

# AugerBot Calculations

Quit

Trial 1: 9/19 - 9/26

Modified Francisco Calculations : 10/18 (Do not use!)

Plotting  $F_x$  to find  $U$  which Balances Forces: 11/1

Plotting  $F_x$  to find  $U$  which Balances Forces 2 : 11/9

Inside Equation for Thrust: 11/16

Backtracking: 11/27

Francisco' s with Chen' s Coefficients: 11/30 (CORRECT ONES)

Abs Val Test: 12/6

Quit;

- Modified convergence method. Abs not needed for Newton - Raphson because the graphs for these when  $F[U_{guess}] < 0$  makes it impossible to solve for a  $U$  which actually projects to a point on the  $F_x$  curve. Automatically skips Newton. Saves computation time. Explains flat line - \_\_ -
- $F_x(U_{guess})$  DNE for  $U_{guess} < 0$  due to natural log term

# Testing Auger Model with Correct Chen Coefficients

## Parameters

### For Helix

```
(*Current param: R = 1.8cm, n = 3.5*)
R = 0.018; (*Screw radius, m*)
n = 1; (*Number of helix turns*)
```

### For Material

```
(*LP poppy Fourier coefficients*)
A00 = 0.051; A10 = 0.047; B11 = 0.053; B01 = 0.083;
Bn11 = 0.020; C11 = -0.026; C01 = 0.057; Cn11 = 0; D10 = 0.025;

 $\beta = (\pi/2) - \phi$ ; (* $\phi$  is symbolic, radians*)
 $\alpha_z = Bn11 \sin[2 \pi (-\beta/\pi)] + A00 \cos[2 \pi \cdot 0] +$ 
 $B01 \sin[2 \pi \cdot 0] + A10 \cos[2 \pi (\beta/\pi)] + B11 \sin[2 \pi (\beta/\pi)];$ 
(*Vertical stress per unit depth, N/m^3*)
 $\alpha_x = Cn11 \cos[2 \pi (-\beta/\pi)] + C01 \cos[2 \pi \cdot 0] +$ 
 $D10 \sin[2 \pi (\beta/\pi)] + C11 \cos[2 \pi (\beta/\pi)];$ 
(*Horizontal stress per unit depth, N/m^3*)

d = 0.05; (*Depth robot buried, m*)

(*Friction coefficients, expressed in terms of  $\phi$ *)
Cn =  $\alpha_x \cdot d$ ; (*N/m^2*)
Ct =  $\alpha_z \cdot d$ ;
```

### For Motor

```
w = 2 * 1000 * (2 * Pi) / 3584; (*Angular velocity with 12V source, rad/s*)
■ (2 ticks/ms)*(1000 ms/s)*(2*Pi rad/rev)*(1 rev/3584 ticks)
```

## Horizontal Thrust Equation (The full one)

```
FxIn[U_] := (2 * Pi * n / Cos[ $\phi$ ]) *
  (((Cn - Ct) * w * Sin[ $\phi$ ] * Cos[ $\phi$ ]) * (((R / (2 * w^2)) * Sqrt[(R * w)^2 + U^2]) +
    (U^2 / (2 * w^3)) * (Log[U] - Log[R * w + Sqrt[(R * w)^2 + U^2]]))) -
  (U * (Ct * Sin[ $\phi$ ]^2 + Cn * Cos[ $\phi$ ]^2) * (Sqrt[(R * w)^2 + U^2] - U) / w^2);
```

