# Operation Research Final Project

# Mask Distribution

Group 7

B06208030 何承諭/ T08902130 何良富/ B06303059 謝君模/ B06303077 江雨柔
B06602043 黃崇晉/ B05704025 黃敬剴/ B06303035 陳冠廷/ B07705045 呂皓恩

## 1 Background and Motivation

Due to the outbreak of COVID-19, people around the globe require plenty of virus prevention tools such as, masks, disinfectant alcohol and swabs. Of which, masks are in short supply due to the need by both healthcare workers and the general population. Stockpiling efforts by governments and hospitals further increased demand towards masks, causing a global severe shortage on face masks. The masks shortage has ended up as a national security issue. Even in the developed nations, such as Europe Union or the United States, there are huge excess demand in contrast to the low supply of masks. In R.O.C.(Taiwan), although there is no infectious spread like Mainland China or the United States, we still face the problem of not having enough mask to provide for our citizens. Hence, the government initially implemented a policy called "7-day-2-mask" for each people in February 2. The government intervention has resulted in public fear and anxiety, and even stimulated the demand for masks, which caused many chaos and hoarded the masks. As the masks manufacturing lines expanded, two months later, in April 9, the government released the masks regulation to "14-day-9-mask". However, the public anxieties and complain still existed. Having witness this situation, our group decided to take a deep look in the distribution of mask, to see if we can some up with a better solution to provide the masks rather than merely equality.

We examine the problem from two perspectives, in this project they are named as the Provider Problem and the Utility Problem. The former is from providers' aspect in which we assume that all the transportation cost for each factory to each town are homogeneous. The latter is from public's aspect. For public aspect, we assume the safety confidence utility for each town and maximize the sum of entire town in Taiwan Province. The reason for measuring safety confidence utility in the unit of town is that in the threat of COVID-19, people tends to not travel far from their place of residence and rarely leaves home except for necessary affairs. In that sense, the people in the town cares about the mask coverage of the town for the reason that the more the coverage is, the more secure for the isolation between individuals. Moreover, the density of the town is also the matter of concern of the residents in the town for that the higher the density is, the more frequently of contacts between individuals in town if the resident are outdoors. Hence, the project examines both the distribution solution of masks and compare the result with the arbitrary equality of the existing policies of the government.

# 2 Data Process

## 2.1 Data Source

1. District data - https://data.gov.tw/dataset/7441) in SHP format.
   The district data is gathered from open access database provided by government. With the assistance of shapefile with geographic attribute table inside it, we are able to visualize Taiwan separated by district, with applying other accompanied methods like choropleth.

2. Population data - https://www.ris.gov.tw
   The population data used in this project generates from data provided by Department of Household Registration, Ministry of the Interior. From the latest population data released in 2019, it provides population of all district with different age and gender in a excel format. By merging different genders into a single data set, with separating age into below and above 12 years old and binding with other info like town id and density, the population data is ready, which is shown as Figure 1.

