(a)

$$\mathcal{U}(X_t, L_t) = \frac{X_t^{1-\gamma} - 1}{1-\gamma} - \chi \frac{N_t^{1+\varphi}}{1+\varphi}$$

$$X_t = \left[(1-\theta) C_t^{1-\nu} + \theta \left(\frac{M_t}{P_t}\right)^{1-\nu}\right]^{\frac{1}{1-\nu}}$$

Lagrangian

$$\mathcal{L}(\{C_t, M_t, N_t, B_t\})$$

$$= \sum^{\infty} \beta^t \Big[\mathcal{U}(X_t, L_t) + \lambda_t \big(W_t N_t + Q_{t-1} B_{t-1} + M_{t-1} + P_t (TR_t + PR_t) - P_t C_t - B_t - M_t \big) \Big]$$

FOC

$[C_t]$ $\quad U_x \, \partial X/\partial C = \lambda_t P_t$

$[M_t]$ $\quad U_x \, \partial X/M + \beta \lambda_{t+1} = \lambda_t$

$[N_t]$ $\quad U_N + \lambda_t W_t = 0$

$[B_t]$ $\quad \lambda_t = \beta \lambda_{t+1} Q_t$

$$U_x = X_t^{-\gamma}$$

$$\partial X/\partial C = X_t^{\nu} (1-\theta) C_t^{-\nu}$$

$$\partial X/\partial M = X_t^{\nu} \theta \left(\frac{M_t}{P_t}\right)^{-\nu} P_t^{-1}$$

$$U_N = -\chi N_t^{\varphi}$$

Put it together

$$\frac{X_t^{\nu-\gamma}(1-\theta) C_t^{-\nu}}{P_t} = \frac{\chi N_t^{\varphi}}{W_t}$$

$$\Rightarrow \frac{W_t}{P_t} X_t^{\nu-\gamma} (1-\theta) C_t^{-\nu} = \chi N_t^{\varphi} \qquad (1)$$

$$\frac{X_t^{\nu-\gamma}(1-\theta) C_t^{-\nu}}{P_t} = \frac{\beta Q_t X_{t+1}^{\nu-\gamma} (1-\theta) C_{t+1}^{-\nu}}{P_{t+1}}$$

$$\Rightarrow X_t^{\nu-\gamma} C_t^{-\nu} \cdot \frac{P_{t+1}}{P_t} = \beta Q_t X_{t+1}^{\nu-\gamma} C_{t+1}^{-\nu} \qquad (2)$$

$[M_t] + [B_t] \rightarrow \quad X_t^{\nu-\gamma} \theta \frac{M_t}{P_t}^{-\nu} \frac{1}{P_t} = \lambda_t \left(1 - \frac{1}{Q_t}\right)$

$$X_t^{v-\gamma}\theta\left(\frac{M_t}{P_t}\right)^{-v}\frac{1}{P_t} = \lambda_t\left(1-\frac{1}{Q_t}\right)$$

$$X_t^{v-\gamma}\theta\left(\frac{M_t}{P_t}\right)^{-v} = X_t^{v-\gamma}(1-\theta)C_t^{-v}\left(1-\frac{1}{Q_t}\right)$$

$$\Rightarrow \left(\frac{M_t}{P_t}\right)^{-v} = \frac{1-\theta}{\theta}C_t^{-v}\left(1-\frac{1}{Q_t}\right) \qquad (3)$$

a) The first order conditions are described by equations 1, 2, and 3.

b) We have money neutrality when $v=\gamma$. This will result in the neutralization of X_t in equations 2 and 3. If we assume no arbitrage, $Q_t = \frac{1}{\beta} = \Pi_t$ leading to a money neutral Euler + intratemporal optimality condition.

