential equations that mathematically model the change in deer and wolf populations over time t:

$$\frac{dD}{dt} = deer\ births - deer\ deaths$$

$$= aD - bDW$$

$$\frac{dW}{dt} = wolf\ births - wolf deaths$$

$$= bcWD - dW$$

Let us now consider the case with a carrying capacity for deer. As illustrated in Figure 2c, the number of deer births is now

$$\text{number of deer births} = a\left(\frac{1-D}{cc}\right)D$$

where cc is the carrying capacity of deer. With the addition of deer carrying capacity to our dynamical system, we have an unchanged dW/dt relationship; however, when we differentiate the deer population size with respect to t we now obtain

$$\frac{dD}{dt} = a\left(\frac{1-D}{cc}\right)D - bDW$$

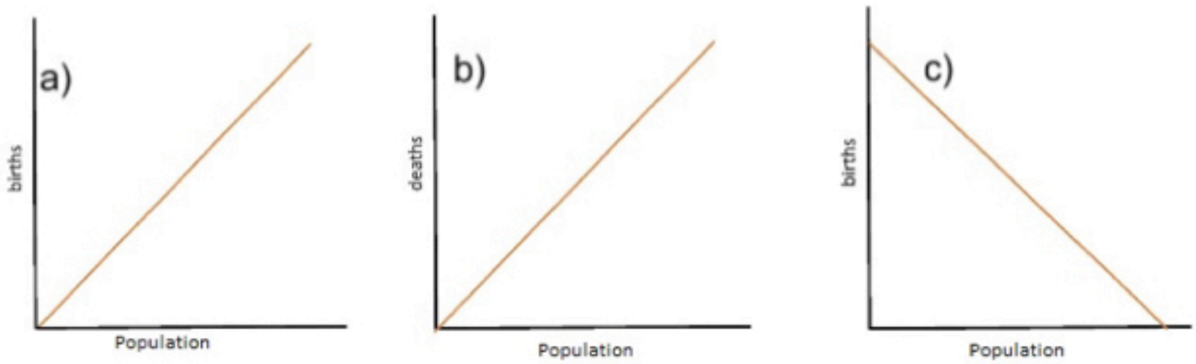

Figure 2. *a) relationship between number of deer births without a deer carrying capacity and deer population size. The relationship between number of wolf births and deer/wolf population sizes is also a positive linear correlation, as it depends on the conversion of prey to predators and the size of both populations. b) relationship between number of deer deaths and deer/wolf population sizes. The graph is similarly correlated for the wolf death rate with and without deer carrying capacity. c) the number of deer births with deer carrying capacity and the deer population is negatively correlated*

Reasonable values for each parameter were determined, summarized in Table 1. *a* was defined as the proportion of the deer population that could give birth multiplied by the average number of young they raised. Approximately one quarter of the deer population can give birth at a given time *t* and they typically raise one fawn at a time (Verme, 1965). Since it is difficult to quantify the proportion of the deer population that is preyed upon each year, *b* was found by researching similar predator-prey models of wolves and deer. *c* is also a complex factor that takes into account both the wolf birth rate and quantifies how consuming a deer contributes to the wolf population; likewise with the *c* value, we consulted similar models within the scientific literature (Mech & Boitani, 2003). Finally, because the average lifespan of wolves is 5 years, we set the wolf death rate $d = (5\text{ years})^{-1}$, or 0.2. The initial wolf population size *W* was small relative to the initial deer population size *D*, which is reflective of the large energy loss between trophic levels.

Parameter	Value
a	0.25
b	0.01
c	0.2
d	0.2
cc	500
D	300
W	10

Table 1. *Values for each parameter used in mathematical model*

In order to formulate our model and define our parameters, we had to make a number of assumptions. First, we assumed that deer were the only species that wolves preyed upon in the wildlife refuge. We assumed that wolves only died due to old age and that other factors—such as natural disasters, hunting or other predators—were negligible. The only source of deer mortality was wolf predation and similarly no deer died of natural causes. The growth rate of wolves only depended on the size of the wolf and deer populations. As well, we assumed that each deer consumed has an equal contribution to the wolf population. The model parameters were also held constant throughout the time period investigated, assuming they did not change over time or with population size.

IV. Results and Discussion

The parameters values in Table 1 and a time step *dt* of 0.01 years were used to construct a model without carrying capacity (Figure 3a) and with carrying capacity (Figure 3b).

