Supermarket Sweep

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Problem Background

- Supermarket Sweep game show: Two shoppers race through a store to collect the most valuable items within a time limit.

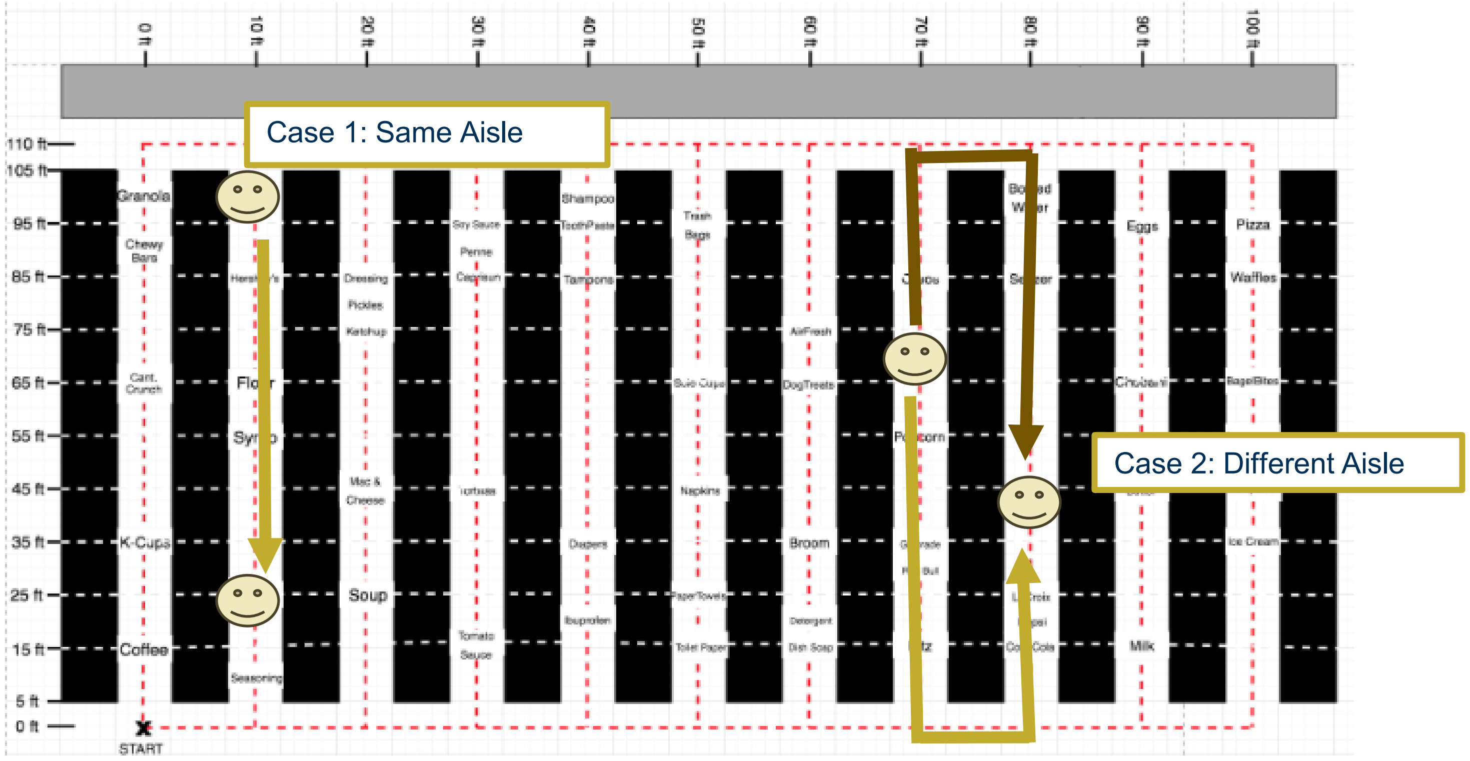
- Given Scenario:
  - 56 grocery items with known values and (x, y) coordinates.
  - Two shoppers start at (0,0) move at 10 ft/s, 60 seconds total to collect and return to the same point.
  - Each can carry ≤10 items; picking each item takes 2 seconds.
  - Each follows **one continuous path, no overlapping items.**

