Stephanie Doan
604981556

HOMEWORK 5

Q1

A. Smoke ⇒ Smoke

Smoke	(Smoke ⇒ Smoke) ≡ (¬Smoke ∨ Smoke)
F	T
T	T

VALID because satisfied by all models

B. Smoke ⇒ Fire

Smoke	Fire	(Smoke ⇒ Fire) ≡ (¬Fire ⇒ ¬Smoke)
F	F	T
F	T	T
T	F	F
T	T	T

NEITHER because satisfied by some, not all and not none of the models

C. Smoke ∨ Fire ∨ ¬Fire

Fire	Smoke	(Smoke ∨ Fire ∨ ¬Fire) ≡ (Smoke ∨ True) ≡ True
F	F	T
F	T	T
T	F	T
T	T	T

VALID because satisfied by all models

D. (Smoke ⇒ Fire) ⇒ (¬Smoke ⇒ ¬Fire)

¬(Smoke ⇒ Fire) ∨ (¬Smoke ⇒ ¬Fire)	Implication elimination
¬(¬Smoke ∨ Fire) ∨ (¬Smoke ⇒ ¬Fire)	Implication elimination
(Smoke ∧ ¬Fire) ∨ (¬Smoke ⇒ ¬Fire)	De Morgan
(Smoke ∧ ¬Fire) ∨ (Smoke ∨ ¬Fire)	Implication elimination

Fire	Smoke	((Smoke ⇒ Fire) ⇒ (¬Smoke ⇒ ¬Fire)) ≡ ((Smoke ∧ ¬Fire) ∨ (Smoke ∨ ¬Fire))
F	F	T
F	T	T
T	F	F
T	T	T

NEITHER because satisfied by some, not all and not none of the models

E. (Smoke ⇒ Fire) ⇒ ((Smoke ∨ Heat) ⇒ Fire)

(Smoke ⇒ Fire) ⇒ (¬(Smoke ∨ Heat) ∨ Fire)

(Smoke ⇒ Fire) ⇒ (Fire ∨ (¬Smoke ∧ ¬Heat))

(Smoke ⇒ Fire) ⇒ ((Fire ∨ ¬Smoke) ∧ (Fire ∨ ¬Heat))

(¬Smoke ∨ Fire) ⇒ ((Fire ∨ ¬Smoke) ∧ (Fire ∨ ¬Heat))

¬(¬Smoke ∨ Fire) ∨ ((Fire ∨ ¬Smoke) ∧ (Fire ∨ ¬Heat))
(¬Fire ∧ Smoke) ∨ ((Fire ∨ ¬Smoke) ∧ (Fire ∨ ¬Heat))

Smoke	Fire	Heat	((Smoke ⇒ Fire) ⇒ (¬Smoke ⇒ ¬Fire)) ≡ (¬Fire ∧ Smoke) ∨ ((Fire ∨ ¬Smoke) ∧ (Fire ∨ ¬Heat))
F	F	F	T
F	F	T	F
F	T	F	T
F	T	T	T
T	F	F	T
T	F	T	T
T	T	F	T
T	T	T	T

NEITHER because satisfied by some, not all and not none of the models

F. ((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire))

((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire))

(¬(Smoke ∧ Heat) ∨ Fire) ⇔ ((¬Smoke ∨ Fire) ∨ (¬Heat ∨ Fire))

((¬Smoke ∨ ¬Heat) ∨ Fire) ⇔ ((¬Smoke ∨ Fire) ∨ (¬Heat ∨ Fire))

(¬Smoke ∨ ¬Heat ∨ Fire) ⇔ (¬Smoke ∨ Fire ∨ ¬Heat ∨ Fire)

$(\neg Smoke \lor \neg Heat \lor Fire) \Leftrightarrow (\neg Smoke \lor \neg Heat \lor Fire)$

Let $(\neg Smoke \lor \neg Heat \lor Fire)$ be A.
$(A \Rightarrow A) \land (A \Rightarrow A)$

$(\neg A \lor A) \land (\neg A \lor A)$
$(True) \land (True)$
True

Smoke	Fire	Heat	$((Smoke \land Heat) \Rightarrow Fire) \Leftrightarrow ((Smoke \Rightarrow Fire) \lor (Heat \Rightarrow Fire))$
F	F	F	T
F	F	T	T
F	T	F	T
F	T	T	T
T	F	F	T
T	F	T	T
T	T	F	T
T	T	T	T

VALID because satisfied by all models

Q2

A. P(A, B, B), P(x, y, z)

Θ = {x/A, y/B, z/B}

B. Q(y, G(A, B)), Q(G(x, x), y)

No unifier exists because I first bind y to G(A,B), and to make y bind to G(x,x), x would have to bind to both A and B which is not possible.

