ISYE 6333 Project
Supermarket Sweep

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1 Shortest travel time

Each item (node) i is located at coordinates (x_i, y_i), with the max possible y-coordinate of $v_l = 110$ ft. The shortest walking distance between items i and j is computed as the below. First case covers when two locations have the same x-coordinate, meaning same aisle. Second case covers when they are in different aisles, so the shopper can either move up or down to switch aisle.

$$distance_{ij} = \begin{cases} |y_i - y_j| & \text{if } x_i = x_j \\ \min\left(y_i + |x_i - x_j| + y_j,\ (v_l - y_i) + |x_i - x_j| + (v_l - y_j)\right) & \text{otherwise} \end{cases}$$

Travel times were then obtained as $distance_{ij}/10$ (ft/s), where 10 ft/s is the walking speed of each shopper. Refer to shortest_times.csv for the shortest time to travel between items and from the start/end location. Refer to Part a) in Supermarket_Sweep_Final.ipynb for the code.

2 Formulation

Parameters:

- $N = \{0, ..., 57\}$ Set of nodes (56 items with start and end)
- $I = \{1, ..., 56\}$ Set of items
- $S = \{1, 2\}$ Shoppers
- v_i = Value of item i
- d_{ij} = Shortest time from node i to node j
- $T = 60$ Total time (seconds) to collect items
- $M = T$ Big M upper-bounded by total time

Decision Variables:

$$x_{ijs} = \begin{cases} 1 & \text{if shopper } s \text{ travels from node } i \text{ to node } j \\ 0 & \text{otherwise} \end{cases} \qquad \forall i, j \in N,\ \forall s \in S$$

$$y_{js} = \text{Time (in seconds) when shopper } s \text{ arrives at node } j \qquad \forall j \in N,\ \forall s \in S$$

$$t_{ijs} = \begin{cases} y_{js} & \text{if shopper } s \text{ travels from node } i \text{ to node } j \\ 0 & \text{otherwise} \end{cases} \qquad \forall i, j \in N,\ \forall s \in S$$

$$g_{is} = \begin{cases} 1 & \text{if shopper } s \text{ grabs item at node } i \\ 0 & \text{otherwise} \end{cases} \qquad \forall i \in I,\ \forall s \in S$$

Objective Function:

$$max \sum_{s=1}^{2} \sum_{i=1}^{56} g_{is} v_i$$

Constraints:

$$\sum_{j=0}^{57} x_{ijs} = g_{is} \qquad \forall i \in I, \forall s \in S \qquad \text{Each grabbed item is left exactly once} \tag{1}$$

$$\sum_{i=0}^{57} x_{ijs} = g_{js} \qquad \forall j \in I, \forall s \in S \qquad \text{Each grabbed item is entered exactly once} \tag{2}$$

$$\sum_{j=1}^{57} x_{0,j,s} = \sum_{i=0}^{56} x_{i,57,s} = 1 \qquad \forall s \in S \qquad \text{Start/end node must be visited} \tag{3}$$

$$\sum_{i=1}^{57} x_{i,0,s} = \sum_{j=0}^{56} x_{57,j,s} = 0 \qquad \forall s \in S \qquad \text{Can't enter start; can't leave end} \tag{4}$$

$$x_{iis} = 0 \qquad \forall i \in N, \forall s \in S \qquad \text{Can't stay at same node} \tag{5}$$

$$t_{ijs} \leq M x_{ijs} \qquad \forall i, j \in N, \forall s \in S \qquad \text{If } x_{ijs} = 0\text{, then } t_{ijs} = 0 \tag{6}$$

$$y_{0,1} = y_{0,2} = 0 \qquad \text{Shoppers start time} = 0 \tag{7}$$

$$y_{js} = \sum_{i=0}^{56} t_{ijs} \qquad \forall j \in [1, 57], \forall s \in S \qquad y \text{ in terms of } t \tag{8}$$

$$\sum_{k=0}^{57} t_{jks} = y_{js} + 2g_{js} + \sum_{k=0}^{57} d_{jk} x_{jks} \qquad \forall j \in I, \forall s \in S \qquad \text{Time to next item} \tag{9}$$

$$\sum_{k=0}^{57} t_{0ks} = y_{0s} + \sum_{k=0}^{57} d_{0k} x_{0ks} \qquad \forall s \in S \qquad \text{Time to next item from start} \tag{10}$$

$$y_{57,s} \leq T \qquad \forall s \in S \qquad \text{Time limit} \tag{11}$$

$$\sum_{i=1}^{56} g_{is} \leq 10 \qquad \forall s \in S \qquad \text{At most 10 items per shopper} \tag{12}$$

$$\sum_{s=1}^{2} g_{is} \leq 1 \qquad \forall i \in I \qquad \text{Item grabbed at most once} \tag{13}$$

$$x_{ijs} \in \{0, 1\} \qquad \forall i, j \in N, \forall s \in S \qquad \text{Binary} \tag{14}$$

$$g_{is} \in \{0, 1\} \qquad \forall i \in I, \forall s \in S \qquad \text{Binary} \tag{15}$$

$$y_{js}, t_{ijs} \geq 0 \qquad \forall i, j \in N, \forall s \in S \qquad \text{Non-negativity} \tag{16}$$

