Operation Research Case 3, Group 11

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1 Formulate a Integer Program

1.1 Parameter

The parameters and trivial notations are,

1. Period p: $p = 1$ refers to the period from 09:00 to 09:30, $p = 2$ refers to the period from 09:30 to 10:00, and so on. The last period is $p = 24$ refers to the period from 20:30 to 21:00. Hence, the domain of p is the integers set from 1 to 24. The set of p is denoted as $\mathbb{P}$.

2. Day d: $d = 1$ refers to the first day of the month. As the number of days defer in different months, the domain of d is the integer set from 1 to A. For example in March, $A = 31$. The set of d is denoted as $\mathbb{M}$.

3. Shift s: s refers to the shift index in the shift table, stating from 0 to 13. The set of the shifts is denoted as $\mathbb{S}$

4. CSR i: denotes the index of the CSRs and $\mathbb{E}$ denotes the set of the CSRs.The relation is given as Table1.

Table 1: Index i and the employees

i	**Name**	**ID**	i	**Name**	**ID**
1	Georgina Stevens	12	2	Cory Grant	19
3	Sasha Blouse	20	4	Sam Burns	23
5	Finley Hamilton	30	6	Lynn Harris	36
7	Avery Mejia	37	8	Rowan Bailey	40
9	Breanna Mckay	45	10	Jay Gardner	49
11	Paloma Blackwell	69	12	Willy Shaw	70
13	Armin Arlart	72	14	Haiden Wood	73
15	Jaylene Waters	74	16	Rose Newman	75
17	Bailey Joseph	84	18	Frankie Adams	90
19	Jackson Gibson	98	20	Richard Mayo	118
21	Alex Pearce	120	22	Braelyn Grant	121
23	Erin Montgomery	122	24	Karen Reed	129
25	Erin Robinson	132	26	Asa Weiss	136
27	Drew Bridges	144	28	Billy Ryan	158
29	Zeke Marquez	170	30	Jo Lucas	171
31	Annie Leonhart	172	32	Toby White	186
33	Conner Johns	190	34	Sienna White	203
35	Denny Simpson	208	36	Rosie Pearson	213
37	Justice Robles	231	38	Lee George	239
39	Val Morse	241	40	Brice Hunt	249

5. $R_{i,s,d}$ is the binary parameter refers to the base-line request of the worker i on shift s at day d. For all $R_{i,s,d}$ where $s = 0$ refers to the leave requests, and for all $R_{i,s,d}$ where $s = 1, 2, ..., 13$ refers to the shift requests. The given parameters are

$$R_{i,s,d} = \begin{cases} 1, & \text{if } (i,s,d) \in \{(27,5,10),(19,6,17),(12,13,27)\} \text{ (shift request)} \\ 1, & \text{if (i, s, d)} \in \{(2,0,1),(2,0,2),(2,0,3),(37,0,14),(18,0,20),(20,0,15) \\ & \text{, (1, 0, 3), (19, 0, 19), (19, 0, 20), (4, 0, 31) \}} \text{ (leave request)} \\ 0, & \text{otherwise} \end{cases}$$

6. $D_{p,d}$ denotes Demands for period p at day d, where $p \in \mathbb{P},\ d \in \mathbb{M}$.

7. Ω_p denotes the set of the shifts that cover the period p. For instance, $\Omega_1 = \{1 \sim 5, 7 \sim 12\}$. Other specific Ω includes

(a) $\Omega_{after} = \{7, 8, 9, 10\}$ denotes the set of afternoon shifts.

(b) $\Omega_{night} = \{11, 12, 13\}$ denotes the set of night shifts.

8. $\Theta_{AMA} = \{2, 3, 4, 10, 13, 17, 19, 20, 22, 31\}$ denotes the set of CSRs' index i that are Assistant Manager or above.

9. $\Theta_M = \{13, 17, 22\}$ denotes the set of CSRs' indices i that are Manager.

10. $\Pi_1 = \mathbb{E} - \{34, 39\}$ denotes the set of CSRs' indices i that have working experience greater or equal to 1 year.

11. $\Pi_2 = \mathbb{E} - \{21, 33, 34, 39\}$ denotes the set of CSRs' indices i that have working experience greater or equal to 2 year.

