Operation Research Case2, Group 11

HO, CHENG-YU B06208030

HO, LEONG HOCK T08902130

CHIANG,YU-JO B06303077

HSIEH, CHUN-MO B06303059

HUANG, CHUNG-CHIN B06602043

Task 1: Formulate a Linear Program

Let's begin with the notation of the variables of our linear program.

1. A denotes the number of days in the month

2. $S_{i,d}$ denotes numbers of CSRs assigned to shift i at day d, where $i = 0, 1, ..., 13$ and $d = 1, 2, ..., A$

3. $D_{p,d}$ denotes Demands for period p at day d, where $p = 1, 2, ..., 24$ and $d = 1, 2, ..., A$

4. $W_{p,d} = \sum_{i \in \Omega_p} S_{i,d}$ denotes numbers of demands that are be met, i.e., the numbers of working CSRs, on period p at day d, where $p = 1, 2, ..., 24$ and $d = 1, 2, ..., A$. Meanwhile, Ω_p denotes the set of the shifts that cover the current period p. Take $p = 1$ for instance, we have

$$W_{1,d} = \sum_{i \in \Omega_1} S_{i,d},\ \Omega_1 = 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12$$

5. $L_{p,d}$ denotes numbers of lack for period p at day d, and $L_{p,d}$ is defined as,

$$L_{p,d} = \begin{cases} D_{p,d} - W_{p,d}, & \text{if } D_{p,d} \geq W_{p,d} \\ 0, & \text{otherwise} \end{cases}$$

However, in term of Linear Program, the function should be rewrite as,

$$L_{p,d} = max\{D_{p,d} - W_{p,d}, 0\}$$

Hence, we can derived the following constraints in linear program

$$L_{p,d} \geq D_{p,d} - W_{p,d} \wedge L_{p,d} \geq 0$$

Besides, the parameters and trivial notations are,

1. Period p: $p = 1$ refers to the period from 09:00 to 09:30, $p = 2$ refers to the period from 09:30 to 10:00, and so on. The last period is $p = 24$ refers to the period from 20:30 to 21:00. Hence, the domain of p is the integers set from 1 to 24.

2. Day d: $d = 1$ refers to the first day of the month. As the number of days defer in different months, the domain of d is the integer set from 1 to A. For example in March, $A = 31$

3. $R_{i,d}$ refers to the base-line request corresponding to each $S_{i,d}$. For all $R_{i,d}$ where $i = 0$ refers to the leave requests, and for all $R_{i,d}$ $i = 1, 2, ..., 13$ refers to the shift requests. The given parameters are

$$R_{i,d} = \begin{cases} 1, & \text{if } (i,d) \in \{(5,10),(6,17),(13,27),(0,1),(0,2),(0,14),(0,15),(0,19),(0,31)\} \\ 2, & \text{if } (i,d) \in \{(0,3),(0,20)\} \\ 0, & \text{otherwise} \end{cases}$$

4. N denotes the numbers of the CSRs.

5. $NIGHT \equiv \lfloor 4.43N \rfloor$ denotes the maximum numbers of nights shifts available in a month

6. $AFTER \equiv \lfloor 8.86N \rfloor$ denotes the maximum numbers of afternoon shifts available in a month

7. M is the total number of days off in a month, and is obtained by $M = 8N$, which is the product of legal monthly 8 days off per person per month with N number of CSRs.

Hence, the linear program is illustrated as following,

$$
\begin{aligned}
\min \quad & \sum_{d=1}^{A}\sum_{p=1}^{24} L_{p,d} && (1)\\
\text{s.t.} \quad & \sum_{d=1}^{A} S_{i=0,d} = M && \text{(monthly days off)}\\
& L_{p,d} \geq D_{p,d} - W_{p,d} \ \forall p = 1,...,24, d = 1,...,A && \text{(max function constraint 1)}\\
& L_{p,d} \geq 0 \ \forall p = 1,...,24, d = 1,...,A && \text{(max function constraint 2)}\\
& S_{i,d} \geq R_{i,d} \ \forall i = 0,...,13, d = 1,...,A && \text{(leave/shift request)}\\
& \sum_{d=1}^{A} \sum_{i\in\Omega_{night}} S_{i,d} \leq NIGHT && \text{(night shift limit)}\\
& \sum_{d=1}^{A} \sum_{i\in\Omega_{afternoon}} S_{i,d} \leq AFTER && \text{(afternoon shift limit)}\\
& \sum_{\delta=0}^{6} S_{i=0,d=\hat{d}+\delta} \geq 40, \forall \hat{d} = 1,2,...,A-6 && \text{(consecutive 7 days limit)}\\
& 40 \geq \sum_{i=1}^{13} S_{i,d} \geq 0, \ \forall d = 1,2,...,A && \text{(reasonable range)}
\end{aligned}
$$