3 Code and solve optimization model

Refer to Part c) in Supermarket_Sweep_Final.ipynb for the code. Refer to Table 1 for the optimal paths and values of items picked up by Shopper 1 and Shopper 2. The total value of the items picked by both shoppers is **$169.30**.

Table 1: Shoppers 1 and 2: Paths, Items, and Values (Including Start and End Nodes)

Shopper 1			**Shopper 2**		
Path (Node)	**Item**	**Value ($)**	**Path (Node)**	**Item**	**Value ($)**
0	Start	0.00	0	Start	0.00
22	Diapers	25.99	32	Detergent	12.99
21	Ibuprofen	5.49	33	Broom	13.99
39	Redbull (4)	7.99	34	Dog Treats	3.99
38	Gatorade (12)	6.99	35	Air Freshner	6.99
40	Ritz	3.99	30	Trash Bags	8.99
41	Coca Cola (12)	5.99	25	Shampoo	8.99
42	LaCroix (12)	5.49	5	Granola	5.49
43	Pepsi (12)	5.99	3	Capt. Crunch	3.99
27	Paper Towels	9.99	2	K-Cups	10.99
26	Toilet Paper	7.99	1	Coffee Beans	6.99
57	End	0.00	57	End	0.00
Total Value		**85.90**			**83.40**

4 How optimal value of the problem vary with respect to the total amount of time allowed

Table 2 shows the results of various time limits on the optimal value of our problem (total value of items grabbed by both shoppers on the same team). After a time limit of 75 seconds, we see that the optimal value plauteaus at $183.50. Figure 1 displays a lineplot of the optimal value versus various total time limits.

Table 2: Optimal Value vs. Total Time Limit

Time Limit (s)	**Total Value ($)**
25	68.44
30	87.42
45	129.17
60	169.30
75	183.50
90	183.50
100	183.50
120	183.50
150	183.50

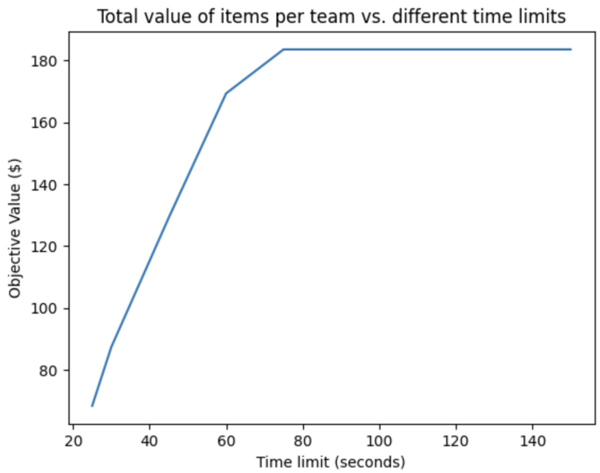

Figure 1: Optimal Value vs. Total Time Allowed

5 Another approach for obtaining a solution to the Supermarket Sweep problem

5.1 Formulate the MIP to determine the path for the first shopper

Parameters:

- $N = \{0, ..., 57\}$ Set of nodes (56 items with start and end)
- $I = \{1, ..., 56\}$ Set of items
- v_i = Value of item i
- d_{ij} = Shortest time from node i to node j
- $T = 60$ Total time (seconds) to collect items
- $M = T$ Big M upper-bounded by total time

Decision Variables:

$$x_{ij} = \begin{cases} 1 & \text{if shopper travels from node } i \text{ to node } j \\ 0 & \text{otherwise} \end{cases} \qquad \forall i, j \in N$$

$$y_j = \text{Time (in seconds) when shopper arrives at node } j \qquad \forall j \in N$$

$$t_{ij} = \begin{cases} y_j & \text{if shopper travels from node } i \text{ to node } j \\ 0 & \text{otherwise} \end{cases} \qquad \forall i, j \in N$$

$$g_i = \begin{cases} 1 & \text{if shopper grabs item at node } i \\ 0 & \text{otherwise} \end{cases} \qquad \forall i \in I$$