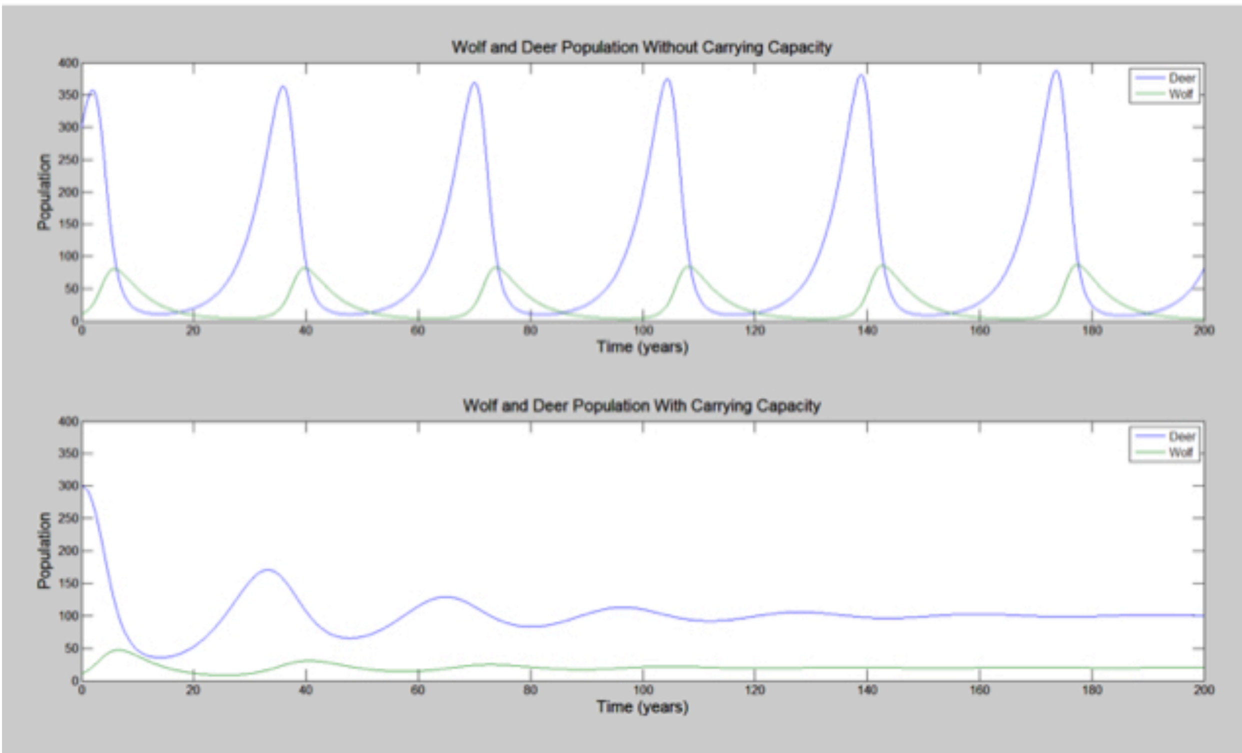

Figure 3. *model run with parameters defined in Table 1 and time step dt = 0.01. a) The top panel shows how wolf and deer populations vary with each other when there is no carrying capacity in the wildlife refuge. b) The bottom panel shows the two populations with carrying capacity: the deer reach a steady-state of approximately 100 individuals after about 150 years, while the wolves reach a steady-state of 20 individuals.*

A *dt* of 0.01 years was chosen as this was smallest time step that was still computationally efficient but didn't cause the populations to explode exponentially. Without a carrying capacity, both the deer and wolf populations cyclically fluctuate over time, with the peak wolf population slightly lagging behind the deer population; thus, both populations are stable. The period of the oscillations is approximately 40 years, with wolves never going above 100 and deer never going above 400 in number. With a carrying capacity, the oscillations stabilize and both populations eventually approach a steady-state equilibrium.

Altering the Time Step

To test the stability of the model, various parameters were altered and the effects on the model were investigated. Figures 4 and 5 shows the changes in the model when the time step *dt* was changed from 0.01 to 0.1 and 0.001 years. For a large time step we see the model without carrying capacity become increasingly unstable as time increases—each spike gets larger and larger as time passes. The carrying capacity model is affected less by the time step but we do see it takes a bit longer to stabilize with a longer time step.