➡ Application of the **Traveling Salesman Problem (TSP)**

Shortest Time to Travel

- Shortest Distance

$$\begin{cases} |y_i - y_j| & \text{if } x_i = x_j \\ \min\left(y_i + |x_i - x_j| + y_j,\ (v_l - y_i) + |x_i - x_j| + (v_l - y_j)\right) & \text{otherwise} \end{cases}$$



Travel Times: distance_ij/10

(10ft/s is the walking speed)

Formulation – Parameters

Parameters:

- $N = \{0, ..., 57\}$ Set of nodes (56 items with start and end)
- $I = \{1, ..., 56\}$ Set of items
- $S = \{1, 2\}$ Shoppers
- v_i = Value of item i
- d_{ij} = Shortest time from node i to node j
- $T = 60$ Total time (seconds) to collect items
- $M = T$ Big M upper-bounded by total time

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Formulation – Decision Variables & Objective Function

$$max \sum_{s=1}^{2} \sum_{i=1}^{56} g_{is} v_i$$

$$x_{ijs} = \begin{cases} 1 & \text{if shopper } s \text{ travels from node } i \text{ to node } j \\ 0 & \text{otherwise} \end{cases} \qquad \forall i, j \in N, \ \forall s \in S$$

$$y_{js} = \text{Time (in seconds) when shopper } s \text{ arrives at node } j \qquad \forall j \in N, \ \forall s \in S$$

$$t_{ijs} = \begin{cases} y_{js} & \text{if shopper } s \text{ travels from node } i \text{ to node } j \\ 0 & \text{otherwise} \end{cases} \qquad \forall i, j \in N, \ \forall s \in S$$

$$g_{is} = \begin{cases} 1 & \text{if shopper } s \text{ grabs item at node } i \\ 0 & \text{otherwise} \end{cases} \qquad \forall i \in I, \ \forall s \in S$$

Formulation – Constraints

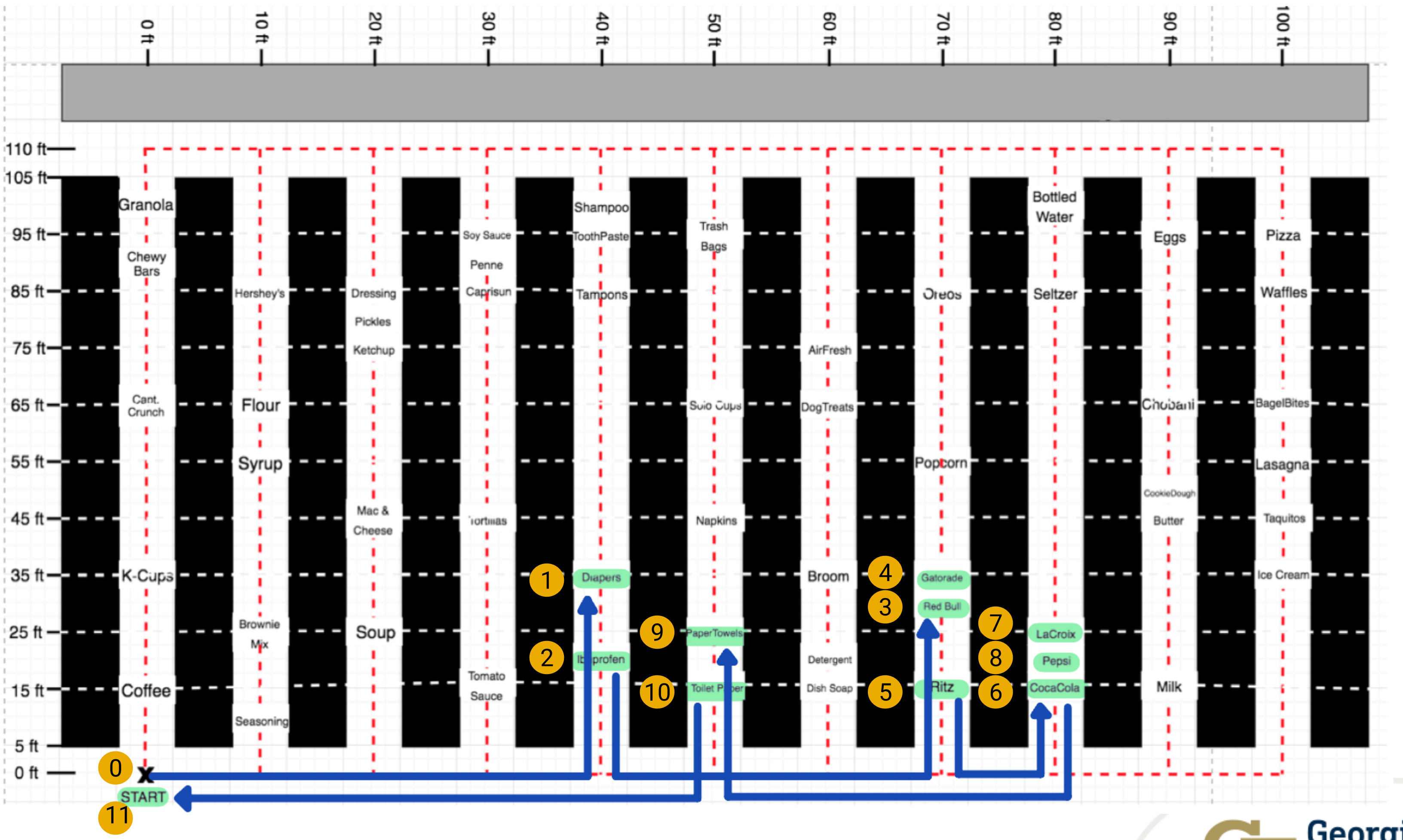
Constraints:

