

Term Project

ACT460: Stochastic Methods for Finance and Actuarial Science Fall 2025

1 Overview

The term project is a group assignment to be completed in teams of four students. Teams of three will only be permitted in exceptional cases when there are not enough students to form a group of four. The project description can be found on page 4 of this document. Each team must answer questions 1 through 5, and then select one option type from those provided for question 6 and answer question 6 accordingly. Note that for question 6, each option type may be chosen by only one team, assigned on a first-come, first-served basis.

Once your team is formed and you have selected your option type for question 6, please sign up here: [sign-up-sheet](#).

- **Sign-Up Deadline:** Monday October 27th, 11:55 pm.

You will have three key deliverables:

1. An in-class presentation on November 25th or 27th
 - **Presentation Submission Deadline:** One submission per team—in PDF or PowerPoint format—due on November 24th, 11:55 pm (10% of final grade).
2. A written group report with accompanying code
 - **Group Report and Code Submission Deadline:** One submission per team—a report in PDF format and code in a Jupyter Notebook—due on November 27th, 11:55 pm (19% of final grade).
3. A written individual contribution statement
 - **Individual Contribution Statement Submission Deadline:** One submission per team member—in PDF format—due on November 27th, 11:55 pm (1% of final grade).

2 Details: Group Report and Individual Contribution Statement

2.1 Goup Report

Deliverables for the report are:

1. A **self-contained report** which addresses all the questions. A report is not simply a printout of numerical results or bullet point answers, but rather a formalized document using LaTeX/Overleaf or Microsoft Word, written in proper academic English.

It is strongly recommended to use LaTeX to typeset your report and especially to use Overleaf so that you can collaborate. A template is provided to get you started: [Link to report template](#). Overleaf has resources to help you learn LaTeX. See [here](#).

You must include the name, student ID, and email address of each team member on the first page.

A well-written report should include a variation of the following:

- an abstract of approximately five sentences, which gives a brief overview of the report and highlights key findings (but not too many) of the project;
- an introduction describing the problem of interest. It must contain a paragraph identifying the sections and structure of the report;
- a description of your work, well-organized in sections and/or subsections, explaining the key steps of your methods. To write your report as accurately as possible, use active language (instead of passive language) and present tense (instead of past tense) when describing your work;
- your interpretations of the results that are supported with plots and tables. All figures must be labeled, easily readable and followed by a descriptive caption. Plots and tables should be informative and not just a collection of outputs;
- a conclusion summarizing your report and emphasizing the key findings. It may include other ideas that were not investigated in the current report and potential future ideas to investigate.

- You must include references for all work that is not yours. Not doing so is an academic offence.

Note: when plotting sample paths, best practice is to simulate a large number of trajectories, and plots some percentiles (ex. 10, 50, 90) as well as a few example trajectories.

2. A well-documented Jupyter notebook containing the code used within the report.

Report Evaluation: Your team will be evaluated using the following rubric.

- Correctness of approach and results: 9%
- Clarity and completeness in describing the approach and results: 5%
- Figures, tables, and writing (readable, properly labeled, informative): 5%

2.2 Individual Contribution Statement

You must submit a brief individual statement (at most one page), which will: (1) Describe what parts of the project you worked on and what parts of the project your teammates worked on, (2) What you learned from the project.

The purpose of the individual statement is to facilitate fair grading and to allow the instructor to understand well what you learned from the project. Note that individual statements will be used to adjust individual grades in exceptional circumstances only (ex. one student did not participate in the project at all despite documented attempts to include them).

3 Details: Presentation

- You and your group will give a joint presentation of **8 minutes** on your topic to the class. Note that all team members must participate in the presentation.
- There will be a 2-minute question period following the presentation, where your group will answer questions from the class.
- Submit slides to the instructor in PDF or Powerpoint format.

Topic: Each team must present **two questions of their choice** from Questions 1–5, as well as **Question 6**, for a total of three questions. This format has been adopted due to time constraints.

Your presentation should include an introduction with relevant background information, a summary of the research question(s), and the results. Be sure to include the outcomes of your numerical experiments, supported by figures and/or tables.

