Modeling Crochet Doilies as Loaded Net/Truss Structures

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Contents

Abstract

Crochet is a unique textile process that can be formed from a single strand using a single needle. However, these potential of these interesting properties have largely been unexplored because of the complexity of the knots and patterns. Even for a simple 2D pattern, the process of looping back to previous knots can result in complex structures, including recursive patterns, which has yet to been replicated/automated by machines. This project develops a discretized, parameterized simplified model of a simple granny-square crochet by assuming that a subset of the elements are sufficient to represent the structure as a net in pure tension and by assuming that the yarn stiffness, yarn friction, knot strength, and knot density can be parameterized as a single overall strength parameter, which is represent in the force density parameter of the Force Density Method (FDM). A MATLAB script was written to generate uniform, random, patterned, or custom geometries based on inputted topologies and model the impact of an applied load on the structure using FDM. The distribution of lengths in the initial and final geometries were analyzed to show that the force density parameter can be used to connect the initial node locations to the loaded performance into the FDM calculations. The structural performance was calculated as the sum of the internal forces in the members weighted by the lengths of the members. These preliminary results show that the structural performance varies significantly less with geometry than it does with topology; from a design perspective, this provides design flexibility in having diverse designs options that all perform similarly well.

Introduction

Hand-crocheting is an art that can produce a diverse variety of 2D and 3D designs, including lace, textiles, clothes, hyperbolic surfaces, and toys from just one continuous strand knotted into interlocking loops. Crochet's unique property of being formed from a single strand and a single needle could offer opportunities for robotic fabrication that doesn't require as much space for machinery as a multi-strand set-up or for temporary structures that can be easily and non-destructively taken-apart for material re-use.

Still, even for a simple 2D pattern, the process of looping back to previous knots can result in complex structures, including recursive patterns, which has yet to been replicated/automated by machines. Due to the large number of nodes and edges and the complex interactions among nodes, it is too challenging to model the patterns exactly, even for doily patterns with greater proportions of negative space. However, because these sparser patterns have many elements – particularly looping or curved elements – that are not under tension, a simplified representation of the crocheted pattern that identifies only the segments under tension could significantly reduce complexity and computational runtime of a structural analysis.

Thus, this project proposes and evaluates a simplified abstract representation of the structural skeleton of sparse, hole-filled 2D crocheted doilies. Furthermore, this project will focus on simple archetypes characterized radial or concentric symmetry. A key assumption is that the yarn stiffness, yarn friction, knot strength, and knot density can be parameterized as a single overall strength parameter; for example, if the yarn material and knot-type are assumed to be constant throughout the crocheted pattern, then it is assumed that the force density represents the knot density. Finally, it is assumed that sparseness in the structural representation is good because that allows more space to incorporate decorative, non-structural elements.

By modeling as a net under compression or equivalently a truss under tension, we can apply existing structural analysis methods, such as force density method, projective constraints method, and direct stiffness method. In this way, we can analyze deformation or stretching resulting from applying a combination of loads normal to the original surface and in-plane with the original surface.

Literature Review

Academic research of crocheted structures has generally been limited to the demonstration of interesting mathematical surfaces, such as hyperbolic spaces [1,2] and Lorenz manifolds [3,4]. Crochet has also been linked to knot theory and textile topology studies that uses other similar forms, such as macramé craft knots [5], bead crochet [6], Celtic knots [7], weaves [8,9], and non-woven fabrics [10]. Various art [11], artistic product design, and architecture [12] have used crochet as a medium or inspiration.

However, few academic studies are of the design or structure of crochet. Matt Gilbert wrote an algorithm to generate asymmetric/irregular paths for crocheting an entire sweater [13], and Alexander Worden investigated digital modeling and design generation inspired by crochet [14]. Kira Street generated crochet designs first by combining base components (e.g. granny squares, flowers, chains) into artistic free-form assemblies and then by randomly selecting stitches for 2D radially-symmetric patterns [15,16], represented by standardized crochet stitch symbols. Nabaei, et al. (2013) draws inspiration from woven textiles to study the deformations on woven timber fabric structures (TFC) using the dynamic relaxation method [17].