12. Mon denotes the set of d that are Monday in the month.

13. Wed denotes the set of d that are Wednesday in the month.

14. Thu denotes the set of d that are Thursday in the month.

15. Fri denotes the set of d that are Friday in the month.

1.2 Variable

The notation of the variables of our linear program are,

1. $W_{i,s,d} = \{0, 1\}$ denotes whether the CSRs i is assigned to shift s at day d, where $i \in \mathbb{E},\ s \in \mathbb{S},\ d \in \mathbb{M}$.

2. $H_{p,d} = \sum_{i \in \mathbb{E}, s \in \Omega_p} W_{i,s,d}$ denotes how many CSRs are supplied for period p at day d where Ω_p denotes the set of the shifts cover the current period p.

3. $L_{p,d}$ denotes numbers of lack for period p at day d, and $L_{p,d}$ is defined as,

$$L_{p,d} = \begin{cases} D_{p,d} - H_{p,d}, & \text{if } D_{p,d} \geq H_{p,d} \\ 0, & \text{otherwise} \end{cases}$$

However, in term of Linear Program, the function should be rewrite as,

$$L_{p,d} = max\{D_{p,d} - H_{p,d}, 0\}$$

Hence, we can derived the following constraints in linear program

$$L_{p,d} \geq D_{p,d} - H_{p,d} \ \ \wedge \ \ L_{p,d} \geq 0$$

1.3 Integer Program

Hence, the integer program is illustrated as following,

$$\min \quad \sum_{d\in\mathbb{M}}\sum_{p\in\mathbb{P}} L_{p,d} \qquad \text{(total lack)}$$

$$\text{s.t.} \quad \sum_{s\in\mathbb{S}} W_{i,s,d} = 1, \quad \forall\, i \in \mathbb{E},\ d \in \mathbb{M} \qquad \text{(one shift per day)}$$

$$\sum_{d\in\mathbb{M}} W_{i,0,d} = 8, \quad \forall\, i \in \mathbb{E} \qquad \text{(eight days off)}$$

$$\sum_{\delta=0,\ s\in\Omega_{night}}^{6} W_{i,s,(\hat{d}+\delta)} \le 1, \quad \forall\, \hat{d} = 1,2,...,25,\ i \in \mathbb{E}$$

(consecutive night shift limit)

$$\sum_{\delta=0,\ s\in\Omega_{afternoon}}^{6} W_{i,s,(\hat{d}+\delta)} \le 2, \quad \forall\, \hat{d} = 1,2,...,25,\ i \in \mathbb{E}$$

(consecutive afternoon shift limit)

$$\sum_{\delta=0}^{6} W_{i,0,(\hat{d}+\delta)} = 1, \forall \hat{d} >= 1,2,...,25,\ i \in \mathbb{E}$$

(at least one day off in consecutive seven days)

$$W_{i,s,d} \ge R_{i,s,d} \quad \forall\, i \in \mathbb{E},\ s \in \mathbb{S},\ d \in \mathbb{M} \qquad \text{(shift and leave request)}$$

$$\sum_{s\in\Omega_{night},\ i\in\Theta_{AMA}} W_{i,s,1} \ge 1 \qquad \text{(manager limit 1)}$$

$$\sum_{s\in\Omega_{afternoon},\ i\in\Theta_{AMA}} W_{i,s,10} \ge 2 \qquad \text{(manager limit 2)}$$

$$\sum_{s\in\Omega_{night},\ i\in\Theta_{AMA}} W_{i,s,15} \ge 1 \qquad \text{(manager limit 3)}$$

$$\sum_{s\in\Omega_{night},\ i\in\Theta_{M}} W_{i,s,22} \ge 1 \qquad \text{(manager limit 4)}$$

$$\sum_{s\in\Omega_{night},\ i\in\Theta_{AMA}} W_{i,s,29} \ge 1 \qquad \text{(manager limit 5)}$$

$$\frac{\sum_{i\in\Pi_1,\ s\in\{11,12,13\}} W_{i,s,d}}{\sum_{i\in\mathbb{E},\ s\in\{11,12,13\}} W_{i,s,d}} \geq 0.45, \quad \forall\ d \in Mon \qquad \text{(senior limit 1)}$$

