MATH 220 — Assignment 2

Stephanie Knill
54882113
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Question 1

Let $U = \{1, 3, 5, \ldots, 15\}$ be the universal set, $A = \{1, 5, 9, 13\}$, and $B = \{3, 9, 15\}$.

(a) $A \cup B = \{1, 3, 5, 9, 13, 15\}$

(b) $A \cap B = \{9\}$

(c) $A - B = \{1, 5, 13\}$

(d) $B - A = \{3, 15\}$

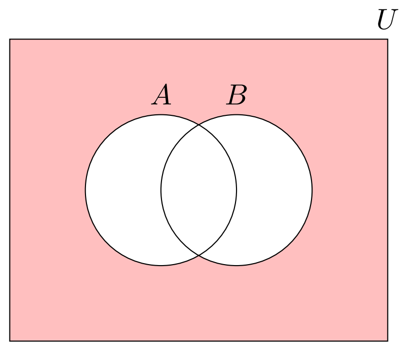
(e) $\overline{A} = U - A = \{3, 7, 11, 15\}$

(f) Since $\overline{B} = \{1, 5, 7, 11, 13\}$, then $A \cap \overline{B} = \{1, 5, 13\}$

Question 2

Let U be the universal set and let A, B be two subsets of U. Then we can express the following sets as Venn diagrams:

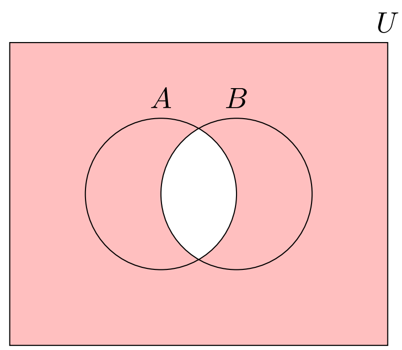
(a) $\overline{A \cup B}$



(b) $\overline{A} \cap \overline{B}$

Since $\overline{A \cup B} = \overline{A} \cap \overline{B}$, the Venn Diagram is identical to that in part a).

(c) $\overline{A \cap B}$



(d) $\overline{A} \cup \overline{B}$

Since $\overline{A \cap B} = \overline{A} \cup \overline{B}$, the Venn Diagram is identical to that in part c).

Question 3

The power set of {1,2,3} is given by

$$\mathcal{P}(\{1,2,3\}) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$$

Looking at the possible permutations for the subsets A and B, we can immediately discard any set of subsets where $A = B$ or when one of the subsets is the empty set. Through some experimentation, we find that all eight set operations are unique if $|A| = |B| = 1$. Without loss of generality, let $A = \{1\}$ and $B = \{2\}$.

While we may be tempted to stop here, upon closer examination of the set operations we can see another pair of subsets C and D. Let $C = \overline{A}$ and $D = \overline{B}$. Then the set operation $A \cup B$ can be rewritten as $\overline{A} \cup \overline{B}$, the set operation $A \cup \overline{B}$ as $\overline{A} \cup B$, and so forth. Thus, we are simply swapping the order of the conditions for the sets A and B. The values for the subset pair $A = \{1\}$ and $B = \{2\}$ and the subset pair $\overline{A} = \{2, 3\}$ and $\overline{B} = \{1, 3\}$ are given in Table 1.

	$A = \{1\}$, $B = \{2\}$	$\overline{A} = \{2, 3\}$, $\overline{B} = \{1, 3\}$
$A \cup B$	$\{1, 2\}$	$\{1, 2, 3\}$
$A \cup \overline{B}$	$\{1, 3\}$	$\{2, 3\}$
$\overline{A} \cup B$	$\{2, 3\}$	$\{1, 3\}$
$\overline{A} \cup \overline{B}$	$\{1, 2, 3\}$	$\{1, 2\}$
$A \cap B$	$\emptyset$	$\{3\}$
$A \cap \overline{B}$	$\{1\}$	$\{2\}$
$\overline{A} \cap B$	$\{2\}$	$\{1\}$
$\overline{A} \cap \overline{B}$	$\{3\}$	$\emptyset$

Table 1: Set operations performed on A and B, and $\overline{A}$ and $\overline{B}$.

Question 4

For a real number r, define S_r to be the interval $[r-1, r+2]$. Let $A = \{1, 3, 4\}$. Then

$$\begin{aligned}\bigcup_{\alpha \in A} S_\alpha &= S_1 \cup S_3 \cup S_4 \\ &= [(1) - 1, (1) + 2] \cup [(3) - 1, (3) + 2] \cup [(4) - 1, (4) + 2] \\ &= [0, 3] \cup [2, 5] \cup [3, 6] \\ &= [0, 6]\end{aligned}$$

and

$$\begin{aligned}\bigcap_{\alpha \in A} S_\alpha &= S_1 \cap S_3 \cap S_4 \\ &= [(1) - 1, (1) + 2] \cap [(3) - 1, (3) + 2] \cap [(4) - 1, (4) + 2] \\ &= [0, 3] \cap [2, 5] \cap [3, 6] \\ &= [3]\end{aligned}$$

Question 5

Let $A_n = \{n, n-1\}$ for every $n \in \mathbb{N}$. Then

$$\begin{aligned} \bigcup_{n\in\mathbb{N}} A_n &= \{1,0\} \cup \{2,1\} \cup \{3,2\} \cup \dots \\ &= \{0\} \cup \mathbb{N} \end{aligned}$$

and

$$\begin{aligned} \bigcap_{n\in\mathbb{N}} A_n &= \{1,0\} \cap \{2,1\} \cap \{3,2\} \cap \dots \\ &= \emptyset \end{aligned}$$

Question 6

For $A = \{1, 2\}$ and $B = \{1\}$, the cartesian product $A \times B$ is given by

$$A \times B = \{(1,1), (2,1)\}$$

Thus we can compute the power set of $A \times B$ to be

$$\mathcal{P}(A \times B) = \{\emptyset, \{(1,1)\}, \{(2,1)\}, \{(1,1), (2,1)\}\ \}$$

Question 7

For $A = \{a \in \mathbb{R} : |a| \leq 1\}$ and $B = \{b \in \mathbb{R} : |b| = 1\}$, we can geometrically describe the points in the xy-plane belonging to $(A \times B) \cup (B \times A)$ as the disk C of radius 1 centered at the origin in the xy-plane. In other words, $x^2 + y^2 \leq 1$, where $x, y \in \mathbb{R}^2$.