Even including these few studies, there has not been any attempt to quantitatively evaluate or to optimize the design and structure of crochet. This project attempts to model a simplified sparse crochet pattern as a net and to reduce the complexity of the crochet pattern in order to apply existing structural analysis methods, such as force density method [18].

Methodology

Simplified Abstract Representation

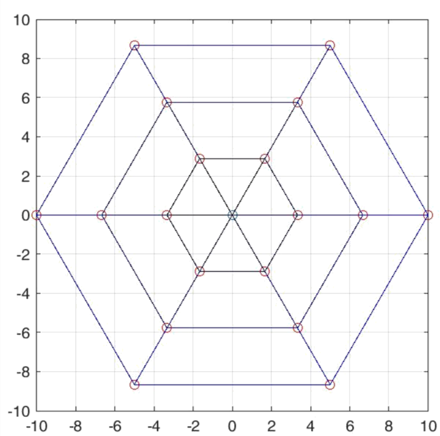
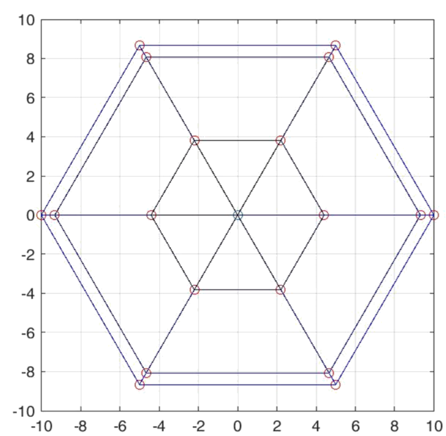
The design space is parameterized by the following topology and loading variables:

- M: Number of rings (concentric segments)
- N: Number of rays (radial segments)
  P: Load scenarios: which nodes are anchored, which nodes have applied loads, magnitude and direction of these applied loads
- Q_i: Force density of each segment

The force density Q_i is assumed to represent a combination of yarn material properties and knot density/strength properties. The radius of the outermost ring (L) is assumed to be constant.

The initial 2D form was generated in Grasshopper parametrically for patterns incorporating uniform spacing. In addition, a MATLAB script was written to automatically generate original nodal positions and node-edge connectivity matrices. This MATLAB script calculates nodal positions and edge connections from the center spiraling outwards, in the same order as a typical granny square crochet pattern. It allows for more diverse patterns that also customize:

- D_i: Spacing in between the rings (concentric asymmetry, default is equal spacing D_i = L/M)
- A_i: Spacing in between the rays (radial asymmetry, default is equal spacing is A_i = 360/N degrees)
- num_nodes: Variable number of nodes in consecutive rings, accounting for branching.

	UNIFORM LENGTHS	RANDOM/CUSTOM LENGTHS
UNIFORM ANGLES	 D = [3.3333, 3.3333, 3.3333] α = [1.0472, 1.0472, 1.0472, 1.0472, 1.0472, 1.0472] rads num_nodes = [1,6,6,6]	 D = [4.4099, 4.9028, 0.6873] α = [1.0472, 1.0472, 1.0472, 1.0472, 1.0472, 1.0472] rads num_nodes = [1,6,6,6]

