Mask Distribution

Team 7

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Background & Motivation

Background

COVID-19 Caused A Shortage Of Face Masks

Due to the outbreak of COVID-19, the demand of mask had a huge increase all over the world including Taiwan, causing a big shortage of masks.

Motivation

Goal : Fix Shortage Problem

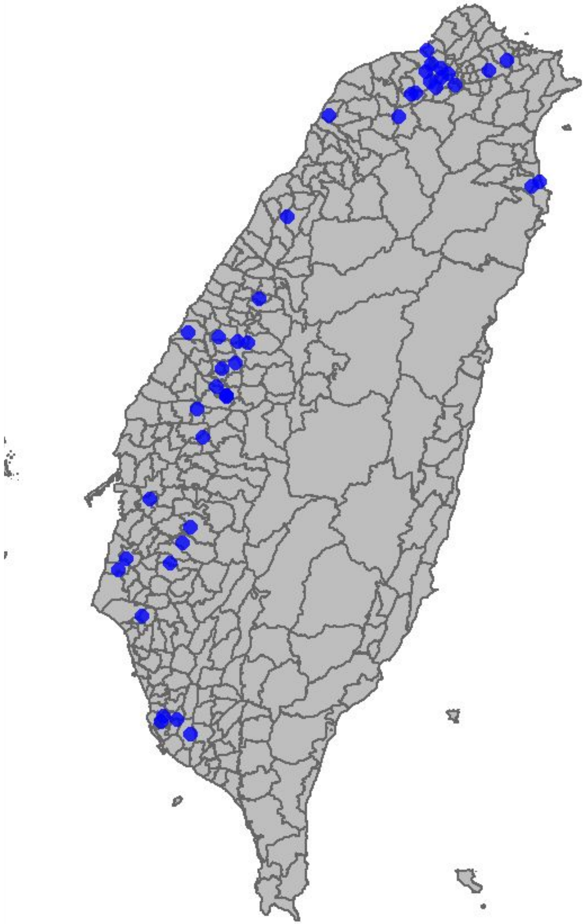
1. Minimize Transportation Cost

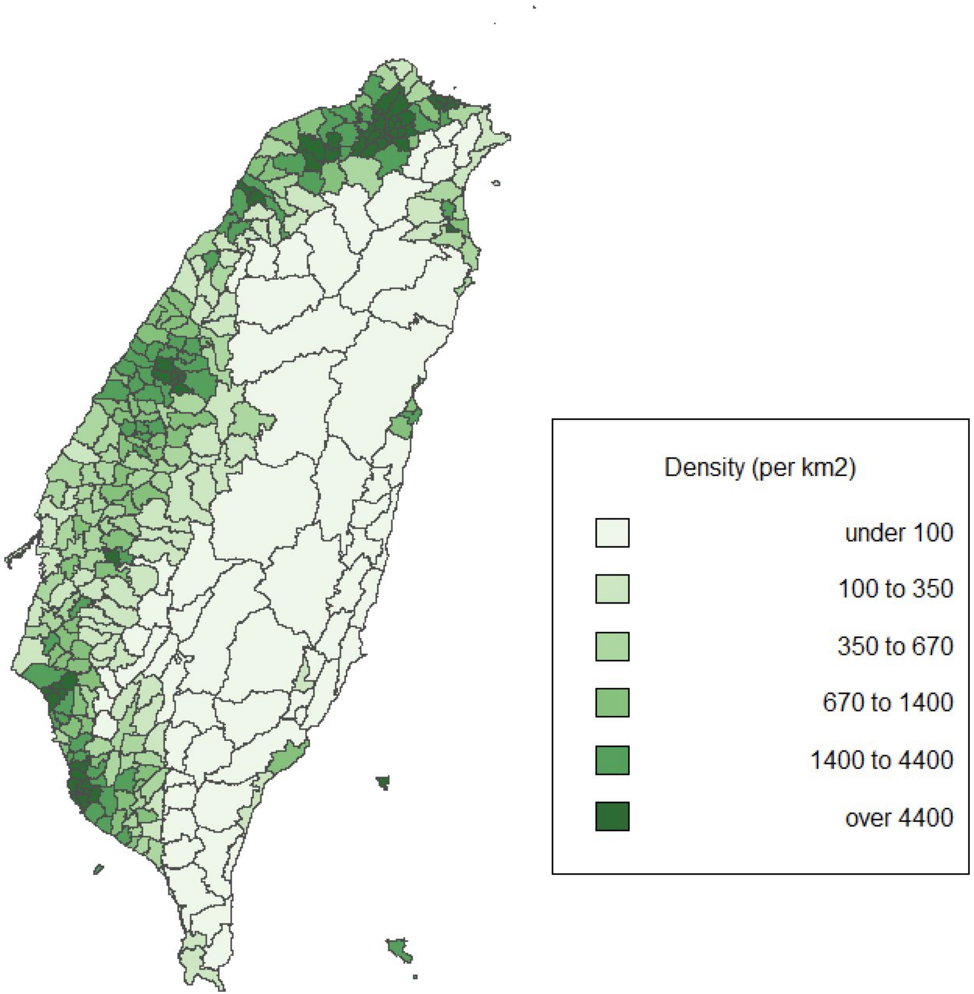
2. Maximize Utility

Data

Data

- Population
  - 內政部戶政司
- Mask Factory
  - 經濟部工業司
- Latitude and Longitude of District
  - 鄉鎮市區(TWD97經緯度) from data.gov.tw
- Derived Distance Matrix
  - From district and factory data




Density (per km2)
under 100
100 to 350
350 to 670
670 to 1400
1400 to 4400
over 4400

Density

Formulation

1. min (Transport Cost)

$$\min \quad \sum_{i=1}^{M}\sum_{j=1}^{N}(X_{Aij} + X_{Yij})D_{ij}C$$

a. Parameter

```
M;                          # Number of factories
N;                          # Number of districts
D{i in 1..M, j in 1..N};    # Distance between Factory i and District j
PA{j in 1..N};              # Adult Population in District j
PY{j in 1..N};              # Children Population in District j
A{i in 1..M};               # 0 if cannot produce children mask, 1 if can
S{i in 1..M};               # Production capacity in Factory i
```

b. Variable & Objective function

Variables