$\sum_{j=0}^{57} x_{ijs} = g_{is}$	$\forall i \in I, \forall s \in S$	Each grabbed item is left exactly once	(1)
$\sum_{i=0}^{57} x_{ijs} = g_{js}$	$\forall j \in I, \forall s \in S$	Each grabbed item is entered exactly once	(2)
$\sum_{j=1}^{57} x_{0,j,s} = \sum_{i=0}^{56} x_{i,57,s} = 1$	$\forall s \in S$	Start/end node must be visited	(3)
$\sum_{i=1}^{57} x_{i,0,s} = \sum_{j=0}^{56} x_{57,j,s} = 0$	$\forall s \in S$	Can't enter start; can't leave end	(4)
$x_{iis} = 0$	$\forall i \in N, \forall s \in S$	Can't stay at same node	(5)
$t_{ijs} \leq M x_{ijs}$	$\forall i, j \in N, \forall s \in S$	If $x_{ijs} = 0$, then $t_{ijs} = 0$	(6)
$y_{0,1} = y_{0,2} = 0$		Shoppers start time = 0	(7)
$y_{js} = \sum_{i=0}^{56} t_{ijs}$	$\forall j \in [1, 57], \forall s \in S$	y in terms of t	(8)
$\sum_{k=0}^{57} t_{jks} = y_{js} + 2g_{js} + \sum_{k=0}^{57} d_{jk} x_{jks}$	$\forall j \in I, \forall s \in S$	Time to next item	(9)
$\sum_{k=0}^{57} t_{0ks} = y_{0s} + \sum_{k=0}^{57} d_{0k} x_{0ks}$	$\forall s \in S$	Time to next item from start	(10)
$y_{57,s} \leq T$	$\forall s \in S$	Time limit	(11)
$\sum_{i=1}^{56} g_{is} \leq 10$	$\forall s \in S$	At most 10 items per shopper	(12)
$\sum_{s=1}^{2} g_{is} \leq 1$	$\forall i \in I$	Item grabbed at most once	(13)
$x_{ijs} \in \{0, 1\}$	$\forall i, j \in N, \forall s \in S$	Binary	(14)
$g_{is} \in \{0, 1\}$	$\forall i \in I, \forall s \in S$	Binary	(15)
$y_{js}, t_{ijs} \geq 0$	$\forall i, j \in N, \forall s \in S$	Non-negativity	(16)

