ECON 210C-2 PS 4

Stephanie Hutson

June 5, 2024

1 Productivity shocks in the Three Equations

(a) See attached notes for the work. Guess $\hat{y}_t = \Psi_{ya}\hat{a}_t$ and $\hat{\pi}_t = \Psi_{\pi a}\hat{a}_t$. Solve for Ψ_{ya} and $\Psi_{\pi a}$.

$$\Psi_{ya} = \frac{\kappa(1-\varphi)(1-\beta\rho)(\sigma\phi_\pi + \rho)}{(\gamma+\varphi)(1-\beta\rho)(\rho-1) - \kappa(\sigma\phi_\pi + \rho)}$$

$$\Psi_{\pi a} = \frac{\kappa(1-\varphi)(1-\beta\rho)(\rho-1)}{(\gamma+\varphi)(1-\beta\rho)(\rho-1) - \kappa(\sigma\phi_\pi + \rho)}$$

(b) IRFs:

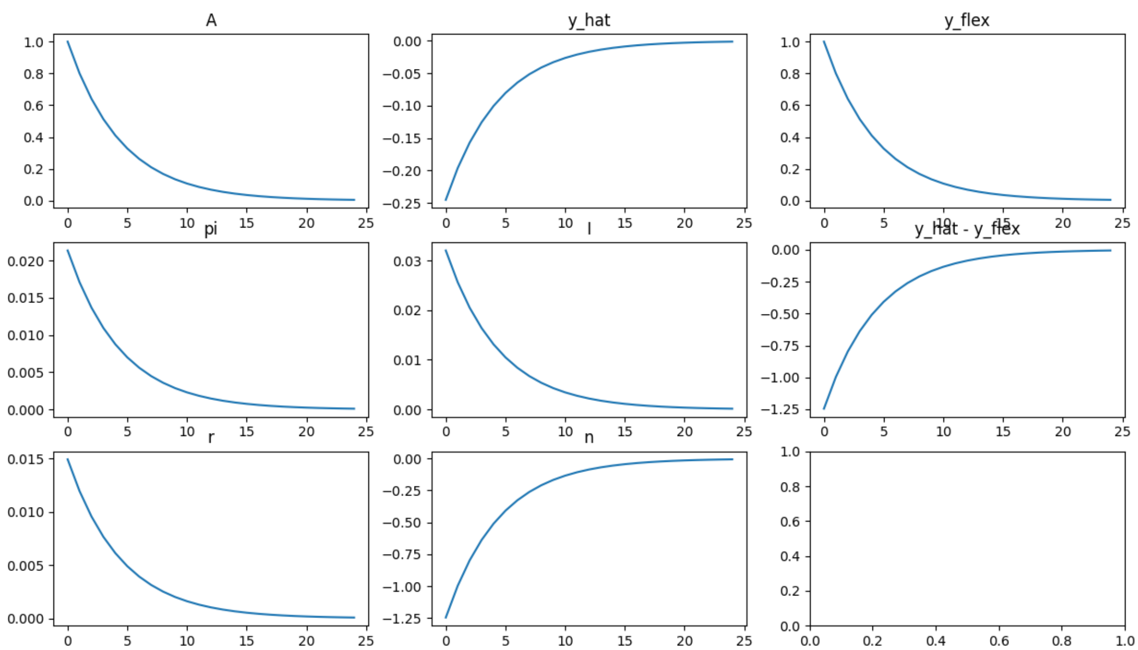

Figure 1: Q1.b

(c) First note that κ is a function of θ which describes what portion of firms can update their prices. A 1 unit increase in $\hat{a}$ naturally causes $\hat{y}$ to go up, because we assumed $\sigma = 1$, we are assuming log utilities, thus there is no intertemporal substitution of labor as a function of A in a flexible environment, so $\hat{y}$ tracks $\hat{a}$ precisely.

As it is, prices are not fully flexible, so $\hat{y}$ goes down, labor by more (the marginal return to labor is greater due to higher $\hat{a}$. $\hat{\pi}$ goes up, because prices are relatively higher. The real interest rate, which is $\hat{r}_t = \hat{i}_t - \mathbb{E}\hat{\pi}_{t+1}$ goes up but not by as much as either inflation or the nominal interest rate.

(d) My IRFs have pi as positive, whereas the irfs in the jupiter notebook. This probably means there is an error in my psi algebra.

2 Non-linear NK model in Jupyter

(a) *Why is the real reset price equation not recursive?* All of the pricing values within the expectation are just a weighting over the sum of all future (relative to price t, not t+s) weights. The denominator for any given s > 0 is backward and forward looking.

