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Let's Play a Love Game

I. Abstract

In the game of love, rational players interact with potential partners in order to find a lasting relationship or to minimize the emotional anguish experienced in the event of failure. This paper seeks to find the optimum strategy to maximize your utility function when 1) you know the opposing player's strategy, or 2) you don't know the opposing player's strategy. Assumptions of classical game theory, evolutionary game theory, infinite number of repeated interactions, mutual levels of feelings between players, and mutual desire for a relationship are used throughout. In the simplified, single interaction game wherein you can either 'Pursue' or 'Not Pursue' the other player, iterated elimination of dominated strategies yields a coordination game. Likewise in the dynamical system analysis, players try to coordinate their strategy to the initial configuration of the population. When the opponent's strategy is unknown, the optimal strategy is 'Pursue'. For the expanded model of infinite number of repeated interactions, you either are indifferent to playing 'Always Pursue' or 'Cautiously Pursue' (opponent plays 'Always Pursue'), play 'Always Pursue' (opponent 'Cautiously Pursue'), play 'Cautiously Pursue' (opponent 'Narcissistic Pursue'), or you are indifferent to playing 'Cautiously Pursue' or 'Always Not Pursue' (opponent 'Always Not Pursue'). When your opponent plays an unknown strategy, risk adverse players should play 'Cautiously Pursue', while risk seeking and risk neutral players should play 'Always Pursue'. The initial assumptions led to shortcomings in the resulting analysis; thus, it is recommended that future research be conducted in the variance of these assumptions.

II. Introduction

We all have that friend—the one that can pick-up any number, the one that has gone on more dates than you dare count, the one that always has the boys or girls chasing after them. He or she may not have the best personality, intelligence, interpersonal skills, personal values, or even physical appearance. So, how exactly do they do it? While long-lasting relationships are often determined by the two individuals' compatibility, the success of the initial courtship may have less to do with desirable characteristics and more to do with luck, ability to read the other person, and strategic optimization.

Thus, this paper strives to answer the age-old question of how to play the game of love: the strategy to maximize your utility function when 1) you know the opposing player's strategy, or 2) you don't know the opposing player's strategy.

Under the assumption that both players have equal feelings for each other and want to be in a relationship, rational players with their own self-interest will interact with other potential partners to form a relationship. The objective of said interaction is to maximize the values of each player's utility function by either finding a lasting relationship or reducing the emotional anguish experienced if the courtship fails. The outcome and subsequent utilities of the interaction(s) will be determined by the players' coordination—or lack thereof—in pursuing each other.

An evolutionary game theory approach will also be used. Successful and unsuccessful relationships have an impact on the evolving *Homo sapiens* population fitness and reproduction, thereby making humans inclined to prefer certain strategies.

Therefore evolutionary and classical forces are intricately at work, yielding a more accurate model of the love game in combination than if examined in isolation.

Assumptions used are summarized here for ease of reference:

- Classical Game Theory: players are rational
  - Higher utilities (which correspond to payoffs) are always preferred
  - Players only care about own utility
- Evolutionary Game Theory:
  - Population with variation ('Pursue' and 'Not Pursue' types)
  - Non-genetic environmental influences on chosen strategies
  - Utility payoffs related to fitness
- Initial Conditions: both players have equal levels of feelings and want to be in a relationship
- Repeated Game Interactions: infinite number of rounds m
- Utilities: $A, B, C \geq 0$, $B > C$, $-kA > -A-C$ (k is a positive non-zero scalar constant)

III. Methodology

Simplified Model

The following 2x2 symmetric payoff matrix (Figure 1)

	Column Player		
Row Player		**Pursue**	**Not Pursue**
	Pursue	(B -½C, B -½C)	(-A - C, -*k*A)
	Not Pursue	(-*k*A, -A -C)	(-*k*A, -*k*A)

Figure 1. *Normal form game (matrix game) for 'Pursue' and 'Not Pursue' strategies. Utilities are determined by A the emotional anguish of love, B the benefit of being in a relationship, and C the cost of pursuing the relationship, where A, B, C ≥ 0, B > C, and k is a positive non-zero scalar constant.*

depicts the case where a player may 'Pursue' or 'Not Pursue' the other player, with utilities determined by the emotional anguish of love (A), the benefit of being in a relationship (B), and the cost of pursuing the relationship (C), where $A, B, C \geq 0$, $B > C$, and $-kA > -A-C$ (see Appendix A for detailed derivation). In the case where both players 'Pursue' each other, the cost of pursuit (time, money, etc.) is distributed evenly between the two players ($-\frac{1}{2}C$) and both players receive the full benefit of the relationship (B). When one player 'Pursue' and the other 'Not Pursue', the pursuer bears the full burden of the cost ($-C$) and experiences the negative emotional payoff of unrequited love ($-A$). The non-pursuer also experiences the negative emotional payoff of love, but this hurt is one of regret—from a love that could've been but which was not pursued ($-kA$[1]). In the final case where both 'Not Pursue', there is neither a cost (no players pursues the other), nor a

[1] Where *k* is a positive non-zero scalar constant

benefit (no relationship formed); however, both experience the negative emotional impact of love not pursued (-kA).

Expanded Model – Repeated Interactions

Although many courtships consist only of a one-time or one-night-stand fleeting encounter, most often they are characterized by a series of interactions involving mutual 'Pursue', mutual 'Not Pursue', or one player 'Pursues' while the other 'Not Pursues'. Building upon our original model, Steven Strogatz's love affairs was used as a framework to derive possible repeated game strategies over an infinite number of rounds *m* that each player may choose (Appendix B):

1. <u>Eager Beaver</u>: always 'Pursue' other player
2. <u>Secure/Cautious Lover</u>: only 'Pursue' if other player first 'Pursue'
    - Retreats from own feelings but encouraged by other player's feelings
3. <u>Narcissistic Nerd</u>: only 'Pursue if other player 'Not Pursue' them
    - Encouraged by own feelings but retreats if the other player pursues them
4. <u>Hermit</u>: always 'Not Pursue' other player

The 'Eager Beaver' strategy is when one 'Always Pursue' and the 'Hermit' is when one 'Always Not Pursue' the other player. Both the 'Securely Cautious' ('Secure/Cautious Lover') and the 'Narcissistic Pursue' ('Narcissistic Nerd') are memory-1 strategies that always 'Not Pursue' in the first round. As a 'Cautiously Pursue' strategist, you will play in subsequent rounds the *same* as the opponent did in the previous round (i.e. if the opponent 'Pursue' last round, you will 'Pursue' in this round; conversely if the opponent 'Not Pursue' last round, you will 'Not Pursue' this round). In colloquial terms, this means that you will only show affection if the other player makes a move. As a 'Narcissistic Pursue' strategist, you will play in subsequent rounds the *opposite* of what the opponent did in the previous round (i.e. if the opponent 'Pursue' last round, you will 'Not Pursue' in this round; conversely if the opponent 'Not Pursue' last round, you will 'Pursue' this round). Often known as playing hard-to-get, this strategy means you will incessantly go for the other player if they don't show interest in you but the moment you have them under your thumb, you become indifferent and withdraw.