Shopper 1 Results

- Total value (shopper 1 + shopper 2) = $169.30

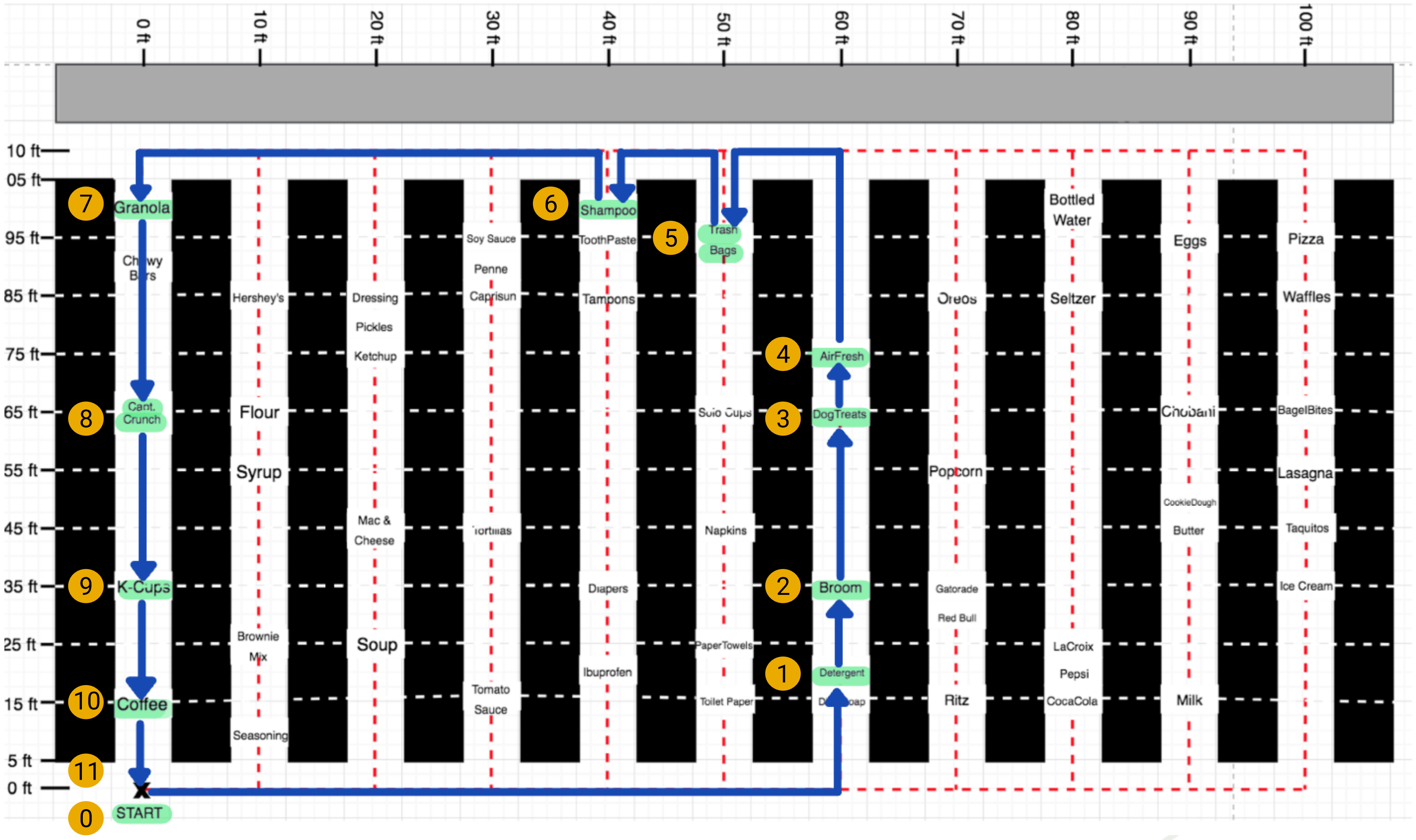
Shopper 1		
Path (Node)	Item	Value ($)
0	Start	0.00
22	Diapers	25.99
21	Ibuprofen	5.49
39	Redbull (4)	7.99
38	Gatorade (12)	6.99
40	Ritz	3.99
41	Coca Cola (12)	5.99
42	LaCroix (12)	5.49
43	Pepsi (12)	5.99
27	Paper Towels	9.99
26	Toilet Paper	7.99
57	End	0.00
Total Value		85.90