| | TOWNID | TOWNCODE | COUNTYNAME | TOWNNAME | TOWNENG | COUNTYID | sum_children | sum_adult | density_106 | z(density) | | total ppl |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | A01 | 63000010 | 台北市 | 松山區 | Songshan District | A | 23729 | 187068 | 22286.0096 | 3.403959 | 0.572035965 | 210797 |
| 2 | A02 | 63000030 | 台北市 | 大安區 | Da'an District | A | 36839 | 281481 | 27282.6412 | 4.280305 | 0.7003155154 | 318320 |
| 3 | A03 | 63000050 | 台北市 | 中正區 | Zhongzheng District | A | 20262 | 141530 | 20981.4515 | 3.175156 | 0.5385437766 | 161792 |
| 4 | A05 | 63000070 | 台北市 | 萬華區 | Wanhua District | A | 16326 | 172657 | 21672.5786 | 3.296371 | 0.5562872248 | 188983 |
| 5 | A09 | 63000060 | 台北市 | 大同區 | Datong District | A | 13152 | 114739 | 22754.2022 | 3.486074 | 0.5840559698 | 127891 |
| 6 | A10 | 63000040 | 台北市 | 中山區 | Zhongshan District | A | 22648 | 212862 | 16862.1776 | 2.452687 | 0.4327888107 | 235510 |
| 7 | A11 | 63000080 | 台北市 | 文山區 | Wenshan District | A | 29051 | 248615 | 8709.38462 | 1.02279 | 0.2234804801 | 277666 |
| 8 | A13 | 63000090 | 台北市 | 南港區 | Nangang District | A | 12827 | 108866 | 5592.56309 | 0.476139 | 0.1434616801 | 121693 |
| 9 | A14 | 63000100 | 台北市 | 內湖區 | Neihu District | A | 33304 | 259178 | 9112.81972 | 1.093548 | 0.2338379524 | 292482 |
| 10 | A15 | 63000110 | 台北市 | 士林區 | Shilin District | A | 28346 | 260273 | 4622.46786 | 0.305997 | 0.1185562258 | 288619 |
| 11 | A16 | 63000120 | 台北市 | 北投區 | Beitou District | A | 26995 | 231335 | 4513.35408 | 0.28686 | 0.1157549253 | 258330 |
| 12 | A17 | 63000020 | 台北市 | 信義區 | Xinyi District | A | 21232 | 204191 | 20142.6698 | 3.028044 | 0.5170095615 | 225423 |
| 13 | B01 | 66000010 | 台中市 | 中區 | Central District | B | 2303 | 16207 | 20958.7641 | 3.171177 | 0.5379613183 | 18510 |
| 14 | B02 | 66000020 | 台中市 | 東區 | East District | B | 7188 | 69033 | 8164.44995 | 0.927216 | 0.2094902603 | 76221 |
| 15 | B03 | 66000030 | 台中市 | 南區 | South District | B | 13765 | 115519 | 18364.9286 | 2.716251 | 0.4713692462 | 129284 |
| 16 | B04 | 66000040 | 台中市 | 西區 | West District | B | 12608 | 105706 | 20270.5024 | 3.050464 | 0.5202914342 | 118314 |
| 17 | B05 | 66000050 | 台中市 | 北區 | North District | B | 14436 | 136361 | 21283.0085 | 3.228045 | 0.5462857115 | 150797 |
| 18 | B06 | 66000060 | 台中市 | 西屯區 | Xitun District | B | 28171 | 207191 | 5697.91225 | 0.494616 | 0.1461663308 | 235362 |
| 19 | B07 | 66000070 | 台中市 | 南屯區 | Nantun District | B | 21836 | 155529 | 5398.97242 | 0.442186 | 0.138491587 | 177365 |
| 20 | B08 | 66000080 | 台中市 | 北屯區 | Beitun District | B | 32853 | 257074 | 4382.84048 | 0.263969 | 0.1124042228 | 289927 |
| 21 | B09 | 66000090 | 台中市 | 豐原區 | Fengyuan District | B | 18571 | 149474 | 4051.98558 | 0.205941 | 0.1039101169 | 168045 |
| 22 | B10 | 66000100 | 台中市 | 東勢區 | Dongshi District | B | 4004 | 45136 | 431.952234 | -0.428967 | 0.01097225646 | 49140 |
| 23 | B11 | 66000110 | 台中市 | 大甲區 | Dajia District | B | 8550 | 68154 | 1328.81174 | -0.271669 | 0.03399751497 | 76704 |
| 24 | B12 | 66000120 | 台中市 | 清水區 | Qingshui District | B | 9271 | 77293 | 1346.71635 | -0.268529 | 0.03445718371 | 86564 |
| 25 | B13 | 66000130 | 台中市 | 沙鹿區 | Shalu District | B | 12574 | 81827 | 2289.76975 | -0.103129 | 0.05866838756 | 94401 |
| 26 | B14 | 66000140 | 台中市 | 梧棲區 | Wuqi District | B | 7022 | 51837 | 3492.52329 | 0.107819 | 0.08954692651 | 58859 |
| 27 | B15 | 66000150 | 台中市 | 后里區 | Houli District | B | 5826 | 48481 | 924.3026 | -0.342615 | 0.02361246864 | 54307 |
| 28 | B16 | 66000160 | 台中市 | 神岡區 | Shengang District | B | 6678 | 58395 | 1870.56457 | -0.176652 | 0.04790604678 | 65073 |
| 29 | B17 | 66000170 | 台中市 | 潭子區 | Tanzi District | B | 11913 | 98115 | 4189.13953 | 0.229996 | 0.1074312985 | 110028 |
| 30 | B18 | 66000180 | 台中市 | 大雅區 | Daya District | B | 11337 | 84663 | 2937.37601 | 0.010453 | 0.07529451624 | 96000 |
| 31 | B19 | 66000190 | 台中市 | 新社區 | Xinshe District | B | 1961 | 21894 | 359.340024 | -0.441702 | 0.009108068261 | 23855 |

Figure 1: The population data

3. Factory data - https://serv.gcis.nat.gov.tw/Fidbweb/index.jsp
   The mask factory data is gathered from Department of Industrial Development Bureau, Ministry of Economic Affairs, with related news. By those resource provided, we are capable of listing out all the factories with their number of machines, location, latitude and longitude for further discussions, which is displayed as Figure 2.