Using these four repeated strategies, a symmetric 4x4 matrix over an infinite number of rounds *m* can be constructed (*Figure 2*):

	Column Player				
Row Player		**Always Pursue**	**Cautiously Pursue**	**Narcissistic Pursue**	**Always Not Pursue**
	Always Pursue	$(B - \frac{1}{2}C, B - \frac{1}{2}C)$	$(B - \frac{1}{2}C, B - \frac{1}{2}C)$	$(-A - C, -kA)$	$(-A - C, -kA)$
	Cautiously Pursue	$(B - \frac{1}{2}C, B - \frac{1}{2}C)$	$(-kA, -kA)$	$(\frac{1}{4}[-(2k+1)A+B-\frac{3}{2}C)], \frac{1}{4}[-(2k+1)A+B-\frac{3}{2}C)])$	$(-kA, -kA)$
	Narcissistic Pursue	$(-kA, -A - C)$	$(\frac{1}{4}[-(2k+1)A+B-\frac{3}{2}C)], \frac{1}{4}[-(2k+1)A+B-\frac{3}{2}C)])$	$(\frac{1}{2}[-kA+B-\frac{1}{2}C], \frac{1}{2}[-kA+B-\frac{1}{2}C)])$	$(-A - C, -kA)$
	Always Not Pursue	$(-kA, -A - C)$	$(-kA, -kA)$	$(-kA, -A - C)$	$(-kA, -kA)$

Figure 2. *Normal form game (matrix game) for 'Always Pursue', 'Cautiously Pursue', 'Narcissistic Pursue', and 'Always Not Pursue' repeated game strategies over an infinite number of rounds m. Utilities are determined by A the emotional anguish of love, B the benefit of being in a relationship, and C the cost of pursuing the relationship, where* $A, B, C \geq 0$, $B > C$, $-kA > -A-C$, *and k is a positive non-zero constant.*

For the mathematically inclined, the utility derivations may be found in Appendix C.

IV. Analysis

Simplified Model

Using iterated elimination of dominated strategies, the 2x2 payoff matrix (Figure 3)

		Column Player	
		Pursue	**Not Pursue**
Row Player	**Pursue**	*S NE, PO* (B -½C, B -½C)	(-A - C, -*k*A)
	Not Pursue	(-*k*A, -A -C)	*S NE* (-*k*A, -*k*A)

Figure 3. *Symmetric coordination game for 'Pursue' and 'Not Pursue' strategies. There are two strict Nash Equilibriums and one Pareto Optimal Outcomes. There are no dominant or dominated strategies.*

is found to be a symmetric coordination game with two strict Nash Equilibriums (when both 'Pursue' or both 'Not Pursue') and one Pareto Optimal Outcome (when both 'Pursue'). There are no dominant or dominated strategies.

Case 1: Opponent Plays Known Strategy

Although both experience the greatest utility when both players 'Pursue' (this Pareto Optimal Outcome is also a strict Nash Equilibrium), if either one suspects the other will 'Not Pursue' then there is a strong incentive to switch strategies and cut their loses from the smallest payoff *-A-C* to the larger payoff *-kA*. In this case wherein both players choose 'Not Pursue', they may stay in this Nash Equilibrium because there is no incentive for either player to unilaterally switch strategy. Thus, in order maximize your utility function it is in your best interest to coordinate strategies with the opposing player.

Dynamical System Analysis

Also taking an evolutionary approach, we wish to explore how a population of *Homo sapiens* will evolve over time and what the resulting population fitness will be. Additionally, we wish to explore whether yourself or other evolved humans you interact with are predisposed to playing certain strategies.

Let x be the frequency of 'Pursue' types. Then plotting *dx/dt* vs. *x* (Figure 4)

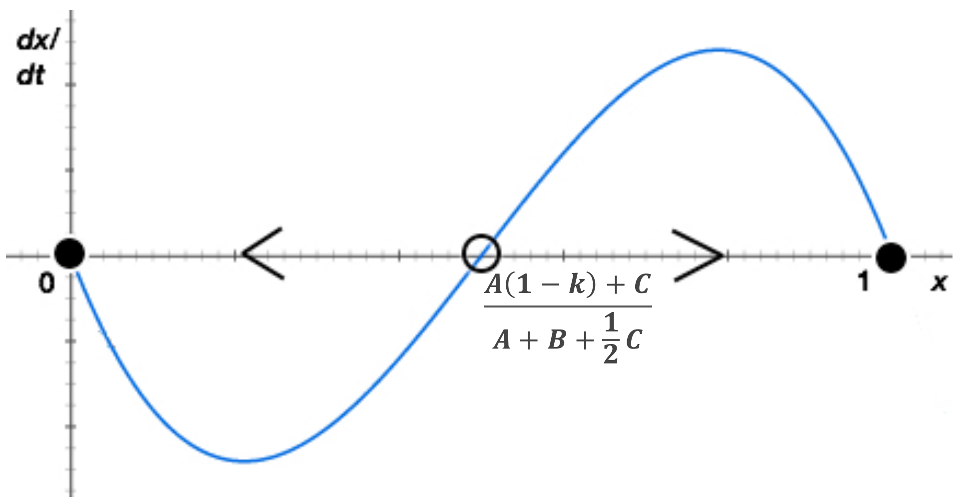

Figure 4. *The differential equation for the Love Game coordination game, where x is the frequency of 'Pursue' types in the population. The rate of change is negative for* $\mathbf{0 < x^* < \frac{A(1-k)+C}{A+B+\frac{1}{2}C}}$ *and positive for* $\mathbf{\frac{A(1-k)+C}{A+B+\frac{1}{2}C} < x^* < 1}$*, so* $x^*=0$ *and* $x^*=1$ *are stable equilibria/fixed points (denoted by closed circles) and* $\mathbf{x^* = \frac{A(1-k)+C}{A+B+\frac{1}{2}C}}$ *is an unstable equilibrium/fixed point (denoted by open circle).*