Objective Function:

$$max \sum_{i=1}^{56} g_i v_i$$

Constraints:

$$\sum_{j=0}^{57} x_{ij} = g_i \qquad \forall i \in I \qquad \text{Each grabbed item is left exactly once} \tag{17}$$

$$\sum_{i=0}^{57} x_{ij} = g_j \qquad \forall j \in I \qquad \text{Each grabbed item is entered exactly once} \tag{18}$$

$$\sum_{j=1}^{57} x_{0,j} = \sum_{i=0}^{56} x_{i,57} = 1 \qquad \text{Start/end node must be visited} \tag{19}$$

$$\sum_{i=1}^{57} x_{i,0} = \sum_{j=0}^{56} x_{57,j} = 0 \qquad \text{Can't enter start; can't leave end} \tag{20}$$

$$x_{ii} = 0 \qquad \forall i \in N \qquad \text{Can't stay at same node} \tag{21}$$

$$t_{ij} \leq M x_{ij} \qquad \forall i, j \in N \qquad \text{If } x_{ij} = 0\text{, then } t_{ij} = 0 \tag{22}$$

$$y_0 = 0 \qquad \text{Shoppers start time} = 0 \tag{23}$$

$$y_j = \sum_{i=0}^{56} t_{ij} \qquad \forall j \in [1, 57] \qquad y \text{ in terms of } t \tag{24}$$

$$\sum_{k=0}^{57} t_{jk} = y_j + 2g_j + \sum_{k=0}^{57} d_{jk} x_{jk} \qquad \forall j \in I \qquad \text{Time to next item} \tag{25}$$

$$\sum_{k=0}^{57} t_{0k} = y_0 + \sum_{k=0}^{57} d_{0k} x_{0k} \qquad \text{Time to next item from start} \tag{26}$$

$$y_{57} \leq T \qquad \text{Time limit} \tag{27}$$

$$\sum_{i=1}^{56} g_i \leq 10 \qquad \text{At most 10 items per shopper} \tag{28}$$

$$x_{ij} \in \{0, 1\} \qquad \forall i, j \in N \qquad \text{Binary} \tag{29}$$

$$g_i \in \{0, 1\} \qquad \forall i \in I \qquad \text{Binary} \tag{30}$$

$$y_j, t_{ij} \geq 0 \qquad \forall i, j \in N \qquad \text{Non-negativity} \tag{31}$$

5.2 Code and solve new MIP

Refer to Part e) in Supermarket_Sweep_Final.ipynb for the code.
Refer to Table 3 for Shopper 1's path and value of items picked up.

Table 3: Shopper 1's Path and Item Values

Path(Node)	**Item**	**Value ($)**
0	Start	0.00
27	Paper Towels	9.99
26	Toilet Paper	7.99
32	Detergent	12.99
33	Broom	13.99
35	Air Freshner	6.99
25	Shampoo	8.99
23	Toothpaste	3.99
24	Tampons	5.29
22	Diapers	25.99
21	Ibuprofen	5.49
57	End	0.00
	Total	101.70

5.3 Formulate new constraints

Let $G = \{i_1, \ldots, i_m\}$ be the set of items grabbed by Shopper 1.

$$g_i = 0 \qquad \forall i \in G \qquad \text{Shopper 2 cannot grab items picked by Shopper 1} \tag{32}$$

5.4 Solve the MIP with new constraints

Refer to Part e) in Supermarket_Sweep_Final.ipynb for the code.
Refer to Table 4 for Shopper 2's path and value of items picked up.

Table 4: Shopper 2's Path and Item Values

Path(Node)	**Item**	**Value ($)**
0	Start	0.00
1	Coffee Beans	6.99
2	K-Cups	10.99
3	Capt. Crunch	3.99
4	Chewy Bars	3.69
5	Granola	5.49
30	Trash Bags	8.99
36	Oreos	3.99
38	Gatorade (12)	6.99
39	Redbull (4)	7.99
40	Ritz	3.99
57	End	0.00
	Total	63.10

5.5 Total values and findings

The total value of the items picked by both shoppers using the sequential approach is **$164.80**, which is lower than the $169.30 achieved when optimizing both shoppers together. This demonstrates that optimizing both shoppers simultaneously results in a better overall solution. In the sequential approach, Shopper 1 is optimized without considering Shopper 2 and may pick items that are better for Shopper 2. Shopper 2 is then limited to choosing from the remaining items, resulting in a globally suboptimal allocation and a lower combined total value. On the other hand, by optimizing both shoppers together, item selection can be coordinated to maximize the total value collected.