XA{i in 1..M, j in 1..N};   # Adult Mask produced by Factory i for District j

XY{i in 1..M, j in 1..N};   # Children Mask produced by Factory i for District j

----

Objective function

minimize cost: sum{i in 1..M, j in 1..N} (XA[i, j] + XY[i, j]) * D[i, j] * C;

c. Constraints

```
factoryLimit { i in 1..M }:                          sum{j in 1..N} (XA[i, j] + XY[i, j]/2) = S[i];
districtAdultRequirement { j in 1..N }:              sum{i in 1..M} XA[i, j] <= PA[j];
districtChildrenRequirement { j in 1..N }:           sum{i in 1..M} XY[i, j] <= PY[j];
factoryChildrenProductionCapability { i in 1..M }:   sum{j in 1..N} XY[i, j] <= 2S[i] * A[i];
XA[i,  j] >= 0;
XY[i,  j] >= 0;
```

d. Formulation

The formulation is

$$
\begin{aligned}
\min \quad & \sum_{i=1}^{M}\sum_{j=1}^{N}(X_{Aij}+X_{Yij})D_{ij}C \\
\text{s.t.} \quad & \sum_{j=1}^{N} X_{Aij}+X_{Yij}=S_i \quad \forall i=1,...,M \quad \text{(Factory Limit)} \\
& \sum_{i=1}^{M} X_{Aij} \leq P_{A,j} \quad \forall j=1,...,N \quad \text{(Adult Requirement)} \\
& \sum_{i=1}^{M} X_{Yij} \leq P_{Y,j} \quad \forall j=1,...,N \quad \text{(Children Requirement)} \\
& \sum_{j=1}^{N} X_{Yij} \leq K\alpha_i \quad \forall i=1,...,M \quad \text{(Children Production Capability)} \\
& X_{Aij} \geq 0 \quad \forall i=1,...,M, j=1,...,N \\
& X_{Yij} \geq 0 \quad \forall i=1,...,M, j=1,...,N \\
& \alpha_i \in \{0,1\} \quad \forall i=1,...,M
\end{aligned}
$$