we see there are stable fixed points at $x^* = 0$, $x^* = 1$ (corresponding to strict Nash Equilibriums) and an unstable fixed point at $x^* = \frac{A(1-k)+C}{A+B+\frac{1}{2}C}$ (see Appendix D for derivation). Depending on the initial value of *x*, we expect an increase in coordination of strategies such that there is only a population of 'Pursue' or 'Not Pursue' types. In other words, above a certain *x* threshold, 'Pursue' is favourable; conversely, below a certain *x* threshold, 'Not Pursue' is favourable. This indicates that people's decision to 'Pursue' or 'Not Pursue' is based on what they perceive the majority of the current population to be playing. Thus, they will try to coordinate their strategy to the initial configuration of the population.

Case 2: Opponent Plays Unknown Strategy

By assuming indifference (i.e. $p^* = q^*$), the mixed Nash Equilibrium (p^*, q^*) is given by

$$p^* = q^* = \frac{A(1-k)+C}{A+B+\frac{1}{2}C}$$

where $0 < p^*, q^* < 1$. Computing the row player's expected utilities for 'Not Pursue' (N), we have

$$\begin{aligned}
U_{Row}(N) &= q^*(-kA) + (1-q^*)(-kA) \\
&= \left(\frac{A(1-k)+C}{A+B+\frac{1}{2}C}\right)(-kA) + \left[1 - \left(\frac{A(1-k)+C}{A+B+\frac{1}{2}C}\right)\right](-kA) \\
&= \left(\frac{A(1-k)+C}{A+B+\frac{1}{2}C}\right)(-kA) + \left(\frac{A(1-k)+C}{A+B+\frac{1}{2}C}\right)(kA) - kA \\
&= -kA
\end{aligned}$$

By indifference, the row player's expected utilities for 'Pursue' (P) is $U_{Row}(P) = U_{Row}(N) = -kA$ (See Appendix E for $U_{Row}(P)$ computation). Assuming rationality for both players, the optimal strategy to play when you don't know the other opponent's strategy can be found from comparing the expected utilities from the three Nash Equilibriums:

$$\begin{array}{ccccc}
-kA & = & -kA & < & B - \frac{1}{2}C \\
U_{Mixed\ NE} & = & U_{(N,N)\ NE} & < & U_{(P,P)\ NE}
\end{array}$$

In the case when the opponent's strategy is unknown, the optimal strategy to play is 'Pursue'.

Expanded Model – Repeated Interactions

Using the same iterated elimination of strictly[2] dominated strategies to analyze the 4x4 payoff matrix (see Appendix F for inequality derivations between utility payoffs), we find that the expanded model (Figure 5)

	Column Player				
Row Player		**Always Pursue**	**Cautiously Pursue**	**Narcissistic Pursue**	**Always Not Pursue**
	Always Pursue	*W NE, PO* (B -½C, B -½C)	*W NE, PO* (B –½C, B -½C)	(-A -C, -*k*A)	(-A -C, -*k*A)
	Cautiously Pursue	*W NE, PO* (B –½C, B -½C)	(-*k*A, -*k*A)	(¼[-(2k+1)A+B-$^{3}/_{2}$C)], ¼[-(2k+1)A+B-$^{3}/_{2}$C)])	(-*k*A, -*k*A)
	Narcissistic Pursue	(-*k*A, -A -C)	(¼[-(2k+1)A+B-$^{3}/_{2}$C)], ¼[-(2k+1)A+B-$^{3}/_{2}$C)])	(½[-kA+B-½C], ½[-kA+B-½C)])	(-A -C, -*k*A)
	Always Not Pursue	(-*k*A, -A -C)	(-*k*A, -*k*A)	(-*k*A, -A -C)	*W NE* (-*k*A, -*k*A)

Figure 5. *Symmetric coordination game for 'Always Pursue', 'Cautiously Pursue', 'Narcissistic Pursue', and 'Always Not Pursue' strategies over an infinite number of rounds m. There are three weak Nash Equilibriums and one Pareto Optimal Outcome. 'Always Not Pursue' is weakly dominated by 'Cautiously Pursue'.*

is a symmetric game with four weak[3] Nash Equilibriums and one Pareto Optimal Outcome. A Pareto Optimal Outcome and Nash Equilibrium occur simultaneously at

[2] We will not eliminate *weakly* dominated strategies. This is to avoid the loss of a weak Nash Equilibrium at (Not Pursue, Not Pursue).

[3] As stated in the Methodology section, utility payoffs are based on an infinite number of rounds *m*. Thus, some of the expected utilities were approximated by limits (Appendix B); this yields a weak Nash Equilibrium at (Always Pursue, Always Pursue). Had this repeated interaction been over a *finite* number of rounds, then the initial interaction would have a reductive effect on the

(Always Pursue, Always Pursue), (Cautiously Pursue, Always Pursue), and (Always Pursue, Cautiously Pursue). There are no strictly dominated strategies; however, 'Always Not Pursue' is weakly dominated by 'Cautiously Pursue'.

Case 1: Opponent Plays Known Strategy

Looking at the above matrix, it becomes apparent that the optimum strategies given your opponent's strategy are

- Opponent plays 'Always Pursue': you are indifferent to playing 'Always Pursue' or 'Cautiously Pursue'
- Opponent plays 'Cautiously Pursue': you play 'Always Pursue'
- Opponent plays 'Narcissistic Pursue': you play 'Cautiously Pursue'
- Opponent plays 'Not Pursue': you are indifferent to playing 'Cautiously Pursue' or 'Always Not Pursue'

Case 2: Opponent Plays Unknown Strategy

In a given population, there are individuals characterized by having risk adverse, risk neutral, or risk seeking preferences. Since we cannot use iterated elimination of strictly dominated strategies to simplify our matrix, let us examine each of these rational preferences to determine the optimum strategy to play when the opponent's strategy is unknown.

