Operations Research
Lecture 2: Introduction to Linear Programming

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1. IEDO Chemical manufactures four chemicals: A, B, C, and D. These chemicals are produced via two production processes. Running process 1 for an hour costs \$7 and yields 3 units of A, 2 of B, 1 of C, and 1 of D. Running process 2 for an hour costs \$3 and produces 1 unit of A and 1 of B. To meet customer demands, at least 12 units of A, 6 of B, 4 of C, and 3 of D must be produced daily.

   (a) Formulate an LP to determine a daily production plan that minimizes the cost of meeting IEDO Chemicals daily demands.

   (b) Graphically solve the LP.

2. Graphically solve the following LP:

$$
\begin{aligned}
\min \quad & x_1 - x_2 \\
\text{s.t.} \quad & x_1 + x_2 \leq 6 \\
& x_1 - x_2 \geq 0 \\
& -2x_1 + x_2 \geq 6 \\
& x_1 \geq 0, x_2 \geq 0.
\end{aligned}
$$

3. Consider the following LP:

$$\begin{aligned} \max \quad & 2x_1 - x_2 \\ \text{s.t.} \quad & 8x_1 - 4x_2 \leq 16 \\ & 3x_1 - 4x_2 \leq 12 \\ & x_1 \geq 0, x_2 \leq 0. \end{aligned}$$

(a) Graphically determine whether it has an unique optimal solution, has multiple optimal solutions, is infeasible, or is unbounded.

(b) Find the binding constraints, if any, at $(x_1, x_2) = (2, 0)$.

(c) Find the binding constraints, if any, at $(x_1, x_2) = (1, 2)$.

(d) For each optimal solution, find the binding constraints at it, if any.

4. For each set below, find the set of extreme points.

(a) The triangle whose vertices are $(1, 0)$, $(0, 1)$, and $(1, 1)$.

(b) $\{x \in \mathbb{R}^2 | x_1^2 + x_2^2 \leq 1, x_1 + x_2 \geq -1\}$.

(c) $\{1, 2, 3, 4, 4.5, 5\}$.

(d) $\{x \in \mathbb{R}^n | x_i \geq 0 \quad \forall i = 1, ..., n\}$.

(e) $\{A \in \mathbb{R}^{n \times n} | A = A^{\mathrm{T}}\}$.

5. Answer the following questions and briefly explain your answers.

(a) True or false: If an LP is unbounded, its feasible region must be unbounded.

(b) True or false: If an LP has an unbounded feasible region, it must be unbounded.

(c) Multiple choice: If an LP has an optimal solution, there must be at least (zero/one/two) binding constraint(s) at that optimal solution.

(d) True or false: If an LP has two optimal solutions, there must be the third optimal solution.

(e) True or false: An LP's optimal solution is always an extreme point.

6. IEDO Oil has refineries in Kaohsiung and Taipei. Currently, the Kaohsiung refinery can refine up to 3 million barrels of oil per year, and the Taipei refinery up to 4 million. Once refined, oil is shipped to two distribution points: Hsinchu and Taichung. IEDO Oil estimates that each distribution point can sell up to 6 million barrels per year. Because of differences in shipping and refining costs, the profit earned per million barrels of oil shipped depends on where the oil was refined and on the point of distribution:

From	To Hsinchu	To Taichung
Kaohsiung	$16,000	$19,000
Taipei	$22,000	$18,000

Formulate an LP that maximizes IEDO's profits for the next year. Intuitively solve the LP.

7. Following from the previous problem, IEDO Oil is now considering expanding the capacity of each refinery. Each million barrels of annual refining capacity that is added will cost \$140,000 for the Kaohsiung refinery and \$180,000 for the Taipei refinery. Capacity can only be added now but can be used in the future ten years. Formulate an LP that maximizes IEDO's profits less expansion costs over a ten-year period.

8. Jay owns a bakery to bake cheesecakes and Black Forest cakes. During any day, he can bake at most 60 cakes. The costs per cake and the demands for cakes, which must be met on time, are listed below. It costs \$0.5 to hold a cheesecake, and \$0.6 to hold a Black Forest cake, in inventory for a day. Formulate an LP that can minimize the total cost of meeting the next three days' demands.

Item	Cost (\$ per units)			Demand (units)		
	Day 1	Day 2	Day 3	Day 1	Day 2	Day 3
Cheesecake	320	330	360	45	30	10
Black Forest	230	270	300	25	35	20

9. Following from the previous problem, now suppose that there are N kinds of cakes and T days. During each day, K cakes may be baked. The per-cake cost and demand for cake i in day t are C_{it} and D_{it}, respectively. The holding cost per cake per day for cake i is H_i. Formulate an LP that can solve the problem.