$\phi$  input must be in radians

## Calculating $U/Rw$ when $F_x = 0$ for Many $\phi$ Cases

```

 $\phi = 10 * \text{Pi} / 180 // \text{N};$  (*Local inclination, radians*)
 $\phi\text{store} = \{\};$ 
 $U\text{store} = \{\};$ 
 $F\text{store} = \{\};$ 
 $F\text{Maxstore} = \{\};$  (*FxIn when  $u = 0.001$ *)

While[ $\phi < 90 * \text{Pi} / 180$ ,
  (*Print statements*)
  (*Print["Let  $\phi = ", \phi * 180 / \text{Pi}, "$  deg"];*)
  Print@
  Plot[{ $F_x\text{In}[U]$ , 0}, { $U$ , 0, 0.1}, PlotLabel → "Inner  $F_x$  vs. Helix Velocity  $U/Rw$ ",
    AxesLabel → {" $U$  (m/s)", " $F_x$  (N/m^2)"}, PlotRange → All];

  (*Finding U intercept: Newton-Raphson Method*)
  guess =  $10^{-6}$ ; (*Reset initial guess*)
  grad = D[ $F_x\text{In}[U]$ , U];

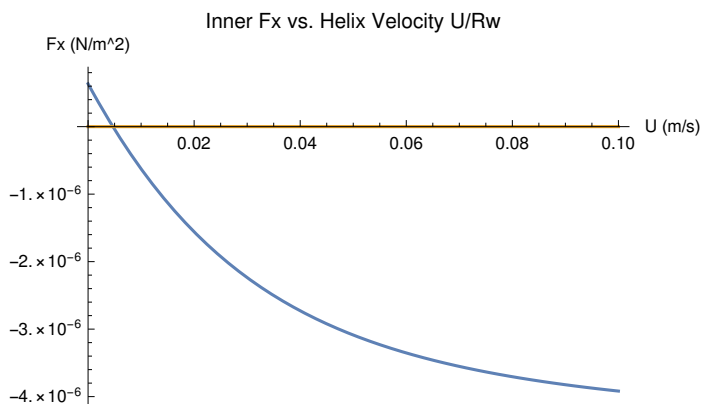
  (*Print[ $F_x\text{In}[guess]$ ];*)

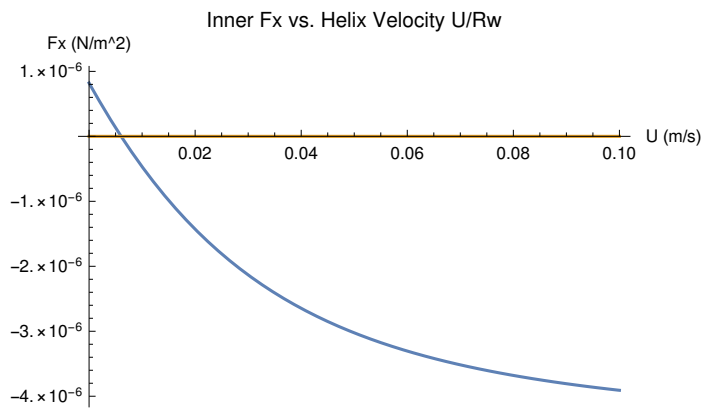