Risk Seeking Preference

A risk seeking player puts a disproportional emphasis on *high* utilities; since *B-½C* is the highest utility, this player will only seek strategies *with* this payoff. Thus, we can eliminate both 'Narcissistic Pursue' and 'Always Not Pursue' from the matrix. Now with a 2x2 matrix consisting of 'Always Pursue' and 'Cautiously Pursue', the player seeks to

average utility of (Cautious Pursue, Always Pursue), making (Always Pursue, Always Pursue) a strict Nash Equilibrium.

maximize receiving the highest payoff (with no regards to lower payoffs). We therefore eliminate 'Cautiously Pursue' as it has only one outcome—rather than two for 'Always Pursue'—with the utility *B-½C*. For risk seeking players, the rational strategy to play when you don't know your opponent's strategy is 'Always Pursue'.

Risk Adverse

A risk adverse player puts a disproportional emphasis on *low* utilities; since *-A-C* is the lowest utility, this player will only seek strategies *without* this payoff. Thus, we can eliminate 'Always Pursue' and 'Narcissistic Pursue' from the matrix. Left with a 2x2 matrix, we notice that 'Cautiously Pursue' is always greater than or equal to 'Always Not Pursue'—the latter is a weakly dominated strategy. Iterated elimination of *weakly* dominated strategies is then used to eliminate 'Always Not Pursue'. We can conclude that for risk adverse players, the rational strategy to play when you don't know your opponent's strategy is 'Cautiously Pursue'. By avoiding the worst outcome at all costs, risk aversive behaviours (most common preference type in a given population) will yield the encounter of two 'Cautiously Pursue' types. Interestingly enough, both players would receive the second lowest payoff *-kA*.

Risk Neutral

A risk neutral player is indifferent to the risk associated with receiving a high or low utility; instead they only seek to maximize their own payoff. Similar to the 2x2 analysis, the optimal strategy to play when you don't know the opponent's strategy can be found from comparing the expected utilities from all Nash Equilibriums. Although computing the 4x4 mixed Nash Equilibrium is essential to our analysis, it is unnecessary to compute. Since *B-½C* is the highest possible payoff given the framework of our game, then

$$U_{Mixed\ NE} \leq B - \frac{1}{2}C$$

By assuming rationality in both players, the optimal strategy to play when you don't know your opponent's strategy is 'Always Pursue'.

V. Discussion

Despite the analysis yielding definitive strategies to utilise in the cases where the opponent's strategy is known or unknown, the assumptions that were made—these included rationality, *m* being an infinite number of rounds, and that both players like each other equally and want to be in relationship—led to shortcomings in the resulting recommended strategies. The reality of relationships is that people like each other an unequal and fluctuating amount, they might not want to be in a relationship, and they may not stay with the same strategy over an infinite number of iterations.

One such limitation was that the merit of the two additional strategies ('Cautiously Pursue' and 'Narcissistic Pursue') was diminished by the above assumptions. Since a player cannot switch strategy after an initial assessment of the other player, this makes it harder to coordinate strategies. If both people 'Cautiously Pursue' (most common preference type), they will end up with the second lowest payoff $-kA$. Had there been the flexibility of switching to 'Always Pursue', they would have the highest payoff $B-½C$ instead. By having an infinite number of rounds, 'Narcissistic Pursue' becomes impractical after several interactions, especially against itself or a 'Cautiously Pursue' type. Moreover, since everyone likes each other equally and wants to be together, there is no benefit in playing games. Had the proportion of affection been variable, these memory-1 strategies may have been useful in coercing your opponent to increase his or her affection towards you. Although rationality is the fundamental assumption of classical game theory, it is important to note the presence of Allais Paradox wherein most people do not act rationally and do not adhere to expected utility theory.

The lifting of these assumptions would lead to deeper and more meaningful insights into the nature of this love game. It is recommended that future research be conducted in the variance of these assumptions.

VI. Summary

Simplified Model

Under the assumptions of rationality, equal feelings between players and wanting to be in a relationship, it is in your best interest to coordinate strategies with the opposing player. This will yield strict Nash Equilibriums when both 'Pursue' (also Pareto Optimal Outcome) or both 'Not Pursue'. Likewise in an evolutionary game theory context, a player is trying to coordinate their strategy to the initial configuration of the population. This occurs because people's decision to 'Pursue' or 'Not Pursue' is based on what they perceive the majority of the current population to be playing. In the case when you don't know the opponent's strategy, then the optimal strategy is the Nash Equilibrium with the highest expected utility—in this case, 'Pursue'.

Expanded Model

Given that your opponent plays:

- 'Always Pursue': you are indifferent to playing 'Always Pursue' or 'Cautiously Pursue'
- 'Cautiously Pursue': you play 'Always Pursue'
- 'Narcissistic Pursue': you play 'Cautiously Pursue'
- 'Not Pursue': you are indifferent to playing 'Cautiously Pursue' or 'Always Not Pursue'

When your opponent plays an unknown strategy, risk adverse players should play ‘Cautiously Pursue’, while risk seeking and risk neutral players should play ‘Always Pursue’.

Note that the assumptions made—rationality, infinite number of rounds, and both players liking each other equally and wanting to be in relationship—led to shortcomings in the resulting analysis. It is recommended that future research be conducted in the variance of these assumptions.

Word Count: 3370
(includes footnotes)

Appendix A

Learning Your A, B, C's[4]

There are two nontrivial (i.e. non-zero) values for A: a player pursues but is unsuccessful, thus unrequited anguish ($-A$); or a player hopes to be in a relationship (initial assumption that both players like each other) but whose strategy meant that they did not pursue the other, thereby leaving them in regret ($-kA$). The result of both scenarios is that a relationship is not formed hence the negative emotional anguish experienced from the interaction. A value of $A=0$ implies no emotional anguish experienced and is omitted from the matrix for clarity.

There is one nontrivial (i.e. non-zero) for B: when both lovers pursue each other and end up together (B). A value of $B=0$ implies that no relationship is formed and is omitted from the matrix for clarity.

Since C represents the extent to which each player pursues the other, there will be three cases: when one player pursues ($-C$) and the other doesn't ($C=0$); when both players pursue equally ($-\frac{1}{2}C$ each); and when neither player pursues the other ($C=0$ each). The case when $C=0$ is omitted from the matrix for clarity.