2. max (Utility)

$$\max \quad \sum_{j=1}^{N} U_j$$

$$U_j = [(PA_{jNorm} + PY_{jNorm})(r_{Aj} + r_{Yj})(dj)]^{\frac{1}{3}}$$

Utility Function

$$U_j = [(PA_{jNorm} + PY_{jNorm})(r_{Aj} + r_{Yj})(dj)]^{\frac{1}{3}}$$

- Factors :
  - population of town j
  - coverage of town j
  - density of town j

- Formulation
  - in porprotion to population, coverage, density
  - diminishing return in marginal utility of coverage (=> pwr <1)
  - normalized three data within [0,1]
  - equalize the effect of three factors (1/3)

a. Parameter

```
PA_Norm{ j in 1..N };          # Adult normalized population in Districe j
PY_Norm{ j in 1..N };          # Children  normalized population in Districe j
```

b. Variable & Objective function

Variables

```
var rA{j in 1..N} >= 0;          # Coverage of adult
var rY{j in 1..N} >= 0;          # Coverage of children
var U{j in 1..N};                # Utility function
var cost;                        # Total cost
```

b. Variable & Objective function

Objective function

maximize utility: sum{j in 1..N} U[j];

c. Constraints

```
subject to factoryLimit {i in 1..M}:                         sum{j in 1..N} (XA[i, j] + XY[i, j] / 2) = S[i];
subject to districtAdultRequirement {j in 1..N}:             sum{i in 1..M} XA[i, j] <= PA[j];
subject to districtChildrenRequirement {j in 1..N}:          sum{i in 1..M} XY[i, j] <= PY[j];
subject to factoryChildrenProductionCapability {i in 1..M}:
                                                             sum{j in 1..N} XY[i, j] <= 2*S[i] * A[i];
subject to nonneg_1 {i in 1..M, j in 1..N}:                   XA[i, j] >= 0;
subject to nonneg_2 {i in 1..M, j in 1..N}:                   XY[i, j] >= 0;
subject to coverage_of_adult {j in 1..N}:                     rA[j] = sum{i in 1..M} XA[i, j] / (PA[j] +
PY[j]);
subject to coverage_of_children {j in 1..N}:                  rY[j] = sum{i in 1..M} XY[i, j] / (PY[j] +
PA[j]);
```

d. Formulation

$$
\begin{aligned}
\max \quad & \sum_{j=1}^{N} U_j \\
\text{s.t.} \quad & r_{Aj} = \frac{\sum_{i=1}^{M} X_{Aij}}{P_{Aj}} && \text{(Coverage of Adult)} \\
& r_{Yj} = \frac{\sum_{i=1}^{M} X_{Yij}}{P_{Yj}} && \text{(Coverage of Children)} \\
& U_j = [(PA_{jNorm} + PY_{jNorm})(r_{Aj} + r_{Yj})(dj)]^{\frac{1}{3}} && \text{(Utility of Town j)} \\
& \sum_{j=1}^{N} (x_{Aij} + \frac{1}{2} X_{Yij}) = S_i, \quad \forall i = 1..M && \text{(Production Capacity)} \\
& \alpha_i = \{0, 1\} && \text{(Factory } i \text{ Capability of Making Children Mask)} \\
& \sum_{j=1}^{N} X_{Yij} \leq 2S_i(\alpha_i), \quad \forall i = 1..M && \text{(Linearize } \alpha_i\text{)} \\
& \sum_{i=1}^{M} X_{Aij} \leq P_{Aj} && \text{(adult demand bound)} \\
& \sum_{i=1}^{M} X_{Yij} \leq P_{Yj} && \text{(children demand bound)}
\end{aligned}
$$

Performance Analysis

1. Min Transportation Cost

Result

The total travel distance is 130 million (km) per day .