| | Factory | Location | latitude | longitude | 產線(機台) | Adult Mask | Child Mask | |
|---|---|---|---|---|---|---|---|---|
| 2 | 台灣康匠製造股份有限公司 | 桃園市桃園區 | 24.971583 | 121.3183584 | 30 | 1 | 1 | |
| 3 | 中國衛生材料生產中心股份有限公司 | 彰化縣彰化市 | 24.0494369 | 120.6160203 | 10 | 1 | 1 | |
| 4 | 永猷股份有限公司(鶯歌) | 新北市鶯歌區 | 24.9750048 | 121.3372133 | 10 | 1 | 1 | |
| 5 | 恆大股份有限公司白河製造廠(台南) | 台南市白河區 | 23.359416 | 120.422647 | 10 | 1 | 0 | |
| 6 | 華新橡膠工業股份有限公司田中廠 | 彰化縣田中鎮 | 23.847619 | 120.569461 | 10 | 1 | 0 | |
| 7 | 康那香企業股份有限公司口寮廠 | 台南市將軍區 | 23.200149 | 120.130691 | 9 | 1 | 0 | |
| 8 | 順易利實業有限公司七堵廠 | 基隆市七堵區 | 25.095131 | 121.70644 | 7 | 1 | 1 | |
| 9 | 舜堡興業股份有限公司(彰化) | 彰化縣田中鎮 | 23.845065 | 120.571282 | 6 | 1 | 1 | |
| 10 | 宣德醫材科技股份有限公司(彰化) | 彰化縣田中鎮 | 23.847559 | 120.569428 | 6 | 1 | 1 | |
| 11 | 格安德工業股份有限公司 | 台南市學甲區 | 23.243293 | 120.16366 | 6 | 1 | 1 | |
| 12 | 香港商麥迪康亞太有限公司台灣分公司(宜蘭 | 宜蘭縣五結鄉 | 24.645559 | 121.837366 | 5 | 1 | 0 | |
| 13 | 易廷企業有限公司(龜山) | 桃園市龜山區 | 25.0549064 | 121.3773392 | 5 | 1 | 1 | |
| 14 | 淨新科技股份有限公司(高雄) | 高雄市三民區 | 22.6552419 | 120.3156663 | 5 | 1 | 1 | |
| 15 | 威欣利實業有限公司 | 新北市林口區 | 25.0824602 | 121.3981711 | 4 | 1 | 1 | |
| 16 | 加利科技有限公司八里廠(新北) | 新北市八里區 | 25.1347353 | 121.3823913 | 4 | 1 | 1 | |
| 17 | 奕綸有限公司(新竹) | 新竹縣新豐鄉 | 24.8901008 | 120.9849256 | 4 | 1 | 1 | |
| 18 | 昭惠實業股份有限公司(嘉義)廠 | 嘉義縣朴子市 | 23.4654854 | 120.2589801 | 4 | 1 | 0 | |
| 19 | 千倍康企業股份有限公司(彰化) | 桃園市大溪區 | 24.8861448 | 121.2691573 | 1 | 1 | 0 | 小廠 |
| 20 | 飛速生物科技有限公司(汐止) | 新北市汐止區 | 25.0588305 | 121.6338917 | 1 | 1 | 1 | 小廠 |
| 21 | 怡賓有限公司(樹林) | 新北市樹林區 | 24.9965666 | 121.4192984 | 1 | 1 | 0 | 小廠 |
| 22 | 台灣舒潔股份有限公司(三重) | 新北市三重區 | 25.0668929 | 121.4362539 | 1 | 1 | 1 | 小廠 |
| 23 | 宏春貿易有限公司前鎮廠(高雄) | 高雄市三民區 | 22.637439 | 120.3035734 | 1 | 1 | 0 | 小廠 |
| 24 | 善竹生物科技股份有限公司(台南) | 台南市六甲區 | 23.224941 | 120.3393155 | 1 | 1 | 0 | 小廠 |
| 25 | 和拓環保科技有限公司(雲林) | 雲林縣西螺鎮 | 23.799827 | 120.448392 | 1 | 1 | 0 | 小廠 |
| 26 | 欣邦有限公司(宜蘭) | 宜蘭縣冬山鄉 | 24.626747 | 121.804859 | 1 | 1 | 0 | 小廠 |
| 27 | 台灣優紙企業有限公司(彰化) | 彰化縣鹿港鎮 | 24.081179 | 120.411918 | 1 | 1 | 1 | 小廠 |
| 28 | 約伯科技有限公司 | 新北市中和區 | 25.003234 | 121.492985 | 1 | 1 | 0 | 小廠 |
| 29 | 台灣精碳有限公司 | 台南市永康區 | 23.029612 | 120.227387 | 1 | 1 | 0 | 小廠 |
| 30 | 健康天使股份有限公司 | 彰化縣田尾鄉 | 23.884328 | 120.526522 | 1 | 1 | 0 | 小廠 |
| 31 | 統億生技實業股份有限公司 | 彰化縣埔心鄉 | 23.951341 | 120.548851 | 1 | 1 | 0 | 小廠 |
| 32 | 宏瑋醫材有限公司 | 台中市烏日區 | 24.046591 | 120.654111 | 1 | 1 | 0 | 小廠 |
| 33 | 通用手套國際股份有限公司 | 台南市東山區 | 23.301753 | 120.39234 | 1 | 1 | 0 | 小廠 |

