OR-lecture-11-In-class

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1. Problem 2

   (a) This is true. When $C_t + H < C_{t-1}$, instead of producing/ordering a positive number of products in period $t1$, it is better to shift the production/ordering to period t. Each unit of shifting will save $C_t + H - C_{t-1} > 0$ dollars. x^t will thus be greater than D_t.

   (b) This is not true. For example, consider the case with $t = 3$, $t' = 1, C_3 = 4, C_2 = 1$, and $C_1 = 10$. Though it is beneficial to shift the production/ordering in period 1 to period 3 (which will make $x^*3 > D_3$, it is even more beneficial to shift it to period 2.

2. Problem 3

   (a) Formulate a dynamic program.
   To formulate a dynamic program, we need to specify two things: the boundary condition (like the base case in mathematical induction) and the optimization problem in each period (like the inductive step in mathematical induction).

      i. The boundary condition: $V_0(y) = 0$ for all y.

      ii. The optimization problem: The value-to-go function is still the same. The demand fulfillment constraint $x_t \geq (D_t y)^+$is still there. Moreover, to incorporate the space limitation, all we need to do is to add a constraint $y + x_t - D_t \leq K$ for period t to ensure that we do not produce/order too many. The complete formulation is thus

$$\begin{array}{ll} \min\limits_{x_t} & V_t(y) = C_t x_t + H(y + x_t - D_t) + V_{t-1}(y + x_t - D_t) \\ s.t. & X_t \geq (D_t - y)^+ \\ & x_t \leq K + D_t - y \end{array}$$

   (b) It is true. As there is no need to produce/order more than $\sum_{t=1}^{T} D_t$, there is no need to store more than that amount. The space is thus always enough, and the storage limitation may thus be ignored.

3. Problem 5

   (a) To find $V_4(0)$, we need to compare all the options of x_4, where the number of options depends on the upper bound of x_4.

   $$x_4 \leq (\sum_{t=1}^{4} D_t - y)^+ = D_{14}$$

   (b) We compare the options from 1 8 , so the complexity is $O(D_{14} - 1)$

   (c) The complexity of finding $V_4(y)$ for all reasonable values of y is

   $$O[D_{14} + (D_{14} - 1) + (D_{14} - 2) + ... + 0] = O[\sum_{k=0}^{D_{14}}] = O(D_{14}^2)$$

4. Problem 6

   (a) False , the complexity of solving the whole DP , should be $O(TD_{1,k}^2)$

   (b) True

   (c) True, while (b) provides a better upper bound.

5. Problem 7

   (a) This is true. When $q_t \geq 1$, we have rp as the expected revenue (where r is the probability of selling one product) and $(q_t y)c$ as the production/ordering cost. When $q_t = 0$, obviously we have no chance to sell anything, and ordering nothing incurs no cost.

   (b) This is true. When qt 1, with probability 1 r we sell nothing, and the ending inventory remains qt. The expected profit from period t 1 to period 1 is thus Vt1(qt). On the contrary, with probability r we sell one unit, the ending inventory becomes qt 1, and the future expected profit is Vt1(qt 1).
   When q0 = 0, the ending inventory will definitely be 0, and the future expected profit is Vt1(0) for sure.

   (c) Derive$V_t(y)$ without using the u function and the expectation operator.
   $\pi t(qt, y) = rp(qty)c + Vt1(qt)(1r) + Vt1(qt1)r$

   $$V_t(y) = \begin{cases} max_{q_t \geq y}\{\pi(q_t, y)\}, & if\ y \geq 1 \\ max\{V_{t-1}(0), max_{q_t \geq}\{\pi(q_t, y)\}\}, & if\ y = 0 \end{cases} \tag{1}$$

   (d) When we consider the probability distribution of D_t and express the problem explicitly in the previous part, everything may be implemented in a computer program.