MATH 302 — Assignment 5

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Question 3: Section 6.1 #8

A royal family has children util it has a boy or until it has three children, whichever comes first. Assuming that each child is a boy with probability 1/2, we can construct a tree diagram for the the sample space $\Omega = \{B, GB, GGB, GGG\}$ (Figure 1):

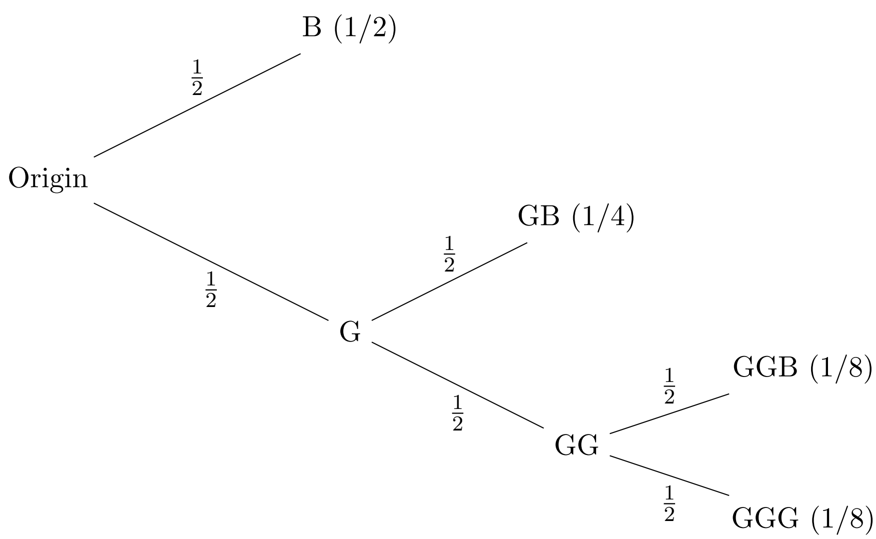

Figure 1: Tree digram for a royal family

Let X be the number of boys in the family. Then the expected value of the random variable X is

$$\begin{aligned}
E[X] &= \sum_x x \cdot p(x) \\
&= 1/2 \cdot (1) + 1/4 \cdot (1) + 1/8 \cdot (1) + 1/8 \cdot (0) \\
&= \frac{7}{8} \\
&= 0.875
\end{aligned}$$

Let Y be the number of girls in the family. Then the expected value of the random variable Y is

$$\begin{aligned}
E[Y] &= \sum_y y \cdot p(y) \\
&= 1/2 \cdot (0) + 1/4 \cdot (1) + 1/8 \cdot (2) + 1/8 \cdot (3) \\
&= \frac{7}{8} \\
&= 0.875
\end{aligned}$$

Question 4: Section 6.2 #2

For a random variable X that has distribution

$$p_X = \begin{pmatrix} 0 & 1 & 2 & 4 \\ 1/3 & 1/3 & 1/6 & 1/6 \end{pmatrix}$$

we can find the expected value, variance, and standard deviation.

Expected Value

$$\begin{aligned} E[X] &= \sum_x x \cdot p(x) \\ &= 1/3 \cdot (0) + 1/3 \cdot (1) + 1/6 \cdot (2) + 1/6 \cdot (4) \\ &= \frac{4}{3} \\ &\approx 1.33 \end{aligned}$$

Variance

$$Var[X] = E[X^2] - E[X]^2$$

Here,

$$\begin{aligned} E[X^2] &= 1/3 \cdot (0)^2 + 1/3 \cdot (1)^2 + 1/6 \cdot (2)^2 + 1/6 \cdot (4)^2 \\ &= 1/3 + 4/6 + 16/6 \\ &= \frac{11}{3} \end{aligned}$$

Substituting this back in gives us

$$\begin{aligned} Var[X] &= E[X^2] - E[X]^2 \\ &= 11/3 - (4/3)^2 \\ &= \frac{17}{9} \\ &\approx 1.89 \end{aligned}$$

Standard Deviation

$$\begin{aligned} D[X] &= \sqrt{Var[X]} \\ &= \sqrt{\frac{17}{9}} \\ &\approx 1.37 \end{aligned}$$

Question 5: Section 6.2 #4

Let X be a random variable with expected value $E[X] = 100$ and variance $Var[X] = 15$. Then we can compute the following

(a) $E(X^2)$

$$\begin{aligned} Var[X] &= E[X^2] - E[X]^2 \\ E[X^2] &= Var[X] + E[X]^2 \\ &= 15 + 100^2 \\ &= 10015 \end{aligned}$$

(b) $E[3X + 10]$

$$\begin{aligned} E[3X + 10] &= 3 \cdot E[X] + 10 \\ &= 3 \cdot 100 + 10 \\ &= 310 \end{aligned}$$

(c) $E[-X]$

$$\begin{aligned} E[-X] &= -1 \cdot E[X] \\ &= -100 \end{aligned}$$

(d) $Var[-X]$

$$\begin{aligned} Var[-X] &= (-1)^2 \cdot Var[X] \\ &= Var[X] \\ &= 15 \end{aligned}$$

(e) $D[-X]$

$$\begin{aligned} D[-X] &= \sqrt{Var[-X]} \\ &= \sqrt{15} \\ &\approx 3.87 \end{aligned}$$