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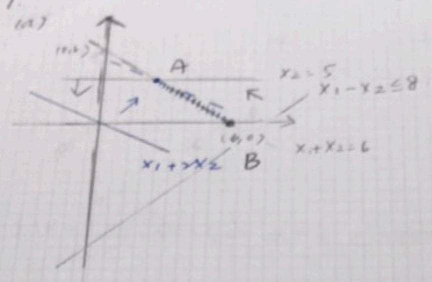
HW1 4/2

1.

(a)



(b) standard form

max x_1+2x_2

s.t. $x_1+x_2=6$

$x_1-x_2+x_3=8$

$x_2+x_4=5$

$x_1\ge 0,\ x_2\ge 0$ #

(c)

Basis	x_1	x_2	x_3	x_4	bfs	
$\{x_1, x_2, x_3\}$	1	5	12	0	✓	A
$\{x_1, x_3, x_4\}$	6	0	2	5	✓	B
$\{x_2, x_3, x_4\}$	0	6	14	-1	×	
$\{x_1, x_2, x_4\}$	7	-1	0	6	×	

(d) no identity matrix → Phase I. Phase II

Phase I.

$$\begin{array}{ccccc|c} 0&0&0&0&-1&0\\ \hline 1&1&0&0&1&x_5=6\\ 1&-1&1&0&0&x_3=8\\ 0&1&0&1&0&x_4=5 \end{array} \rightarrow \begin{array}{ccccc|c} 1&1&0&0&0&6\\ \hline \boxed{1}&1&0&0&1&x_5=6\\ 1&-1&1&0&0&x_3=8\\ 0&1&0&1&0&x_4=5 \end{array} \rightarrow \begin{array}{cccc|c} 0&0&0&0&0\\ \hline 1&1&0&0&x_1=6\\ 0&-2&1&0&x_3=2\\ 0&1&0&1&x_4=5 \end{array}$$

$x^0=(0,0,8,5,6)$

feasible basis $\{x_1, x_3, x_4\}$

$x^1=\{6,0,2,5\}$

Phase II.

$$\begin{array}{cccc|c} -1&-2&0&0&0\\ \hline 1&1&0&0&x_1=6\\ 0&-2&1&0&x_3=2\\ 0&1&0&1&x_4=5 \end{array} \rightarrow \begin{array}{cccc|c} 0&-1&0&0&6\\ \hline 1&1&0&0&x_1=6\\ 0&-2&1&0&x_3=2\\ 0&\boxed{1}&0&1&x_4=5 \end{array} \rightarrow \begin{array}{ccc|c} 0&0&0&11\\ \hline 1&0&0&x_1=1\\ 0&0&1&x_3=12\\ 0&1&0&x_2=5 \end{array}$$

bfs (1,5,12,0) #

initial bfs: (6,0,2,5)

optimal bfs: (1,5,12,0) #

(e) $\begin{array}{cccc|l} -1 & 2 & 0 & 0 & 0 \\ \hline 1 & 1 & 0 & 0 & x_1=6 \\ 0 & -2 & 1 & 0 & x_3=2 \\ 0 & 1 & 0 & 1 & x_4=5 \end{array} \rightarrow \begin{array}{cccc|l} 0 & 3 & 0 & 0 & 0 \\ \hline 1 & 1 & 0 & 0 & x_1=6 \\ 0 & -2 & 1 & 0 & x_3=2 \\ 0 & 1 & 0 & 1 & x_4=5 \end{array}$ ← no negative

There is no optimal bfs.

$\{6, 0, 2, 5\}$ is the only bfs.

2.

(a) The set of extreme points is $\{(x_1, x_2) \mid x_1^2 + x_2^2 = 81\}$

(b) $(11, 0)$ $(4, 7)$ $(4, -7)$

(c) $(4, -15)$ $(4, 7)$

(d) $(4, 7)$ $(4, -15)$

(e) $\{(x_1, x_2) \mid x_1^2 + x_2^2 = 81 \wedge x_1 \geq 4)$

被分配的量.

(b) $(11, 0)$ $(4, 7)$ $(4, -7)$

(c) $(4, -15)$ $(4, 7)$

(d) $(4, 7)$ $(4, -15)$

(e) $\{(x_1, x_2) \mid x_1^2 + x_2^2 = 81 \wedge x_1 \geq 4)$

3.

(a) x_i = agent i 被分配的量.

$$\max \sum_{j=1}^{n} \sum_{i=1}^{m} \frac{b_j}{q_j} \times x_i$$

$$\text{s.t.} \quad x_i \times C_i \leq k_i \quad \forall i = 1 \ldots m$$

$$\sum_{i=1}^{m} x_{ij} \leq \sum_{j=1}^{n} q_j$$

$$x_{ij} \geq 0,\ C_i \geq 0,\ k_i \geq 0,\ b_j \geq 0,\ q_j \geq 0$$

$$\forall i,\ 1 \ldots m \quad \forall j\ 1 \ldots n \quad \emptyset \in [0, 1]$$

(b) $$\text{Mean} = \left[\sum_{j=1}^{n} \sum_{i=1}^{m} (1-\emptyset) \frac{b_j}{q_j} \times x_{ij}\right] \frac{1}{m}$$

$$B = \left| \sum_{j=1}^{n} \sum_{i=1}^{m} (1-\emptyset) \frac{b_j}{q_j} x_{ij} - \text{Mean} \right|$$

$$\min \sum_{i=1}^{m} B_i$$

$$\text{s.t.} \quad B \geq \sum_{j=1}^{n} (1-\emptyset) \frac{b_j}{q_j} x_{ij} - \text{Mean} \quad \forall i = 1 \ldots m$$

$$B \geq \text{Mean} - \sum_{j=1}^{n} (1-\emptyset) \frac{b_j}{q_j} x_{ij} \quad \forall i = 1 \ldots m$$

這兩行再加上上面這些限制

3.

(c) $\max \quad \sum_{j=1}^{n} \sum_{i=1}^{m} \frac{b_j}{q_j} x_{ij} - \sum_{i=1}^{m} B_i$

s.t.

$$\sum_{i=1}^{m} x_{ij} \le \sum_{j=1}^{n} q_j$$
$$x_i \times c_i \le k_i \quad \forall i = 1 \dots m$$
$$x_{ij} \ge 0 \quad \forall i = 1 \dots m. \quad \forall j = 1 \dots n$$
$$B_i \ge \sum_{j=1}^{n} \frac{b_j}{q_j} x_{ij} - \text{Mean}.$$
$$B_i \ge \text{Mean} - \sum_{j=1}^{n} \frac{b_j}{q_j} x_{ij}$$
$$x_{ij} \ge 0, \ c_i \ge 0, \ k_i \ge 0, \ b_j \ge 0, \ q_j \ge 0$$
$$\varnothing \in [0,1]$$

To satisfy both efficient and fairness, I try to maximize the profit as the first goal and by subtracting the difference between every workers wage and the mean wage can make this outcome more fair.
To make B smaller, one must make every worker to earn the same amount as possible, this is how I try to achieve fairness while also looking for high profit.