MATH 220 — Assignment 1

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Question 1

Let $S = \{-2, -1, 0, 1, 2, 3\}$. Then we can describe the sets A, B, C, D as

(a) $A = \{1, 2, 3\} = \{\, x \in S \mid x \geq 1 \,\} = \{\, x \in S \mid x \text{ is positive} \,\}$

(b) $B = \{0, 1, 2, 3\} = \{\, x \in S \mid x \geq 0 \,\} = \{\, x \in S \mid x \text{ is nonnegative} \,\}$

(c) $C = \{-2, -1\} = \{\, x \in S \mid x \leq -1 \,\} = \{\, x \in S \mid x < 0 \,\} = \{\, x \in S \mid x \text{ is negative} \,\}$

(d) $D = \{-2, 2, 3\} = \{\, x \in S \,:\, |x| \geq 2 \,\} = \{\, x \in S \,:\, x \geq 2 \textit{ and } x \leq -2 \,\}$

Question 2

(a) $A = \{\, n \in \mathbb{Z} \mid -4 < n \leq 4 \,\} = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\} = \{-4, -3, -2, \ldots, 4\}$

(b) $A = \{\, n \in \mathbb{Z} \mid n^2 < 5 \,\} = \{-2, -1, 0, 1, 2\}$

(c) $A = \{\, n \in \mathbb{Z} \mid n^3 < 100 \,\} = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\} = \{-4, -3, -2, \ldots, 4\}$

(d) $A = \{\, x \in \mathbb{R} \mid x^2 - x = 0 \,\} = \{0, 1\}$

(e) $A = \{\, x \in \mathbb{R} \mid x^2 + 1 = 0 \,\} = \{\,\} = \emptyset$

Question 3

(a) The sets $A = \{1, 2\}, B = \{1, 2\}, C = \{1, 2, 3, 4\}$ have the property $A \subseteq B \subset C$.

(b) Let $A = \{1,2\}, B = \{\,\{1,2\},3\}, C = \{\,\{\{1,2\},3\},4,5\}$. Then $A \in B$, $B \in C$, and $A \notin C$.

(c) Let $A = \emptyset, B = \{\emptyset,1,2\}, C = \{-1,0,100\}$. Then $A \in B$ and $A \subset C$

Question 4

For the set $A = \{0, \emptyset, \{\emptyset\}\}$, we have

$$\mathcal{P}(A) = \{\emptyset, \{0\}, \{\emptyset\}, \{\{\emptyset\}\}, \{0,\emptyset\}, \{0,\{\emptyset\}\}, \{\emptyset,\{\emptyset\}\}, \{0,\emptyset,\{\emptyset\}\}\,\}$$

and $|\mathcal{P}(A)| = 2^{|A|} = 2^3 = 8$

Question 5

(a) Conjecture: If $\{1\} \in \mathcal{P}(A)$, then $1 \in A$.

Proof

By definition, the power set of $\mathcal{P}(A)$ is the set consisting of all subsets of A. If $\{1\} \in \mathcal{P}(A)$, then 1 is an element in A, hence $1 \in A$. ■

(b) Conjecture: If $\{1\} \in \mathcal{P}(A)$, then $1 \notin A$.

False. Let $\mathcal{P}(A) = \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$, then $A = \{1,2\}$, Since $1 \in A$ is true, the statement $1 \notin A$ is false. Therefore the conjecture is also false.

(c) Conjecture: If four sets A, B, C, D are subsets of $\{1,2,3\}$ such that $|A| = |B| = |C| = |D| = 2$, then at least two of these sets are equal.

Proof

The power set of {1, 2, 3} is given by

$$\{1,2,3\} = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$$

Since $|A| = |B| = |C| = |D| = 2$, we are only interested in the 2-elements sets in the power series. In this case, there are 3 sets $\{1,2\}, \{1,3\}$, and $\{2,3\}$. Since there are only 3 possible subsets to be assigned to the 4 sets A, B, C, and D, then either all the sets are equal, three sets are equal or two sets are equal. Thus we can conclude that at least two of the sets must be equal. ■

(d) Conjecture: $A \subset \mathcal{P}(B)$ and $|A| = 2$, then B has at least two elements.

Proof

Since A is a proper subset of $\mathcal{P}(B)$ that contains 2 elements, then $|\mathcal{P}(B)| \geq 4$. Using the definition of the power set, we have that $|\mathcal{P}(B)| = 2^{|B|} \geq 4$, or $|B| \geq 2$. Thus we can conclude that the set B has at least two elements. ■