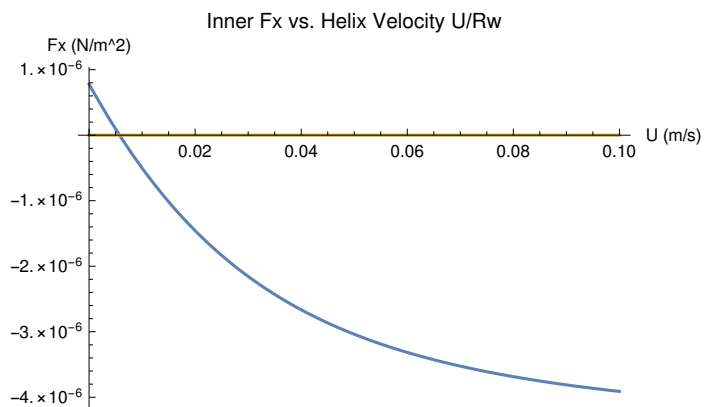
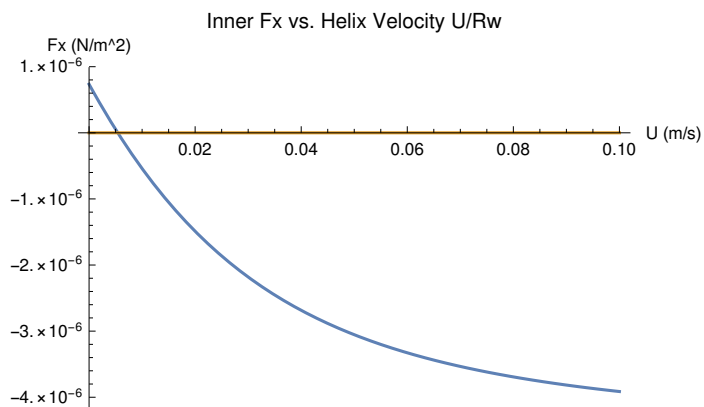
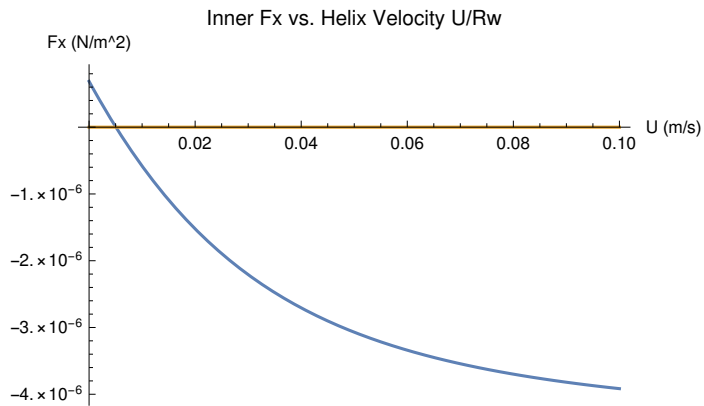
  While[ $F_x\text{In}[guess] > 10^{-12}$ , (*Keep iterating until  $F_x\text{In} \approx 0$ *)
    gradEval = grad /. U → guess; (*Find  $F_x\text{In}'(guess)$ *)
    guess = guess -  $F_x\text{In}[guess] / \text{gradEval}$  (* $u_{i+1} = u_i - F_x\text{In}(u_i) / F_x\text{In}'(u_i)$ *)
  ];
  Uint = guess; (*U intercept found*)