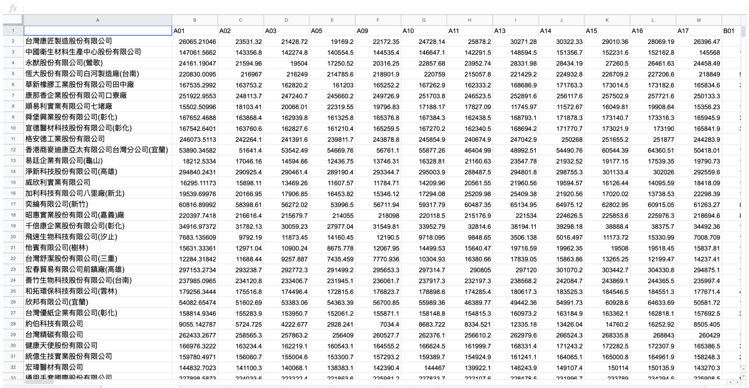
Figure 2: The factory data

## 2.2 Data Process

### 2.2.1 Generate the Variables of Concern

Given all source data needed, the following step is to generate variables of concern. Since the vision of this project is to find some better mask allocation solution to deal with this situation, the distance between each factory and district matters. However, the transportation time and cost may differ between transportation time, factories and districts, which is case-by-case sensitive. Therefore, we try to evaluate the transportation cost by using some intuitive way, which is the concept of Euclidean Distance, to generate the comparison. In practice, we utilize the formula of The Great Circle Distance provided by Professor Wen[1] in lecture of Spacial Analysis, to deal with the distance calculation. After the calculation process, the result is shown as Figure 5