$$\frac{\sum_{i\in\Pi_2,\ s\in\{1\sim5,7\sim10,12\}} W_{i,s,d}}{\sum_{i\in\mathbb{E},\ s\in\{1\sim5,7\sim10,12\}} W_{i,s,d}} \geq 0.55, \quad \forall\ d \in Wed \qquad \text{(senior limit 2)}$$

$$\frac{\sum_{i\in\Pi_1,\ s\in\{11,12,13\}} W_{i,s,d}}{\sum_{i\in\mathbb{E},\ s\in\{11,12,13\}} W_{i,s,d}} \geq 0.3, \quad \forall\ d \in Wed \qquad \text{(senior limit 3)}$$

$$\frac{\sum_{i\in\Pi_2,\ s\in\{11,12,13\}} W_{i,s,d}}{\sum_{i\in\mathbb{E},\ s\in\{11,12,13\}} W_{i,s,d}} \geq 0.3, \quad \forall\ d \in Thu \qquad \text{(senior limit 4)}$$

$$\frac{\sum_{i\in\Pi_2,\ s\in\{11,12,13\}} W_{i,s,d}}{\sum_{i\in\mathbb{E},\ s\in\{11,12,13\}} W_{i,s,d}} \geq 0.3, \quad \forall\ d \in Fri \qquad \text{(senior limit 5)}$$

$$L_{p,d} \geq D_{p,d} - H_{p,d} \qquad \text{(maximization of } L_{p,d} \text{ 1)}$$

$$L_{p,d} \geq 0 \qquad \text{(maximization of } L_{p,d} \text{ 2)}$$

$$W_{i,s,d} = \{0, 1\} \qquad \text{(binary variable)}$$

2 AMPL

The optimization task is solve by ampl(cplex). The complete codes are in the attached files, case3.mod and case3.dat. The optimal solution resultes in total lack of 222.

3 Summary of the Result

The total lack of the optimal shift table is 222. The suggested shift table is exhibited in Table 2 and Table 3 where the number in title's header from the second column refers to the date of March. to the last one

Table 2: Shift Table for 3/1 3/15

Name	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Georgina Stevens	2	12	0	7	3	4	8	2	11	0	10	0	2	4	0
Cory Grant	0	0	0	11	0	3	8	2	2	10	11	0	2	2	4
Sasha Blouse	2	2	2	2	11	9	0	0	7	2	4	2	11	0	2
Sam Burns	11	3	10	2	2	0	4	11	2	0	8	9	0	4	11
Finley Hamilton	2	2	11	4	0	4	10	2	10	11	0	0	4	0	2
Lynn Harris	2	9	0	2	11	2	7	4	0	2	0	11	3	0	7
Avery Mejia	11	9	0	2	2	10	2	11	2	0	0	10	0	0	11
Rowan Bailey	11	2	4	2	4	4	0	11	4	2	7	2	0	0	11
Breanna Mckay	2	10	2	4	11	0	3	3	8	2	9	11	0	0	2
Jay Gardner	7	0	2	11	2	10	0	7	3	0	12	2	10	0	7
Paloma Blackwell	10	9	11	2	3	0	0	0	2	11	0	0	2	9	2
Willy Shaw	0	0	9	0	13	4	9	2	0	10	2	13	2	10	2
Armin Arlart	9	6	7	11	2	0	4	5	7	7	5	2	0	13	0
Haiden Wood	7	11	0	2	2	7	6	10	11	0	4	0	10	0	7
Jaylene Waters	11	9	0	7	4	6	4	11	0	10	0	7	3	4	11
Rose Newman	2	9	3	11	2	7	0	2	2	10	11	0	2	0	2
Bailey Joseph	9	11	1	0	2	2	8	0	11	6	0	3	0	10	2
Frankie Adams	2	2	11	7	0	9	6	2	2	11	0	0	0	8	10
Jackson Gibson	6	0	2	13	9	7	2	4	0	1	2	11	0	10	2
Richard Mayo	2	13	9	9	2	0	2	4	13	9	4	0	7	2	0
Alex Pearce	10	2	2	6	7	0	13	5	2	0	7	2	8	13	0
Braelyn Grant	11	0	7	2	2	0	0	11	2	7	3	0	0	10	11
Erin Montgomery	10	4	7	4	11	6	0	8	0	7	0	3	11	2	10
Karen Reed	4	10	0	2	2	13	7	4	2	0	0	7	13	1	2
Erin Robinson	2	11	0	7	10	0	4	2	11	0	0	7	0	7	2
Asa Weiss	11	4	0	1	7	7	4	11	0	2	0	10	0	4	11
Drew Bridges	2	10	2	11	3	6	0	5	10	5	11	10	2	0	2
Billy Ryan	3	10	2	11	0	4	7	2	2	0	11	10	0	7	2
Zeke Marquez	7	2	0	7	4	11	3	2	0	7	0	0	13	0	7
Jo Lucas	7	2	11	2	8	2	0	4	2	11	0	0	0	7	2
Annie Leonhart	11	6	7	2	0	2	2	11	0	7	2	0	10	2	11
Toby White	8	2	0	11	2	2	7	0	2	7	13	0	0	7	2
Conner Johns	9	10	2	2	11	4	0	9	10	0	2	13	0	0	2
Sienna White	8	11	2	2	2	10	0	8	13	2	0	0	9	0	4
Denny Simpson	7	11	0	2	2	4	4	7	11	0	8	5	0	6	2
Rosie Pearson	2	0	11	0	4	7	7	0	6	11	0	2	10	7	2
Justice Robles	13	10	2	9	0	2	0	13	9	0	9	2	2	0	11
Lee George	2	11	4	7	2	0	4	7	11	2	0	7	0	6	4
Val Morse	2	10	11	2	2	7	0	0	0	11	2	0	2	6	2
Brice Hunt	7	3	7	2	13	4	0	2	2	7	0	1	11	7	2

