OR-LectureProblem 8

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1. Problem 1

   (a) Let the sides be x,y.

   The formulation is:

$$\begin{aligned} max \quad & xy \\ \\ s.t. \quad & 2x + 2y = 1 \\ & x, y \geq 0 \end{aligned}$$

   (b) Let the sides be x,y.

   The formulation is:

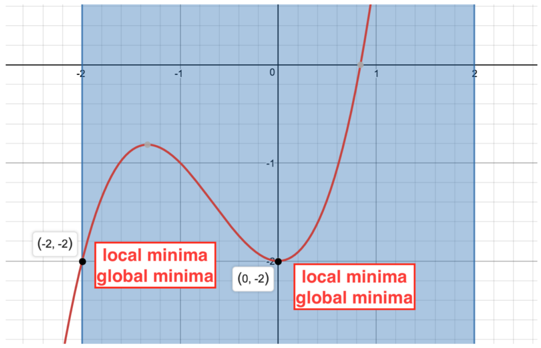
$$\begin{aligned} min \quad & 2x + 2y \\ \\ s.t. \quad & xy = 1 \\ & x, y \geq 0 \end{aligned}$$

2. Problem 2

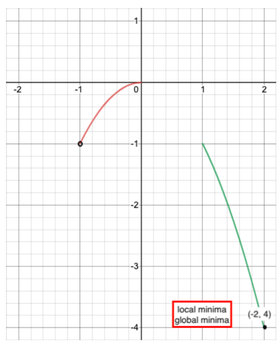
   Ans: (a),(c),(e)

3. Problem 4

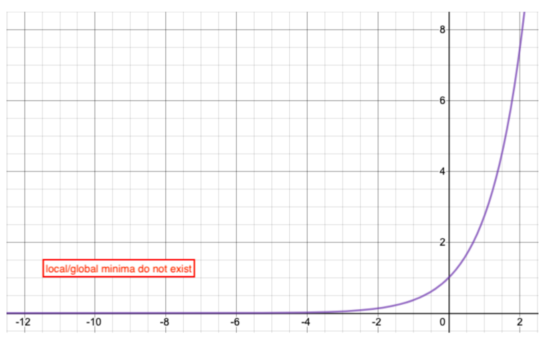
   (a) The solution is shown in Figure 1. There are two local (and global) minimax = 0 and 2.



(b) A local minimum locates at x = 2. It is also a global minimum. There is no local minimum in the interval $(1, 0]$.



(c) There is no local minimum.



4. Problem 5

(a) Concave. Because the second derivative is less than zero over the domain. The function is concave over the domain.

(b) Convex. Because the second derivative is greater than zero over the domain. The function is convex over the domain.

(c) Convex. Because the second derivative is greater than zero over the domain. The function is convex over the domain.

(d) It is true that the sum of two twice-differentiable convex functions is still a convex function. This can be proved by adding their second-order derivatives.

5. Problem 7

   (a) the formulation

$$\begin{aligned} max \quad & q(a - bq - c) \\ s.t. \quad & q \geq 0 \end{aligned}$$

   (b) solution
   $f(q) = aq - bq^2 - qc$, let $f'(q)$=0.
   so $f'(q) = a - 2bq - c = 0$, so $q = (c - a)/2b$.
   The obejective value of $\pi = ((a - c)^2)/4b$

   (c) q goes up when a goes up and b or c goes down. π goes up when a goes up and b or c goes down.

   (d) discuss one by one

   - a indicates the upper bound of the price. When a goes up, people are willing to pay more to buy the product. We should charge them a higher price and earn more money.
   - b indicates price sensitivity. When b goes up, people are more sensitive to quantity changes, which give us less room to increase our quantity. Therefore, we should charge a lower quantity and earn less money from each product.
   - c indicates the cost. When c goes up, obviously the equilibrium price should go up, and thus the quantity should do down. The equilibrium profit also go down.

6. Problem 8
   We have

   D = annual demand = 48000 gallons.
   K = unit ordering cost = 50 dollars.
   h = unit holding cost per year = 0.3 dollars.
   p = unit purchasing cost = 0.7 dollars.

   We want to determine q, the order quantity per order.

   (a) In EOQ model, The NLP is:

$$min \quad \frac{KD}{q} - pD + \frac{hq}{2}$$

Let $TC(q) = \frac{KD}{q} + \frac{hq}{2}$ to be our objective function and get q*. As q* $= \sqrt{\frac{2KD}{h}} = 4000$, one order should contain 4000 gallons.

(b) As D $= \frac{48000}{4000} = 12$, 12 orders should be placed annually.

(c) As $\frac{q}{D} = \frac{1}{12}$ year = 1 month, it will be one month between orders.

(d) We should order the gasoline earlier before the station's tank is empty. The reorder point R should be the selling slope times the ordering lead time $D * L$ and get$R = LD$. Therefore, we should reorder while the on-hand inventory level is LD.

(e) $LD = \frac{1}{2} * \frac{1}{12} * 48000 = 2000$.

(f) $LD = \frac{22}{10} * \frac{1}{12} * 48000 = 8800$. As this is above q = 4000, we divide 8800 by 4000 and take the remainder. We should order when we have 880 gallons remains. In other words, R = 880.

7. Problem 9

(a) First, we have $TC(q*) = \sqrt{2KDh}$. We then have

$$\begin{aligned} TC(rq*) &= \frac{2KD}{rq*} + \frac{hrq*}{2} \\ &= (\frac{r+\frac{1}{2}}{2}) * \sqrt{2KDh} \end{aligned}$$

This implies $f(r) = (\frac{r+\frac{1}{2}}{2})$

(b) $f(\frac{1}{2}) = f(2) = \frac{4}{5}$

(c) The wrong EOQ, q' is $\sqrt{\frac{2(2K)D}{h}} = \sqrt{2}q*$.

(d) We may plug in $r = 2$ into $f(r)$ and obtain $f(\sqrt{2}) \approx 1.06$. In other words, we lose about 6% by using the wrong EOQ. As the original error is around 41%, Such a huge error in the decision only results in such a relatively small error in the outcome. We thus say the EOQ model is a robust decision model.