[1] Wen T.H., Professor, Department of Geography, National Taiwan University



| | A01 | A02 | A03 | A05 | A09 | A10 | A11 | A13 | A14 | A15 | A16 | A17 | B01 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 台灣康匠製造股份有限公司 | 26065.21046 | 23531.32 | 21428.72 | 19169.2 | 22172.35 | 24728.14 | 25878.2 | 30271.28 | 30322.33 | 29010.36 | 28069.19 | 26396.47 | |
| 中國衛生材料生產中心股份有限公司 | 147061.5662 | 143356.8 | 142274.8 | 140554.5 | 144535.4 | 146647.1 | 142291.5 | 148594.5 | 151356.7 | 152231.6 | 152162.8 | 145568 | |
| 永猷股份有限公司(鶯歌) | 24161.19047 | 21594.96 | 19504 | 17250.52 | 20316.25 | 22857.68 | 23952.74 | 28331.98 | 28434.19 | 27260.5 | 26461.63 | 24458.49 | |
| 恆大股份有限公司白河製造廠(台南) | 220830.0095 | 216967 | 216249 | 214785.6 | 218901.9 | 220759 | 215057.8 | 221429.2 | 224932.8 | 226709.2 | 227206.6 | 218849 | |
| 華新橡膠工業股份有限公司田中廠 | 167535.2992 | 163753.2 | 162820.2 | 161203 | 165252.2 | 167262.9 | 162333.2 | 168686.9 | 171763.3 | 173014.5 | 173182.6 | 165834.6 | |
| 康那香企業股份有限公司口寮廠 | 251922.9553 | 248113.7 | 247240.7 | 245660.2 | 249726.9 | 251703.8 | 246523.5 | 252891.6 | 256117.6 | 257502.9 | 257721.6 | 250133.3 | |
| 順易利實業有限公司七堵廠 | 15502.50996 | 18103.41 | 20066.01 | 22319.55 | 19796.83 | 17188.17 | 17827.09 | 11745.97 | 11572.67 | 16049.81 | 19908.64 | 15356.23 | |
| 舜堡興業股份有限公司(彰化) | 167652.4688 | 163868.4 | 162939.8 | 161325.8 | 165376.8 | 167384.3 | 162438.5 | 168793.1 | 171878.3 | 173140.7 | 173316.3 | 165945.9 | |
| 宣德醫材科技股份有限公司(彰化) | 167542.6401 | 163760.6 | 162827.6 | 161210.4 | 165259.5 | 167270.2 | 162340.5 | 168694.2 | 171770.7 | 173021.9 | 173190 | 165841.9 | |
| 格安德工業股份有限公司 | 246073.5113 | 242264.1 | 241391.6 | 239811.7 | 243878.8 | 245854.9 | 240674.9 | 247042.9 | 250268 | 251655.2 | 251877 | 244283.9 | |
| 香港商麥迪康亞太有限公司台灣分公司(宜蘭) | 53890.34582 | 51641.4 | 53542.49 | 54669.76 | 56761.1 | 55877.26 | 46404.99 | 48992.51 | 54490.76 | 60544.39 | 64360.51 | 50418.01 | |
| 易廷企業有限公司(龜山) | 18212.5334 | 17046.16 | 14594.66 | 12436.75 | 13746.31 | 16328.81 | 21160.63 | 23547.78 | 21932.52 | 19177.15 | 17539.35 | 19790.73 | |
| 淨新科技股份有限公司(高雄) | 294840.2431 | 290925.4 | 290461.4 | 289190.4 | 293344.7 | 295003.9 | 288487.5 | 294801.8 | 298755.3 | 301133.4 | 302026 | 292559.6 | |
| 威欣利實業有限公司 | 16295.11173 | 15898.11 | 13469.26 | 11607.57 | 11784.71 | 14209.96 | 20561.55 | 21960.56 | 19594.57 | 16126.44 | 14095.59 | 18418.09 | |
| 加利科技有限公司八里廠(新北) | 19539.69976 | 20166.95 | 17906.85 | 16453.82 | 15346.12 | 17294.08 | 25209.98 | 25409.38 | 21920.56 | 17020.02 | 13738.53 | 22298.39 | |
| 奕綸有限公司(新竹) | 60816.89992 | 58398.61 | 56272.02 | 53996.5 | 56711.94 | 59317.79 | 60487.35 | 65134.95 | 64975.12 | 62802.95 | 60915.05 | 61263.27 | |
| 昭惠實業股份有限公司(嘉義)廠 | 220397.7418 | 216616.4 | 215679.7 | 214055 | 218098 | 220118.5 | 215176.9 | 221534 | 224626.5 | 225853.6 | 225976.3 | 218694.6 | |
| 千倍康企業股份有限公司(彰化) | 34916.97372 | 31782.13 | 30059.23 | 27977.04 | 31549.81 | 33952.79 | 32814.6 | 38194.11 | 39298.18 | 38888.4 | 38375.7 | 34492.36 | |
| 飛速生物科技有限公司(汐止) | 7683.135609 | 9792.19 | 11873.45 | 14160.45 | 12190.5 | 9718.095 | 9848.65 | 3506.138 | 5016.497 | 11173.72 | 15330.99 | 7008.709 | |
| 怡賓有限公司(樹林) | 15631.33361 | 12971.04 | 10900.24 | 8675.778 | 12067.95 | 14499.53 | 15640.47 | 19716.59 | 19962.35 | 19508 | 19518.45 | 15837.81 | |
| 台灣舒潔股份有限公司(三重) | 12284.31842 | 11688.44 | 9257.887 | 7435.459 | 7770.936 | 10304.93 | 16380.66 | 17839.05 | 15863.86 | 13265.25 | 12199.47 | 14237.41 | |
| 宏春貿易有限公司前鎮廠(高雄) | 297153.2734 | 293238.7 | 292772.3 | 291499.2 | 295653.3 | 297314.7 | 290805 | 297120 | 301070.2 | 303442.7 | 304330.8 | 294875.1 | |
| 善竹生物科技股份有限公司(台南) | 237985.0965 | 234120.8 | 233406.7 | 231945.1 | 236061.7 | 237917.3 | 232197.3 | 238568.2 | 242084.7 | 243869.1 | 244365.5 | 235997.4 | |
| 和拓環保科技有限公司(雲林) | 179256.3444 | 175516.8 | 174496.4 | 172815.6 | 176823.7 | 178898.6 | 174285.4 | 180617.3 | 183525.3 | 184546.5 | 184551.3 | 177671.4 | |
| 欣邦有限公司(宜蘭) | 54082.65474 | 51602.69 | 53383.06 | 54363.39 | 56700.85 | 55989.36 | 46389.77 | 49442.36 | 54991.73 | 60928.6 | 64633.69 | 50581.72 | |
| 台灣優紙企業有限公司(彰化) | 158814.9346 | 155283.9 | 153950.7 | 152061.2 | 155871.1 | 158148.8 | 154815.3 | 160973.2 | 163184.9 | 163362.1 | 162818.1 | 157692.5 | |
| 約伯科技有限公司 | 9055.142787 | 5724.725 | 4222.677 | 2928.241 | 7034.4 | 8683.722 | 8334.521 | 12335.18 | 13426.04 | 14760.2 | 16252.92 | 8505.405 | |
| 台灣精碳有限公司 | 262433.2677 | 258565.3 | 257863.2 | 256409 | 260527.7 | 262376.1 | 256610.2 | 262979.6 | 266524.3 | 268335.8 | 268843 | 260429 | |
| 健康天使股份有限公司 | 166976.3222 | 163234.4 | 162219.1 | 160543.1 | 164555.2 | 166624.5 | 161999.7 | 168331.4 | 171243.2 | 172282.5 | 172307.9 | 165386.5 | |
| 統億生技實業股份有限公司 | 159780.4971 | 156060.7 | 155004.6 | 153300.7 | 157293.2 | 159389.7 | 154924.9 | 161241.1 | 164065.1 | 165000.8 | 164961.9 | 158248.3 | |
| 宏瑋醫材有限公司 | 144832.7023 | 141100.3 | 140068.1 | 138383.1 | 142390.4 | 144467 | 139922.1 | 146243.9 | 149107.4 | 150114 | 150135.9 | 143270.3 | |