Table 3: Shift Table for 3/16 3/31

Name	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
Georgina Stevens	11	0	8	9	2	0	2	11	0	4	7	2	0	2	12	10
Cory Grant	2	4	9	0	11	2	7	2	2	9	0	11	0	2	2	2
Sasha Blouse	0	4	4	0	12	10	2	4	0	8	2	11	2	0	2	0
Sam Burns	2	2	2	0	4	3	11	0	10	4	0	7	2	12	0	0
Finley Hamilton	10	11	10	0	2	4	2	0	12	2	7	0	0	10	2	12
Lynn Harris	2	3	2	12	2	0	0	2	0	4	11	4	0	2	7	10
Avery Mejia	2	4	10	0	2	8	12	4	0	8	3	0	10	11	4	2
Rowan Bailey	4	0	2	4	7	0	12	2	0	9	0	10	2	11	2	0
Breanna Mckay	2	0	10	11	4	2	0	2	9	10	0	12	0	2	4	0
Jay Gardner	0	2	11	2	0	9	0	10	2	12	2	0	2	10	9	2
Paloma Blackwell	9	11	0	3	4	2	2	0	11	4	10	2	4	10	0	12
Willy Shaw	0	3	4	13	4	0	4	2	0	0	4	13	9	2	10	4
Armin Arlart	4	4	5	0	2	13	0	3	2	6	9	0	13	0	0	2
Haiden Wood	11	0	0	6	10	2	10	11	4	0	6	4	10	7	11	0
Jaylene Waters	0	7	1	7	2	0	13	6	7	0	7	0	0	13	3	7
Rose Newman	0	4	13	7	7	2	2	0	3	2	12	0	0	2	0	7
Bailey Joseph	13	4	0	2	2	2	9	11	0	0	2	2	10	0	2	13
Frankie Adams	4	11	0	2	0	8	2	2	12	0	4	10	1	0	4	13
Jackson Gibson	4	6	0	0	0	13	3	7	0	5	0	2	11	2	10	2
Richard Mayo	13	4	0	3	0	0	0	13	9	7	5	0	2	2	13	10
Alex Pearce	2	4	9	0	10	13	4	0	2	2	0	0	13	10	0	7
Braelyn Grant	2	0	7	4	2	4	11	0	2	10	3	4	2	11	0	10
Erin Montgomery	0	7	2	6	13	0	4	2	10	8	2	0	13	0	0	7
Karen Reed	0	7	0	7	13	4	2	2	0	10	4	13	0	10	0	4
Erin Robinson	11	4	0	3	2	7	0	11	7	2	3	4	0	7	11	7
Asa Weiss	0	7	0	7	2	4	13	2	7	0	7	0	2	13	6	7
Drew Bridges	2	0	0	13	9	0	10	2	0	2	12	0	0	9	2	9
Billy Ryan	0	2	11	7	0	2	2	0	7	13	7	5	0	4	0	7
Zeke Marquez	2	7	0	2	13	4	7	0	10	0	2	13	6	7	6	9
Jo Lucas	2	11	2	0	7	2	7	2	13	2	0	0	0	10	10	13
Annie Leonhart	2	0	2	2	2	0	11	2	0	0	10	8	4	11	0	2
Toby White	2	2	0	7	13	2	2	0	0	4	0	8	7	6	11	4
Conner Johns	0	4	4	13	2	0	2	10	2	0	13	2	8	0	2	10
Sienna White	13	4	7	0	2	10	5	11	2	0	2	10	5	0	12	0
Denny Simpson	11	7	0	0	3	2	2	11	4	0	7	0	0	2	10	12
Rosie Pearson	6	11	0	0	7	8	4	6	12	2	0	2	7	7	0	13
Justice Robles	2	9	2	8	0	4	13	0	2	9	0	2	0	13	2	10
Lee George	11	0	2	7	2	4	0	11	2	0	0	2	0	10	11	2
Val Morse	10	11	10	0	2	0	2	10	13	2	0	2	4	8	0	11
Brice Hunt	0	0	2	3	7	13	0	2	7	0	0	7	12	4	0	7