Task 2: AMPL

The optimization task is solve by ampl(cplex). The complete codes are in the attached files, case2.mod and case2.dat.

Task 3: Summary of the Result

The ampl solver suggests that the optimal shift plan is illustrated in following table. The minimum total lack according to the shift plan is 243. The first column of the table denotes the Date in the month from 1^{st} to 31^{st}. The following 14 columns are the suggested number of CSRs assigned to the corresponding shift on each day. The S_i denotes the shift i, and S_0 refers to the days off, $i = 1, 2, .., 6$ refers to the morning shifts, $i = 7, 8, 9, 10$ refers to the afternoon shifts, and $i = 11, 12, 13$ refers to the night shifts.

Date	S_0	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}
1	3	12	0	0	9	5	0	0	3	0	0	0	0	8
2	13.5	4	0	2	6.5	0	3	2	3.5	0	0	5.5	0	0
3	10	3	0	0	3	7	3	5	0	3	0	0	1	5
4	0	0	0	0	0	12	6	9	0	5	0	8	0	0
5	15	0	0	0	0	0	0	6	5	6	1	4	0	3
6	10	0	0	0	3	7	6	5	0	3	0	5	1	0
7	18	0	8	3	0	0	0	2	0	0	4	5	0	0
8	8	7	8	0	0	0	0	0	0	0	9	5	0	3
9	15	4	0	2	5	0	0	2	5	0	0	7	0	0
10	14.5	0	4.5	2	2	3.5	0.5	6	0	1	0	6	0	0
11	17	0	0	3	3	2	0	2	7	0	0	6	0	0
12	11	6	0	0	3	1	9	2	0	0	8	0	0	0
13	17	0	7	0	3	2	0	2	0	3	0	3	0	3
14	16	6	5	0	2	0	0	1	0	0	5	0	1	4
15	1	8	0	0	0	11	2	4	1	0	5	0	0	8
16	10	4	0	2	3	5	5	2	0	0	2	7	0	0
17	16	0	6	3	0.5	0	1	5.5	0	0	3.5	3	0.5	1
18	13	6	0	0	3	6	1	2	0	3	0	0	0	6
19	11	0	0	0	0	6	4	8	0	6	0	0	0	5
20	2	3	2	0	0	10	6	4	0	0	5	5	0	3
21	10	4	0	0	0	3	7	0	0	2	7	5	2	0
22	10	0	13	0	0	0	0	4	0	0	7	6	0	0
23	6	0	0	2	0	9	9	2	0	0	5	4	0	3
24	10	3	0	3	0	8	2	6	0	0	2	0	0	6
25	13	0	0	0	9	0	7	8	0	3	0	0	0	0
26	5	0	0	0	0	7	15	2	0	0	11	0	0	0
27	14	0	1	0	8	0	0	13	0	3	0	0	0	1
28	15	0	12	0	0	0	0	0	0	0	8	0	0	5
29	0	0	0	0	0	12	6	9	0	5	0	3	0	5
30	15	0	0	0	0	0	0	6	5	6	1	4	0	3
31	1	0	1	0	0	11	7	7	0	3	2	5	0	3

Meanwhile, for the sake of providing a robust proposal, the following table illustrates the detailed distribution of the lack in demand of each period at each day. The other ignored periods has no lack in demand. The notation $19A$ refers to 19:00 19:30, $19B$ refers to 19:30 20:00, etc.