Figure 3: The Great Circle Distance between factories and districts

### 2.2.2 Data Normalization

In the utility problem, we tried to design a function for evaluating the performance of different solution. However, the relative data input is given in different unit, which means it may affect the weight of different data and leads to distortion in the result. Thus, the data of density and population are normalized within the range of 0 and 1. Given the series of data, X, for each data $i$, the normalized value $Z_i$ is,

$$Z_i = \frac{X_i - \max(X)}{\max(X) - \min(X)}$$

## 2.3 Obtaining Data Matrix

To obtain the distance matrix between the factories and each district, we used R to import the District data in SHP format using the readOGR() function in rgdal R package. The factory data is also imported using the read_csv() function in readr R Package and converted to an sf object using st_as_sf() function is sf R Package.Then we plot the factory and District data to ensure that the coordinates are correct. After that, we extract the coordinates of the factory and District data (centroid) into a matrix and use the distm() function from geosphere R Package to obtain the distance matrix. The distance calculated is based on the geodesic using WGS84 ellipsoid. The resulting distance matrix is output as a csv file for later use to solve the model.

# 3 Problem Description

## 3.1 Provider Problem

The Provide problem is to minimize the total transportation cost for the summation of delivery from factory $i$ to town $j$. The unit of the transportation is mask times unit transportation cost per meter. The parameters include

1. $M$: the number of Factory
2. $N$: the number of District
3. $D_{i,j}$: the distance between Factory $i$ and District $j$ for $i = 1, ..., M, j = 1, ..., N$
4. $P_j^A$ be the number of adult population in District $j$ for $j = 1, ..., N$
5. $P_j^Y$ be the number of children population in District $j$ for $j = 1, ..., N$
6. $C$: the cost of delivering one mask per meter
7. $S_i$ be the production capacity of Factory $i$ for $i = 1, ..., M$
8. $K$ be the maximum production of the factory

The variables include

1. $\alpha_i$ be 0 if Factory $i$ cannot produce children mask and 1 if Factory $i$ can produce children mask for $i = 1, ..., M$. The formal definition is

$$\begin{cases} \sum_{j=1}^{N} X_{i,j}^Y = 0, & \text{if } \alpha = 0 \\ \sum_{j=1}^{N} X_{i,j}^Y \geq 0, & \text{otherwise} \end{cases}$$

2. $X_{i,j}^A$: the number of adult mask produced by Factory $i$ for District $j$ for $i = 1, ..., M, j = 1, ..., N$
3. $X_{i,j}^Y$: the number of children mask produced by Factory $i$ for District $j$ for $i = 1, ..., M, j = 1, ..., N$

The complete linear model formulation is illustrated below

$$
\begin{aligned}
\min \quad & \sum_{i=1}^{M}\sum_{j=1}^{N}(X_{i,j}^{A}+X_{i,j}^{Y})D_{i,j}C \\
\text{s.t.} \quad & \sum_{j=1}^{N} X_{i,j}^{A}+X_{i,j}^{Y}=S_i \quad \forall i=1,...,M \quad \text{(Factory Limit)} \\
& \sum_{i=1}^{M} X_{i,j}^{A} \leq P_j^{A} \quad \forall j=1,...,N \quad \text{(Adult Requirement)} \\
& \sum_{i=1}^{M} X_{i,j}^{Y} \leq P_j^{Y} \quad \forall j=1,...,N \quad \text{(Children Requirement)} \\
& \sum_{j=1}^{N} X_{i,j}^{Y} \leq K\alpha_i \quad \forall i=1,...,M \quad \text{(Children Production Capability)} \\
& X_{i,j}^{A} \geq 0 \quad \forall i=1,...,M, j=1,...,N \\
& X_{i,j}^{Y} \geq 0 \quad \forall i=1,...,M, j=1,...,N \\
& \alpha_i \in \{0,1\} \quad \forall i=1,...,M
\end{aligned}
$$

### 3.2 Utility Problem

The Utility problem is to maximize the safety confidence score of all the towns in Taiwan Province. The project assumes the utility function, $U_j(\cdot)$ as the measurement of the safety confidence score of all the residences in town $j$. The reason is that during the period of COVID-19, people tend not to leave far away from their place of residence. All of the necessary affairs are done within the neighborhood. Thus, the project assumes that all the residence in town $j$ shares the same utility of safety. Meanwhile, the utility function form is proposed on the basis of public thoughts and rationales. Those considerations includes,

1. Density: People in Taiwan complains the "7-day-2-mask" or "14-day-9-mask" about that the policies ignores the difference between each town, especially the crowd are difference between town and town. For those live in town that has relatively high density, they need to have wear masks more frequently, just in case of the highly infected possibility from massive contact opportunity. Thus, many complains that the urban areas with high population density should have relatively more mask per day in contrast to the rural area that has lower population density. As a result, the utility function should make the density of the town positively affects the utility, i.e., the denser, the higher need of masks.