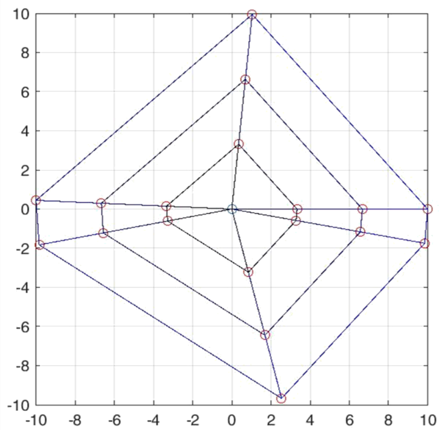
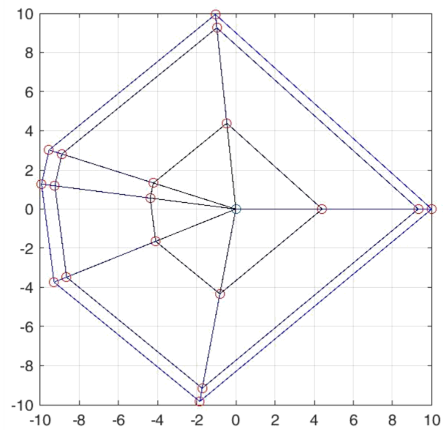
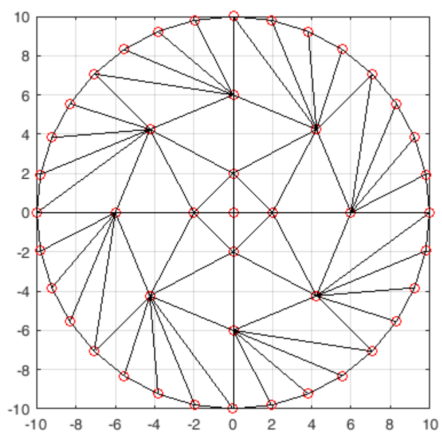
RANDOM/CUSTOM ANGLES	 D = [3.3333, 3.3333, 3.3333] α = [1.4665, 1.6304, 0.2286, 1.6440, 1.1382, 0.1756] rads num_nodes = [1,6,6,6]	 D = [4.4099, 4.9028, 0.6873] α = [1.6750, 1.1597, 0.1789, 0.5107, 1.0029, 1.7560] rads num_nodes = [1,6,6,6]

Figure 1: Example designs of various levels of customization vs uniformity



D = [2,4,4]
num_nodes = [1, 4, 8, 32]

Figure 2: Example of branching structure

Form Finding

For assumed topology and load scenarios, FDM was implemented as a Python script in Grasshopper to calculate final static equilibrium positions of the nodes.

Structural Analysis

Internal forces calculated using structural analysis library Karamba in Grasshopper.

Performance Evaluation

For each topology and load case, the amount of material can be considered the sum of the internal forces under our assumption that the force density is directly related to the knot density and therefore yarn used; future research assuming a different relationship between the knot density, amount of material used per knot, and force density could estimate of the amount of material using a more complicated function.

$$T = \int_{\text{edges}} L_e Q_e = \int_{\text{edges}} F_e \quad \text{(Eqn 1.)}$$

The degree of symmetry will be calculated based on the distribution of lengths and angles, for considering concentric and radial symmetry, respectively. For example for concentric symmetry, the variance in length and the difference between the average length of a radial segment and the length of a radial segment if the geometry had equally-spaced rings.

$$DOS = Var(L_e) + \left((Mean(L_e) - \frac{L}{M} \right) \quad \text{(Eqn 2.)}$$

Finally, the sum of internal forces of all segments in each structure will be weighted not only by segment length, but also by segment force density. This penalizes forces with high density, which represents the usage of more material.

$$F_T = \int_{\text{edges}} |F_e L_e| \quad \text{(Eqn 3.)}$$

In conclusion, the objective space comprises of the following metrics:

- T: Amount of material based on integrating the length and force density
- S: Degree of symmetry based on the distribution (variance and mean) of lengths and angles
- F_T: Net internal forces weighted by length and force density

Design Space Exploration

This project investigated a specific area of the design space that is characterized by:

- Radial symmetry in the topology
- Constant number of rings (M = 3)
- Number of nodes in each ring in the form of num_node = [1, n_i, 2*n_i, k*n_i], for $n_i \in$ {3,4,5,6} and k $\in$ {1, 2, 3, 4, 5}
- Fixed points are the [(M-1]/2] nodes in the –y hemisphere of the outermost ring
- Uniform load of p = √(2)/2 *[0, 1,-1] on all free (not-fixed) nodes
- Force densities of edges proportional to the length of the initial geometry

The number of nodes in the first non-center ring was varied from 3 to 6; this parameter will referred to in as Ni. For each value of Ni, the initial topology was set by choosing a random value of k from 2 to 5 and according to rule-based MATLAB code that calculated how many inter-ring edges to apply to ensure topological circular symmetry. The initial geometry was set by uniformly spacing the nodes within each ring (radial symmetry) and by choosing a random inter-ring spacing (concentric asymmetry).