C. Older(Father(y), y), Older(Father(x), John)

Θ = {y/John, x/John}

D. Knows(Father(y),y), Knows(x,x).

No unifier exists because to unify these two sentences, we would have to bind x to both Father(y) and y which is not possible.

Q3

A. Translate these sentences into formulas in first-order logic.

John likes all kinds of food.	**$\forall$x Food(x) $\Rightarrow$ Likes(John, x)**
Apples are food.	**Food(Apples)**
Chicken is food.	**Food(Chicken)**
Anything anyone eats and isn't killed by is food.	**$\forall$x ($\exists$y Eats(y,x) $\wedge$ ¬KilledBy(y,x)) $\Rightarrow$ Food(x)**
If you are killed by something, you are not alive.	**$\forall$x ($\exists$y KilledBy(y,x)) $\Rightarrow$ ¬Alive(y)**
Bill eats peanuts and is still alive.*	**Eats(Bill, Peanuts) $\wedge$ ¬KilledBy(Bill, Peanuts)**
Sue eats everything Bill eats.	**$\forall$x (Eats(Bill, x) $\Rightarrow$ Eats(Sue, x))**

B. Convert the formulas of part (a) into CNF (also called clausal form).

R1	$\forall$x Food(x) $\Rightarrow$ Likes(John, x)	**¬Food(x) $\vee$ Likes(John,x)**
R2	Food(Apples)	**Food(Apples)**
R3	Food(Chicken)	**Food(Chicken)**
R4	$\forall$x ($\exists$y Eats(y,x) $\wedge$ ¬KilledBy(y,x)) $\Rightarrow$ Food(x)	**¬Eats(y,x) $\vee$ KilledBy(y,x) $\vee$ Food(x)**
R5	$\forall$x ($\exists$y KilledBy(y,x)) $\Rightarrow$ ¬Alive(y)	**¬KilledBy(y,x) $\vee$ Alive(y)**
R6	Eats(Bill, Peanuts) $\wedge$ ¬KilledBy(Bill, Peanuts)	**Eats(Bill, Peanuts)**
R7		**¬KilledBy(Bill, Peanuts)**
R8	$\forall$x (Eats(Bill, x) $\Rightarrow$ Eats(Sue, x))	**¬Eats(Bill,x) $\vee$ Eats(Sue,x)**

C. Prove that John likes peanuts using resolution.
Conclusion: *Likes(John,Peanuts)*
Prove by contradiction by showing that ¬Likes(John,Peanuts) is unsatisfiable.

R9	Negate conclusion	**¬Likes(John,Peanuts)**
R10	R1 and R8, {x/Peanuts}	**¬Food(Peanuts)**
R11	R10 and R4, {x/Peanuts}	**¬Eats(y,Peanuts) $\vee$ KilledBy(y,Peanuts)**
R12	R11 and R6, {y/Bill}	**KilledBy(Bill,Peanuts)**
R13	R12 and R7, {}	$\perp$

Therefore, we conclude ***Likes(John,Peanuts)***.

D. Use resolution to answer the question, “What food does Sue eat?”

Prove by contradiction by showing that ¬Eats(Sue,Peanuts) is unsatisfiable.

R14	Negate conclusion	**¬Eats(Sue,Peanuts)**
R15	R8 and R14, {x/Peanuts}	**¬Eats(Bill,Peanuts)**
R16	R6 and R15, {x/Peanuts}	⊥

Therefore, we conclude ***Eats(Sue, Peanuts)***.

E. Use resolution to answer the question, “What food does Sue eat?” if, instead of the axiom marked with an asterisk above, we had: (1) If you don’t eat, you die. (2) If you die, you are not alive. (3) Bill is alive.