2. Coverage: The coverage of the masks of the town must positively effects the utility without a doubt. That is, the more masks, the more safety the town is, thus results in high utility in safety confidence.

3. Population: Given all others equal, the more the population of the town $j$ is, the safer the town gets the masks. Hence, the population should positively effects the utility.

4. Diminishing Marginal Returns: As a basic idea in the utility function(and many other functions in Economics), the rational function must be assume to be exhibited with diminishing marginal returns. Hence, the utility function of the town's safety confidence score should exist the property of diminishing marginal returns. Specifically speaking, the value of the second partial derivatives should be negative for the three factors mentioned above.

5. Equal Weight of Factors: From the basic viewpoint, the project assumes that the three factors are equally important. In order to equalize the effect of the three factors, all of the factors, $r_j$, $P_j$, $d_j$ are normalized within the range of 0 and 1. Meanwhile, considering both diminishing marginal return and equal effects, the project assumes the utility function as the cubic root of the product of the three factors.

The utility function of the town $j$ is described as,

$$U_j(r_j, P_j, d_j) = (P_j \cdot r_j \cdot d_j)^{\frac{1}{3}}$$

The entire non-linear model is exhibited below, all the notations are inherited from Section 3.1.

$$
\begin{aligned}
\max \quad & \sum_{j=1}^{N} U_j \\
\text{s.t.} \quad & r_j^A = \frac{\sum_{i=1}^{M} X_{i,j}^A}{P_j^A} && \text{(Coverage of Adult)} \\
& r_j^Y = \frac{\sum_{i=1}^{M} X_{i,j}^Y}{P_j^Y} && \text{(Coverage of Children)} \\
& r_j = r_j^A + r_j^Y && \text{(Total Coverage of town } j\text{)} \\
& P_j = P_j^A + P_j^Y && \text{(Total Population in town } j\text{)} \\
& U_j = (P_j \cdot r_j \cdot d_j)^{\frac{1}{3}} && \text{(Utility of Town } j\text{)} \\
& \sum_{j=1}^{N} (x_{i,j}^A + \frac{1}{2} X_{i,j}^Y) = S_i, \quad \forall i = 1..M && \text{(Production Capacity)} \\
& \alpha_i = \{0, 1\} && \text{(Factory } i \text{ Capability of Making Children Mask)} \\
& \sum_{j=1}^{N} X_{i,j}^Y \leq 2S_i(\alpha_i), \quad \forall i = 1..M && \text{(Linearize } \alpha_i\text{)} \\
& \sum_{i=1}^{M} X_{i,j}^A \leq P_j^A && \text{(adult demand bound)} \\
& \sum_{i=1}^{M} X_{i,j}^A \leq P_j^Y && \text{(children demand bound)} \\
& X_{i,j}^A \geq 0 \quad \forall i = 1, ..., M, j = 1, ..., N \\
& X_{i,j}^Y \geq 0 \quad \forall i = 1, ..., M, j = 1, ..., N
\end{aligned}
$$

# 4 Performance Analysis

## 4.1 Provider Problem

1. Result

- We maximize the production of the mask to 16 million, and lower the travel distance by half comparing to the governments "14-day-9-mask" policy.

$$
\begin{cases}
\text{Our: 130 million (km) per day.} \\
\text{Gov: 348 million (km) per day.}
\end{cases}
$$

The calculation of governments travel distance is by making the coverage of every district regardless of adult or children equal to $\frac{9}{14}$, which is the "14-day-9-mask" policy, therefore we can get $X_{ij}^{A}$ and $X_{ij}^{Y}$, hence we can get the travel distance.

- The district closer to the factories are the first to be satisfied with the demands of masks.

2. Benefit

- The production of face masks is maximized since we made all the factories to produce the mask at their most. So we can get more masks then what governments provided.
- We still manage to cut down the cost, while providing more masks to different in different districts.

3. Drawbacks

- We still don't have enough mask for everyone, since we can only produce 1.6 million mask per day, it is still lower than our total population which is approximately 2.3 million.
- We did not maximize the total utility, and it is not a fair allocation of masks because we first provide to the closest districts, leaving the places with no factories in there neighborhood with no masks provided. In the picture below we can see that the east side has no factory at all, this means that half of Taiwan may not be provided with masks.

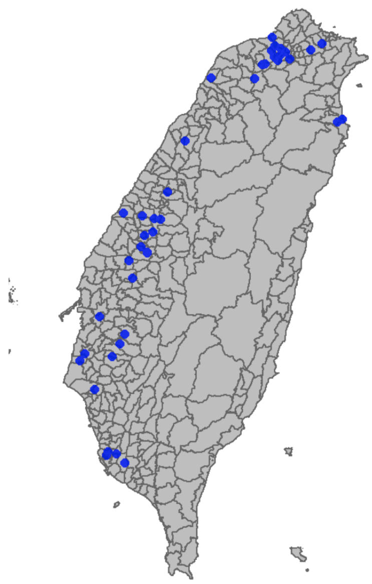


Figure 4: The Mask Factories in Taiwan Province

### 4.2 Utility Problem

1. Result

   We provide a higher coverage of mask in high density and high population areas comparing to the government.