(b) see figure 2

2 b) $F_{2t} = \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} (P_{t+k}/P_t)^{\varepsilon-1} Y_{t+k}$

$\theta^0 \Lambda_{t,t} P_t/P_t Y_t = Y_t$

$= Y_t + \theta \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+1+k} \left(\frac{P_{t+1+k}}{P_t}\right)^{\varepsilon-1} Y_{t+1+k}$

$= Y_t + \theta \left(\frac{P_{t+1}}{P_t}\right)^{\varepsilon-1} \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+1+k} \left(\frac{P_{t+1+k}}{P_{t+1}}\right)^{\varepsilon-1} Y_{t+1+k}$

$= Y_t + \theta \Pi_{t+1}^{\varepsilon-1} \Lambda_{t,t+1} \sum_{k=0}^{\infty} \theta^k \Lambda_{t+1,t+1+k} \left(\frac{P_{t+1+k}}{P_{t+1}}\right)^{\varepsilon-1} Y_{t+1+k}$

$= Y_t + \theta \Pi_{t+1}^{\varepsilon-1} \Lambda_{t,t+1} F_{2t+1}$

Figure 2: 2.b

(c) see figure 3

(d) see figure 4

(e) When $p^* > 1$ then

$$
\begin{aligned}
1 &> \theta \Pi_t^{\epsilon-1} + (1-\theta) \\
\theta &< \theta \Pi_t^{\epsilon-1} \\
1 &> \Pi_t^{\epsilon-1} \\
1 &< \Pi_t
\end{aligned}
$$

for $\epsilon < 1$

(f) (See Code)

(g) (See Figure 5)

(h) If theta is close to zero, then the impulse response functions perfectly map to the changes in productivity. The prices nearly always change and are flexible. Where theta is close to 1, prices are very sticky, so inflation is practically zero.

(i) The same shock in an RBC model without capital would look like $\theta = 0.9999$ (or infact, $\theta = 1$). The main difference is that in RBC you have competitive

2c) $F_{1t} \equiv (1+\mu)\sum_{s=0}^{\infty} \theta^s \Lambda_{t,t+s} Y_{t+s} \left(P_{t+s}/P_t\right)^{\varepsilon-1} \frac{W_{t+s}/P_t}{A_{t+s}}$

$$= (1+\mu) Y_t \frac{W_t/P_t}{A_t} + (1-\mu)\theta \Pi_{t+1}^{\varepsilon-1} \sum_{s=0}^{\infty}\theta^s \Lambda_{t,t+1+s} Y_{t+1+s} \left(\frac{P_{t+s+1}}{P_{t+1}}\right)^{\varepsilon-1} \frac{W_{t+s}/P_t}{A_{t+1+s}}$$

$$= (1-\mu)Y_t \frac{W_t/P_t}{A_t} + \left[(1-\mu)\theta\Pi^{\varepsilon-1}\Lambda_{t,t+1} \sum_{s=0}^{\infty} \theta^s \Lambda_{t+1,t+1+s} Y_{t+1+s}\left(\frac{P_{t+s+1}}{P_{t+1}}\right)\frac{W_{t+1+s}/P_{t+1}}{A_{t+1}} \cdot \frac{P_{t+1}}{P_t}\right]$$

$$= (1-\mu)Y_t \frac{W_t/P_t}{A_t} + \Big[(1-\mu)\theta\Pi^{\varepsilon}\Lambda_{t,t+1} \cdot \sum_{s=0}^{\infty} \theta^s \Lambda_{t+1,t+1+s} Y_{t+1+s}\left(\frac{P_{t+1+s}}{P_{t+1}}\right)\frac{W_{t+1+s}/P_{t+1}}{A_{t+1}}$$

$$= (1-\mu)Y_t\frac{W_t/P_t}{A_t} + (1-\mu)\theta\Pi^{\varepsilon}\Lambda_{t,t+1}F_{1t+1}$$

Figure 3: 2.c

2d: $P_t = [\theta P_{t-1}^{1-\varepsilon} + (1-\theta)P_t^{*\,1-\varepsilon})^{1/1-\varepsilon}$

$$P_t^{1-\varepsilon} = \theta P_{t-1}^{1-\varepsilon} + (1-\theta)P_t^{*\,1-\varepsilon}$$

$$1 = \theta\Pi^{\varepsilon-1} + (1-\theta)\frac{P_t^{*\,1-\varepsilon}}{P_t^{1-\varepsilon}}$$

how does

$\frac{P_t^*}{P_t} = p_t^*$?