Convert new statements to CNF:

If you don’t eat, you die	∀x (∀y ¬Eat(y,x) ⇒ Dies(y))	**Eat(y,x)** ∨ **Dies(y)**
If you die, you are not alive	∀y Dies(y) ⇒ ¬Alive(y)	**¬Dies(y)** ∨ **¬Alive(y)**
Bill is alive.	Alive(Bill)	**Alive(Bill)**

New Knowledge Base:

R1	∀x Food(x) ⇒ Likes(John, x)	**¬Food(x)** ∨ **Likes(John,x)**
R2	Food(Apples)	**Food(Apples)**
R3	Food(Chicken)	**Food(Chicken)**
R4	∀x (∃y Eats(y,x) ∧ ¬KilledBy(y,x)) ⇒ Food(x)	**¬Eats(y,x)** ∨ **KilledBy(y,x)** ∨ **Food(x)**
R5	∀x (∃y KilledBy(y,x)) ⇒ ¬Alive(y)	**¬KilledBy(y,x)** ∨ **Alive(y)**
R6	∀x (Eats(Bill, x) ⇒ Eats(Sue, x))	**¬Eats(Bill,x)** ∨ **Eats(Sue,x)**
R7	∀x (∀y ¬Eat(y,x) ⇒ Dies(y))	**Eat(y,x)** ∨ **Dies(y)**
R8	∀y Dies(y) ⇒ ¬Alive(y)	**¬Dies(y)** ∨ **¬Alive(y)**
R9	Alive(Bill)	**Alive(Bill)**

Resolution to answer “What does Sue eat?”

R10	Negate conclusion	**¬Eats(Sue,Apples)**
R11	R10 and R7, {x/Apples}	**Dies(Sue)**
R12	R11 and R8, {y/Sue}	**¬Alive(Sue)**
R13	R9 and R5, {y/Bill}	**¬KilledBy(Bill,x)**
R14	R10 and R6, {x/Apples}	**¬Eats(Bill,Apples)**

R15	R14 and R7, {y/Bill}, {x/Apples}	**Dies(Bill)**
R16	R15 and R8, {y/Bill}	**¬Alive(Bill)**
R17	R16 and R9	$\perp$

Therefore, we conclude ***Eats(Sue,Apples)***.

Q4

A. Represent the above information using a propositional logic knowledge base (set of sentences in propositional logic).

If the unicorn is mythical, then it is immortal	**Mythical(x) ⇒ Immortal(x)**
If it is not mythical, then it is a mortal mammal	**¬Mythical(x) ⇒ (¬Immortal(x) ∧ Mammal(x))**
If the unicorn is either immortal or a mammal, then it is horned	**(Immortal(x) ∨ Mammal(x)) ⇒ Horned(x)**
The unicorn is magical if it is horned.	**Horned(x) ⇒ Magical(x)**

⇒ ∨ ¬ ∧

B. Convert the knowledge base into CNF

R1	Mythical(x) ⇒ Immortal(x)	**¬Mythical(x) ∨ Immortal(x)**
R2	¬Mythical(x) ⇒ (¬Immortal(x) ∧ Mammal(x))	Mythical(x) ∨ (¬Immortal(x) ∧ Mammal(x)) **(Mythical(x) ∨ ¬Immortal(x)) ∧ (Mythical(x) ∨ Mammal(x))**
R3	(Immortal(x) ∨ Mammal(x)) ⇒ Horned(x)	¬(Immortal(x) ∨ Mammal(x)) ∨ Horned(x) Horned(x) ∨ (¬Immortal(x) ∧ ¬Mammal(x)) **(Horned(x) ∨ ¬Immortal(x)) ∧ (Horned(x) ∨ ¬Mammal(x))**
R4	Horned(x) ⇒ Magical(x)	**¬Horned(x) ∨ Magical(x)**

C. Can you use the knowledge base to prove that the unicorn is mythical? How about magical? Horned?

Prove that unicorn is mythical by contradiction of ¬Mythical(x)

R5	Negation	**¬Mythical(x)**
R6	R5 and R1	**Immortal(x)**
R7	R6 and R2	Immortal(x) ∨ Mythical(x) ∨ (¬Immortal(x) ∧ Mammal(x)) Mythical(x) ∨ ((Immortal(x) ∨ ¬Immortal(x)) ∧ (Immortal(x) ∨ Mammal(x))) Mythical(x) ∨ Immortal(x) ∨ Mammal(x)
R8	R7 and R5	Immortal(x) ∨ Mammal(x)

Cannot prove that unicorn is Mythical.