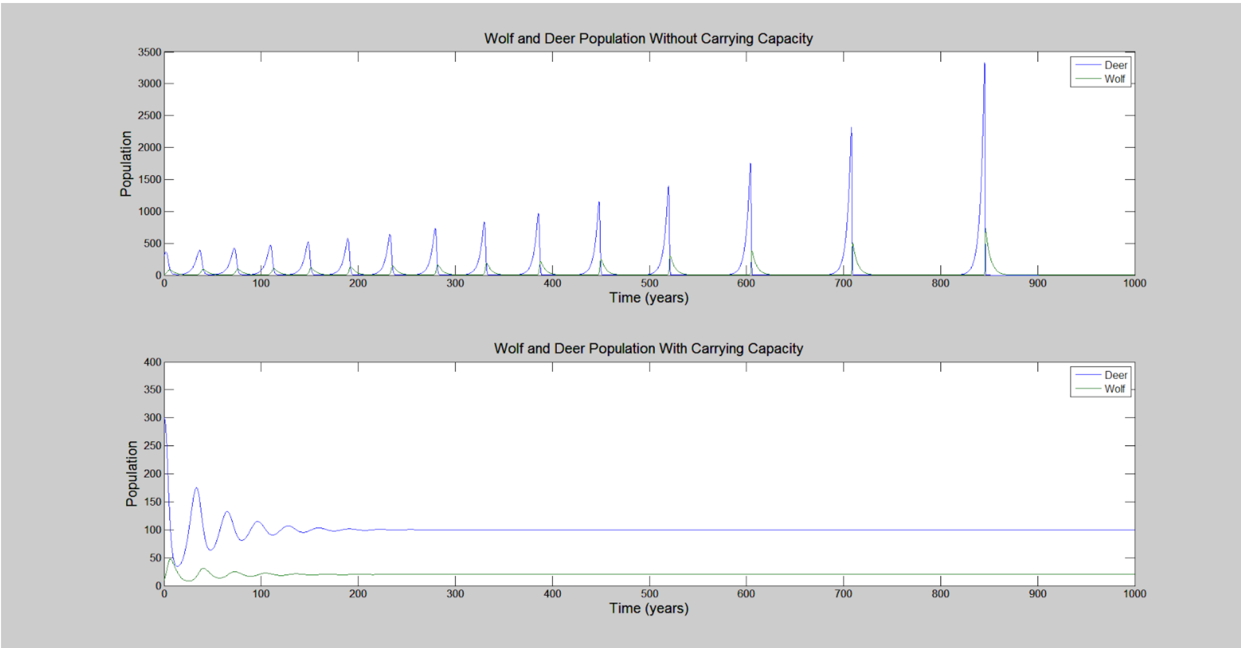

Figure 4. *model run with time step dt = 0.1 years. a) This larger time step causes the top panel to become unstable, with the deer populations increasing with each oscillation. This demonstrates the sensitivity of the initial parameters and this larger time step does not accurately reflect what is occurring in the system. b) The larger time step does not have a large effect on the bottom panel, with steady-state levels the same as the original model run (note the difference in the x-axis scale relative to Figure 3).*