For each initial topology and geometry, a uniform load of $p = \sqrt{(2)}/2 *[0, 1,-1]$ was applied on all free (not-fixed) nodes.

Results

Structural Performance and Variation of Lengths

According to the FDM method, the initial locations of the non-fixed points (and therefore the initial lengths of the edges) are independent of the calculation of the final locations under the input load case. However, because this model assumes that the force density is proportional to the initial edge lengths, there is a trend in the best performing designs having longer initial lengths and shorter final lengths. As shown in the top graphs of Figure 3, the highest performing designs (green) have greater lengths overall than the lowest performing designs (red). This occurs because a longer initial length translates to a higher force density, which suggests that the same tension can be held by a shorter edge. In practice, having the nodal positions get closer would likely break the assumption that these edges are in tension. Thus, the force density should be formulated to be negatively proportional to the initial length.

This trend is less noticeable for higher N_i (such as $N_i = 6$ in the bottom plots of Figure 3) because as the number of total edges in the structure increases, the average length of the edges decreases for both high-performing and low-performing designs.

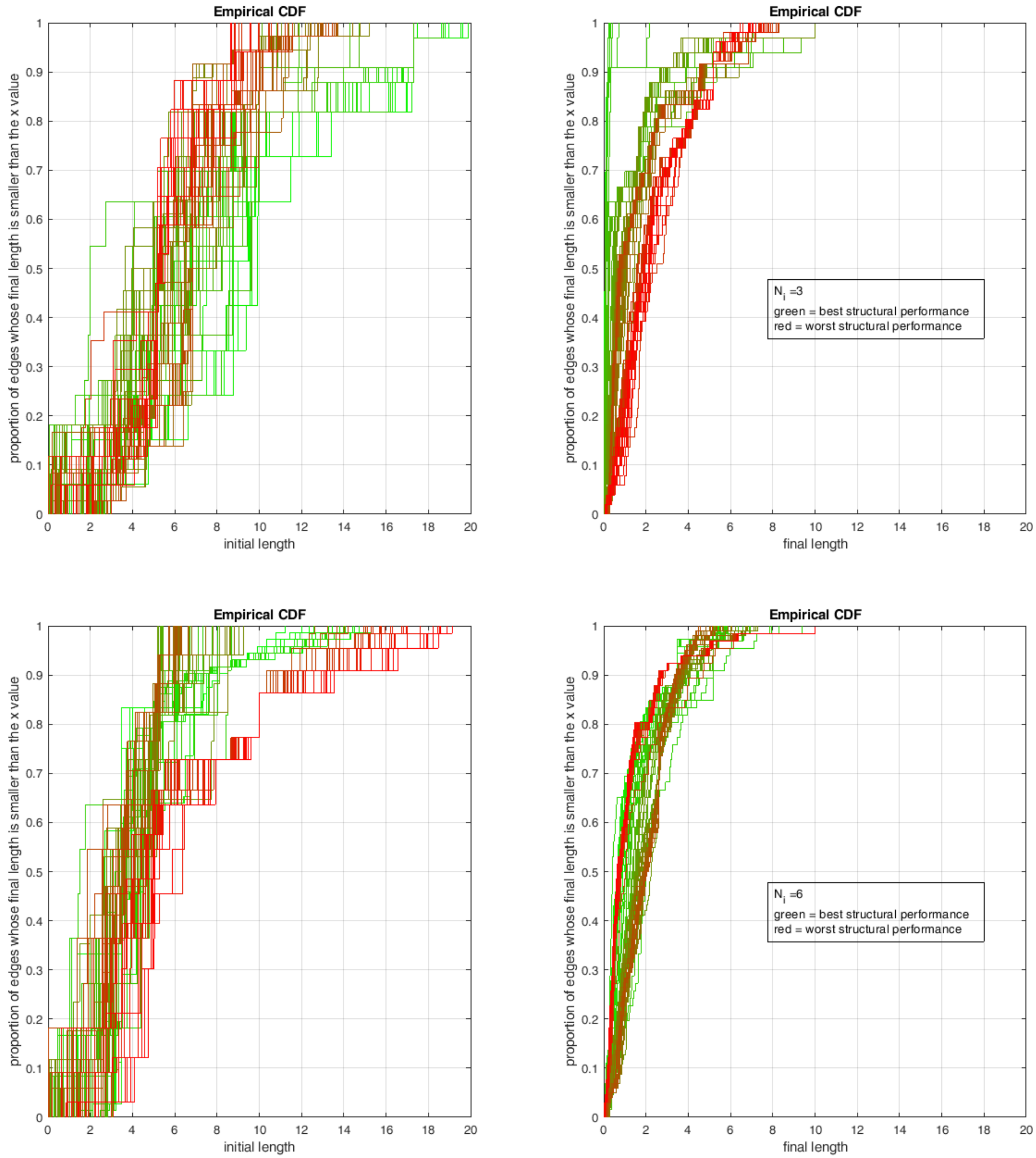


Figure 3: Cumulative Distribution Function for the initial lengths (left plots) and final lengths (right plots) of designs where Ni = 3 (top plots) or Ni = 6 (bottom plots). Green indicates a lower weighted sum of internal forces (i.e. better structural performance; Red indicates a higher weighted sum of internal forces (i.e. worse structural performance).

Diversity

Our initial results suggest that diversity in the initial geometry does not affect performance significantly. In a CDF plot of the weighted sum of internal forces, there are jumps in the value of total internal forces that correspond with changes in topology, whereas plateaus of similar performance in the plot are for groups of designs with the same structural topologies (Figure 4).