[4] Remark: assume that $A, B, C \geq 0$, $B > C$, and $-kA > -A-C$

Appendix B

Steven H. Strogatz's love affairs from "Nonlinear Dynamics and Chaos: With Applications To Physics, Biology, Chemistry, And Engineering" (Strogatz, 1994) was used as the framework to construct a simplified array of the possible repeated game strategies that each player may choose. Let the two players' (denoted by *R* and *C* for row and column players respectively) love or hate for the other player at time *t* be represented by $R(t)$ and $C(t)$, where $R(t), C(t) > 0$ indicates love and $R(t), C(t) < 0$ indicates hate. To indicate the romantic style of each player, coefficients *a, b* will be used for the row player and coefficients *c, d* for the column player. This can be expressed by the two differential equations,

$$\frac{dR}{dt} = aR + bC$$

$$\frac{dC}{dt} = cR + dC$$

where the coefficients *a* (and *c*) represent the extent the row player (column player) is encouraged/discouraged by his or her *own* feelings and the coefficients *b* (and *d*) represent the extent the row player (column player) is encouraged/discouraged by the *other's* feelings.

When $a, c < 0$ the player is *encouraged* by his or her own feelings; $a, c > 0$ the player is *discouraged* by his or her own feelings; $a, c = 0$ the player is *oblivious* to his or her own feelings. When $b, d < 0$ then the player is *encouraged* by the other's feelings; $b, d > 0$ the player is *discouraged* by the other's feelings; $b, d = 0$ the player is *oblivious* to the other's feelings. Combining the coefficients for each player, there are four different romantic styles that each player my use (the edge cases of $a, b, c, d = 0$ are omitted for simplicity):

1. Eager Beaver ($a > 0, b > 0; c > 0, d > 0$): always 'Pursue' other player
   - Encouraged by own and other's feelings
2. Secure/Cautious Lover ($a < 0, b > 0; c < 0, d > 0$): only 'Pursue' if other player first 'Pursue'
   - Retreats from own feelings but encouraged by other player's feelings
3. Narcissistic Nerd ($a > 0, b < 0; c > 0, d < 0$): only 'Pursue if other person 'Not Pursue' them
   - Encouraged by own feelings but retreats if the other player pursues them
4. Hermit ($a < 0, b < 0; c < 0, d < 0$): always 'Not Pursue' other player
   - Discouraged by own and other's feelings

Appendix C

Below are the derivations for the expanded 4x4 repeated interactions matrix[5]. Since the edge cases were already described in the 2x2 matrix, we will only examine the additional payoffs that result from the addition of the 'Cautiously Pursue' and 'Narcissistic Pursue' strategies

(Cautiously Pursue, Cautiously Pursue)

In a repeated interaction of infinite rounds m where each player can choose to either 'Pursue' (P) or 'Not Pursue' (N), a game between a 'Cautiously Pursue' and another 'Cautiously Pursue' would be:

'Cautiously Pursue':	N	N	…
'Cautiously Pursue':	N	N	…

From here we can derive the average payoffs for each 'Cautiously Pursue' player to be

$$Average\ Payoff\ (Caut.) = \frac{m(NN)}{m}$$

$$= -kA$$

$$\therefore Average\ Payoff = (-kA, -kA)$$

(Cautiously Pursue, Narcissistic Pursue)

In a repeated interaction of infinite rounds m where each player can choose to either 'Pursue' (P) or 'Not Pursue' (N), a game between a 'Cautiously Pursue' and a 'Narcissistic Pursue' would be:

'Cautiously Pursue':	N	N	P	P	N

[5] Strategies and payoffs in format (Row, Column)

‘Narcissistic Pursue’: N P P N N

thus forming another infinite cycle (Figure 6):

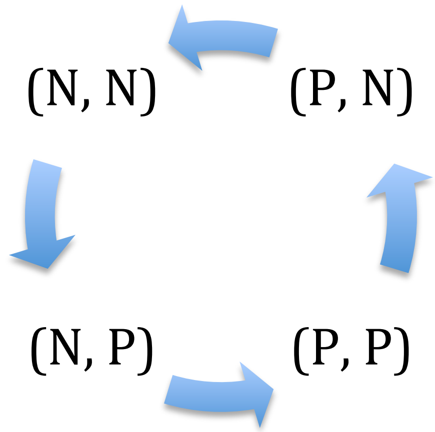

Figure 6. *Repeated game interaction between ‘Cautiously Pursue’ and ‘Narcissistic Pursue’, where ‘Pursue’ and ‘Not Pursue’ are denoted by P and N respectively. Each interaction is shown in format (Row, Column), i.e. (Cautiously Pursue, Narcissistic Pursue).*

From here we can derive the average payoff for the ‘Cautiously Pursue’ (row player) player to be:

$$Average\ Payoff\ (Caut.) = \frac{\boldsymbol{N}N + \boldsymbol{N}P + \boldsymbol{P}P + \boldsymbol{P}N}{4}$$

$$= \frac{(-kA) + (-kA) + \left(B - \frac{1}{2}C\right) + (-A - C)}{4}$$

$$= \frac{-(2k+1)A + B - \frac{3}{2}C}{4}$$

and the average payoff for the ‘Narcissistic Pursue’ player

$$Average\ Payoff\ (Nar.) = \frac{N\boldsymbol{N} + N\boldsymbol{P} + P\boldsymbol{P} + P\boldsymbol{N}}{4}$$

$$= \frac{(-kA) + (-A - C) + \left(B - \frac{1}{2}C\right) + (-kA)}{4}$$

$$= \frac{-(2k+1)A + B - \frac{3}{2}C}{4}$$

$$\therefore Average\ Payoff\ ('Cautiously\ Pursue')$$

$$= \ Average\ Payoff('Narcissistic\ Pursue)$$

$$\therefore Average\ Payoff = \left(\frac{-(2k+1)A + B - \frac{3}{2}C}{4}, \quad \frac{-(2k+1)A + B - \frac{3}{2}C}{4} \right)$$

(Narcissistic Pursue, Narcissistic Pursue)

In a repeated interaction of infinite rounds *m* where each player can choose to either 'Pursue' (P) or 'Not Pursue' (N), a game between a 'Narcissistic Pursue' and another 'Narcissistic Pursue' would be:

'Narcissistic Pursue':	N	P	N	…
'Narcissistic Pursue':	N	P	N	…

thus forming an infinite cycle (Figure 7):