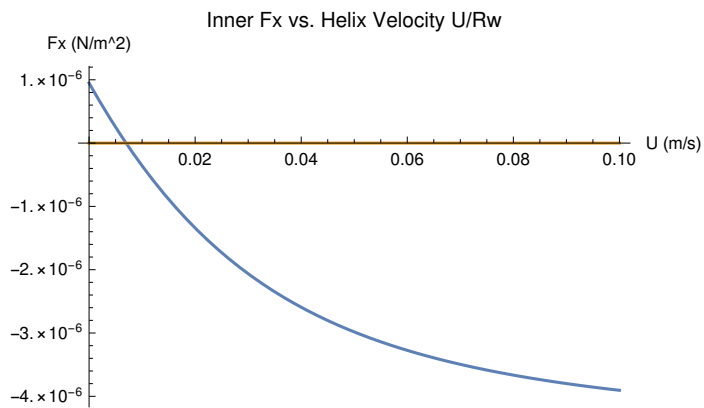
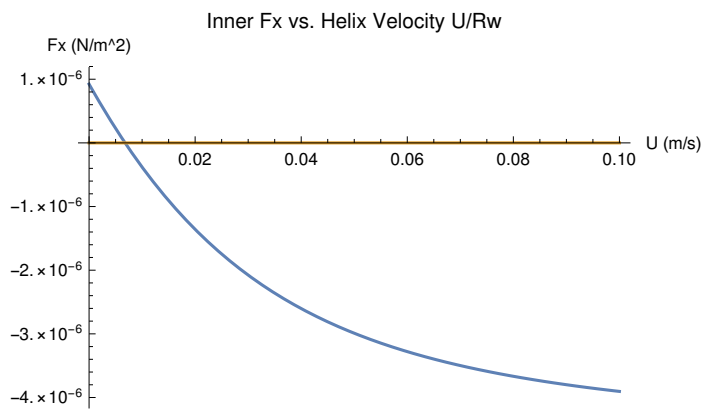
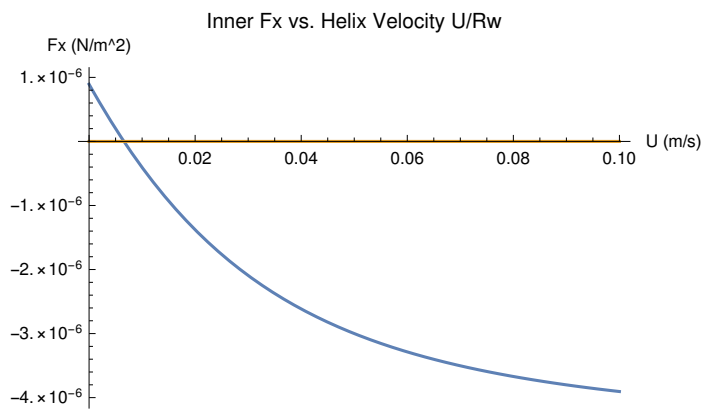
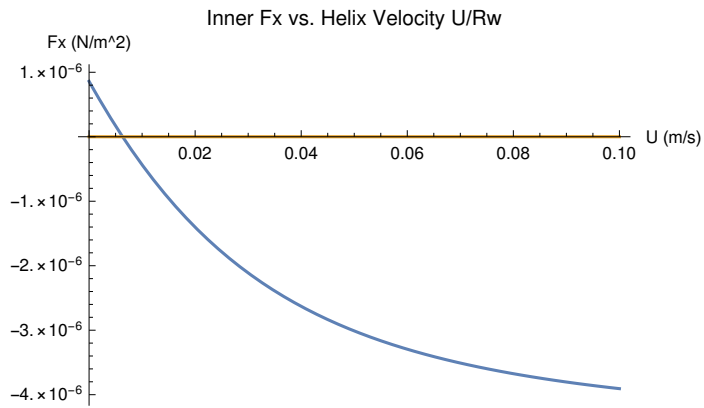
  (*Storing data in arrays*)
   $\phi\text{store} = \text{Join}[\phi\text{store}, \{\phi\}];$  (*Storing  $\phi$  in Radians*)
   $F\text{Maxstore} = \text{Join}[F\text{Maxstore}, \{F_x\text{In}[10^{-3}]\}];$  (* $F_x\text{In}(0.001)$ *)