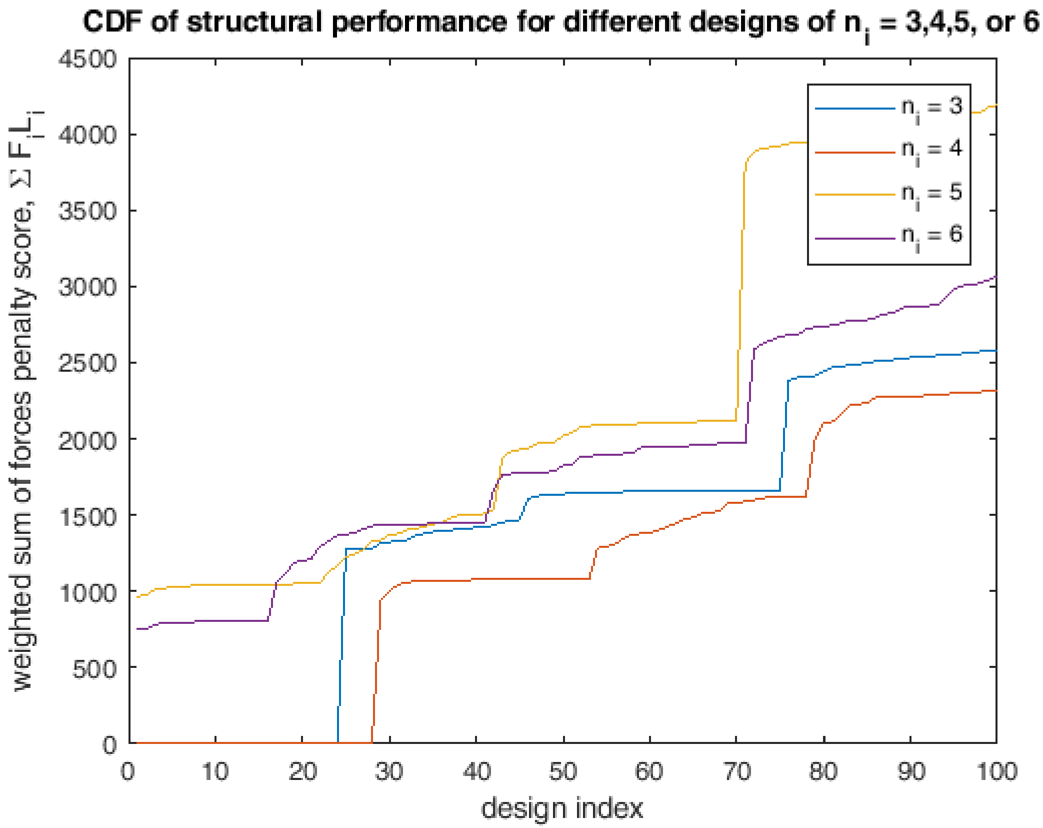


Figure 4: CDF Plot of summed weighted internal forces of entire structure

This results in part by design of the model; by distinguishing structural elements from aesthetic elements, this abstraction gives designers a lot of flexibility and diversity in the geometry and force density properties of the crochet pattern in-practice with the same structural topology in the model. Still, the topological diversity of the model could be expanded by allowing for greater customization of the branching between rings in terms of the number and placement of inter-ring