Lecture 5 - In-class.

1.

min $5y_1 + 3y_2 + 6y_3 + 4y_4$.

s.t.

$y_1 + 2y_2 + y_4 \le 4$

$y_1 + y_2 + 2y_3 = -1$

$y_3 + y_4 = 2$.

$y_1 \ge 0$, $y_2 \ge 0$, $y_3 \le 0$, y_4 urs. #

3.

	-3	-1	-5	0	0	0	
	1	1	1	1	0	$x_4 = 50$
	2	3	1	0	1	$x_5 = 100$

$\Rightarrow$

	2	-2	0	5	0	$x_3 = 50$
	1	1	1	1	0	$x_3 = 50$
	1	2	0	-1	1	$x_5 = 50$

$\Rightarrow$

	3	0	0	4	0	3 00	
	$\frac{1}{2}$	0	1	$\frac{3}{2}$	$-\frac{1}{2}$	$x_3 = 25$	
	1	1	0	-1	1	$x_2 = 50$	25
	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	$\frac{1}{2}$		

(a)

(b) the primal optimal solution is $(0, 25, 25, 0, 0)$

Yes, it is unique.

(c)

min $50y_1 + 100y_2$

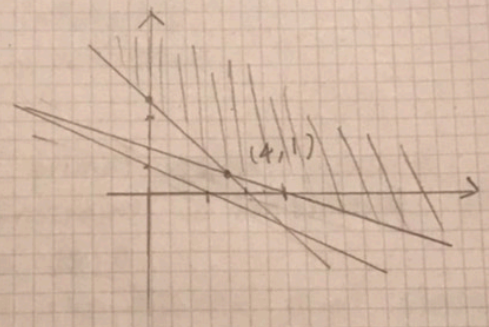
s.t.

$y_1 + 2y_2 \ge 3$

$y_1 + 3y_2 \ge 7$

$y_1 + y_2 \ge 5$

$y_1, y_2 \ge 0$.



(d) $C^T = [5\ 7]$ $A_B = \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}$.

$C_B^T A_B^{-1} = [5\ 7] \begin{bmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} = [4, 1]$ #

4.

(a) max $6x_1 + 10x_2 + 9x_3 + 20x_4$.

s.t.

$4x_1 + 9x_2 + 7x_3 + 10x_4 \le 600$.

$x_1 + x_2 + 3x_3 + 40x_4 \le 400$.

$3x_1 + 4x_2 + 2x_3 + x_4 \le 500$.

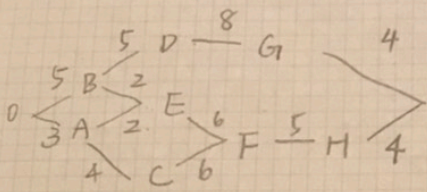
$x_i \ge 0 \quad \forall i = 1 \dots 4$.

(b) $\min\ 600y_1 + 400y_2 + 500y_3$

$4y_1 + y_2 + 2y_3 \geq 6$
$9y_1 + y_2 + 4y_3 \geq 10$
$2y_1 + 2y_2 + 3y_3 \geq 9$
$10y_1 + 40y_2 + y_3 \geq 20$
$y_i \geq 0 \quad i = 1, 2, 3$
objective value $= z^* = \frac{3800}{3}$

(c) the third constraint $2x_1 + 4x_2 + 2x_3 + x_4 \leq 500$ is not binding at the optimal solution $(\frac{400}{3}, 0, 0, \frac{20}{3})$. As there is still unused glass, additional supply of glass contributes nothing to sales revenue. Therefore the shadow price of this constraint is 0, which is $y_3 = 0$.

7. (a)



<BDGIJ>
<BEFHIJ>
cost 28

(b) A starting 0 complete ≥

E	=	3	= 5
J	=	22	= 28

8. (a) $X_E \geq X_A + 2, \quad X_E \geq X_B + 2$

(b) $X_A \geq X_0 + 3,\ X_B \geq X_0 + 5,\ X_C \geq X_A + 4,\ X_E \geq X_A + 2,\ X_E \geq X_B + 2$
$X_D \geq X_B + 5,\ X_G \geq X_D + 8,\ X_F \geq X_E + 6,\ X_F \geq X_C + 6,\ X_H \geq X_F + 5$
$X_I \geq X_G + 4,\ X_I \geq X_H + 4,\ X_J \geq X_I + 6,\ X_I \geq X_J$

(c) $\min\ X_1$
$s.t\ X_0 = 0$

(d) N = set of nodes. A = set of ars. p_i = time of task i

$\min\ X_1$
$s.t\ X_0 = 0$
$x_j \geq x_i + p_j \quad \forall (i,j) \in A$
$x_i \geq 0 \quad \forall i \in N$