Date	$18B$	$19A$	$19B$	$20A$	$20B$
1	0	0	0	1	0
2	2.5	3.5	2.5	2.5	1.5
3	0	0	2	1	0
4	0	0	0	1	0
5	1	2	1	1	0
6	0	0	2	1	0
7	2	3	2	2	1
8	0	0	0	1	0
9	1	2	1	1	0
10	0	0	2	1	0
11	1	2	1	1	0
12	6	6	5	6	6
13	1	2	1	1	0
14	1	1	0	1	1
15	0	0	0	1	0
16	1	2	1	1	0
17	1.5	1.5	3.5	2.5	1.5
18	1	2	1	1	0
19	1	1	0	1	1
20	0	0	0	1	0
21	1	2	1	1	0
22	2	2	2	3	2
23	1	2	1	1	0
24	0	0	2	1	0
25	7	8	7	7	6
26	6	6	5	6	6
27	6	7	6	6	5
28	1	1	0	1	1
29	0	0	0	1	0
30	1	2	1	1	0
31	0	0	0	1	0

Task 4: Comparison and Suggestion on the number of CSRs

In order to compare the difference with respect to different amounts of CSRs, we can simply alter the value of N in the linear program. Meanwhile, the other parameters such as $M, AFTER, NIGHT$ will simultaneously be altered. The relationship between total lacks and N is demonstrated as Table 1 and Figure 1

N	$NIGHT$	$AFTER$	$Total\ Lack$
35	155	310	353
36	159	318	333
37	163	327	313
38	168	336	288
39	172	345	268
40	177	354	243
41	181	363	223
42	186	372	198
43	190	380	178
44	194	389	158
45	199	398	133

Table 1: The corresponding total lack with respect to N

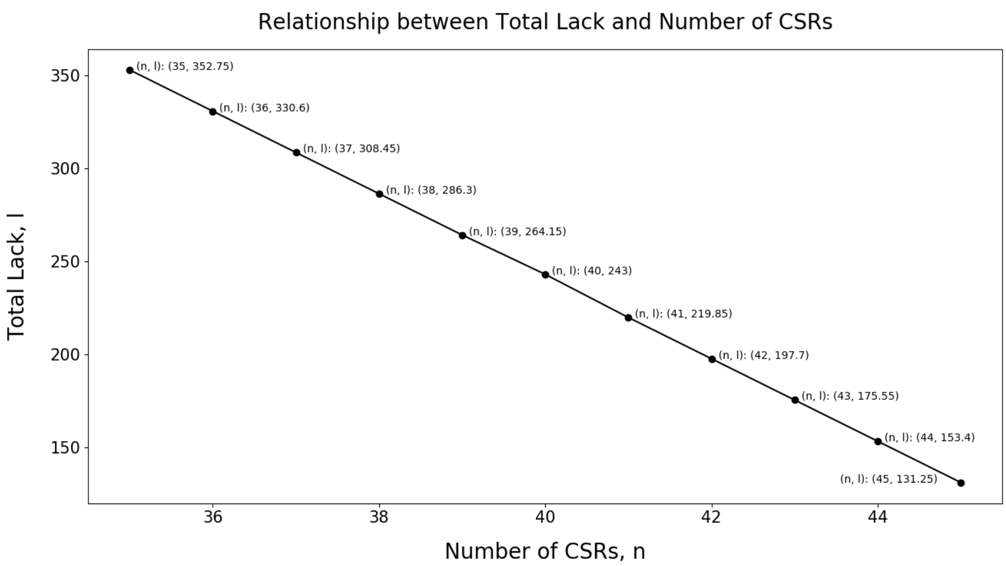

Figure 1: The relationship between total lack and N

The conclusion is that, with mere perspective of minimize the total lack, the suggestion is to hire more CSRs. From the figure, we can find that with the increment of the amount of CSRs, the corresponding total lack goes down gradually. Based on Figure 1, each increase if 1 CSR results in lack decreasing between 20 to 25. The conclusion is simple, for that the lack in demand is exactly stemmed from the lack in CSRs. Last but not least, if the linear program includes other issues, such as labor costs, the suggestion may differ. However, in this situation we have no data(parameter) in the linear program. Hence, the more CSRs, the less lack in demand.

In addition, we discovered that most of the lack is in the time period 18:30 to 21:00. Therefore instead of solely increasing the number of CSRs, an alternative is to allow CSRs to work additional night shifts in a month for more pay or hire some part timer CSRs to work solely at night. These are more cost effective ways to reduce the amount of lack.