   $U\text{store} = \text{Join}[U\text{store}, \{Uint\}];$  (*U found when  $F_x\text{In} < 10^{-6}$ *)
   $F\text{store} = \text{Join}[F\text{store}, \{F_x\text{In}[Uint]\}];$  (* $F_x\text{In}$  val at U-intercept*)

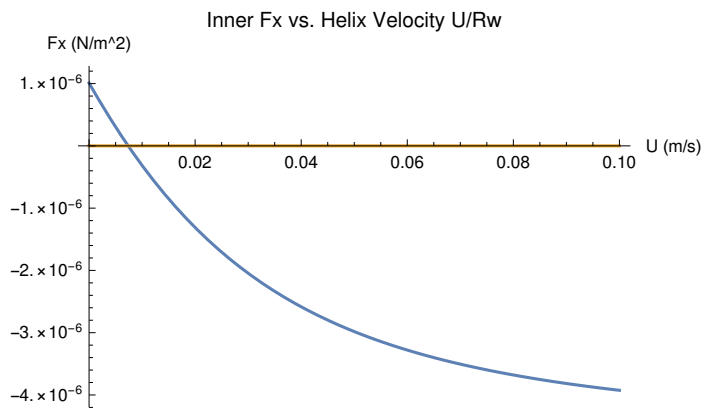
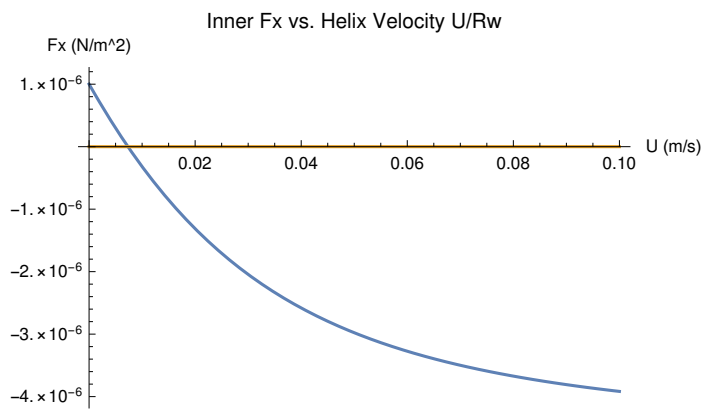
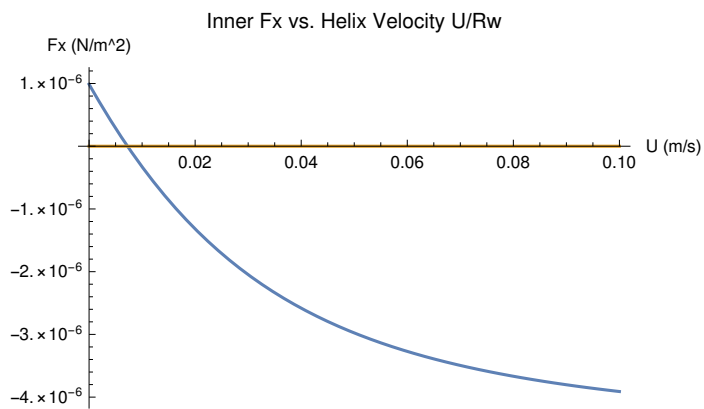
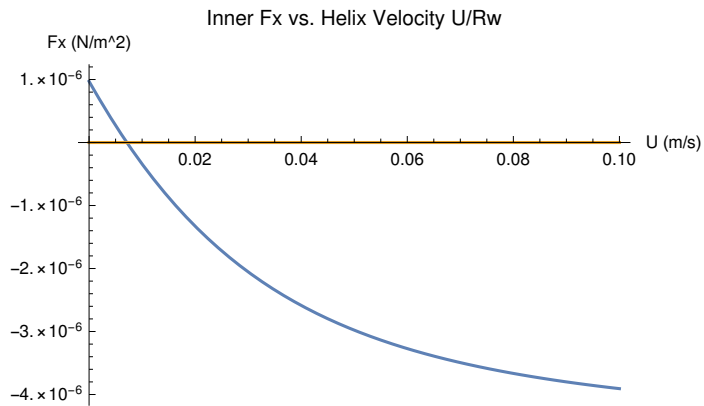
   $\phi = \phi + (1 * \text{Pi} / 180)$  (*Increment by 1 deg*)
]

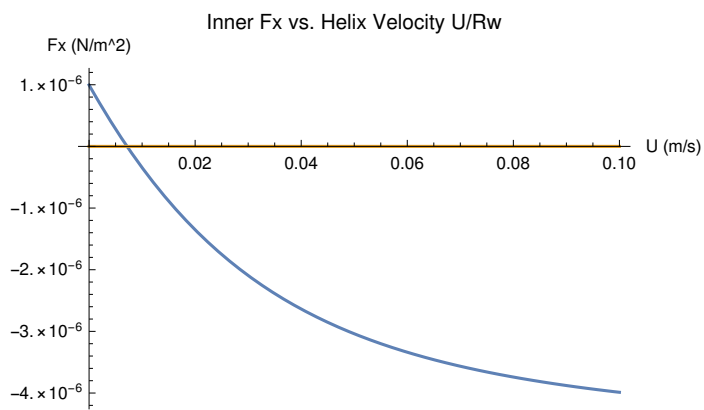