Prove that unicorn is magical by contradiction of ¬Magical(x), with same original KB from R1 to R4.

R5	Negation	**¬Magical(x)**

R6	R5 and R4	**¬Horned(x)**
R7	R6 and R3	**¬(Immortal(x)** ∨ **Mammal(x))**
R8	R7 and R2	Mythical(x) ∨ ((¬Immortal(x) ∧ Mammal(x)) ∨ ¬(Immortal(x) ∨ Mammal(x))) Mythical(x) ∨ ((¬Immortal(x) ∧ Mammal(x)) ∨ (¬Immortal(x) ∧ ¬Mammal(x))) Mythical(x) ∨ (¬Immortal(x) ∧ (Mammal(x) ∨ ¬Mammal(x))) **Mythical(x)** ∨ **¬Immortal(x)**
R9	R8 and R1	Mythical(x) ∨ ¬Immortal(x) ∨ ¬Mythical(x) ∨ Immortal(x) **Empty**

Therefore, we conclude **Magical(x).**

Prove that unicorn is horned by contradiction of ¬Horned(x), with same original KB from R1 to R4.

R5	Negation	**¬Horned(x)**
R6	R5 and R3	**¬(Immortal(x)** ∨ **Mammal(x))**
R7	R6 and R2	Mythical(x) ∨ ((¬Immortal(x) ∧ Mammal(x)) ∨ ¬(Immortal(x) ∨ Mammal(x))) Mythical(x) ∨ ((¬Immortal(x) ∧ Mammal(x)) ∨ (¬Immortal(x) ∧ ¬Mammal(x))) Mythical(x) ∨ (¬Immortal(x) ∧ (Mammal(x) ∨ ¬Mammal(x))) **Mythical(x)** ∨ **¬Immortal(x)**
R8	R7 and R1	Mythical(x) ∨ ¬Immortal(x) ∨ ¬Mythical(x) ∨ Immortal(x) **Empty**

Therefore, we conclude **Horned(x)**.

Q5

Prove each of the following assertions:

A. α is valid if and only if True |= α.

True is true in all models, and since α is valid in all models that True is true, α is also true in all models, which is the definition of valid. Therefore, α is valid.

B. For any α, False |= α.

False is true in no models, so the set of models in which False is true is the empty set. α is true in all models of the empty set, which is a vacuous truth.

C. α |= β if and only if the sentence (α ⇒ β) is valid.

We must prove in that (1) if (α |= β), then (α ⇒ β) and (2) if (α ⇒ β), then (α |= β)

(1) To prove that if α |= β, then α ⇒ β: α |= β means that in models in which α is true, β is true. So, in models in which α is true, α ⇒ β holds because both α and β are true. In models in which α is false, α ⇒ β also holds because there cannot exist the counterexample of α ⇒ ¬β because α is false in the first place.

(2) To prove that if α ⇒ β, then α |= β: α ⇒ β means that if α is true, then β is also true, and if α is false, then β is false. This fits the definition of entailment, that in every model in which α is true, β is also true.

D. α |= β if and only if the sentence α ∧ ¬β is unsatisfiable.

We must prove that (1) if α |= β, then α ∧ ¬β is unsatisfiable and (2) if α ∧ ¬β is unsatisfiable, then α |= β.

(1) To prove that if α |= β, then α ∧ ¬β is unsatisfiable: Entailment α |= β means that in every model in which α is true, β is also true. There cannot exist a model such that α is true and β is not true, which is described by α ∧ ¬β, so this statement is unsatisfiable.

(2) To prove that if α ∧ ¬β is unsatisfiable, then α |= β: Unsatisfiability of α ∧ ¬β means that there exists no model in which α is true and β is false, which means that in every model in which α is true, β is also true. There can exist only models in which α is true and β is true and models in which α is false and β is false. This is the definition of entailment: α |= β.