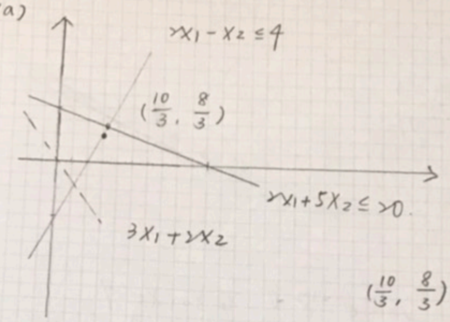
Lecture 7 In-class

1.

(a)



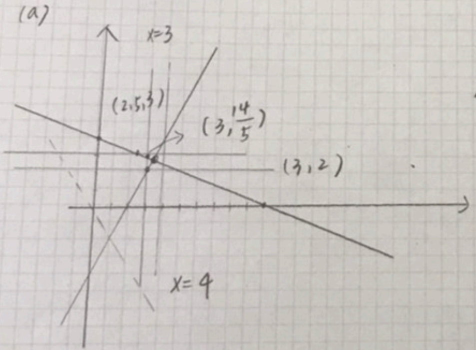
$(\frac{10}{3}, \frac{8}{3})$ objective value $\frac{46}{3}$

(b) like the figure above $(3,2)$ objective value 13
We can show that linear relaxation provides an upper bound with comparing objective value.

(c) four points $(3,2)$ $(3,3)$ $(4,3)$ $(4,2)$
only round down x which is $(3,2)$ is IP feasible.

2.

(a)



$x_1 \ge 4$. no solution
$x_1 \le 3$. solution $(3, \frac{14}{5})$

None #

(b) There is one candidate solution $(3,2)$.

(c)

$(x_1, x_2) = (\frac{10}{3}, \frac{8}{3})$
$z_1 = \frac{46}{3}$

$x_1 \le 3$ | $x_1 \ge 4$

$(x_1, x_2) = (3, \frac{28}{10})$ | INF
$z_2 = \frac{73}{5}$

$x_2 \le 2$ | $x_2 \ge 3$

$(x_1, x_2) = (3, 2)$ | $(x_1, x_2) = (\frac{5}{2}, 3)$
$z_3 = 13$ | $z_4 = \frac{27}{2}$

$x_1 \le 2$ | $x_1 \ge 3$

$(x_1, x_2) = (2, \frac{16}{5})$ | INF
$z_5 = \frac{62}{5}$

We don not have to branch more because $\frac{62}{5} \le 13$.

(d) $(x_1, x_2) = (\frac{10}{3}, \frac{8}{3})$

$Z_1 = \frac{46}{3}$

$x_2 \le 2$ $x_2 \ge 3$

$(x_1, x_2) = (3, 2)$ $(x_1, x_2) = (2.5, 3)$

$Z_2 = 13$ $Z_3 = 13.5$

$x_1 \le 2$ $x_1 \ge 3$

$(x_1, x_2) = (2, 3.2)$ INF

$Z_4 = 12.4$

X

4.

(a) $r\{1..5\}$ $\frac{2}{4}$, $\frac{3}{5}$, $\frac{4}{3}$, $\frac{1}{1}$, $\frac{3}{4}$

First choose $x_3 \to x_4 \to x_5$ value : $8 < x_1, x_3, x_5$ 9

Which means the algorithm can not find an optimal solution.

(b) No, we use above algorithm and got $(0, 0.6, 1, 1, 1)$.

The algorithm does not always find an optimal solution because x_2 is either 0 or 1.

5.

(a) $x_2 \le 0$, $x_2 \ge 1$ are added.

(b) With $x_2 \le 0$ $(x_1, x_2, x_3, x_4, x_5) = (0.75, 0, 1, 1, 1)$ $Z_2 = 9.5$

With $x_2 \ge 1$ $(0, 1, 1, 1, 0.5)$ $Z_3 = 9.5$

(c) $(0, 0.6, 1, 1, 1)$

$Z_1 = 9.8$

$x_2 \le 0$ $x_2 \ge 1$

$(0.75, 0, 1, 1, 1)$ $(0, 1, 1, 1, 0.5)$

$Z_2 = 9.5$ $Z_3 = 9.5$

$x_1 \le 0$ $x_1 \ge 1$ $x_5 \le 0$ $x_5 \ge 1$

$(0, 0, 1, 1, 1)$ $(1, 0, 1, 1, \frac{3}{4})$ $(0.5, 1, 1, 1, 0)$ $(0, 1, \frac{2}{3}, 0, 1)$

$Z_4 = 8$ $Z_5 = 9.25$ $Z_7 = 9$ $Z_6 = 8.76$

X

$x_5 \le 0$ $x_5 \ge 1$ $x_1 \le 0$ $x_1 \ge 1$

$(1, 0, 1, 1, 0)$ $(1, 0, 1, 0, 1)$ $(0, 1, 1, 1, 0)$ $(1, 1, \frac{2}{3}, 0, 0)$

$Z_8 = 7$ $Z_9 = 9$ $Z_{10} = 8$ $Z_{11} = \frac{23}{3}$

$x_3 \le 0$ $x_3 \ge 1$

$(1, 1, 0, 1, 0)$ INF

$Z_{12} = 6$

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6.

(a) The node with highest objective value has the highest potential to lead to an IP feasible solution with high objective value.

(b) We can find the solution faster.

7.

(a) number of product 1 x_1

number of product 2 x_2

$$w_i = \begin{cases} 0 & \text{if } x_i = 0 \\ 1 & \text{if } x_i > 0 \end{cases} \quad \forall i = 1, 2$$

$$\max \quad 5x_1 - 20w_1 + 7x_2 - 40w_2$$

s.t.

$$9x_1 + 7x_2 \le 120$$

$$x_1 \le \frac{120}{9} w_1$$

$$x_2 \le \frac{120}{7} w_2$$

(b)

$$z_0 = \begin{cases} 0 & w_1 = 0 \text{ or } w_2 = 0 \\ 1 & w_1 = 1 \text{ and } w_2 = 1 \end{cases}$$

$$\max \quad 5x_1 - 20w_1 + 7x_2 - 40w_2 + 20z$$

s.t.

$$9x_1 + 7x_2 \le 120.$$

$$x_1 \le \frac{120}{9} w_1$$

$$x_2 \le \frac{120}{7} w_2$$

$$z \le w_1$$

$$z \le w_2$$

$$x_i \ge 0 \quad \forall i = 1, 2$$

$$w_i \in \{0, 1\} \quad \forall i = 1, 2.$$

$$z \in \{0, 1\}$$