c) Steady State: $A_t = 1$

Firm Problem:

$$\max_{N_t} (1)N_t - \frac{W_t}{P_t}N_t$$

FOC: $1 = \frac{W_t}{P_t} \Rightarrow W_t = P_t$

Gov't BC: $B_t + M_t = P_t Tr_t + Q_{t-1}B_{t-1} + M_{t-1}$

Market clearing:

$$N_t^f = N_t^h$$
$$B_t^g = B_t^h$$
$$M_t^g = M_t^h$$
$$Y_t = C_t$$

$$\frac{W_t}{P_t} X_t^{v-\gamma}(1-\theta) C_t^{-v} = \chi N_t^{\varphi} \quad (1)$$

$$X_t^{v-\gamma} C_t^{-v} \cdot \frac{P_{t+1}}{P_t} = \beta Q_t X_{t+1}^{v-\gamma} C_{t+1}^{-v} \quad (2)$$

$$\left(\frac{M_t}{P_t}\right)^{-v} = \frac{1-\theta}{\theta} C_t^{-v}\left(1-\frac{1}{Q_t}\right) \quad (3)$$

$$SS \Rightarrow \quad C_t = C_{t+1} = C_{ss}$$

$$X_t = X_{t+1} = X_{ss}$$

$$\frac{M_t}{P_t} = \frac{M_{t+1}}{P_{t+1}} = m_{ss}$$

we have $\quad W_t = P_t$

$$X_{ss}^{v-\gamma}(1-\theta) C_{ss}^{-v} = \chi N_t^{\varphi}$$

Let $P_{t+1}/P_t = \Pi_{ss}$

$$* \quad m_{ss}^{-v} = \frac{1-\theta}{\theta} C_{ss}^{-v}\left(1-\frac{1}{Q_{ss}}\right) = \frac{1-\theta}{\theta} C_{ss}^{-v}(1-\beta)$$

$$m_{ss} = \left[\left(\frac{1-\theta}{\theta}\right)(1-\beta)\right]^{-1/v} C_{ss}$$

$$Y_{ss} = (1) N_{ss} = C_{ss}$$

$$\Rightarrow \quad X_{ss}^{v-\gamma}(1-\theta) C_{ss}^{-v} = \chi C_{ss}^{\varphi}$$

$$** \quad \Leftrightarrow \quad X_{ss}^{v-\gamma} \frac{(1-\theta)}{\chi} = C_{ss}^{\varphi+v}$$

plug in $*$ to $**$

$$\left[(1-\theta) C_{ss}^{1-v} + \theta\, m_{ss}^{1-v}\right]^{\frac{v-\gamma}{1-v}} \frac{1-\theta}{\chi} = C_{ss}^{\varphi+v}$$

$$\left[(1-\theta) C_{ss}^{1-v} + \theta\left[\frac{1-\theta}{\theta}(1-\beta)\right]^{\frac{-(1-v)}{v}} C_{ss}^{1-v}\right]^{\frac{v-\gamma}{1-v}} \frac{1-\theta}{\chi} = C_{ss}^{\varphi+v}$$

$$C_{ss}^{v-\gamma}\left[(1-\theta) + \theta\left(\frac{1-\theta}{\theta}\right)(1-\beta)^{-\frac{(1-v)}{v}}\right]^{\frac{v-\gamma}{1-v}} \frac{1-\theta}{\chi} = C_{ss}^{\varphi+v}$$

$$\star \quad C_{ss} = \left[\frac{1-\theta}{\chi}\right]^{\frac{1}{\varphi+\gamma}}\left[(1-\theta) + \theta\left[\frac{1-\theta}{\theta}(1-\beta)\right]^{-\frac{(1-v)}{v}}\right]^{\left(\frac{v-\gamma}{1-v}\right)\left(\frac{1}{\varphi+\gamma}\right)}$$

d) Algorithm to solve for S.S.

1 First Parameterize all exogenous variables,
$\varphi, \theta, v, \chi, \beta, \gamma$

2 Next allow $Q_{ss} = \frac{1}{\beta} = \Pi_{ss}$.
This pins down the money supply.

3 Solve for C_{ss} from M_{ss} eq. (3)

4 $C_{ss} = N_{ss}$.

5 Select some Tr and check that Bonds clear.

e) Calibrate θ given v

$$C_{ss} = \left[\frac{1-\theta}{\chi}\right]^{\frac{1}{\varphi+\gamma}} \left[(1-\theta) + \theta\left(\frac{1-\theta}{\theta}(1-\beta)\right)^{-\frac{(1-v)}{v}} \right]^{\left(\frac{v-\gamma}{1-v}\right)\left(\frac{1}{\varphi+\gamma}\right)}$$

We need the moment for C_{ss}.
That or the moments for

$$(3) \quad \left(\frac{M_t}{P_t}\right)^{-v} = \frac{1-\theta}{\theta} C_t^{-v}\left(1 - \frac{1}{Q_t}\right)$$

Then we solve for θ.

