Stephanie Doan
ID: 604981556

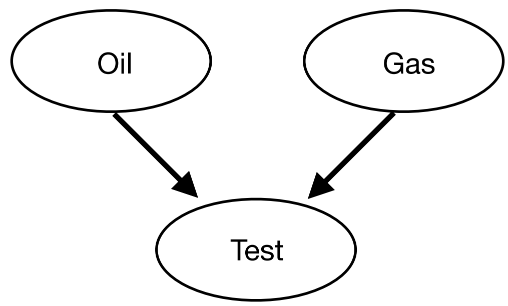
HOMEWORK 6

Q1

A. Bayesian network over Oil, Gas, and Test

Oil	**P(Oil)**
T	0.3
F	0.7

Gas	**P(Gas)**
T	0.2
F	0.8



Test	**Oil**	**Gas**	**P(Test)**
T	T	T	0
T	T	F	0.8
T	F	T	0.2
T	F	F	0.1

B. Probability that oil is present

$$P(Oil\,|\,Test) = \frac{P(Test\,|\,Oil)P(Oil)}{P(Test)}$$

$$P(Test) = P(Test\,|\,Oil)P(Oil) + P(Test\,|\,\neg Oil)P(\neg Oil)$$

$$P(Test) = (0.8 \times 0.3) + ((0.2 + 0.1)) \times 0.7) = 0.45$$

$$P(Oil\,|\,Test) = \frac{0.8 \times 0.3}{0.45} = \mathbf{0.533}$$

Q2

A. P(A, B, C, D, E, F, G, H) = **P(A)*P(B)*P(C|A)*P(D|A,B)*P(E|B)*P(F|C,D)*P(G|F)*P(H|F,E)**

B. $P(E, F, G, H) = \Sigma_{A,B,C,D} P(A, B, C, D, E, F, G, H)$

$P(E, F, G, H) = \Sigma_{A,B,C,D} P(G \mid F) * P(H \mid F, E) * P(F \mid C, D) * P(E \mid B) * P(D \mid A, B) * P(C \mid A) * P(B) * P(A)$

1. Sum out D: $f_{10}(A, B, C, F) = \Sigma_D f_3(C, D, F) \times f_5(A, B, D)$

   $P(E, F, G, H) = \Sigma_{A,B,C} f_1(F, G) \times f_2(F, E, H) \times f_{10}(A, B, C, F) \times f_4(B, E) \times f_6(A, C) \times f_7(B) \times f_8(A)$

2. Sum out C: $f_{11}(A, B, F) = \Sigma_C f_{10}(A, B, C, F) \times f_6(A, C)$

   $P(E, F, G, H) = \Sigma_{A,B} f_1(F, G) \times f_2(F, E, H) \times f_{11}(A, B, F) \times f_4(B, E) \times f_7(B) \times f_8(A)$

3. Sum out B: $f_{12}(A, E, F) = \Sigma_B f_{11}(A, B, F) \times f_4(B, E) \times f_7(B)$

   $P(E, F, G, H) = \Sigma_A f_1(F, G) \times f_2(F, E, H) \times f_{12}(A, E, F) \times f_8(A)$

4. Sum out A: $f_{13}(E, F) = \Sigma_A f_{12}(A, E, F) \times f_8(A)$

   $\mathbf{P(E, F, G, H) = f_1(F, G) \times f_2(F, E, H) \times f_{13}(E, F)}$

C. P(a, ¬b, c, d, ¬e, f, ¬g, h) = P(h|¬e, f)*P(¬g|f)*P(f|c,d)*P(¬e|¬b)*P(d|a,¬b)*P(c|a)*P(¬b)*P(a)
= P(h|¬e, f)*P(¬g|f)*P(f|c,d)*0.2*0.6*P(c|a)*0.4*0.1
= **0.0048*P(h|¬e, f)*P(¬g|f)*P(f|c,d)*P(c|a)**

D. P(¬a,b) = P(¬a)*P(b) = 0.9*0.6 = **0.54**
Since A and B are independent, we can simply multiply their individual probabilities, since P(X,Y) = P(X)*P(Y)

P(¬e|a) = P(¬e,a)/P(a) = P(¬e)P(a)/P(a) = P(¬e)
P(¬e) = P(¬e, b) + P(¬e, ¬b) = P(¬e|b)P(b) + P(¬e|¬b)P(¬b)
= (0.9 * 0.6) + (0.2 * 0.4) = 0.54 + 0.08 = **0.62**

Product rule was applied to the first transformation. Then, since E is dependent on only B and A is independent, E and A are conditionally independent, allowing us to multiply their individual probabilities and then calculating P(¬e) as sum of its probability in the presence of B and in the absence of B.

E. Markovian assumption: given its parents, a node is independent of its non-descendants

A⊥B A⊥E	B⊥C B⊥A	C⊥D | A C⊥E | A C⊥B | A	D⊥E | A,B D⊥C | A,B	E⊥C | B E⊥D | B E⊥F | B E⊥G | B E⊥A | B	F⊥E | C,D F⊥A | C,D F⊥B | C,D	G⊥H | F G⊥A | F G⊥C | F G⊥D | F G⊥E | F G⊥B | F	H⊥G | E,F H⊥C | E,F H⊥D | E,F H⊥A | E,F H⊥B | E,F

F. Markov blanket for variable D are **{A,B,F,C}**: the children, parents, and children's parents of node D.

G. Multiply the factors for Pr(D|AB) and Pr(E|B).

A	B	D	**P(D|A,B)**	B	E	**P(E|B)**	A	B	D	E	**P(D|A,B) * P(E|B)**
0	0	0	0.2	0	0	0.2	0	0	0	0	0.04 = 0.2*0.2
0	0	1	0.8	0	1	0.8	0	0	0	1	0.16 = 0.2*0.8
0	1	0	0.8	1	0	0.9	0	0	1	0	0.16 = 0.8*0.2
0	1	1	0.2	1	1	0.1	0	0	1	1	0.64 = 0.8*0.8
1	0	0	0.4				0	1	0	0	0.72 = 0.8*0.9
1	0	1	0.6				0	1	0	1	0.08 = 0.8*0.1
1	1	0	0.3				0	1	1	0	0.18 = 0.2*0.9
1	1	1	0.7				0	1	1	1	0.02 = 0.2*0.1
							1	0	0	0	0.08 = 0.4*0.2
							1	0	0	1	0.32 = 0.4*0.8
							1	0	1	0	0.12 = 0.6*0.2
							1	0	1	1	0.48 = 0.6*0.8
							1	1	0	0	0.27 = 0.3*0.9
							1	1	0	1	0.03 = 0.3*0.1
							1	1	1	0	0.63 = 0.7*0.9
							1	1	1	1	0.07 = 0.7*0.1

H. Sum out D

A	B	E	$\mathbf{\Sigma_d f(A, B, D, E) = f(A, B, E)}$
0	0	0	0.2 = 0.04+0.16
0	0	1	0.8 = 0.16+0.64
0	1	0	0.9 = 0.72+0.18
0	1	1	0.1 = 0.08+0.02
1	0	0	0.2 = 0.08+0.12
1	0	1	0.8 = 0.32+0.48
1	1	0	0.9 = 0.27+0.63
1	1	1	0.1 = 0.03+0.07