Slides: You can prepare your slides using either Beamer, PowerPoint or any other software that produces a PDF output. You should make sure that:

- Mathematical equations are properly formatted,
- Figures are outputted with appropriate font size, line width, etc., to be visible on the slide,
- If tables are included, they are readable on the slide, with appropriate highlighting.

You may optionally use LaTeX to make your presentation: [Link to Beamer presentation template](#). This is *not* required.

Participation mark: The presentations will be organized into three sessions to take place during the lecture time in the week of November 24th. Groups will be randomly assigned to a session. The presentation order within each session will also be determined at random during the session. You are required to attend all other presentations in your session and prepare at least one question for each group.

Presentation Evaluation: Your team will be evaluated using the following rubric.

- Clarity of presentation: 2%
- Slide quality: 2%
- Correctness: 2%
- Answers to and asking of questions: 2%
- Adherence to time limits: 2%

4 Academic integrity

Academic integrity is fundamental to learning and scholarship at the University of Toronto. As a result, U of T treats cases of cheating and plagiarism very seriously. Please read the University of Toronto's Code of Behaviour on Academic Matters available at: <http://www.governingcouncil.utoronto.ca/policies/behaveac.htm>.

In particular, **all written work, slides, and code must be your own work.** The use of paid services, tutoring, or generative AI to create any deliverable for this project is not allowed under any circumstances.

5 Project Description: Path-dependent options

1. An Asian option is an option where the payoff depends on the average level of the underlying asset (compared with just the terminal value of the underlying in a European option). For example, an Asian call option using the arithmetic average of asset prices has payoff

$$(\bar{S} - K)_+, \quad \text{where } \bar{S} = \frac{1}{m} \sum_{i=1}^m S_{t_i}.$$

Estimate the price of an Asian call using the Black-Scholes model and Monte Carlo simulation. Use the initial (annualized) parameters:

$$S_0 = 100, \quad K = 100, \quad r = 0.05, \quad \sigma = 0.4, \quad T = 1.$$

2. Explore the effects of model parameters on the price of the Asian option.
3. Explore the effect of discretization error on the estimated prices of the Asian option for the parameter values provided in Question 1.
4. Estimate the following Greeks: the delta $\frac{\partial Y}{\partial S_0}$, the vega $\frac{\partial Y}{\partial \sigma}$, and the rho $\frac{\partial Y}{\partial r}$ for the Asian option. Use both finite difference methods and pathwise estimators. Compare the approaches for the parameter values provided in Question 1.
5. Now consider an Asian call option written on the geometric average,

$$\bar{S} = \left(\prod_{i=1}^m S_{t_i} \right)^{1/m}.$$

Following [Glasserman], derive a closed-form expression for the price of this Asian call option. For the parameter values provided in Question 1, compare the analytic price with Monte Carlo price estimate for the geometric average Asian call option.

6. Other types of path-dependent options include barrier options and lookback options (see [Hull], Chapter 26, for descriptions of several barrier and lookback options). For example, a down-and-out call with barrier b , strike K , and expiration T has payoff

$$\mathbb{1}_{\{\tau_b > T\}}(S_T - K)_+, \quad \text{where } \tau_b := \inf\{t : S_t < b\}.$$

Choose one of the following option types. Each option type may be selected by only one team on a first-come, first-served basis. Explore the selected option and answer the following questions. You may use the parameter values provided in

Option Number	Option Type
1	Up-and-In Call
2	Up-and-In Put
3	Up-and-Out Call
4	Up-and-Out Put
5	Down-and-In Call
6	Down-and-In Put
7	Down-and-Out Call
8	Down-and-Out Put
9	Fixed-Strike Lookback Call
10	Fixed-Strike Lookback Put
11	Floating-Strike Lookback Call
12	Floating-Strike Lookback Put

Table 1: List of barrier and lookback option types.

Question 1 where applicable.

- (a) Describe the option and its potential uses.
- (b) Implement a Monte Carlo scheme to estimate the price of the option and compare it to the analytic solution given in [Hull].

References

- [1] Hull, J. C. (2022). *Options, Futures, and Other Derivatives* (11th ed.). Pearson Education.
[2] Glasserman, P. (2004). *Monte Carlo Methods in Financial Engineering*. Springer.