f. Given knowledge of other parameters.

we set M s.t $P=1$

from eq 3:

$$\left(\frac{M_t}{P_t}\right)^{-v} = \frac{1-\theta}{\theta} C_t^{-v} \left(1-\frac{1}{\beta}\right)$$

$$M^* = \left(\frac{1-\theta}{\theta}\right)^{-1/v} C^* \left(1-\frac{1}{\beta}\right)^{-1/v}$$

and we have $C_{ss} = C^*$ calculated in an earlier problem.

g) Solution to Log linearization

$$\hat{y}_t = \hat{c}_t$$

$$\hat{m}_t = \hat{p}_t - \hat{c}_t + 1/v \left(\frac{\beta}{1-\beta} \right) \hat{q}_t = 0$$

$$\varphi \hat{n}_t - (\hat{\omega}_t - \hat{p}_t) - (v - \gamma) \hat{x}_t + v \hat{c}_t = 0$$

$$(\gamma - \gamma) \hat{x}_t - v \hat{c}_t + \hat{p}_{t+1} - \hat{p}_t - \hat{q}_t$$
$$- (v - \gamma) \hat{x}_{t+1} + v \hat{c}_{t+1} = 0$$

$$(\hat{x}_t - \hat{c}_t)\left((1-\theta) c_t^{*(1-v)} \right) + (\hat{x}_t - \hat{m})(\theta m_t^{*1-v})$$
$$= \theta \hat{p}_t$$

$$\hat{n} = \hat{y}$$
$$\hat{p} = \hat{\omega}$$

for derivations and scratch work continue to the "Appendix" found after solutions for i and j.

i) Intuitive interpretation of results.

Graph 1 I believe is prices.

The responsiveness of prices to money supply is perfectly negatively correlated for all r.

We see in Graph 4 the importance of a change in money supply slip away as r increases. This is also true of Graph 5 and 2.

j) Low r means consumption and holding money is complementary, and high r means they are substitutes.

$r = 1$ means money has no real impact.

An increase in money supply $\Rightarrow$ prices $\searrow$ $\Rightarrow$ consumption increases $\Leftrightarrow$ $r > 1$.

Appendix

Not all work is good, and probably not all work is correct, but here is my scratch work.

g Log linearization

$$\frac{W_t}{P_t} X_t^{v-\gamma}(1-\theta) C_t^{-v} = \chi N_t^{\varphi} \quad (1)$$

$$X_t^{v-\gamma} C_t^{-v} \cdot \frac{P_{t+1}}{P_t} = \beta Q_t X_{t+1}^{v-\gamma} C_{t+1}^{-v} \quad (2)$$

$$\left(\frac{M_t}{P_t}\right)^{-v} = \frac{1-\theta}{\theta} C_t^{-v}\left(1 - \frac{1}{Q_t}\right) \quad (3)$$

(1) $f(W, P, X, C) = g(N)$

$= f(e^{\ln W}, e^{\ln P}, e^{\ln X}, e^{\ln C}) = g(e^{\ln N})$

$\Leftrightarrow \ln f(\cdot) = \ln g(\cdot)$

Center around SS. (*)

$\ln f(e^{\ln W}, e^{\ln P}, e^{\ln X}, e^{\ln C})$

$$\approx \ln f(W^*, P^*, X^*, C^*) + \frac{1}{f(W^*, P^*, X^*, C^*)}\Big[\partial f/\partial W \; W \; (\ln W - \ln W^*) + \partial f/\partial P \cdot P[\ln P - \ln P^*] + \partial f/\partial X \cdot X[\ln X - \ln X^*] + \partial f/\partial C \cdot C[\ln C - \ln C^*]\Big]$$

$$\approx \ln g(N^*) + \frac{1}{g(N^*)} \partial g/\partial N \cdot N(\ln N - \ln N^*)$$

note: $\ln g^* = \ln f^*$)

$$\Rightarrow \hat{N}\varphi g(\cdot) = \hat{W} f(\cdot) + \hat{X}(v-\gamma) f(\cdot) - \hat{P} f(\cdot) + \hat{C}(-v) f(\cdot)$$

$$\Rightarrow \varphi \hat{N}_t = \hat{w}_t - \hat{P}_t + (v - \gamma)\hat{X}_t - v\hat{C}_t \qquad *$$