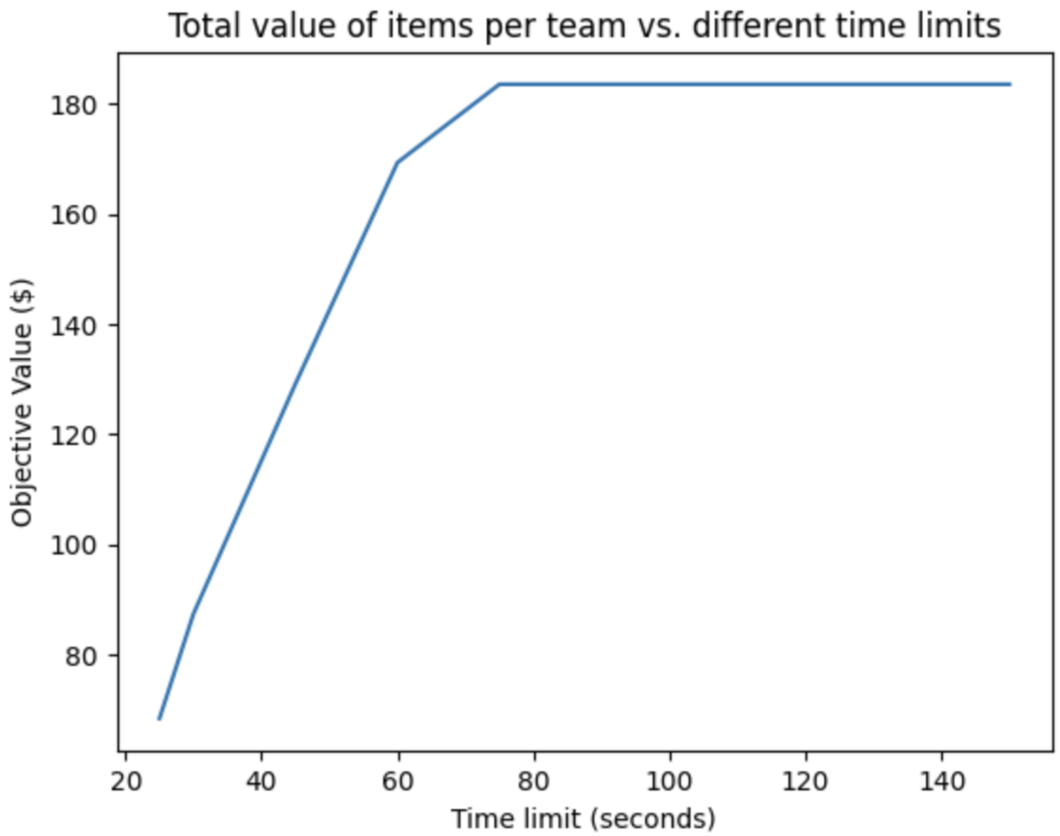
Shopper 2 Results

- Total value (shopper 1 + shopper 2) = $169.30

Shopper 2		
Path (Node)	Item	Value ($)
0	Start	0.00
32	Detergent	12.99
33	Broom	13.99
34	Dog Treats	3.99
35	Air Freshner	6.99
30	Trash Bags	8.99
25	Shampoo	8.99
5	Granola	5.49
3	Capt. Crunch	3.99
2	K-Cups	10.99
1	Coffee Beans	6.99
57	End	0.00
		83.40



How does the time limit affect the optimal value?



Time Limit (s)	Total Value ($)
25	68.44
30	87.42
45	129.17
60	169.30
75	183.50
90	183.50
100	183.50
120	183.50
150	183.50

Approach 2: Sequential Shopper Path

1. Optimize Shopper 1’s path to maximize total item value

2. Optimize Shopper 2’s path using only items remaining after Shopper 1

Formulation - Parameters

- $N = \{0, ..., 57\}$ Set of nodes (56 items with start and end)
- $I = \{1, ..., 56\}$ Set of items
- v_i = Value of item i
- d_{ij} = Shortest time from node i to node j
- $T = 60$ Total time (seconds) to collect items
- $M = T$ Big M upper-bounded by total time