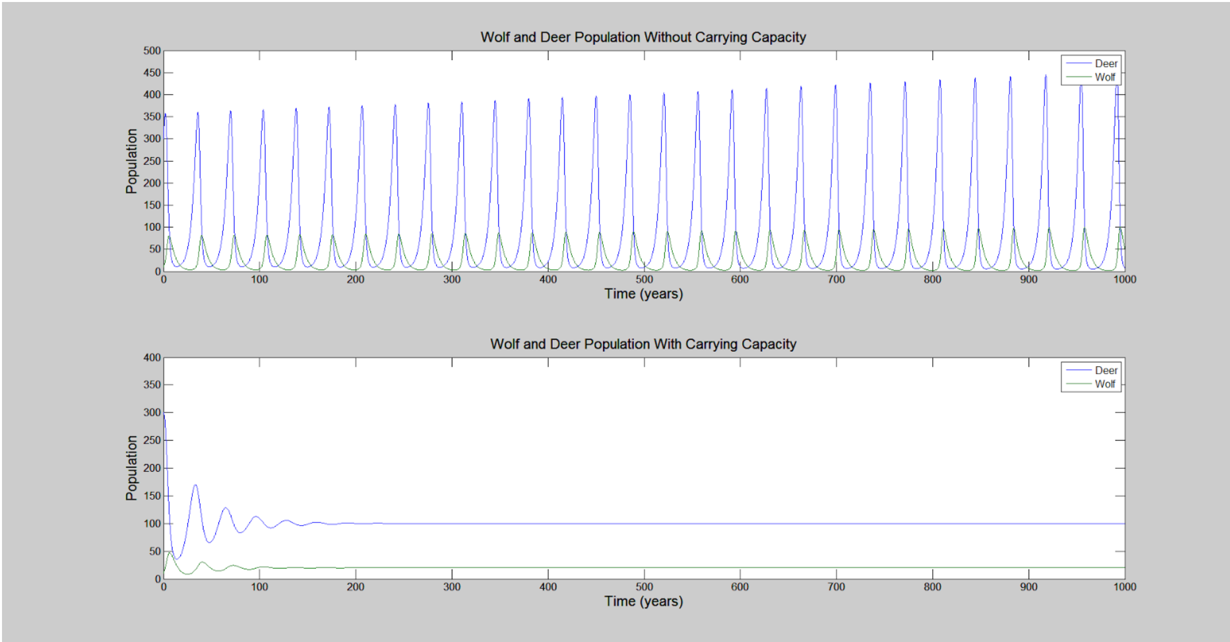

Figure 5. *the original model run (dt = 0.01 years) with the expanded x-axis in order to compare with Figure 4. Without carrying capacity, this smaller time step does not result in large increases in the deer population with each oscillation*

Altering Parameter Values

The various model parameters were halved and doubled, and the resulting effects on the model are shown in Figures 6 and 7. Figure 6 shows the model run with no carrying capacity. Overall, smaller parameter values cause the period of population oscillations to decrease. Smaller oscillations also allow the deer population to reach higher maximum numbers (up to 475) and higher minimum values (down to 75, as compared to near-zero values with larger parameters). The wolf population is steadier with smaller parameter values, with the maximum and minimum population numbers being closer together. Figure 7 shows the same altered parameters but with carrying capacity. Different steady-states are reached, with lower parameter values favouring the deer (who reach higher steady-state numbers, while the wolves reach a lower steady-state compared to the initial parameters).

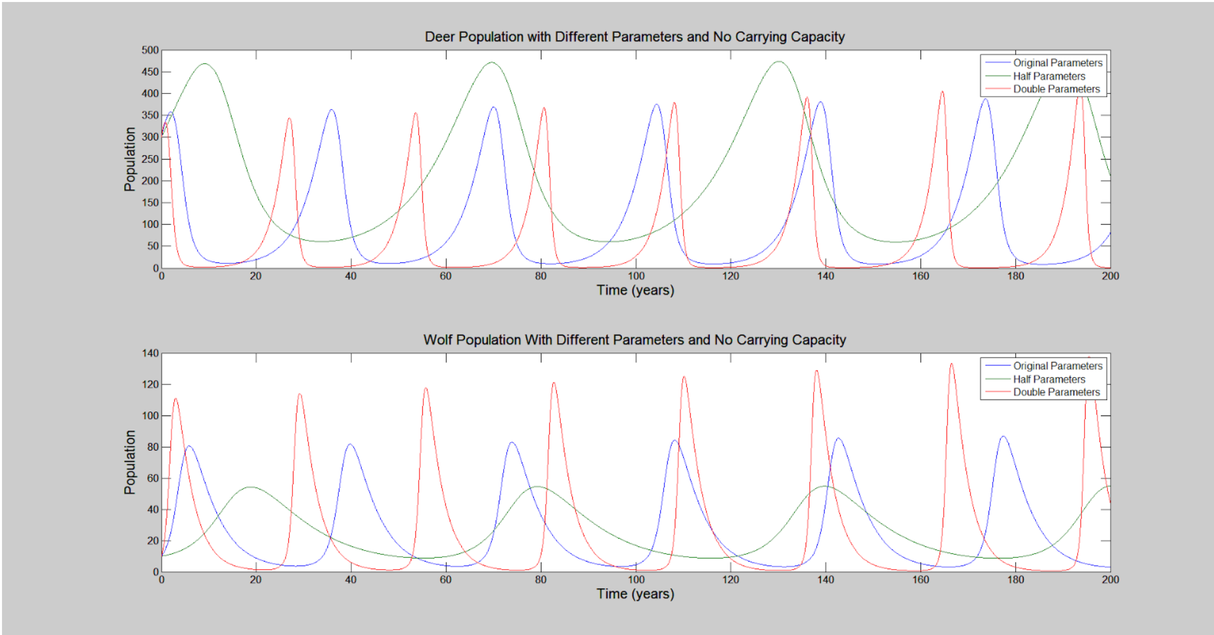

Figure 6. *Running the non-carrying capacity model with various parameter values over 200 years. Blue indicates the initial values, green indicates halved-values and red indicates doubled values.*