$$X_t^{v-\gamma} C_t^{-v} \cdot \frac{P_{t+1}}{P_t} = \beta Q_t X_{t+1}^{v-\gamma} C_{t+1}^{-v} \qquad (2)$$

$$** \ (v-\gamma)\hat{X}_t - v\hat{C}_t + \hat{p}_{t+1} - \hat{p}_t = \hat{q}_t + (v-\gamma)X_t - v\hat{C}_{t+1}$$

$$\underbrace{\left(\frac{M_t}{P_t}\right)^{-v}}_{(a)} = \underbrace{\frac{1-\theta}{\theta} C_t^{-v}\left(1 - \frac{1}{Q_t}\right)}_{(b)} \qquad (3)$$

$$\hat{X} = \frac{X_t - X^*}{X^*}$$

(a) $-v(\hat{m}_t - \hat{p}_t)$

(b) $\ln\left(\frac{1-\theta}{\theta} C_t^{-v}\left(1 - \frac{1}{Q_t}\right)\right)$

$$= \underbrace{\ln\left(\frac{1-\theta}{\theta}\right)}_{0} \underbrace{- v \ln C_t}_{-vC_t^*} + \underbrace{\ln\left(1 - \frac{1}{Q_t}\right)}_{(c)}$$

(c) $\ln\left(1 - \frac{1}{Q_t}\right) = \ln\left(1 - \frac{1}{Q^*}\right)$

$$+ \underbrace{\frac{1}{1 - \frac{1}{Q^*}} \cdot \left(\frac{1}{Q^{*2}}\right)(Q_t - Q^*)}_{\frac{(Q_t - Q_*)}{Q^{*2}}\left(\frac{Q^*}{Q^* - 1}\right)}$$

$$= \hat{q}_t \left(\frac{1}{Q^* - 1}\right)$$

$$\therefore (3) \ -v(\hat{m}_t - \hat{p}_t) = -v\hat{C}_t + \hat{q}_t\left(\frac{1}{Q^* - 1}\right)$$

$$\varphi \hat{N}_t = \hat{w}_t - \hat{p}_t + (v-\gamma)\hat{x}_t - v\hat{c}_t$$

$$(v-\gamma)\hat{x}_t - v\hat{c}_t + \underbrace{\hat{p}_{t+1} - \hat{p}_t}_{\pi} = \hat{q}_t + (v-\gamma)x_{t+1} - v\hat{c}_{t+1}$$

$$-v(\hat{m}_t - \hat{p}_t) = -v\hat{c}_t + \hat{q}_t\left(\frac{1}{Q^* - 1}\right)$$

log linearized.

$$\frac{W_t}{P_t} x_t^{v-\gamma}(1-\theta)C_t^{-v} = \chi N_t^{\varphi} \qquad (1)$$

$$x_t^{v-\gamma} C_t^{-v} \cdot \frac{P_{t+1}}{P_t} = \beta Q_t x_{t+1}^{v-\gamma} C_{t+1}^{-v} \qquad (2)$$

$$\left(\frac{M_t}{P_t}\right)^{-v} = \frac{1-\theta}{\theta} C_t^{-v}\left(1 - \frac{1}{Q_t}\right) \qquad (3)$$

g) SS:

$$C^* = N^*$$

$$Q^* = \frac{1}{\beta}$$

$$P^* = 1 \implies \pi_{ss} = 1$$

$$W = P = 1$$

$$(3) \quad M^{-v} = \frac{1-\theta}{\theta} C^{-v}\left(1 - \frac{1}{\beta}\right)$$

$$\frac{M^*}{C^*} = \left[\frac{1-\theta}{\theta}\left(1 - \frac{1}{\beta}\right)\right]^{-1/v}$$

Log Linearization

$$X_t^{1-\nu} = (1-\theta)\, C_t^{1-\nu} + \theta\left(\frac{M_t}{P_t}\right)^{1-\nu}$$

$$f(e^{\log x}) = g(e^{\log c}, e^{\log M}, e^{\log P})$$

$$f(X^*) + X_t^{*\,1-\nu}(1-\nu)(\log x_t - \log x^*)$$

$$\cancel{f(X^*)} + f(X^*)(1-\nu)\hat{x}_t$$

$$= \cancel{g(C^*, M^*, P^*)} + g_1(\cdot^*)\, e^{\log c^*}(\log c - \log c^*)$$
$$+ g_2(\cdot^*)\, e^{\log m^*}(\log m - \log m^*)$$
$$+ g_3(\cdot^*)\, e^{\log p^*}(\log p - \log p^*)$$