Figure 4: 2.d

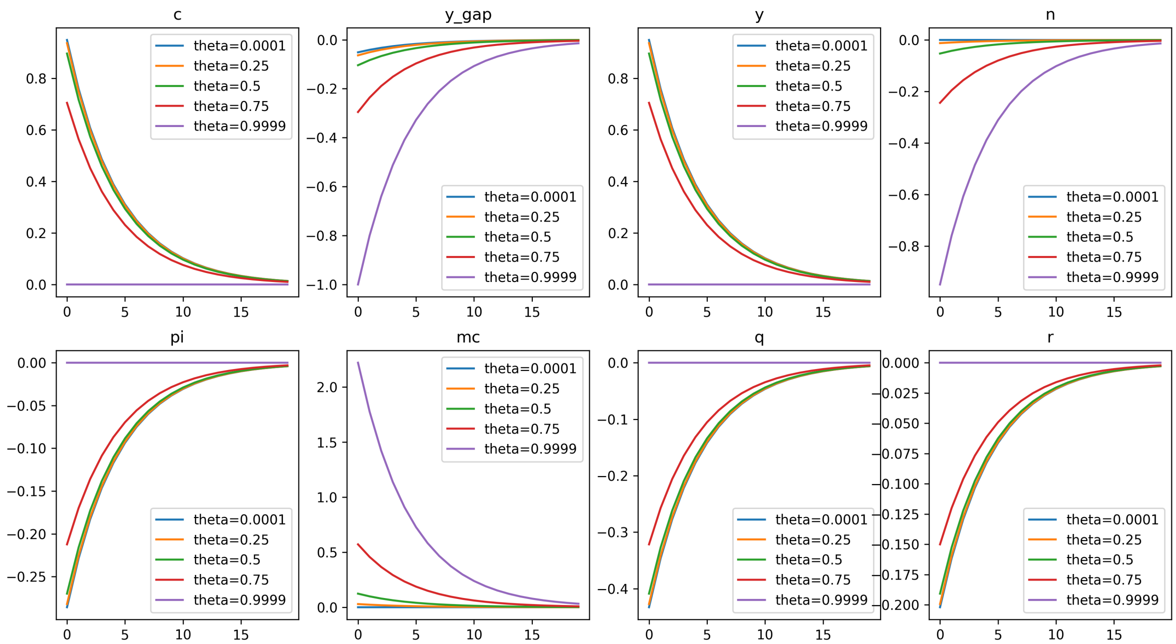
c
y_gap
y
n
pi
mc
q
r
theta=0.0001
theta=0.25
theta=0.5
theta=0.75
theta=0.9999

Figure 5: Q2.g

Stephanie Hutson

Problem Set 4.

1) $\hat{y}_t = -\sigma[\hat{i}_t - \mathbb{E}\{\hat{\pi}_{t+1}\}] + \mathbb{E}\{\hat{y}_{t+1}\}$

$\hat{\pi}_t = \kappa(\hat{y}_t - \hat{y}_t^{flex}) + \beta\mathbb{E}\{\hat{\pi}_{t+1}\}$

$\hat{i}_t = \phi_\pi \pi_t + v_t$

$\hat{a}_t = \rho_a \hat{a}_{t-1} + \varepsilon_t$

guess $\hat{y}_t = \Psi_{ya}\hat{a}_t$

$\hat{\pi}_t = \Psi_{\pi a}\hat{a}_t$

$\Psi_{ya}\hat{a}_t = \hat{y}_t = -\sigma[\phi_\pi \Psi_{\pi a}\hat{a}_t - \mathbb{E}[\Psi_{\pi a}\hat{a}_{t+1}]$

$+ \mathbb{E}\Psi_{ya}a_{t+1}$

$= \hat{a}_t[-\sigma\phi_\pi\Psi_{\pi a} - \Psi_{\pi a}\rho + \Psi_{ya}\rho]$

$\Psi_{ya} = -\sigma\phi_\pi\Psi_{\pi a} - \Psi_{\pi a}\rho + \Psi_{ya}\rho$

$\Psi_{ya} = \dfrac{\Psi_{\pi a}(\sigma\phi_\pi + \rho)}{\rho - 1}$

$\Psi_{\pi a}\hat{a}_t = \kappa[\Psi_{ya}\hat{a}_t - \frac{1+\varphi}{\gamma+\varphi}\hat{a}_t] + \beta\rho\hat{a}_t\Psi_{\pi a}$

$\Psi_{\pi a}(1-\beta\rho) = \kappa[\Psi_{ya} - \frac{1+\varphi}{\gamma+\varphi}]$

$\Psi_{\pi a} = \left[\dfrac{\kappa\Psi_{\pi a}(\sigma\phi_\pi + \rho)}{\rho-1} - \kappa\frac{1+\varphi}{\gamma+\varphi}\right] / 1-\beta\rho$