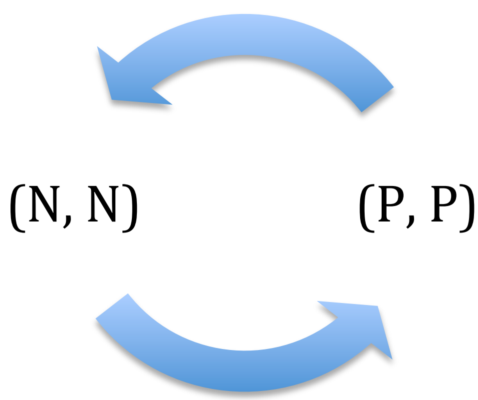

Figure 7. *Repeated game interaction between 'Narcissistic Pursue' and 'Narcissistic Pursue', where 'Pursue' and 'Not Pursue' are denoted by P and N respectively. Each interaction is shown in format (Row, Column), i.e. (Narcissistic Pursue, Narcissistic Pursue)*

From here we can derive the average payoff for each 'Narcissistic Pursue' player to be:

$$Average\ Payoff\ (Nar.) = \frac{NN + PP}{2}$$

$$= \frac{-kA + B - \frac{1}{2}C}{2}$$

$$\therefore Average\ Payoff = \left(\frac{-kA + B - \frac{1}{2}C}{2}, \quad \frac{-kA + B - \frac{1}{2}C}{2} \right)$$

(Always Pursue, Narcissistic Pursue)

In a repeated interaction of infinite rounds *m* where each player can choose to either 'Pursue' (P) or 'Not Pursue' (N), a game between an 'Always Pursue' and a 'Narcissistic Pursue' would be:

'<u>Always Pursue</u>':	P	P	…
'<u>Narcissistic Pursue</u>':	N	N	…

From here we can derive the average payoff for the 'Always Pursue' (row player) player to be:

$$Average\ Payoff\ (Always\ Pursue) = \frac{m(\boldsymbol{P}N)}{m}$$

$$= -A - C$$

and the average payoff for the 'Narcissistic Pursue' player

$$Average\ Payoff\ (Nar.)\ = \frac{m(P\boldsymbol{N})}{m}$$

$$= -A - C$$

$$\therefore Average\ Payoff\ ('Narcissistic\ Pursue')$$

$$=\ Average\ Payoff('Narcissistic\ Pursue)$$

$$\therefore Average\ Payoff = (-A - C, \qquad -A - C)$$

(Always Pursue, Cautiously Pursue)

In a repeated interaction of infinite rounds m where each player can choose to either 'Pursue' (P) or 'Not Pursue' (N), a game between an 'Always Pursue' and a 'Cautiously Pursue' would be:

'<u>Always Pursue</u>':	P	P	P	…
'<u>Cautiously Pursue</u>':	N	P	P	…

From here we can derive the average payoff for the 'Always Pursue' (row player) player to be:

$$Average\ Payoff\ \ (Always\ Pursue) = \frac{(\boldsymbol{P}N) + (m-1)(\boldsymbol{P}P)}{m}$$

$$= \frac{-A - C + (m-1)\left(B - \frac{1}{2}C\right)}{m}$$

$$\therefore \lim_{m\to\infty} \frac{-A - C + (m-1)\left(B - \frac{1}{2}C\right)}{m} = B - \frac{1}{2}C$$

and the average payoff for the 'Cautiously Pursue' player

$$Average\ Payoff\ \ (Caut.) = \frac{(P\boldsymbol{N}) + (m-1)(P\boldsymbol{P})}{m}$$

$$= \frac{-kA + (m-1)\left(B - \frac{1}{2}C\right)}{m}$$

$$\therefore \lim_{m\to\infty} \frac{-kA + (m-1)\left(B - \frac{1}{2}C\right)}{m} = B - \frac{1}{2}C$$

$$\therefore Average\ Payoff = \left(B - \frac{1}{2}C, \qquad B - \frac{1}{2}C\right)$$

* Remark: the above average payoffs are over an infinite number of rounds, thus the initial interaction may be omitted from the average payoffs. For a finite number of rounds, it will affect the average payoff, resulting in a value < *B-½C*.

(Always Not Pursue, Cautious Pursue)

In a repeated interaction of infinite rounds *m* where each player can choose to either 'Pursue' (P) or 'Not Pursue' (N), a game between an 'Always Not Pursue' and a 'Cautiously Pursue' would be:

'Always Not Pursue':	N	N	…
'Cautiously Pursue':	N	N	…

From here we can derive the average payoff for the 'Always Not Pursue' (row player) player to be:

$$Average\ Payoff\ (Always\ Not\ Pursue) = \frac{m(\boldsymbol{N}N)}{m}$$

$$= -kA$$

and the average payoff for the 'Cautiously Pursue' player

$$Average\ Payoff\ (Caut.) = \frac{m(N\boldsymbol{N})}{m}$$

$$= -kA$$

$$\therefore Average\ Payoff\ (Always\ Not\ Pursue)$$

$$= Average\ Payoff(Cautiously\ Pursue)$$

$$\therefore Average\ Payoff = (-kA, -kA)$$

(Always Not Pursue, Narcissistic Pursue)

In a repeated interaction of infinite rounds m where each player can choose to either 'Pursue' (P) or 'Not Pursue' (N), a game between an 'Always Not Pursue' and a 'Narcissistic Pursue' would be:

'<u>Always Not Pursue</u>':	N	N	N	…
'<u>Narcissistic Pursue</u>':	N	P	P	…

From here we can derive the average payoff for the 'Always Not Pursue' (row player) player to be:

$$Average\ Payoff\ (Always\ Not\ Pursue) = \frac{\boldsymbol{N}N + (m-1)(\boldsymbol{N}P)}{m}$$

$$= \frac{-kA + (m-1)(-kA)}{m}$$

$$= -kA$$

and the average payoff for the 'Narcissistic Pursue' player

$$Average\ Payoff\ (Nar.) = \frac{N\boldsymbol{N} + (m-1)(N\boldsymbol{P})}{m}$$

$$= \frac{-kA + (m-1)(-A-C)}{m}$$

$$\therefore \lim_{m\to\infty} \frac{-kA + (m-1)(-A-C)}{m} = -A - C$$

$$\therefore Average\ Payoff = (-kA, \qquad -A - C)$$

*Remark: the above average payoffs are over an infinite number of rounds, thus the initial interaction for the 'Narcissistic Pursue' player may be omitted from the average payoffs. For a finite number of rounds, it will affect the average payoff, resulting in a value < -*A*-*C*.