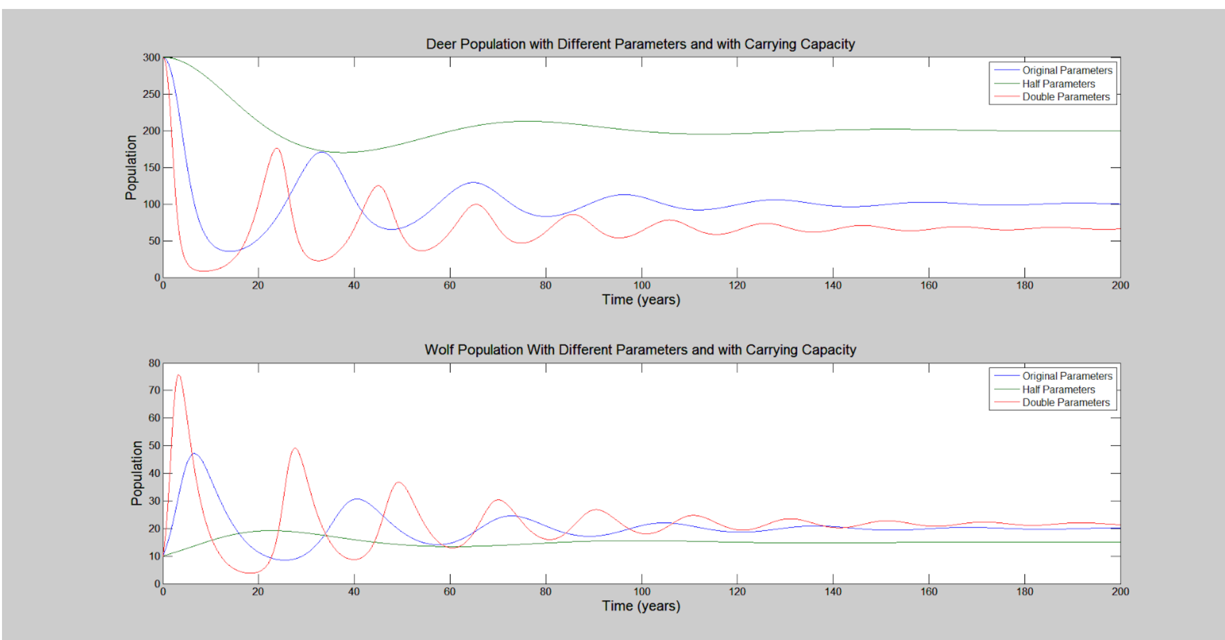

Figure 7. *Running the carrying capacity model with various parameter values over 200 years. Blue indicates the initial values, green indicates halved-values and red indicates doubled values***.**

As shown in Figures 6 and 7, the model is quite sensitive to the initial parameters. For the model without carrying capacity, we see the spikes differ in amplitude, period and frequency when we vary the initial conditions. Under some initial conditions we see the wolf and deer populations dip to near zero levels. The model is not stable for greatly different parameters since some of the populations will reach zero; for parameters close to our original parameters our model is stable.

For the model with carrying capacity we see the same variations in the wave parameters as before with different steady-states getting reached depending on the initial conditions. Some steady-states for various parameters are listed below in Table 2. We see the steady-states vary by a large margin with deer steady-states ranging from 50-100 and wolves by 15-90. Since the populations always reach a sustainable steady-state, our model with carrying capacity is stable with different parameters.

Parameter *a*	**Parameter *b***	**Parameter *c***	**Parameter *d***	**Steady-state pop. for deer**	**Steady-state pop. for wolves**
0.25	0.01	0.2	0.2	100	20
0.5	0.02	0.4	0.4	60	22
0.125	0.005	0.1	0.1	200	15
0.125	0.005	0.2	0.2	200	15
0.5	0.02	0.1	0.1	100	22
0.5	0.005	0.4	0.1	50	90

Table 2. *Steady-states of wolf and deer populations with varying parameters. The initial population of deer is 300 and for wolves is 10. The carrying capacity is 500 for deer.*

Altering the Carrying Capacity

Changing the carrying capacity *cc* value (Figure 8Figure 9) we see a change in the wave amplitude. The steady-state reached by the deer is unaffected by carrying capacity, whereas the wolf steady-state interestingly does change with different carrying capacities. Our model is stable for different carrying capacities since the steady-state always reaches a stable population.