$$f(x^*)\cancel{(1-\nu)}\hat{x}_t = (1-\nu)(1-\theta)\, C^{*\,1-\nu}(\hat{C}_t)$$
$$+ (1-\nu)(\theta)\, m^{*\,1-\nu}(\hat{m}_t)$$
$$- (1-\nu)(\theta)\, p^{*\,\nu-1}\hat{p}_t$$

$$= \cancel{(1-\nu)}\left[(1-\theta)c^{*\,1-\nu}\hat{c}_t + \theta m^{*\,1-\nu}\hat{m}_t - \theta p^{*\,\nu-1}\hat{p}_t\right]$$

$$\hat{X}_t = \frac{(1-\theta)c^{*\,1-\nu}\hat{c}_t + \theta m^{*\,1-\nu}\hat{m}_t - \theta p^{*\,\nu-1}\hat{p}_t}{X_t^{*\,1-\nu}}$$

Firm: $\hat{y}_t = \hat{n}_t$

MC: $\hat{y}_t = \hat{c}_t$

$$\hat{x}_t \left[(1-\theta) c^{*\,1-v} + \theta \left(\frac{M^*}{P^*} \right)^{1-v} \right]$$

$P^* = 1$

$$(\hat{x}_t - \hat{c}_t) \left[(1-\theta) c^{*\,1-v} \right] + (\hat{x}_t - \hat{m}_t) \left[\theta m^{*\,1-v} \right] = \theta \hat{p}_t$$

DAG

exogenous

$A = 1$

unknown

n, m

n

firm

$Y = N$ $\quad$ $w = P$

n
m

households

$c = n$

$c + m - p = w - p + n + m_{-1} - p$

c x

p

bond market

y

q

market clearing

$Y = c$

$m - p = c - \frac{1}{\nu} \frac{\beta}{1-\beta} q$

Probably terrible

$$\phi_{1,m} = v\, I_T$$

$$\phi_{1,p} = -v\, I_T$$

$$\phi_{1,c} = -v\, I_T$$

$$\phi_{1,q} = \frac{\beta}{1-\beta} I_T$$

$$\phi_2\, x = (v-\gamma) I_T$$

$$\phi_2\, x_{+1} = -(v-\gamma) J_T$$

$$\phi_2\, c = -v\, I_T$$

$$\phi_2\, c_{+1} = v\, J_T$$

$$\phi_2\, p = -I_T$$

$$\phi_2\, p_{+1} = J_T$$

$$\phi_2\, q = -I_T$$

$$\phi_3\, n = \varphi\, I_T$$

$$\phi_3\, w = -I_T$$

$$\phi_3\, p = I_T$$

$$\phi_3\, x = -(v-\gamma) I$$

$$\phi_3\, c = v\, I_T$$

$$\phi_{4y} = I_T$$

$$\phi_{4n} = -I_T$$

let $J_T = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & \ddots & \vdots \\ & & & 0 & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$