Formulation – Decision Variables & Objective Function

$$max \sum_{i=1}^{56} g_i v_i$$

$$x_{ij} = \begin{cases} 1 & \text{if shopper travels from node } i \text{ to node } j \\ 0 & \text{otherwise} \end{cases} \qquad \forall i, j \in N$$

$$y_j = \text{Time (in seconds) when shopper arrives at node } j \qquad \forall j \in N$$

$$t_{ij} = \begin{cases} y_j & \text{if shopper travels from node } i \text{ to node } j \\ 0 & \text{otherwise} \end{cases} \qquad \forall i, j \in N$$

$$g_i = \begin{cases} 1 & \text{if shopper grabs item at node } i \\ 0 & \text{otherwise} \end{cases} \qquad \forall i \in I$$

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Formulation – Constraints

Constraints:

$$\sum_{j=0}^{57} x_{ij} = g_i \quad \forall i \in I \quad \text{Each grabbed item is left exactly once} \tag{17}$$

$$\sum_{i=0}^{57} x_{ij} = g_j \quad \forall j \in I \quad \text{Each grabbed item is entered exactly once} \tag{18}$$

$$\sum_{j=1}^{57} x_{0,j} = \sum_{i=0}^{56} x_{i,57} = 1 \quad \text{Start/end node must be visited} \tag{19}$$

$$\sum_{i=1}^{57} x_{i,0} = \sum_{j=0}^{56} x_{57,j} = 0 \quad \text{Can't enter start; can't leave end} \tag{20}$$

$$x_{ii} = 0 \quad \forall i \in N \quad \text{Can't stay at same node} \tag{21}$$

$$t_{ij} \leq M x_{ij} \quad \forall i,j \in N \quad \text{If } x_{ij} = 0\text{, then } t_{ij} = 0 \tag{22}$$

$$y_0 = 0 \quad \text{Shoppers start time} = 0 \tag{23}$$

$$y_j = \sum_{i=0}^{56} t_{ij} \quad \forall j \in [1, 57] \quad y \text{ in terms of } t \tag{24}$$

$$\sum_{k=0}^{57} t_{jk} = y_j + 2g_j + \sum_{k=0}^{57} d_{jk} x_{jk} \quad \forall j \in I \quad \text{Time to next item} \tag{25}$$

$$\sum_{k=0}^{57} t_{0k} = y_0 + \sum_{k=0}^{57} d_{0k} x_{0k} \quad \text{Time to next item from start} \tag{26}$$

$$y_{57} \leq T \quad \text{Time limit} \tag{27}$$

$$\sum_{i=1}^{56} g_i \leq 10 \quad \text{At most 10 items per shopper} \tag{28}$$

$$x_{ij} \in \{0, 1\} \quad \forall i,j \in N \quad \text{Binary} \tag{29}$$

$$g_i \in \{0, 1\} \quad \forall i \in I \quad \text{Binary} \tag{30}$$

$$y_j, t_{ij} \geq 0 \quad \forall i,j \in N \quad \text{Non-negativity} \tag{31}$$