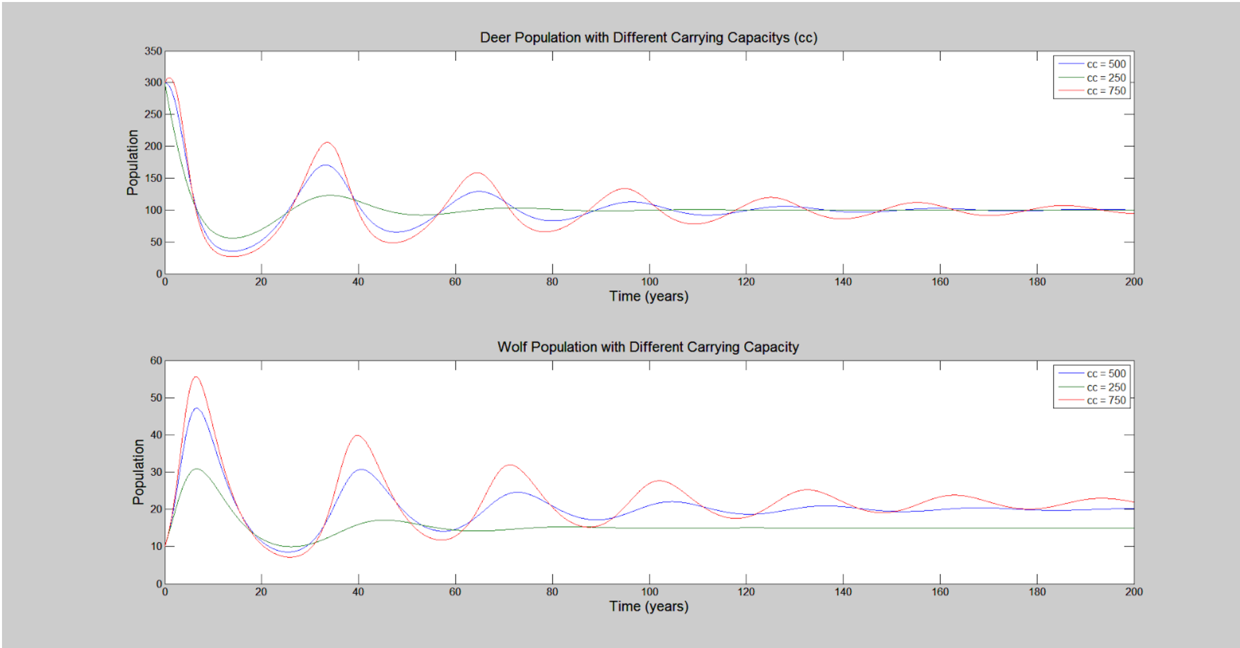

Figure 8. *deer population a) and wolf population b) over time with deer carrying capacity values cc = 250 (green), 500 (blue), 750 (red)*

Altering Initial Population Sizes

Changing the initial population sizes will affect the model without carrying capacity (Figure 9). Relatively large wolf populations will cause the deer populations to crash to near zero, thus becoming unsustainable. Our model is sensitive to initial conditions for this reason. To remain sustainable the initial wolf population needs to be low relative to the initial deer population.