Appendix D

Dynamical System Analysis

Let x be the frequency of 'Pursue' types

Then

$$\frac{dx}{dt} = x(1-x)(f_P - f_N)$$

$$= x(1-x)[ax + b(1-x) - cx - d(1-x)]$$

Setting $dx/dt = 0$, the fixed points (x^*) are

$$x^* = 0, 1, \frac{d-b}{a-b-c+d}$$

Bounds on fixed point $x^* = \frac{d-b}{a-b-c+d}$

Lower Bound

$$x^* = \frac{-kA + A + C}{B - \frac{1}{2}C + A + C + kA - kA}$$

$$= \frac{A(1-k) + C}{A + B + \frac{1}{2}C}$$

Since $-kA > -A - C$

$$-kA + A > -C$$

$$A(1-k) > -C$$

Then the numerator

$$A(1-k)+C>0$$

Since $A, B, C \geq 0$, then the denominator

$$A+B+\frac{1}{2}C>0$$

Therefore

$$x^{*}=\frac{A(1-k)+C}{A+B+\frac{1}{2}C}>0$$

Upper Bound

$$x^{*}=\frac{-kA+A+C}{B-\frac{1}{2}C+A+C+kA-kA}$$

Comparing the numerator and the denominator, we have

$$-kA+A+C<A+B+\frac{1}{2}C$$

$$-kA+\frac{1}{2}C<B$$

Since $B > C$, and $A, B, C \geq 0$, the above inequality holds true

Thus

$$x^{*}=\frac{-kA+A+C}{B-\frac{1}{2}C+A+C+kA-kA}<1$$

$$\therefore 0<x^{*}<1$$

Graphical Representation of *dx/dt*

$$\frac{dx}{dt} = x(1-x)[ax + b(1-x) - cx - d(1-x)]$$

For $x^* = \frac{d-b}{a-b-c+d}$

$$ax + b(1-x) - cx - d(1-x)$$

$$x\left(A + B + \frac{1}{2}C\right) - [A(1-k) + C]$$

Since $0 \leq x \leq 1$ and $A + B + \frac{1}{2}C$

Then

$$\frac{dx}{dt} \propto x(-x)(+x)$$

$$\propto x^3$$

Plotting *dx/dt* vs. *x* (Figure 4)

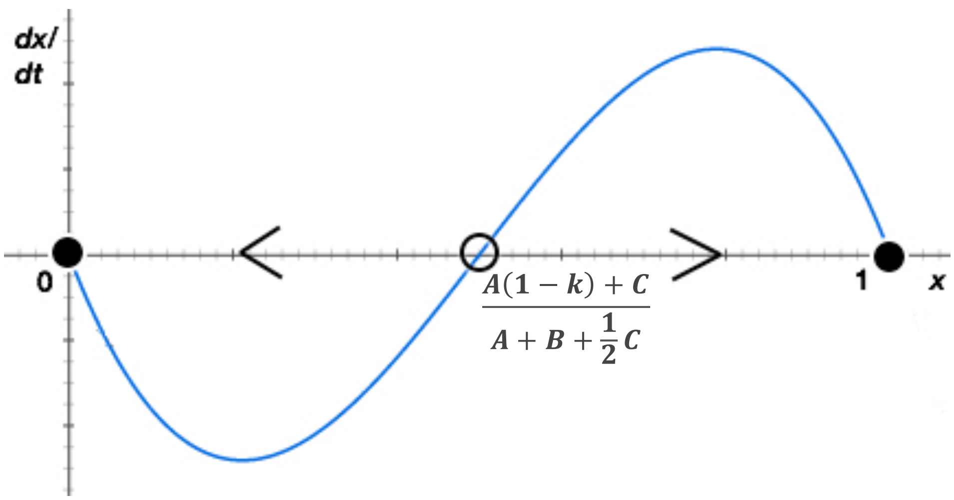

Figure 4. *The differential equation for the Love Game coordination game, where x is the frequency of 'Pursue' types in the population. The rate of change is negative for* $\mathbf{0 < x^* < \frac{A(1-k)+C}{A+B+\frac{1}{2}C}}$ *and positive for* $\mathbf{\frac{A(1-k)+C}{A+B+\frac{1}{2}C} < x^* < 1}$*, so* $x^*=0$ *and* $x^*=1$ *are stable equilibria/fixed points (denoted by closed circles) and* $\mathbf{x^* = \frac{A(1-k)+C}{A+B+\frac{1}{2}C}}$ *is an unstable equilibrium/fixed point (denoted by open circle).*

Appendix E

Assuming indifference, the expected utilities of the row player playing 'Pursue' (P) and 'Not Pursue' (N) for the 2x2 matrix should equal, i.e. $U_{Row}(P) = U_{Row}(N)$. Calculating the expected utility for 'Pursue', we find that

$$U_{Row}(P) = q^*\left(B - \frac{1}{2}C\right) + (1 - q^*)(-A - C)$$

$$= \left(\frac{A(1-k)+C}{A+B+\frac{1}{2}C}\right)\left(B - \frac{1}{2}C\right) + \left[1 - \left(\frac{A(1-k)+C}{A+B+\frac{1}{2}C}\right)\right](-A - C)$$

$$= \frac{[A(1-k)+C]\left(B-\frac{1}{2}C\right) + (-A-C)\left(A+B+\frac{1}{2}C\right) + [A(1-k)+C](A+C)}{A+B+\frac{1}{2}C}$$

$$= \frac{\left[AB(1-k) + BC - \frac{1}{2}AC(1-k) - \frac{1}{2}C^2\right] + \left[-A^2 - AB - \frac{1}{2}AC - AC - BC - \frac{1}{2}C^2\right] + [A^2(1-k) + AC + AC(1-k) + C^2]}{A+B+\frac{1}{2}C}$$

$$= \frac{AB(1-k) + \frac{1}{2}AC(1-k) - A^2 - AB - \frac{1}{2}AC + A^2(1-k)}{A+B+\frac{1}{2}C}$$

$$= \frac{AB - kAB + \frac{1}{2}AC - \frac{1}{2}kAC - A^2 - AB - \frac{1}{2}AC + A^2 - A^2k}{A+B+\frac{1}{2}C}$$

$$= \frac{-kAB - \frac{1}{2}kAC - A^2k}{A+B+\frac{1}{2}C}$$

$$= \frac{-kA\left(B + \frac{1}{2}C + A\right)}{A+B+\frac{1}{2}C}$$

$$= -kA$$

which is equivalent to $U_{Row}(N)$. Thus we can support our assumption that the row player (and by symmetry, the column player) is indifferent.

