OR-final

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1. I certify that I have carefully read the exam rules listed on Page 1. If I violate any of the rules, I agree to be penalized accordingly.

2. In the absence of degeneracy the objective value increases in each iteration, because all basic sets are different. The number of different sets B is bounded by [m,n+m] since in each B we have to chooses m basic variables among n+m variables. Thus the procedure must stop after a finite number of iterations. We conclude that when a linear program is not unbounded, the simplex method can always find an optimal solution after a finite number of iterations if the smallest index is applied.

3. (a) Using the dual simplex method we can find the final basis at the beginning which can leave a lot of steps behind while using branch and bound.

   (b) Yes, it works. First , we check if the current optimal solution satisfies the new constraint.If it is satisfied, the inclusion of the constraints has no effect on the current optimal solution, the solution remains feasible and optimal. If it does not satisfy, the current solution is not feasible, then we use the dual simplex method to find new optimal solution.

4. (a) Let $f(x) = f + n - \beta(xa_1)^2 - w(xa_0)^2$The optimal solution should satisfy $f'(x) = 0$, $f'(x) = -2\beta(x - a_1) - 2w(x - a_0)$, we can get $x^* = \frac{wa_0+\beta a_1}{w+\beta}$

   (b) Let $q(x) = (p - c)(m - (a_1 - x^*)^2 - p) - f$.
   The ∇q is $\begin{bmatrix} m - (a_1 + x^*)^2 - 2p + c \\ -1 \end{bmatrix}$
   We can get $(p^*, f^*) = (\frac{m-(a_1-x^*)^2+c}{2}, \beta(x^* - a_1)^2 + w()x^* - a_0)^2 - n$

5. (a) the linear program.
   Let $D = 1$ if demand is satisfied , 0 otherwise.

$$\begin{array}{ll} max & \sum_{i=1}^{n_B} \sum_{k=1}^{m} Dp_i d_{ik} \\ s.t. & \sum_{i=1}^{n_B} d_{ik} \leq h_k \quad \forall k = 1..m \\ & D \in [0,1] \quad \forall i = 1..n_B \\ & d_{ik}, h_k, p_i, \geq 0 \quad \forall i = 1,..,5 \end{array}$$

(b) the linear program.
Let S be 1 if price is $\geq q_j$,0 otherwise.

$$\begin{array}{ll} max & \sum_{i=1}^{n_B} \sum_{k=1}^{m} Dp_i d_{ik} \\ s.t. & \sum_{i=1}^{n_B} d_{ik} \leq S s_{jk} \quad \forall k = 1..m, j = 1..n_S \\ & S \in [0,1] \quad \forall j = 1...n_S \\ & d_{ik}, h_k, p_i, s_{jk}, \geq 0 \quad \forall i = 1,..,n_B, \quad \forall j = 1,..,n_S, \quad \forall k = 1,..,m \end{array}$$

(c) the linear program.
Let S be 1 if price is $\geq q_j$,0 otherwise.

$$\begin{array}{ll} max & \sum_{i=1}^{n_B} \sum_{k=1}^{m} Dp_i d_{ik} \\ s.t. & \sum_{i=1}^{n_B} d_{ik} \leq \sum_{j=1}^{n_S} S s_{jk} \quad \forall k = 1..m \\ & S \in [0,1] \quad \forall j = 1...n_S \\ & d_{ik}, h_k, p_i, s_{jk}, \geq 0 \quad \forall i = 1,..,n_B, \quad \forall j = 1,..,n_S, \quad \forall k = 1,..,m \end{array}$$

6. (a) the hessian matrix is
$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$
It is a PSD , so the function is convex.

(b) The first derivative function is
$$\begin{bmatrix} -p(a) \\ -1 + p(a) \end{bmatrix}$$
We can see that the maximization of the objective function has nothing to do with the wage rate. And since the utility function is nonnegative,we don't need $w_1 = w_0$ to satisfy the constraint,therefore $w_1 = w_0$ is not in an optimal solution.

7. dynamic program
$\mathrm{V}_t(x) = min_{x_t \leq L}\{E[V_{t-1}R]\}$
$= min_{x_t \leq L}\{\sum_{y=0}^{x_t} RV_{t-1}(p_{t-1} + p + t)Pr(y) + \sum_{y=x_t}^{N} RV_{t-1}(0)Pr(y) + 1\}$