The lack of each period at day d is exhibited in Table 4 and Table 5 where the number in title's header from the second column refers to the date of March.

Table 4: Lack Table for 3/1 3/15

Period	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
09:00 09:30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
09:30 10:00	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10:00 10:30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10:30 11:00	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11:00 11:30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11:30 12:00	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12:00 12:30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12:30 13:00	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13:00 13:30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13:30 14:00	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14:00 14:30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14:30 15:00	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15:00 15:30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15:30 16:00	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16:00 16:30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16:30 17:00	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17:00 17:30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17:30 18:00	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18:00 18:30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18:30 19:00	0	0	0	0	1	4	6	0	0	0	1	1	2	4	0
19:00 19:30	0	1	0	0	2	4	7	0	1	0	2	1	3	4	0
19:30 20:00	0	0	2	0	1	6	6	0	0	2	1	0	2	3	0
20:00 20:30	1	0	1	1	1	5	6	1	0	1	1	1	2	4	1
20:30 21:00	0	0	0	0	0	4	5	0	0	0	0	1	1	4	0

Table 5: Lack Table for 3/16 3/31

Period	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
09:00 09:30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
09:30 10:00	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10:00 10:30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10:30 11:00	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11:00 11:30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11:30 12:00	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12:00 12:30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12:30 13:00	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13:00 13:30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13:30 14:00	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14:00 14:30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14:30 15:00	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15:00 15:30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15:30 16:00	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16:00 16:30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16:30 17:00	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17:00 17:30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17:30 18:00	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18:00 18:30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18:30 19:00	0	0	4	1	2	4	0	0	0	5	2	1	1	0	1	0
19:00 19:30	1	0	5	1	2	5	0	1	0	6	2	2	1	0	2	0
19:30 20:00	0	2	4	0	2	4	0	0	2	5	1	1	0	0	1	0
20:00 20:30	0	1	4	1	3	4	1	0	1	5	2	1	1	1	1	1
20:30 21:00	0	0	3	1	2	3	0	0	0	4	2	0	1	0	0	0

4 Opinions and Thoughts

Our team feels that we have learnt a lot from the three case assignments. It makes us understand the real-world applications of Operations Research. We like that all the three case assignments flows like a story, and the requirements changes according to our syllabus. From that we can see how the same data can give different insights using the different methods we learnt in this course.