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1. 1 = cheesecake 2. Black Forest

$$\min. \rightarrow 20x_{11} + 30x_{12} + 60x_{13} + 20y_{11} + 20y_{12} + 50y_{13} + 0.5(x_{11} + x_{12} + x_{13} + y_{11} + y_{12} + y_{13})$$

$$\begin{aligned} \text{s.t.} \quad & x_{11} + 20 - 45 = y_{11} \\ & y_{11} + x_{12} - 30 = y_{12} \\ & y_{12} + x_{13} - 10 = y_{13} \\ & x_{21} + 20 - 25 = y_{21} \\ & y_{21} + x_{22} - 35 = y_{22} \\ & y_{22} + x_{23} - 20 = y_{23} \\ & x_{ij} \geq 0, y_{ij} \geq 0 \quad \forall i=1,2, \forall j=1,2 \end{aligned}$$

2.  $\min 8x_1 + 12x_2 + 13x_3 + 15x_4$

$$\begin{aligned} \text{s.t.} \quad & x_1 + x_2 + x_3 + x_4 = 1000 \\ & 0.04x_1 + 0.06x_2 + 0.1x_3 + 0.11x_4 \leq 50 \\ & 0.02x_1 + 0.05x_2 + 0.03x_3 + 0.09x_4 \leq 40 \\ & 0.01x_1 + 0.01x_2 + 0.03x_3 + 0.04x_4 \geq 20 \\ & x_2 \geq 100 \\ & x_1, x_3, x_4 \geq 0 \end{aligned}$$

3. shifts  $x_{12}, x_{13}, x_{14}, x_{23}, x_{24}, x_{34}$

$$\min. 9600(x_{12} + x_{23} + x_{34} + x_{14}) + 12000(x_{13} + x_{24})$$

$$\begin{aligned} \text{s.t.} \quad & x_{12} + x_{13} + x_{14} \geq 12 \\ & x_{12} + x_{23} + x_{24} \geq 6 \\ & x_{13} + x_{23} + x_{34} \geq 14 \\ & x_{14} + x_{24} + x_{34} \geq 17 \\ & x_{ij} \geq 0 \quad \forall i=1, \dots, 3, j=i+1, \dots, 4 \end{aligned}$$

5.  $\min 3w$

$$\begin{aligned} \text{s.t.} \quad & x_1 + 2x_2 \geq 5 \\ & x_1 + x_2 \geq x_1 + x_2 \\ & 2x_1 + 3x_2 = 4 \\ & w \geq x_1 - 5 \\ & w \geq 5 - x_2 \\ & x_i = 0 \quad \forall i=1,2 \end{aligned}$$

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4 b.

(a)  $x_t$  = products produced  
 $y_t$  = inventory remained

$$\max \sum_{t=1}^T p_t D_t - \sum_{t=1}^T x_t C_t - H \sum_{t=1}^T y_t$$

$$\text{s.t. } y_0 = 0$$

$$y_{t-1} + x_t - D_t = y_t \quad \forall t = 1 \dots T$$

$$x_t \leq k_t \quad \forall t = 1 \dots T$$

$$x_t \geq 0, y_t \geq 0 \quad \forall t = 1 \dots T$$

(b)  $w_t$  = the number sold.

$$\max \sum_{t=1}^T w_t P_t - \sum_{t=1}^T x_t C_t - H \sum_{t=1}^T y_t$$

$$\text{s.t. } y_0 = 0$$

$$y_{t-1} + x_t - w_t = y_t \quad \forall t = 1 \dots T$$

$$x_t \leq k_t \quad \forall t = 1 \dots T$$

$$w_t \leq D_t \quad \forall t = 1 \dots T$$

$$x_t \geq 0, y_t \geq 0, w_t \geq 0 \quad \forall t = 1 \dots T$$

7.

$x_t$  = product produced.

$y_t$  = inventory remained

$$\max \sum_{t=1}^T D_t P_t - \sum_{t=1}^T x_t C_t - H \sum_{t=1}^T \max\{y_t, 0\} - s \sum_{t=1}^T \max\{-y_t, 0\}$$

$$\text{s.t. } y_0 = 0$$

$$y_{t-1} + x_t - D_t = y_t \quad \forall t = 1 \dots T$$

$$x_t \leq k_t \quad \forall t = 1 \dots T$$

$$x_t \geq 0, y_t \text{ urs. } \forall t = 1 \dots T$$

(linearized)

$$\Rightarrow \max \sum_{t=1}^T D_t P_t - \sum_{t=1}^T x_t C_t - H \sum_{t=1}^T y_t^+ - s \sum_{t=1}^T y_t^-$$

$$\text{s.t. } y_0 = 0$$

$$y_{t-1} + x_t - D_t = y_t \quad \forall t = 1 \dots T$$

$$y_t^+ \geq y_t \quad \forall t = 1 \dots T$$

$$y_t^+ \geq 0$$

$$y_t^- \geq -y_t \quad \forall t = 1 \dots T$$

$$y_t^- \geq 0$$

$$x_t \leq k_t \quad \forall t = 1 \dots T$$

$$x_t \geq 0, y_t \text{ urs. } \forall t = 1 \dots T$$



9. The model file contains

```
var x1 >= 0;
var x2 >= 0;

minimize obj: 3 * x2;

subject to constraint1:    x1 + 2 * x2 >= 6;
subject to constraint2: 2 * x1 + 3 * x2 = 14;
```

An optimal solution is  $(x_1^*, x_2^*) = (7, 0)$ . The associated objective value is  $z^* = 0$ .

```
context:  >>> solve; <<<
[ampl: model file.mod;

file.mod, line 1 (offset 5):
      x1 is already defined
context:  var >>> x1 <<< >= 0;
[ampl: solve
[ampl? solve;

solve is not defined
context:  >>> solve; <<<
[ampl: option solver './cplex';
[ampl: solve;
CPLEX 12.9.0.0: optimal solution; objective 0
0 dual simplex iterations (0 in phase I)
[ampl: display x1 x2;

syntax error
context:  display x1 >>> x2; <<<
[ampl: display x1, x2;
x1 = 7
x2 = 0

ampl: █
```

## 10. The model file contains

```
param N;
param T;

param H;
param K;
param I{i in 1..N};
param D{i in 1..N, t in 1..T};
param C{i in 1..N, t in 1..T};

var x{i in 1..N, t in 1..T} >= 0;
var y{i in 1..N, t in 0..T} >= 0;

minimize cost: sum{i in 1..N, j in 1..T} (C[i, t] * x[i, t] + H * y[i, t]);

subject to capacity{j in 1..T}: sum{i in 1..N} x[i, t] <= K;
subject to inv{i in 1..N, t in 1..T}: y[i, t - 1] + x[i, t] - D[i, t] = y[i, t];
subject to initInv{i in 1..N}: y[i, 0] = I[i];
```

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## The data file contains

```
param N := 2;
param T := 3;
param K := 60;
param H := 0.5;
param I :=
  1  20
  2  20;
param D :
  1  2  3 :=
  1  45 30 10
  2  25 35 20;
param C :
  1  2  3 :=
  1  320 330 360
  2  230 270 300;
```