$$\phi_{5y} = I_T$$

$$\phi_{5c} = -I_T$$

$$\phi_6\, w = I_T$$

$$\phi_6\, p = -I_T$$

DAG q_+

exogenous m

unknown q_+

$\downarrow$ $\qquad\qquad$ $\Big\downarrow q$

$$\frac{m_{-1}}{P_{-1}} = \frac{m}{P}$$

$/p$ $\qquad \xrightarrow{mp}$

firm

$p = w$ $\qquad \xrightarrow{w}$

hh

$$vm - vp - vc + q\frac{\beta}{1-\beta}$$

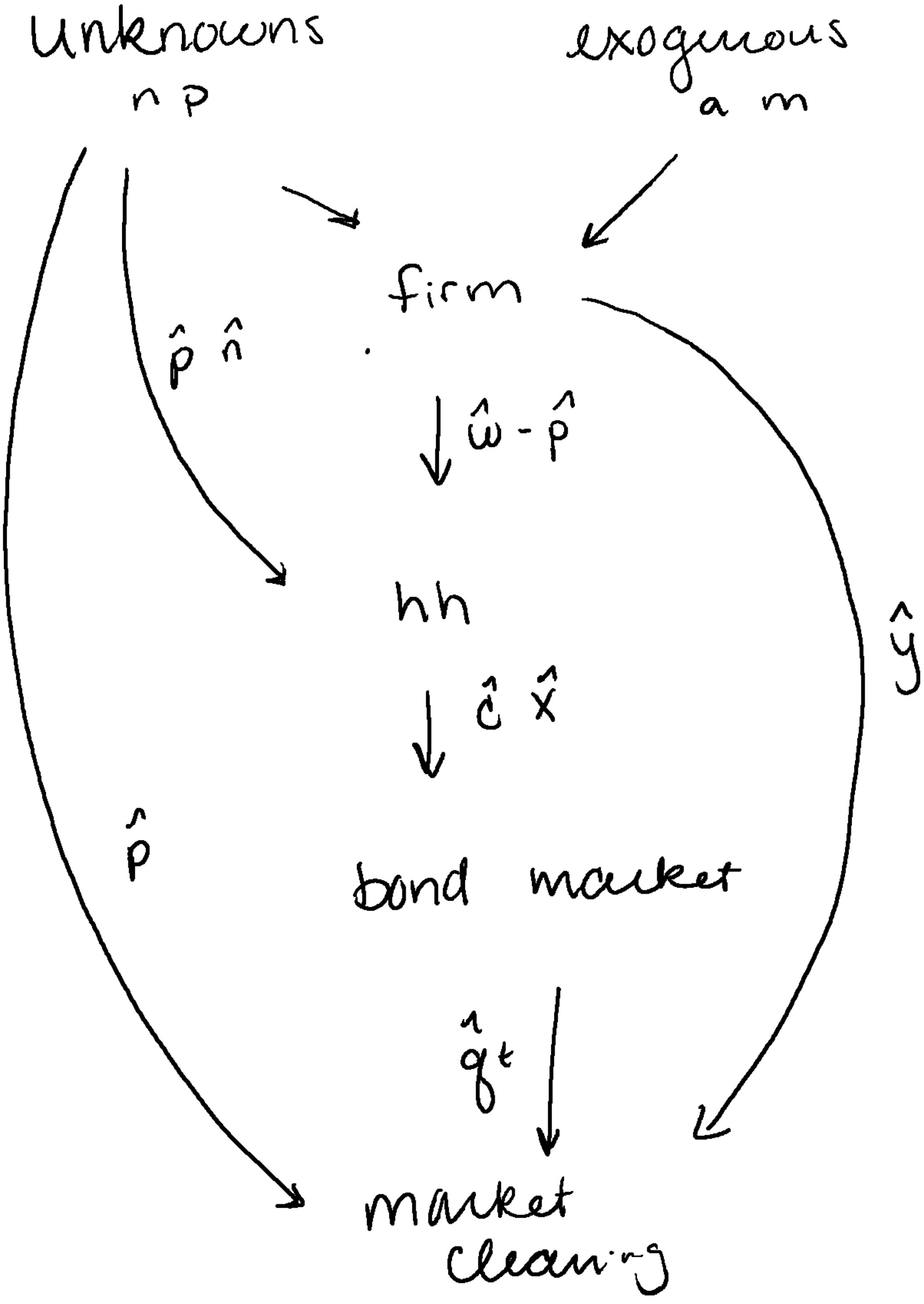
$$\varphi\hat{n}_t - \hat{w}_t + \hat{p}_t - (v-\gamma)\hat{x}_t + v\hat{c}_t$$

$\swarrow n, c$

mc

$y = c$

$U = \{q \quad c \quad p\}$



$$\hat{y}_t = \hat{c}_t$$

$$\hat{m}_t - \hat{p}_t = \hat{c}_t - \frac{1}{\nu}\frac{\beta}{1-\beta}\hat{Q}_t$$

$Y = \{ c, n, y, x, q, w-P, m, R$

$$H : \begin{bmatrix} mc \\ mon \end{bmatrix} \qquad \begin{array}{l} u = [n, P] \\ z = [m] \end{array}$$

$\partial H / \partial Y =$ mc block

$$\partial Y / \partial u = \begin{bmatrix} \partial Yf / \partial u \\ \partial Yn / \partial u \end{bmatrix}$$

$$\partial Y / \partial z = \begin{bmatrix} \partial Yf / \partial z \\ \partial Yn / \partial z \end{bmatrix}$$

$\partial Yf / \partial u =$