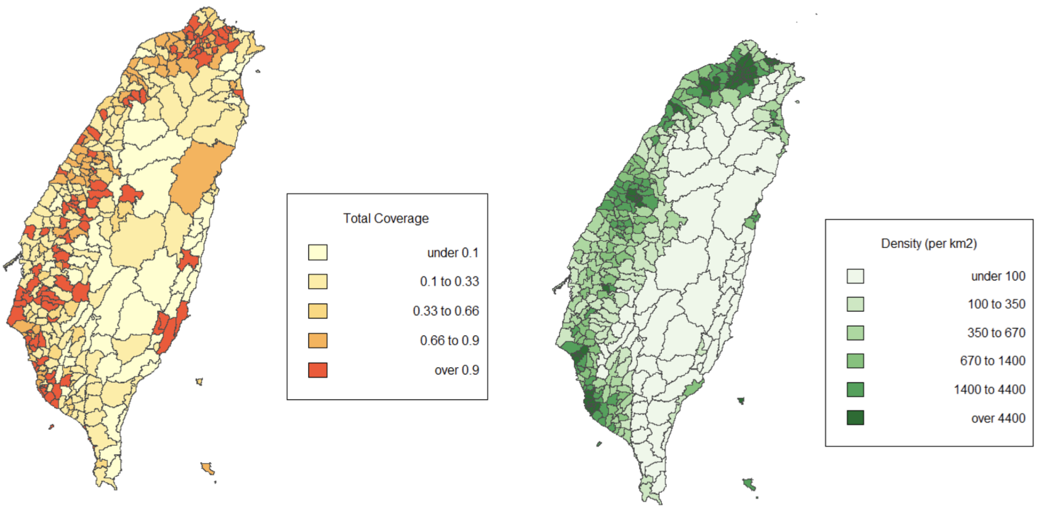



Figure 5: Comparison between the Coverage of the Mask and Density

2. Benefits

   Large city with high population and high density has higher utility which means the coverage is higher. The risk of getting infected is lower, since more mask is provided.

3. Drawbacks

   - The total travel distance is up to 579 million which is way higher than 130 million. This means that the cost will be super high even comparing to governments policy we are still 1.5 times higher.
   - We sacrificed fairness in order to gain efficiency. So some districts may be left with no mask provided.

## Conclusion and Future Works

### 4.3 Conclusion

Looking back at our formulation, not only did we out-performed governments policy, we also obtained some benefits.

1. The way of mask distribution is closer to public thoughts. This means that people are more willing to accept our policy, hence with better cooperation with the people,we can be more efficient at controlling the epidemic.

2. We lower people's frustration towards the disease. Since people with high risk of getting infected are provided with more masks, they should feel much safer and protected. Once feel secured, the fear of the COVID-19 no longer exists.

3. We maximize the social welfare. With all the benefits above, we can gain great control of the epidemic, and in this case the faster we can get back to our normal lives the higher social welfare we get.

### 4.4 Future Works

1. Application of Nations
   The project is expected to expand the application situation not only considering our country but also the country in the globe. Recently, the COVID-19 disease is under well-control in R.O.C.(Taiwan) due to the border regulation and the 14 days quarantine policy. Nevertheless, the others countries, especially the United States and Brazil are still out of control. The masks distribution is indeed an existing crisis for them to distribute the goods and make the citizens safe and confident. Thus, the project is looking forward to expand to some foreign nations to aid in mask distribution.

2. Virus Prevention Stuffs of Concern
   The current virus prevention stuff of concern is merely mask. However, there are other stuffs that are less severe but still important. Take protective clothing, alcohols, tissues for instance, the project can extend the virus prevention stuff of concern to other stuffs.

3. Rare Resource Reallocation
   There are a lot of things that are not unlimited access for all of us. Most things are considered limited, or even rare. By using our formulation, we can distribute these resources in a better way, to those who are in need, instead of controlled by only few people. We might start with public goods provided by the governments, as for private goods the market itself should solve the problem.

4. Improvements

   - Calculation of Population
     The data we gathered is based on administrative district, but to make the model more

realistic, we should change it to living quarters. The administrative district gathers only the birth and the death population, we did not take the commuters in account, which is the most important cause of spreading the virus. The population flow should definitely be considered if we are trying to make the formulation more precise.

- Calculation of Distance
  The method we use to calculate the distance is simply using the longitude and latitude. This will underestimate the cost and may cause considerable errors for some special cases. To make the model more precise, we should consider the traffic routes in real world.
- Calculation of Travel Distance
  The method we use to calculate the travel distance is designing the driver to visit only one district, we did this to simplify our formulation. So to let the model be more precise, we can design the route to be visiting multiple districts at once, also adding the capacity limit to the truck, last but not least the number of truck drivers should be limited too.