edges. A higher variance force density formulation could also impact the diversity of performance for the same topology.

Figure 5 shows an example of two designs with the same structural performance, but different topologies and relatively different initial and final geometries.

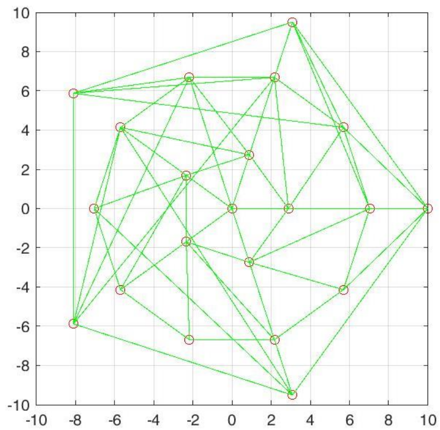
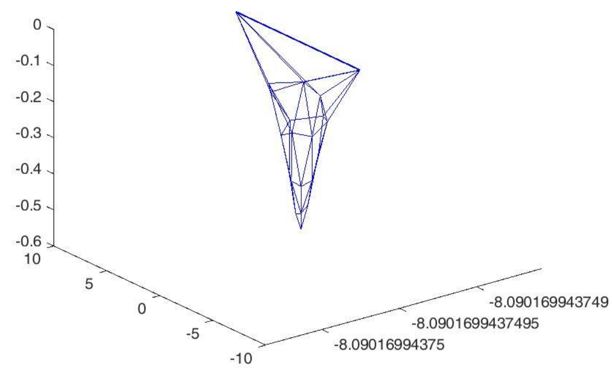
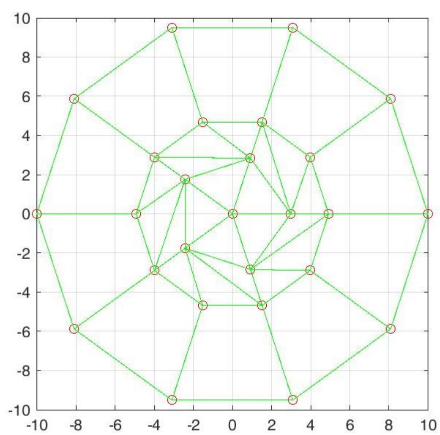
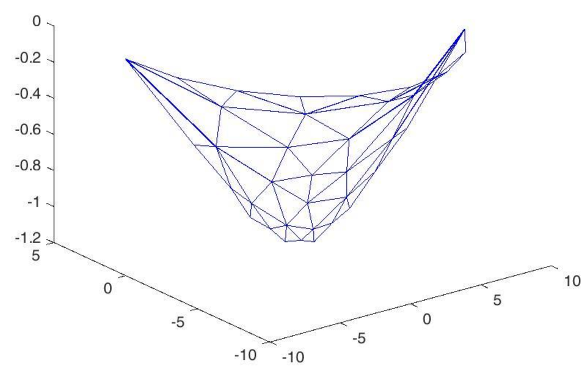
	Initial Geometry	Final Geometry
N_i = 5, Design #9	 D = [2.8824, 4.1526, 2.9651]	 F = 1039
N_i = 5, Design #11	 D = [2.9861, 1.9331, 5.0808]	 F = 1040