$\Psi_{\pi a}\left(1 - \dfrac{\kappa(\sigma\phi_\pi+\rho)}{(1-\beta\rho)(\rho-1)}\right) = \kappa\dfrac{1+\varphi}{\gamma+\varphi}$

☆ $\Psi_{\pi a} = \dfrac{\kappa(1+\varphi)(1-\beta\rho)(\rho-1)}{(\gamma+\varphi)((1-\beta\rho)(\rho-1) - \kappa(\sigma\phi_\pi+\rho))}$

$\Psi_{ya} = \dfrac{\kappa(1+\varphi)(1-\beta\rho)(\sigma\phi_\pi+\rho)}{(\gamma+\varphi)((1-\beta\rho)(\rho-1) - \kappa(\sigma\phi_\pi+\rho))}$

2 b) $F_{2t} \equiv \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} (P_{t+k}/P_t)^{\varepsilon-1} Y_{t+k}$

$$\theta^0 \Lambda_{t,t}\, P_t/P_t\, Y_t = Y_t$$

$$= Y_t + \theta \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+1+k} \left(\frac{P_{t+1+k}}{P_t}\right)^{\varepsilon-1} Y_{t+1+k}$$

$$= Y_t + \theta \left(\frac{P_{t+1}}{P_t}\right)^{\varepsilon-1} \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+1+k} \left(\frac{P_{t+1+k}}{P_{t+1}}\right)^{\varepsilon-1} Y_{t+1+k}$$

$$= Y_t + \theta \Pi_{t+1}^{\varepsilon-1} \Lambda_{t,t+1} \sum_{k=0}^{\infty} \theta^k \Lambda_{t+1,t+1+k} \left(\frac{P_{t+1+k}}{P_{t+1}}\right)^{\varepsilon-1} Y_{t+1+k}$$

$$= Y_t + \theta \Pi_{t+1}^{\varepsilon-1} \Lambda_{t,t+1} F_{2t+1}$$

2 c) $F_{1t} \equiv (1+\mu) \sum_{s=0}^{\infty} \theta^s \Lambda_{t,t+s} Y_{t+s} (P_{t+s}/P_t)^{\varepsilon-1} \frac{W_{t+s}/P_t}{A_{t+s}}$

$$= (1+\mu) Y_t \frac{W_t/P_t}{A_t} +$$

$$(1-\mu)\theta \Pi_{t+1}^{\varepsilon-1} \sum_{s=0}^{\infty} \theta^s \Lambda_{t,t+1+s} Y_{t+1+s} \left(\frac{P_{t+s+1}}{P_{t+1}}\right)^{\varepsilon-1} \frac{W_{t+1+s}/P_t}{A_{t+1+s}}$$

$$= (1-\mu) Y_t \frac{W_t/P_t}{A_t} + \Big[(1-\mu)\theta \Pi^{\varepsilon-1} \Lambda_{t,t+1}$$

$$\sum_{s=0}^{\infty} \theta^s \Lambda_{t+1,t+1+s} Y_{t+1+s} \left(\frac{P_{t+s+1}}{P_{t+1}}\right) \frac{W_{t+1+s}/P_{t+1}}{A_{t+1}} \cdot \frac{P_{t+1}}{P_t} \Big]$$

$$= (1-\mu) Y_t \frac{W_t/P_t}{A_t} + \Big[(1-\mu)\theta \Pi^{\varepsilon} \Lambda_{t,t+1} \cdot$$

$$\sum_{s=0}^{\infty} \theta^s \Lambda_{t+1,t+1+s} Y_{t+1+s} \left(\frac{P_{t+1+s}}{P_{t+1}}\right) \frac{W_{t+1+s}/P_{t+1}}{A_{t+1}}$$

$$= (1-\mu) Y_t \frac{W_t/P_t}{A_t} + (1-\mu)\theta \Pi^{\varepsilon} \Lambda_{t,t+1} F_{1t+1}$$

2d: $P_t = [\theta P_{t-1}^{1-\varepsilon} + (1-\theta) P_t^{*\,1-\varepsilon})^{1/1-\varepsilon}$

$$P_t^{1-\varepsilon} = \theta P_{t-1}^{1-\varepsilon} + (1-\theta) P_t^{*\,1-\varepsilon}$$

$$1 = \theta \Pi^{\varepsilon-1} + (1-\theta) \frac{P_t^{*\,1-\varepsilon}}{P_t^{1-\varepsilon}}$$

how does

$$\frac{P_t^*}{P_t} = p_t^* ?$$

This doesn't track with other log linearization we have done...