Appendix F

Using the four repeated game strategies, a symmetric 4x4 matrix was constructed. For ease of reference, we will restate it here (*Figure 2*).

	Column Player				
Row Player		**Always Pursue**	**Cautiously Pursue**	**Narcissistic Pursue**	**Always Not Pursue**
	Always Pursue	(B -½C, B -½C)	(B –½C, B -½C)	(-A -C, -*k*A)	(-A -C, -*k*A)
	Cautiously Pursue	(B –½C, B -½C)	(-*k*A, -*k*A)	(¼[-(2k+1)A+B-$^3/_2$C)], ¼[-(2k+1)A+B-$^3/_2$C)])	(-*k*A, -*k*A)
	Narcissistic Pursue	(-*k*A, -A -C)	(¼[-(2k+1)A+B-$^3/_2$C)], ¼[-(2k+1)A+B-$^3/_2$C)])	(½[-kA+B-½C], ½[-kA+B-½C)])	(-A -C, -*k*A)
	Always Not Pursue	(-*k*A, -A -C)	(-*k*A, -*k*A)	(-*k*A, -A -C)	(-*k*A, -*k*A)

Figure 2 *Normal form game (matrix game) for 'Always Pursue', 'Cautiously Pursue', 'Narcissistic Pursue', and 'Always Not Pursue' repeated game strategies over an infinite number of rounds m. Utilities are determined by A the emotional anguish of love, B the benefit of being in a relationship, and C the cost of pursuing the relationship, where A, B, C ≥ 0, B > C, -kA > -A-C, and k is a positive non-zero constant.*

While most utilities can be compared by inspection alone, there are three notable cases that require some algebraic manipulation.

(Pursue, Cautiously Pursue) vs. (Narcissistic Pursue, Cautiously Pursue)

$$(Pursue, Cautiously\ Pursue) > (Nacissistic\ Pursue, Cautiously\ Pursue)$$

$$B - \frac{1}{2}C \ > \ -\frac{1}{4}(2k+1)A + \frac{1}{4}B - \frac{3}{8}C$$

$$\frac{3}{4}B \ > \ -\frac{1}{4}(2k+1)A + \frac{1}{8}C$$

$$3B \;>\; -(2k+1)A + \frac{1}{2}C$$

By $B > C$ and $A, B, C \geq 0$, the above *inequality* is true.

(Cautiously Pursue, Nar. Pursue) vs. (Nar. Pursue, Nar. Pursue)

$$(Cautiously\ Pursue, Nar.Pursue) > (Nar.Pursue, Nar.Pursue)$$

$$\frac{1}{4}\left[-(2k+1)A + B - \frac{3}{2}C\right] \;>\; \frac{1}{2}\left[-kA + B - \frac{1}{2}C\right]$$

$$\frac{1}{2}\left[-(2k+1)A + B - \frac{3}{2}C\right] \;>\; -kA + B - \frac{1}{2}C$$

$$-kA - \frac{1}{2}A + \frac{1}{2}B - \frac{3}{4}C \;>\; -kA + B - \frac{1}{2}C$$

$$\frac{1}{2}B \;>\; -\frac{1}{2}A - \frac{1}{4}C$$

$$B \;>\; -A - \frac{1}{2}C$$

By $A, B, C \geq 0$, the above inequality is true.

(Nar. Pursue, Nar. Pursue) vs. (Always Not Pursue, Nar. Pursue)

$$(Nar.Pursue, Nar.Pursue) > (Always\ Not\ Pursue, Nar.Pursue)$$

$$\frac{1}{2}\left[-kA + B - \frac{1}{2}C\right] \;>\; -kA$$

$$-kA + B - \frac{1}{2}C \;>\; -2kA$$

$$B \;>\; -kA + \frac{1}{2}C$$

By $A, B, C \geq 0$ and $B > C$, the above *inequality* is true.

(Cautiously Pursue, Nar. Pursue) vs. (Always Not Pursue, Nar. Pursue)

$$\frac{1}{4}\left[-(2k+1)A + B - \frac{3}{2}C\right] > \frac{1}{2}\left[-kA + B - \frac{1}{2}C\right] > -kA$$

$$\therefore \frac{1}{4}\left[-(2k+1)A + B - \frac{3}{2}C\right] > -kA$$

$$\therefore (Cautiously\ Pursue, Nar.Pursue) > (Always\ Not\ Pursue, Nar.Pursue)$$

Representing the relationship between utility values for the row player with arrows and equal signs, the findings can be summarized in the following 4x4 matrix (Figure 8):

		Column Player			
		Always Pursue	**Cautiously Pursue**	**Narcissistic Pursue**	**Always Not Pursue**
Row Player	**Always Pursue**	*W NE, PO* B -½C	*W NE, PO* B –½C	-A -C	-A -C
	Cautiously Pursue	*W NE, PO* B –½C	-*k*A	¼[-(2k+1)A+B-$^3/_2$C)]	-*k*A
	Narcissistic Pursue	-*k*A	¼[-(2k+1)A+B-$^3/_2$C)]	½[-kA+B-½C]	-A -C
	Always Not Pursue	-*k*A	-*k*A	-*k*A	*W NE* -*k*A

Figure 8. *Symmetric coordination game for 'Always Pursue', 'Cautiously Pursue', 'Narcissistic Pursue', and 'Always Not Pursue' strategies over an infinite number of rounds m. Arrows and equal signs represent the relationship between utilities of the row player down a column. There are three weak Nash Equilibriums and one Pareto Optimal Outcome. 'Always Not Pursue' is weakly dominated by 'Cautiously Pursue'.*

Literature Cited

Steven H. Strogatz. *Nonlinear dynamics and chaos: with applications to physics, biology, chemistry and engineering*. Westview Press, 1994