Problem 1

1. There will be no hospital for town3.
2. The solution is feasible. There must be two hospital so that that the constraint can be satisfied.

Problem 2
Let $x_i = 1$ if a hospital is built in town i or 0 otherwise. We denote the connectivity among towns by an n n matrix A. Let $A_{ij} = A_{ji} = 1$ if a road between towns i an j exists or 0 otherwise. Moreover, let $A_i = 1$ for all i.

$$
\begin{array}{ll}
min & \sum_{i=1}^{n} x_i \\
\\
s.t. & \sum_{j=1}^{n} A_{ij}x_j \geq 1 \quad \forall i = 1...n \\
 & x_i \in \{\ 0, 1\} \quad \forall i = 1...n
\end{array}
$$

Problem 3
Let $x_j = 1$ if a park is built at location j or 0 otherwise , and $y_{ij} = 1$ if the park at location j is the closest one for town i.

$$
\begin{array}{ll}
min & \sum_{i=1}^{m} \sum_{j=1}^{n} h_i d_{ij} y_{ij} \\
\\
s.t. & \sum_{j \in J} y_{ij} \quad \forall i = 1...m \\
 & y_{ij} \leq x_{ij} \quad \forall i = 1...m, j = 1...n \\
 & \sum_{j \in J} x_j \leq p \\
 & x_j \in \{\ 0, 1\} \quad \forall j = 1...n \\
 & y_{ij} \in \{\ 0, 1\} \quad \forall i = 1...m, j = 1...n
\end{array}
$$

Problem 6
Let J = 1,...,n be the set of jobs, I = 1,...,m be the set of machines. Let x_{ij} =1 if $jobj \in J$ is assigned to machine $i \in I$ or 0 otherwise; let w be the benefit earned by the machine learning the least benefit.

$$
\begin{array}{ll}
max & w \\
\\
s.t. & w \leq \sum_{j \in J} b_j x_{ij} \quad \forall i \in J \\
 & \sum_{i \in j} x_{ij} \leq 1 \quad \forall i \in J \\
 & \sum_{j \in J} p_j x_{ij} \leq K] \quad \forall i \in I \leq p \\
 & x_{ij} \in \{\ 0, 1\} \quad \forall j \in I, \quad \forall j \in J
\end{array}
$$

Problem 7

1. The Hamiltonian cycle problem can be reduced to the traveling salesperson problem. To see this, consider a Hamiltonian cycle instance G = (V,E), where G is an incomplete graph. We will construct a traveling salesperson instance H = (U, A), where H is a complete graph and d_{ij} is the distance of edge $[i, j] \in A$, in the following three steps:

   (a) For each node in V, add a node into U.

   (b) For each edge $[i, j] \in E$,add an edge into A and set d_{ij} =1.

   (c) For each pair of nodes $i \in V$ and $j \in V$ such that the edge $[i, j] \in /E$,add an edge into A and set d_{ij} =2.

   We may then solve the instance H to see whether an optimal solution exists and its objective value is equal to the number of nodes in V . If this is the case, we know in G there is a Hamiltonian cycle; otherwise, we know a Hamiltonian cycle does not exist.

2. We will formulate the traveling salesperson problem based on the above reduction. Let $x_{ij} = 1$ if arc $[i, j] \in A$ is selected in an optimal tour or 0 otherwise.

$$
\begin{array}{ll}
min & \sum_{j \in J} y_{ij} + 2 \sum_{[i,j] \notin E} x_{iJ} \\
 & \\
s.t. & \sum_{i \in V, i \neq k} x_{ik} = 1 \sum_{[i,j]} \quad \forall k \in V \\
 & \sum_{k \in V, k \notin j} x_{kj} = 1 \quad]\forall k \in V \\
 & \sum_{i \in S, j \in S, i \neq j} x_{ij} \leq |S| - 1 \quad \forall SV, \quad |S| \geq 2 \\
 & x_{iJ} \in \{\ 0, 1\} \quad \forall [i, j] \in A
\end{array}
$$

3. Problem 8

   We will formulate this problem based on the traveling salesperson problem formulation. Let the post office be at node 0. The only difference is that now for node 0 there can be multiple entering and leaving edges. The only requirement is that the number of entering and leaving edges must be identical. Therefore, Let $x_{ij} = 1$ if arc $[i, j] \in A$ is selected in an optimal tour or 0 otherwise.

$$
\begin{array}{ll}
min & \sum_{[i,j]\in E} d_{ij}x_{ij} \\
 & \\
s.t. & \sum_{i\in V\setminus\{\ k\}\} x_{ik}=1 \quad \forall k\in V\setminus 0} \\
 & \sum_{j\in V\setminus\{\ k\}}\ x_{kj} = 1 \quad \forall k \in V \setminus \{\ 0\} \\
 & \sum_{j\in V\setminus\{\ 0\}}\ x_{i,0} = \sum_{j\in V\setminus 0} x_{0,j} \\
 & \sum_{i\in S, j\in S, i\neq j} x_{ij} \leq |S| - 1 \quad \forall S \setminus V, |S| \geq 2, 0 \notin S \\
 & x_{ij} \in \{\ 0, 1\} \quad \forall [i, j] \in A
\end{array}
$$