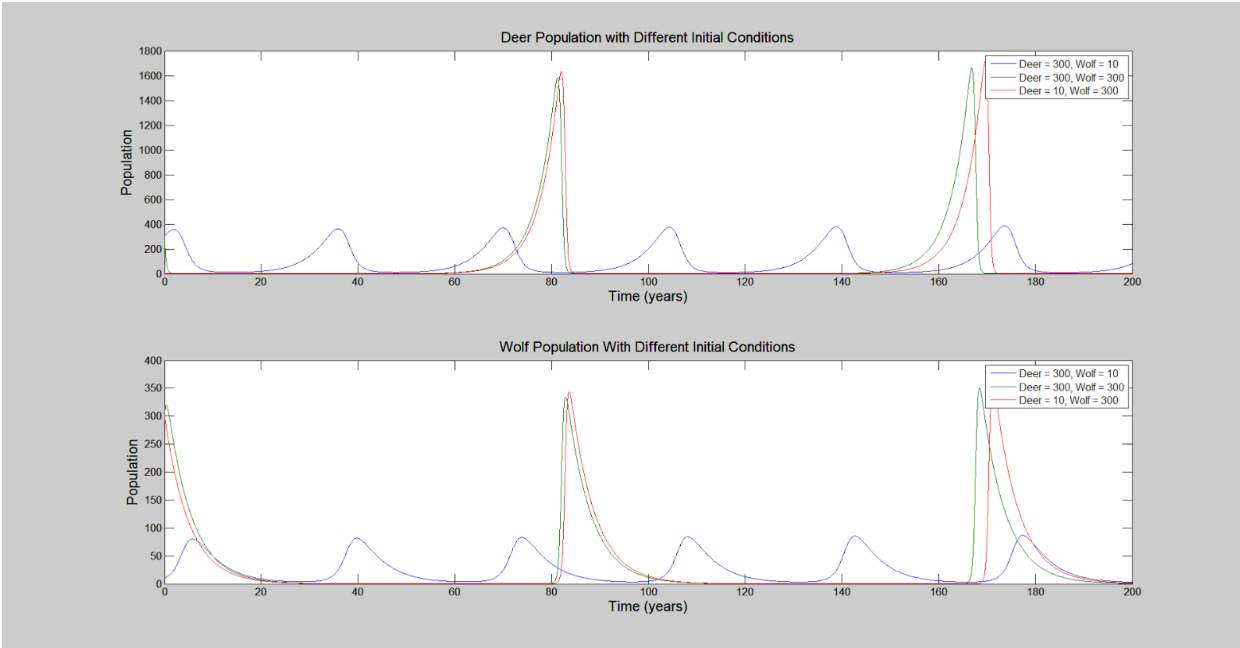

Figure 9. *deer population a) and wolf population b) over time without deer carrying capacity. Initial population sizes of D = 300, W =300 (green), D = 300, W = 10 (blue), D = 10, W = 300 (red).*

Our model with carrying capacity (Figure 10) is affected by the initial conditions early in the time period, but will always reach the same steady-state regardless of initial conditions. When the initial wolf population is large relative to the deer population, we see the deer and wolf populations crash to near zero levels early in the model run. Although they will rebound to healthy steady-states, the populations are not sustainable for approximately 100 years. The model is stable for different initial conditions.

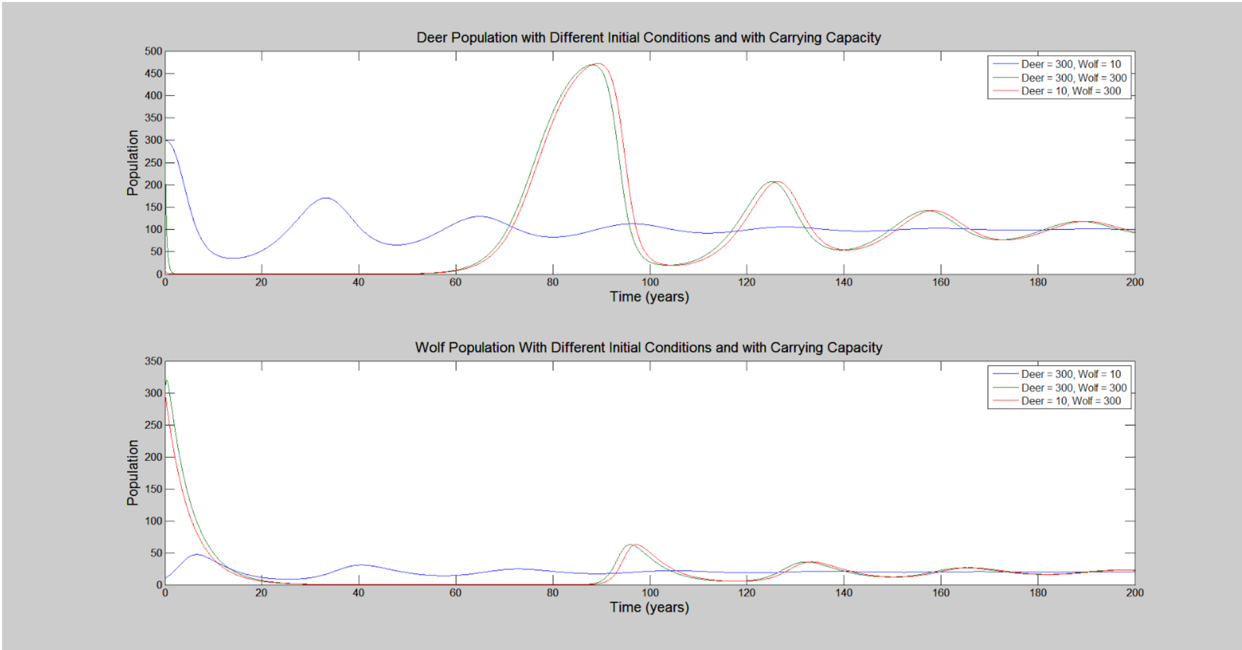

Figure 10. *deer population a) and wolf population b) over time with deer carrying capacity. Initial population sizes of D = 300, W =300 (green), D = 300, W = 10 (blue), D = 10, W = 300 (red).*

Impact of Hunting

We simulated the presence of hunting by doubling the wolf death rate parameter (Figure 11). This resulted in a new deer population steady-state that was much higher. As well, the deer population minimum prior to stabilization was also much higher. The effect of doubling the wolf death rate on the wolf population was minimal with the wolves reaching similar steady-states in both scenarios. Based on this model, if the deer population were at a dangerously low level, it would be recommended to hunt wolves—this will have minimal affect on the long-term steady-state of the wolf population, but it will increase the deer population steady-state level.

