OR-HW2

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1. Problem 1

   (a) dual LP

$$
\begin{aligned}
min \quad & 15y_1 + 18y_2 + 20y_3 \\
s.t. \quad & 2y_1 + 3y_2 + 2y_3 \leq 6 \\
& 4y_1 + 3y_2 + 2y_3 \leq 4 \\
& 3y_1 + 2y_2 + 3y_3 \leq 5 \\
& y_1 + 2y_2 + 4y_3 \leq 7 \\
& y_1 \geq 0 \\
& y_2 \leq 0 \\
& y_3 \geq 0
\end{aligned}
$$

   (b) i. $\bar{x}$ is feasible to the primal LP , because it satisfies all the given constraints.

   ii. Since $\bar{x}$ is $(4, 0, 0, 3)$, the first constraint is non-binding , so $\bar{y}_1$ should be zero , and with complementary theory $2y_1+3y_2+2y_3 = 6$, $y_1 + 2y_2 + 4y_3 = 7$ should be satisfied , $\bar{y}_2$ can't all be negative,it violates the constraint ,so $\bar{x}$ is not the optimal solution.

   (c) i. $\bar{y}$ is feasible to the primal LP , because it satisfies all the given constraints.

   ii. $\bar{y}$ is $(\frac{5}{3}, 0, \frac{4}{3})$, the first and the third constrain of the primal LP must be satisfied. And for the dual LP , the second and the third constraint is non-binding , so $\bar{x}_2, \bar{x}_3$ must be zero. We can get $\bar{x} = (\frac{20}{3}, 0, 0\frac{5}{3})$, and since $\bar{x}_1, and\bar{x}_4$ are non-zero , the first and the forth constraint of $\bar{y}$ must be binding. It's satisfied so $\bar{y}$ is the optimal solution.

   (d) $\bar{y}$ is the optimal solution of the dual LP, the shadow price is vector is $(\frac{5}{3}, 0, \frac{4}{3})$.

(e) the new dual LP

$$
\begin{array}{ll}
min & 15y_1 + 18y_2 + 20y_3 \\
s.t. & 2y_1 + 3y_2 + 2y_3 \leq 6 \\
& 4y_1 + 3y_2 + 2y_3 \leq 4 \\
& 3y_1 + 2y_2 + 3y_3 \leq 5 \\
& y_1 + 2y_2 + 4y_3 \leq 7 \\
& y_i \geq 0 \quad \forall i = 1...3 \\
& y_3 \geq 0
\end{array}
$$

Since $\bar{y}_2$ can be positive now , it satisfies all the constraints in the dual LP, $\bar{x}$ is the optimal solution of the primal LP.

2. Problem 2

(a) linear relaxation

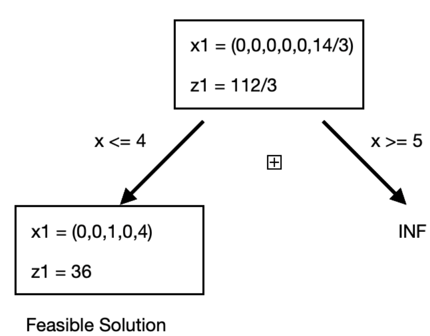
$$
\begin{array}{ll}
max & 5x_1 + 2x_2 + 4x_3 + 3x_4 + 8x_5 \\
s.t. & 3x_1 + 3x_2 + 2x_3 + 6x_4 + 3x_5 \leq 14 \\
& x_i \geq 0 \quad \forall i = 1...5 \\
& x_i \in R \quad \forall i = 1...5
\end{array}
$$

(b) greedy algorithm

objects	x_1	x_2	x_3	x_4	x_5
CP value	$\frac{5}{3}$	$\frac{2}{3}$	2	$\frac{1}{2}$	$\frac{8}{3}$

By this chart , we choose the fifth item , we acquire $\frac{14}{3}$ of x_5.

(c) branch and bound



We get the solution of$(0, 0, 0, 1, 0, 4)$.

(d) If we try the greedy algorithm , we will insure the items put into the knapsack has the highest weight-value, so there must be at most two items, because if there were more than two this implies there is still capacity for the items that were first put into , we should keep putting the items until it is over the capacity.

3. Problem 3

(a) minimize total travel time
Let k_{ij} be 1, if engineer travels from client i to client j , 0 otherwise , and u_i as an auxiliary variable to eliminate subtour.

$$
\begin{array}{ll}
min & \sum_{i=1}^{m}\sum_{j=1}^{m} d_{ij}k_{ij} \\
s.t. & \sum_{i=1}^{m} k_{ij} = 1 \quad \forall i,j = 1..m \\
 & x_i \leq s_i \quad \forall i = 1..m \\
 & x_i + p_i \leq t_i \quad \forall i = 1..m \\
 & u_i - u_j + mk_{ij} \leq m - 1 \quad i \neq j, i, j = 2..m \\
 & d_{ij}, x_i, s_i, t_i \geq 0 \\
 & k_{ij} \in (0,1) \quad \forall i,j = 1..m \\
 & u_i \geq 0 \quad \forall i = 1..m
\end{array}
$$

(b) minimize total travel time
Let e_t be 1 if engineer t was sent to client i, 0 otherwise.

$$
\begin{array}{ll}
min & \sum_{t=1}^{n}\sum_{i=1}^{m}\sum_{j=1}^{m} e_t d_{ij}k_{ij} \\
s.t. & \sum_{i=1}^{m} k_{ij} = 1 \quad \forall i,j = 1..m \\
 & x_i \leq s_i \quad \forall i = 1..m \\
 & x_i + p_i \leq t_i \quad \forall i = 1..m \\
 & u_i - u_j + mk_{ij} \leq m - 1 \quad i \neq j, i, j = 2..m \\
 & d_{ij}, x_i, s_i, t_i \geq 0 \\
 & k_{ij} \in (0,1) \quad \forall i,j = 1..m \\
 & e_t \in (0,1) \quad \forall t = 1..n \\
 & u_i \geq 0 \quad \forall i = 1..m
\end{array}
$$

(c) minimize penalties

$$
\begin{array}{ll}
min & \sum_{t=1}^{n}\sum_{i=1}^{m}\sum_{j=1}^{m} e_t d_{ij} k_{ij} c(s_i x_i) + d(x_i + p_i t_i) \\
min & \sum_{t=1}^{n}\sum_{i=1}^{m}\sum_{j=1}^{m} e_t d_{ij} k_{ij} \\
s.t. & \sum_{i=1}^{m} k_{ij} = 1 \quad \forall i, j = 1..m \\
 & x_i \leq s_i \quad \forall i = 1..m \\
 & x_i + p_i \leq t_i \quad \forall i = 1..m \\
 & u_i - u_j + m k_{ij} \leq m - 1 \quad i \neq j, i, j = 2..m \\
 & d_{ij}, x_i, s_i, t_i \geq 0 \\
 & k_{ij} \in (0,1) \quad \forall i, j = 1..m \\
 & e_t \in (0,1) \quad \forall t = 1..n \\
 & u_i \geq 0 \quad \forall i = 1..m
\end{array}
$$

4. Problem 4

   (a) Shadow price is the difference of value when you increase one unit of the RHS on constraints. By solving the shadow price we can decide whether we want to either one of the resources.

   (b) Yes, branch and bound algorithms are used to solve discrete optimization problems. We make the decision of going or not going to destination i , than one by one we choose the road that is the shortest.