OR-HW3

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1. Problem 1

   (a) For the function to be convex , the $\nabla^2 f$ should be PSD.
   the $\nabla^2 f$ is

   $$\begin{bmatrix} 6x_1 + 2\frac{x_2}{x_1^3} & \frac{-1}{x_1^2} & 2 \\ \frac{-1}{x_1^2} & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix}$$

   The second principle cannot be positive, so

   (b) the function is convex for all $x \in R$.

2. Problem 2

   (a) The $\nabla^2 f$ is

   $$\begin{bmatrix} 1 & 1 \\ 1 & 6 \end{bmatrix}$$

   It is a PSD , so the function is convex.

   (b) The FOC requires $x_1 + x_2 = 0$ and $6x_2 + x_1 = 0$, we can get the optimal solution is $\bar{x} = (0, 0)$.
   The constraint is $x_1 \leq 3$, so the optimal solution is infeasible.

   (c) Choosing the feasible point we set x_1 to be 3, and to be closest to (0,0),we let x_2 be zero , $\tilde{x} = (3, 0)$. If the optimal is $\tilde{x}$ we cannot get λ , so $\tilde{x}$ is not the optimal solution.

   (d) The lagrangian is
   $L(x|\lambda) = \frac{1}{2}x_1^2 + 3x_2^2 + x_1x_2 - x_1 - 2x_2 - \lambda(3 - x_1)$

By solving

$$x_1 + x_2 - \lambda = 0$$
$$6x_2 + x_1 = 0$$

We can get the optimal solution $(3, -\frac{1}{2})$.
$\lambda = \frac{5}{2} \geq 0$, $\lambda(3 - x_1) = 0$,and $x_1 \geq 3$, satisfies the KKT condition so it is optimal.

3. Problem 3

(a) the gradient $\nabla f(x)$ is

$$\begin{bmatrix} 2x_2 + x_2 - 1 \\ 6x_2 + x_1 - 2 \end{bmatrix}$$

the hessian is $\nabla^2 f(x)$

$$\begin{bmatrix} 2 & 1 \\ 1 & 6 \end{bmatrix}$$

(b) $x^0 = (1, 1)$, we have $\nabla f(x) = (2, 5)$ and $\nabla^2 f(x) = \begin{bmatrix} 2 & 1 \\ 1 & 6 \end{bmatrix}$.

Therefore , we have

$$\begin{aligned} x^1 &= x^0 - [\nabla^2 f(x^0)]^{-1} \nabla f(x^0) \\ &= \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \frac{1}{11} \begin{bmatrix} 6 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \end{bmatrix} \\ &= \begin{bmatrix} \frac{4}{11} \\ \frac{3}{11} \end{bmatrix} \end{aligned}$$

(c) $x^0 = (1, 1)$. $f(x^0) = 2$. $\nabla f(x^0) = (2, 5)$
$x^1 = (1, 1) - \frac{1}{2}(2, 5) = (0, \frac{-3}{2})$
$f(x^1) = \frac{39}{4}$,and $f(x^0) = 2$.
$f(x^0) - f(x^1) = -7\frac{3}{7}$ we can see that the improvement is negative.

(d) $a_0 = argmin_{a \geq 0} f(x^0 - a(x^0))$, so

$$\begin{aligned} f(x^0 - a(x^0)) &= f(1 - 2a, 1 - 5a) \\ &= 89a^2 - 29a + 2 \end{aligned}$$

We can get $a_0 = 0.16292134$

$x^1 = (1,1) - \frac{29}{178}(2,5) = (\frac{120}{178}, \frac{33}{178})$

$f(x^1) = -0.36235955 \approx -0.4$

the result is improved by 2.4

(e) The optimal function is acquired by 3(b). And because the function is a convex function, the local min is a global min. Hence, $(\frac{4}{11}, \frac{3}{11})$ is the optimal solution.

4. Problem 4

(a) The cost table is below

Table 1: $V_t(y)$

t	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	13	9	0	1	2	3	4	5	6	7	8	9	10	11
2	24	13	10	2	4	6	8	10	12	14	16	18	20	22
3	30	26	22	14	12	5	8	11	14	17	20	23	26	29
4	56	49	42	30	27	24	17	16	10	14	18	22	26	30
5	74	68	62	56	50	44	33	31	29	23	23	18	23	28
6	68	65	62	59	56	53	50	47	39	38	37	32	33	29

(b) changing S effects

i. $V_6(0)$ change by S

Table 2: $V_6(0)$

S	0	5	10	15
	54	68	78	83

We can see that the cost rises with S, with nothing on hand at first,the production cost will certainly rise due the increase of setup cost.

ii. optimal output change by S

Table 3: $S = 0$

t	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	2	1	0	0	0	0	0	0	0	0	0	0	0	0
2	1	0	0	0	0	0	0	0	0	0	0	0	0	0
3	3	2	1	0	0	0	0	0	0	0	0	0	0	0
4	3	2	1	0	0	0	0	0	0	0	0	0	0	0
5	3	2	1	0	0	0	0	0	0	0	0	0	0	0
6	8	7	6	5	4	3	2	1	0	0	0	0	0	0

Table 4: $S = 5$

t	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	2	1	0	0	0	0	0	0	0	0	0	0	0	0
2	1	0	0	0	0	0	0	0	0	0	0	0	0	0
3	5	4	3	0	0	0	0	0	0	0	0	0	0	0
4	3	2	1	0	0	0	0	0	0	0	0	0	0	0
5	6	5	4	0	0	0	0	0	0	0	0	0	0	0
6	8	7	6	5	4	3	2	1	0	0	0	0	0	0

Table 5: $S = 10$

t	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	2	1	0	0	0	0	0	0	0	0	0	0	0	0
2	3	0	0	0	0	0	0	0	0	0	0	0	0	0
3	5	4	3	0	0	0	0	0	0	0	0	0	0	0
4	3	2	1	0	0	0	0	0	0	0	0	0	0	0
5	6	5	4	0	0	0	0	0	0	0	0	0	0	0
6	8	7	6	5	4	3	2	1	0	0	0	0	0	0

Table 6: $S = 15$

t	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	2	1	0	0	0	0	0	0	0	0	0	0	0	0
2	3	0	0	0	0	0	0	0	0	0	0	0	0	0
3	5	4	3	0	0	0	0	0	0	0	0	0	0	0
4	3	2	1	0	0	0	0	0	0	0	0	0	0	0
5	6	5	4	0	0	0	0	0	0	0	0	0	0	0
6	13	12	11	10	9	8	7	6	0	0	0	0	0	0

We can see that the optimal production changed more from day 3, the higher setup cost is, the firm is more likely to produce at once, and store the stocks for later use.

5. Problem 5

$$
\begin{aligned}
V_t(0) &= max_{q_t \geq y}\{u(q_t, 0) + E[V_{t-1}(q_t - D_t)]\} \\
&= max_{q_t \geq y}\{[\sum_{x=0}^{q_t} xPr(x) + \sum_{x=q_t+1}^{N} q_t Pr(x)]p - q_t c + \sum_{x=0}^{q_t} V_{t-1}(q_t - x)Pr(x) + \sum_{x=q_t+1}^{N} V_{t-1}(0)Pr(x)\}
\end{aligned}
$$