Figure 5: Initial (left) and final (left) geometries for designs with different topologies, but similar structural performance.

Applications and future research

Types of Symmetry

Although symmetry is a key design aspect for many architectural styles, it is not necessarily the only metric nor a universal metric for aesthetics, so a future area of study could be comparing the aesthetic appeal of fully uniform designs, regularly-patterned designs, lateral symmetry, or circular symmetry. This analysis suggests that the tendency for structures to distributed forces evenly (resulting in similar lengths and patterns throughout the geometry the final loaded structure) in the case of relatively similar force densities may have historically influenced our perception of aesthetics based on these patterns.

Furthermore, it is likely that symmetry in one part of the design may be more or less impactful than symmetry in another part of the design. For example, people may prefer uniform spacing between the rings more than uniform spacing between the rays (as contrasted in Figure 1).

Optimization

For a given load case and target amount of material, this approach can be used to study the tradeoff in density of rings and rays and propose optimal structures based on a combination of performance metrics (F_T, δ, T) and aesthetic metrics (S, subjective ranking) and a penalization on excessive material usage.

Another approach would be to focus on continuous design variables (such as spacing) rather than integer variables (such as the number of nodes) so that gradient-based optimizations methods could be used.

Polar Coordinates

Because of the circular and radial nature of crocheted granny squares, an interesting future direction would be to implement this model and the FDM calculations using polar coordinates.

Experimental Calibration and Validation

The biggest limitation is the assumption that force density can represent knot density for crocheted patterns with the same yarn type and knot type. Physical prototyping and experimentation would be necessary to verify this approximation and to check that non-uniformity of yarn or knot (due to human imperfection and material imperfections) doesn’t introduce too significant of variation and uncertainty in the performance. Furthermore, physical prototyping could help understand the angle at which strands enter and exit knots to extend the representation for more complex bent knots.

Conclusion

The goals of this project are to develop a discretized, parameterized model of a crochet net and to identify trends between the design parameters and the aesthetic and structural metrics. I made a simplified parametric representation that is extendable to more complex geometries (such as branching) and diverse designs (including repeating patterns, randomness, and complete customization). A key simplification is that the yarn stiffness, yarn friction, knot strength, and knot density can be parameterized as a single overall strength parameter. Experimentation on actual crocheted stitches may be necessary to understand which of these properties contributes the most to overall strength. A more general metric for measuring symmetry and patterns within the design is necessary for accounting for branching structures with more diversity of length in the original initial geometry.

Although FDM deviates from a realistic analogy in the way the initial and final positions of the free non-fixed nodes can be arbitrarily distant, relating the length of the initial lengths of the edges to the force densities can make the final equilibrium position depend on the initial nodal positions. More importantly, FDM is an appropriate method for computational optimization because it is a deterministic matrix calculation method that is fast enough to evaluate a large catalog of designs.

The performance of these structures does not vary significantly with geometry for the current formulation of force density, and only varies somewhat with topology. This is appropriate because most crochet applications do not require much structural performance, and the overall similarity in performance increases the design space for aesthetic purposes.

This results of this project could inspire new quantitatively-informed approaches to generating crochet designs, such as performing topological optimization and then choosing stitches based on the thickness.

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