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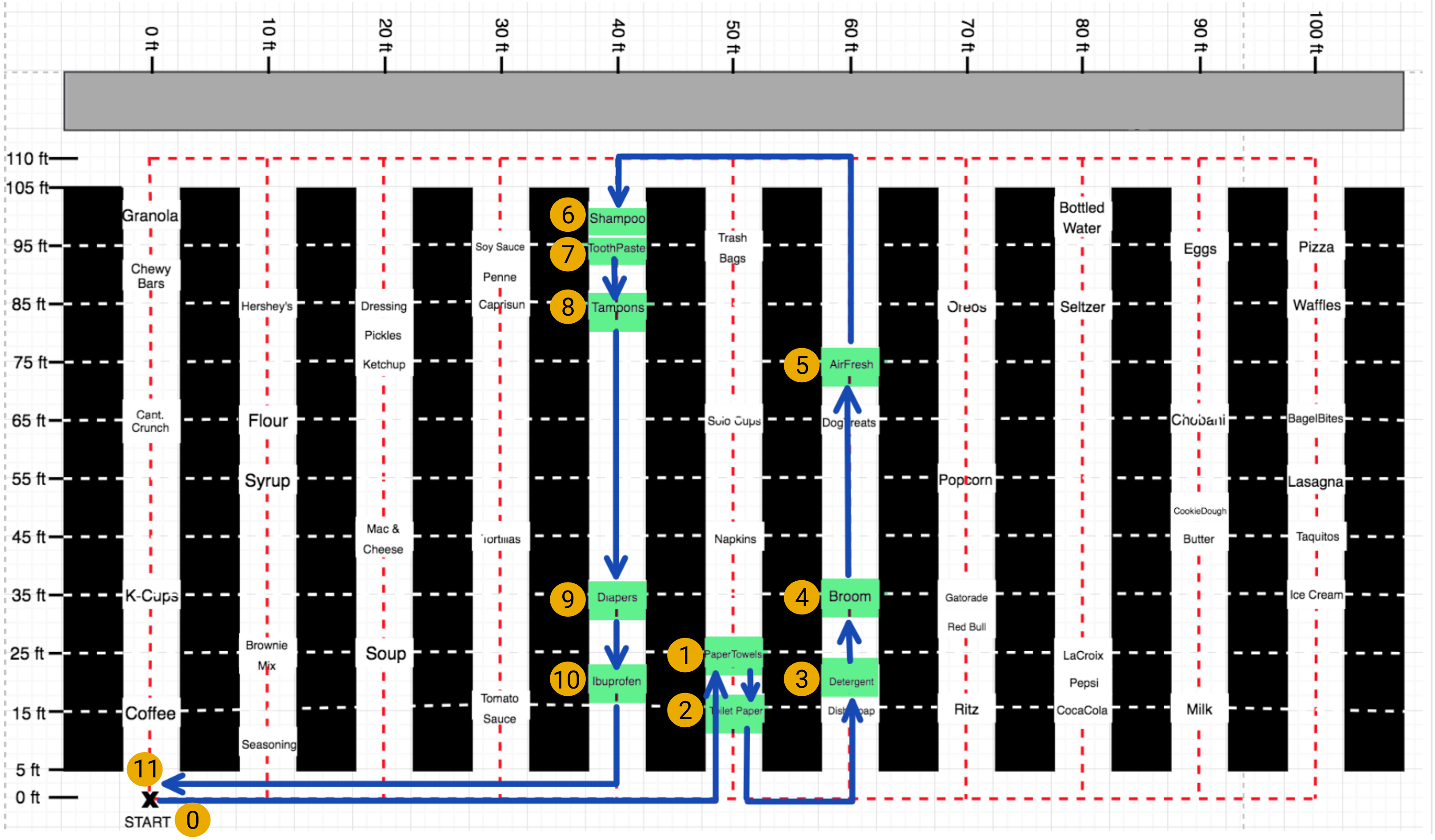
Shopper 1 Results

- Total value: $101.70

Path(Node)	Item	Value ($)
0	Start	0.00
27	Paper Towels	9.99
26	Toilet Paper	7.99
32	Detergent	12.99
33	Broom	13.99
35	Air Freshner	6.99
25	Shampoo	8.99
23	Toothpaste	3.99
24	Tampons	5.29
22	Diapers	25.99
21	Ibuprofen	5.49
57	End	0.00
	Total	101.70