1. Some district have 0 mask a day. (eg. 台北市松山區 is too far for factory.)
2. Those district closer to the factory will be delivered first.

1. Min Transportation Cost

Benefits

1. Lock the total mask at 16,000,000, which maximize the amount of masks distributed.
2. Minimize the transportation cost as well comparing to government's cost of 348,012,699,800.

1. Min Transportation Cost

[Calculation of the govt's cost]

$$\min \quad \sum_{i=1}^{M} \sum_{j=1}^{N} (X_{Aij} + X_{Yij}) D_{ij} C$$

$$r_{Aj} = \frac{\sum_{i=1}^{M} X_{Aij}}{P_{Aj}} \qquad \text{(Coverage of Adult)}$$

$$r_{Yj} = \frac{\sum_{i=1}^{M} X_{Yij}}{P_{Yj}} \qquad \text{(Coverage of Children)}$$

1. Min Transportation Cost

Drawbacks

1. We do not have enough mask for every people per day.
2. We did not optimize the utility.

2. Max Utility

Result

The total utility is 58.12596265.

2. Max Utility

Benefits

1. Large city with large population and high density gets larger utility.

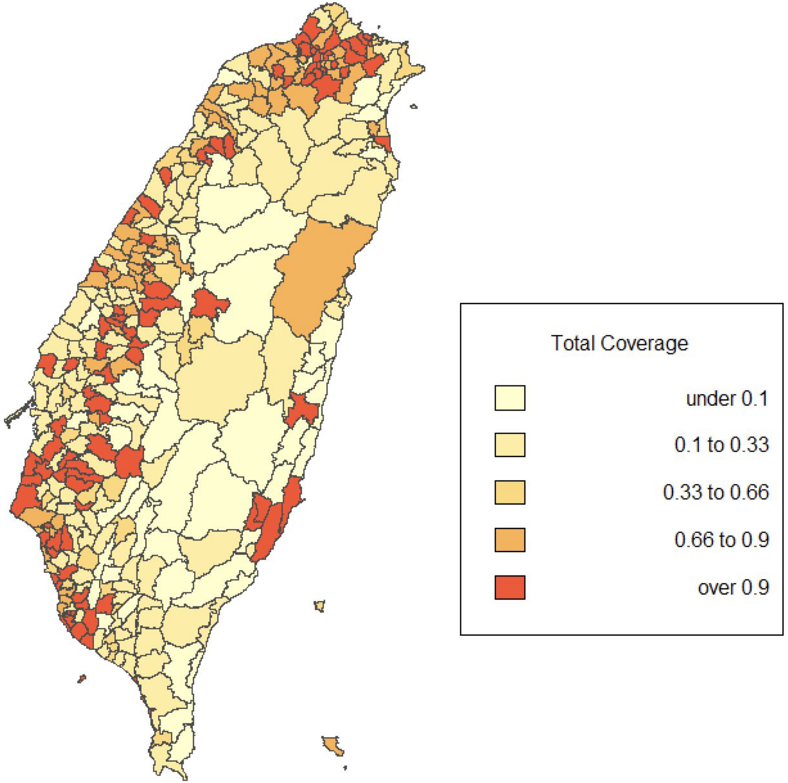
Drawbacks

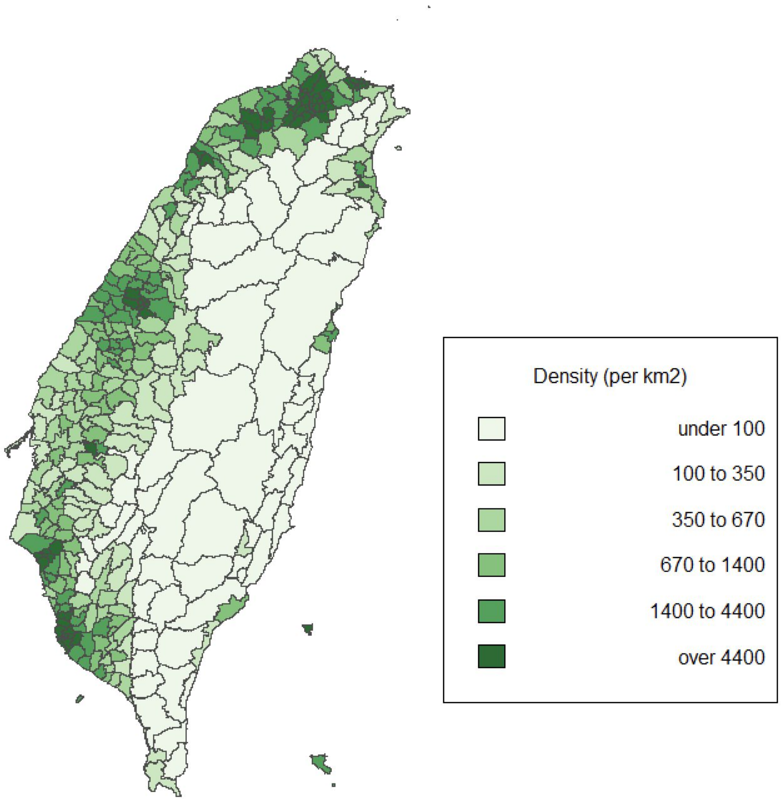
1. The total travel distance is 579 million (km) per day, which is 4.5 times of method 1.
2. Some district have 0 mask a day.

Conclusion

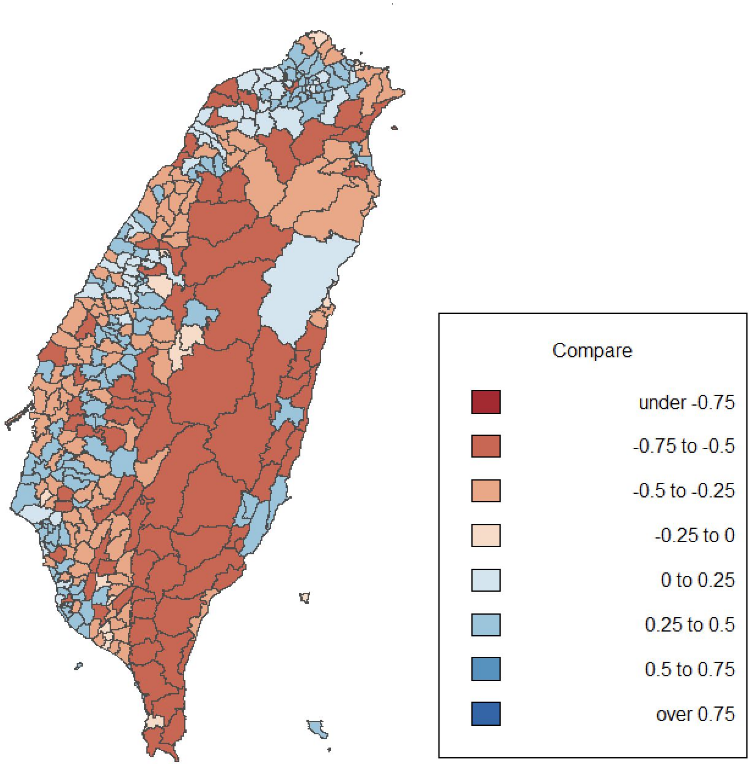
Conclusion

- Closer to public thoughts
- Lower people's worry about the disease
- Maximize social welfare





Coverage of our proposal



compare to ' 14 days - 9 masks ' policy

Future

- Resource Management for Government
- Distribute Resource with Different Objective
- Able to Apply to Another Crisis
- Consider the workers in and out