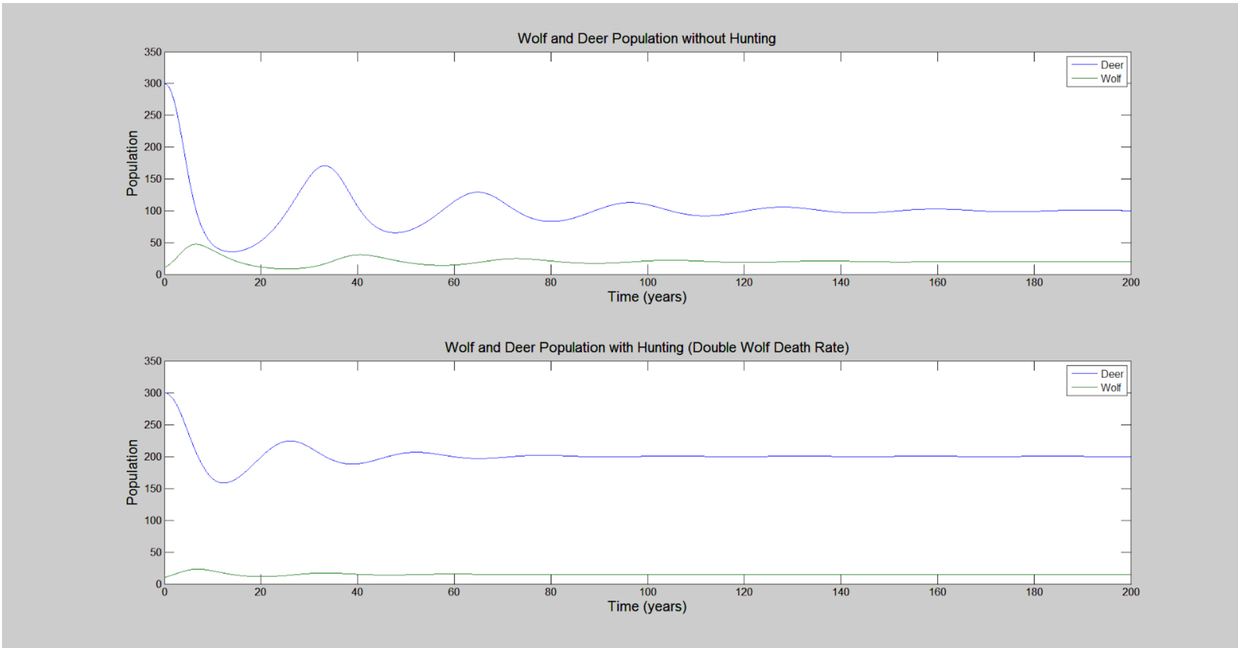

Figure 11. *a) carrying capacity model with unaltered parameter values; depicted for reference. b) the carrying capacity model with doubled wolf death rate d to simulate the presence of wolf hunting.*

V. Conclusion

Our model shows that the predator-prey relationship between deer and wolves is sensitive to differing parameter and initial conditions, but ultimately stable when we introduce a carrying capacity in our model. Without a carrying capacity, we see that large changes in parameter and initial conditions can cause the deer and wolf populations to crash to zero; however, our model remains stable over reasonable time limits and small time steps. Our model shows that initial wolf populations must remain small compared to deer populations for both populations to not crash to near zero levels. With a large time step we see our model will not remain stable without a carrying capacity. When we introduce a carrying capacity into our model, large—but reasonable—changes to initial conditions, model parameters, and carrying capacity will not cause the deer and wolf populations to become unstable. If the deer population becomes endangered, our model shows that hunting wolves is a good strategy to help the deer recover.

Works Cited

Mech, D.L., Boitani, L. (2003). *Wolves: Behaviour, Ecology and Conservation*. University of Chicago Press

Verme, L.J. (1965). *Reproduction Studies on Penned White-Tailed Deer*. The Journal of Wildlife Management, 29(1): 74 - 79

Group Contributions

- Nils: wrote the Matlab code, created figures; helped write the discussion and the conclusion
- April: wrote the abstract and background
- Derek: conceptual model; all the methods and most of the results
- Stephanie: editing of each section; formatting and compiling into Word