Formulate New Constraint

Shopper 2 cannot grab items already picked by Shopper 1

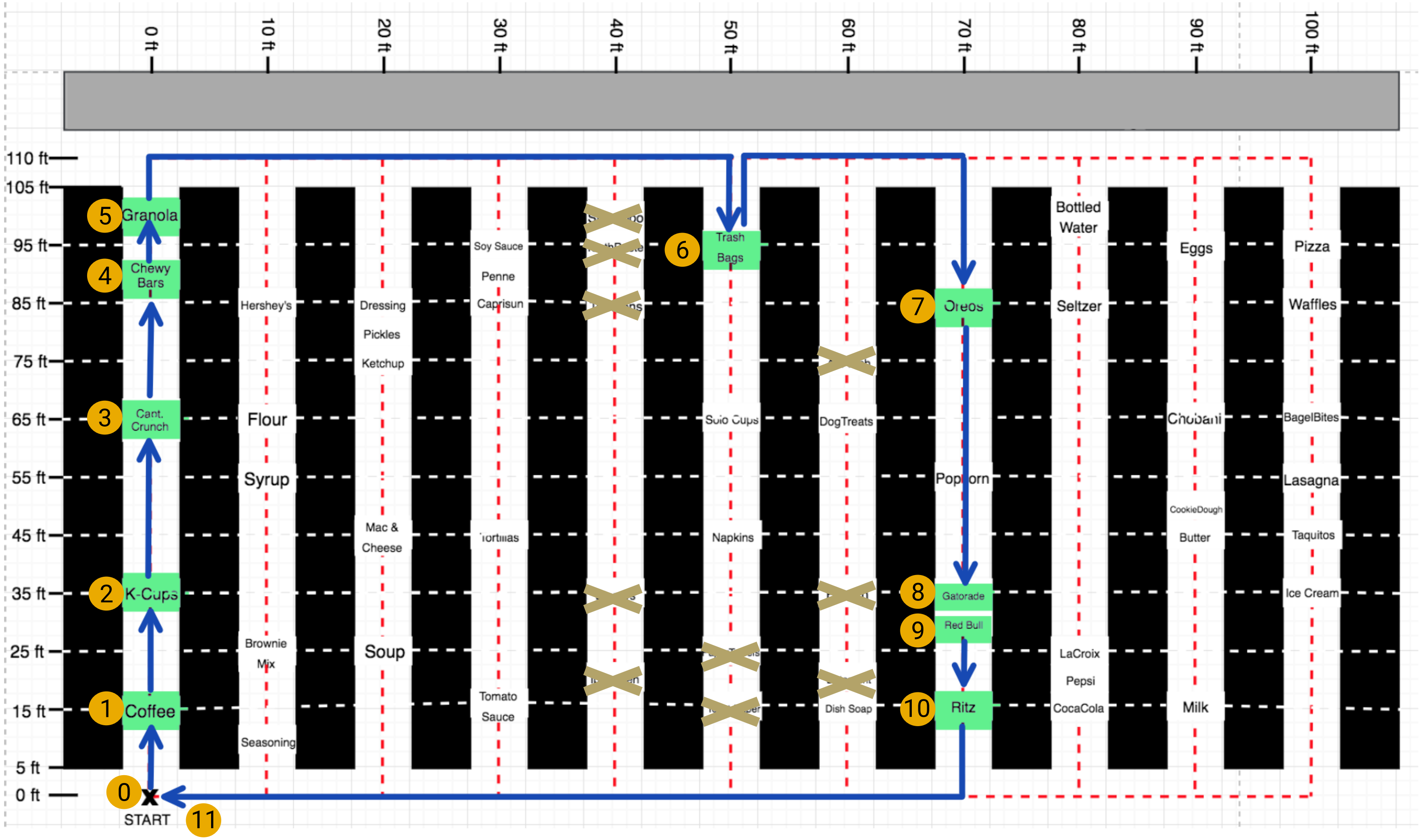
Let $G = \{i_1, \ldots, i_m\}$ be the set of items grabbed by Shopper 1.

$$g_i = 0 \qquad\qquad \forall i \in G$$

Shopper 2 Results

- Total value: $63.10

Path(Node)	Item	Value ($)
0	Start	0.00
1	Coffee Beans	6.99
2	K-Cups	10.99
3	Capt. Crunch	3.99
4	Chewy Bars	3.69
5	Granola	5.49
30	Trash Bags	8.99
36	Oreos	3.99
38	Gatorade (12)	6.99
39	Redbull (4)	7.99
40	Ritz	3.99
57	End	0.00
	Total	63.10



= Items picked by Shopper 1

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Total Value and Findings

- **Approach 1: Optimize shopper paths simultaneously**
  - Coordinated item selection maximizes total value across both shoppers
- **Approach 2: Optimize shopper paths sequentially**
  - Shopper 1 optimized without considering Shopper 2
  - Shopper 2 limited to remaining items, leading to suboptimal allocation

Shopper	**Appr. 1 Total Value ($)**	**Appr. 2 Total Value ($)**
Shopper 1	85.90	101.70
Shopper 2	83.40	63.10
Total	169.30	164.80

Appr. 1 – Total Value
$169.30

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Appr. 2 - Total Value
$164.80

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Questions?

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