

$$2x_1 + x_2 = 100$$

$$x_1 + x_2 + x_3 = 70$$

$$x_1 - x_4 = 40$$

# 3 條式子, 3 個 basis.

$x_1 \ x_2 \ x_3 \ x_4$  basis. bfs.

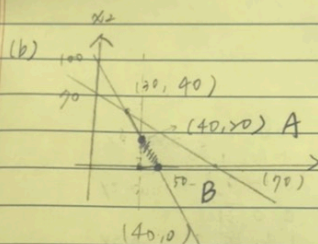
→  $40 \ 70 \ 10 \ 0 \ \{x_1, x_2, x_3\} \checkmark \ A$

$70 \ 40 \ 0 \ -10 \ \{x_1, x_2, x_4\}$

$0 \ 100 \ -70 \ -40 \ \{x_2, x_3, x_4\}$

$50 \ 0 \ 70 \ 10 \ \{x_1, x_3, x_4\} \checkmark \ B$

4 bfs, 2 bfs #



5

(a)

-4	1	0	0	0	0	
2	1	1	0	0	0	$x_2 = 8$
0	1	0	1	0	0	$x_4 = 5$
1	-1	0	0	1	0	$x_5 = 4$
-4	0	0	-1	0	-5	
2	0	1	0	0	0	$x_3 = 3$
0	1	0	1	0	0	$x_2 = 5$
1	0	0	1	1	0	$x_5 = 9$

$z = -5, x^* = (0, -5)$

- (b) ① 因為 min, 所以選大於 0 的那一行  
做 ratio test, 踢掉  $x_4$  ( $x_5 < 0$  不看)
- ② 將 pivot 那行上下變零, pivot 乘上 -1 加上  $z$  那行
- ③ 皆小於 0 所以此為 optimal solution  
又  $x_2 \leq -5$ , 答案為  $(0, -5)$

LP  $\begin{cases} \text{infeasible.} \\ \text{feasible} \end{cases} \begin{cases} \text{unbounded. 無限多解} \\ \text{bounded} \end{cases} \begin{cases} \text{唯一解} \\ \text{多解} \end{cases}$

2. (a) No, because there is no improving direction  
(b) No, there is just one optimal solution.  
If there are multiple solution, a non basic variable can enter the basis, but in this case there is none, so the solution is unique.

10.  $0 \rightarrow 0 \mid 0 \rightarrow 0 \ 0 \ 0 \ -1 \ -1 \mid 0$  此單位矩陣, 上面要為  $(0,0)$

$1 \ 2 \ -1 \mid 6 \rightarrow 1 \ 2 \ -1 \mid 0 \ x_4 = 6$

$2 \rightarrow 0 \mid 4 \rightarrow 2 \ 3 \ 0 \ 0 \ 1 \mid x_5 = 4$

沒有單位矩陣 加上 artificial var  $x^0 = \{0, 0, 0, 6, 4\}$

$x_1 \rightarrow 0 \ \frac{1}{2} \ -1 \ 0 \mid 4$

$x_5 \leftarrow 0 \ \frac{1}{2} \ -1 \ 1 \mid x_4 = 4$

$1 \ \frac{3}{2} \ 0 \ 0 \mid x_1 = 2$

$x_2 \text{ enter } -\frac{1}{3} \ 0 \ -1 \ 0 \mid \frac{10}{3}$

$\Rightarrow -\frac{1}{3} \ 0 \ -1 \ 1 \mid x_4 = \frac{10}{3}$

$x_1 \text{ leave } \frac{2}{3} \ 1 \ 0 \ 0 \mid x_2 = \frac{4}{3}$

$x^1 = \{2, 0, 0, 4, 0\}$

$x^2 = \{0, \frac{4}{3}, 0, \frac{10}{3}, 0\}$

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