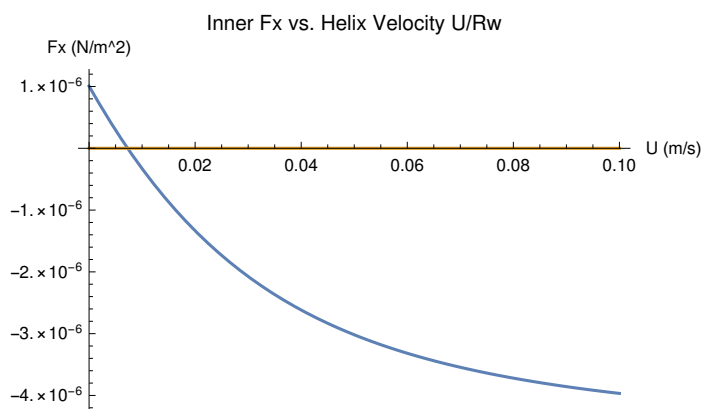
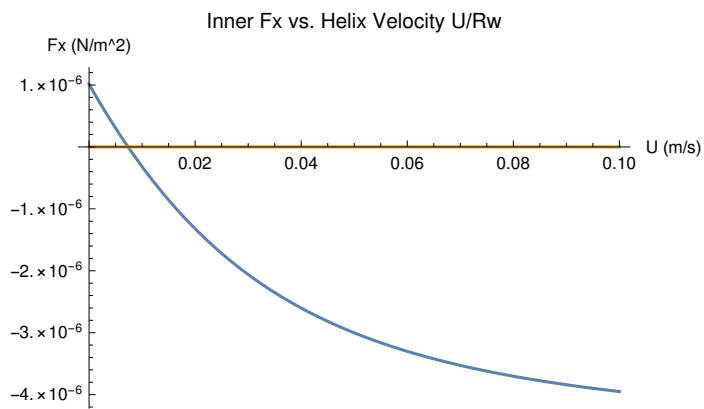
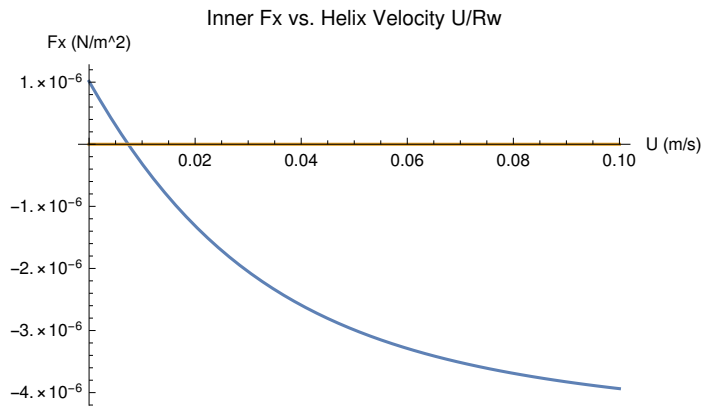
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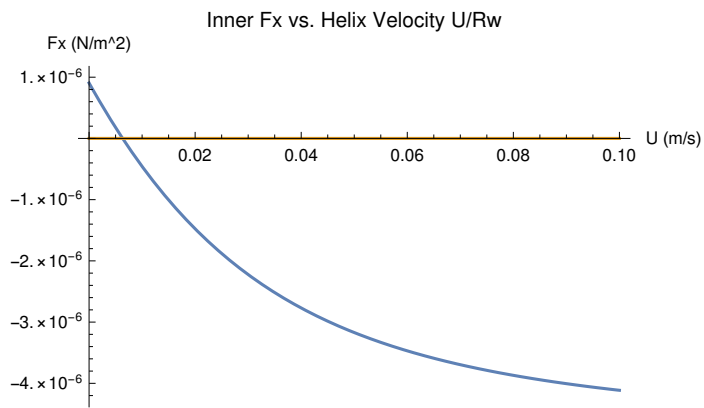
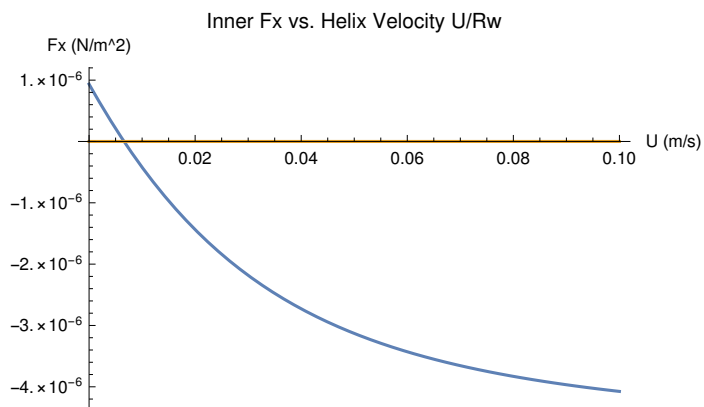
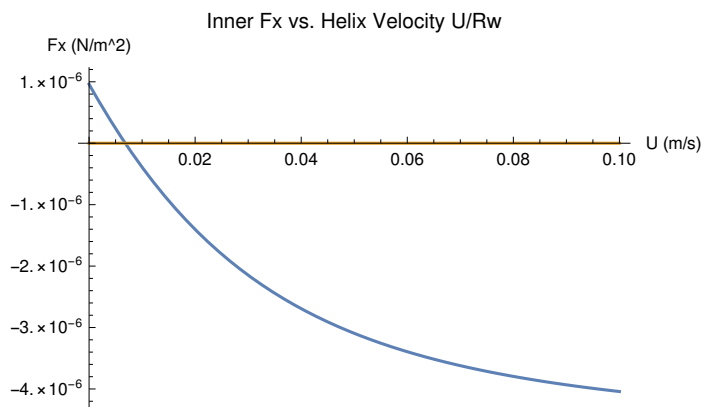
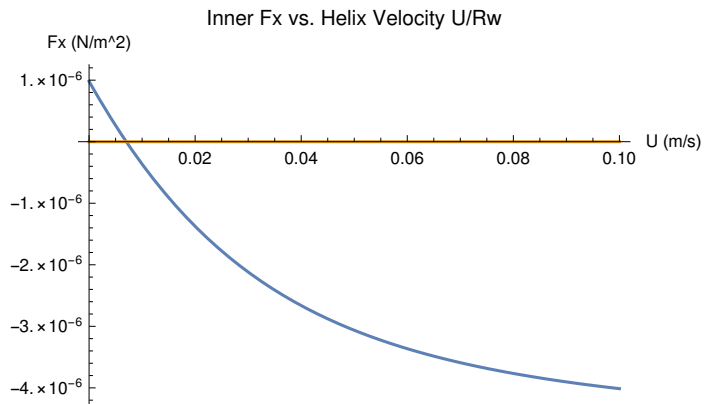




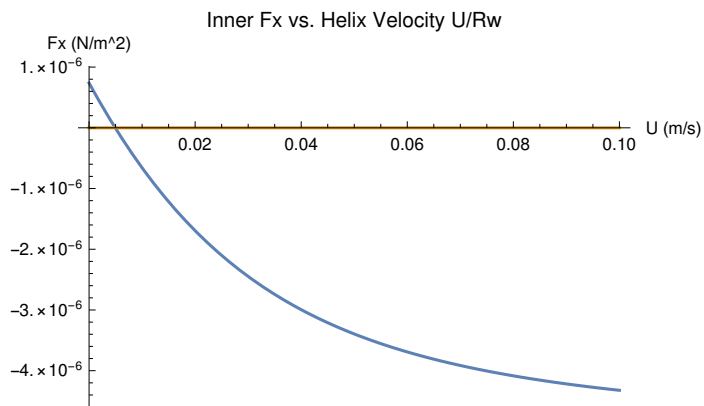
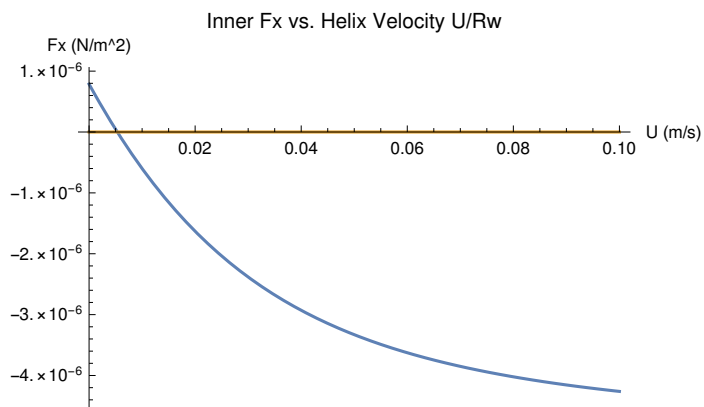
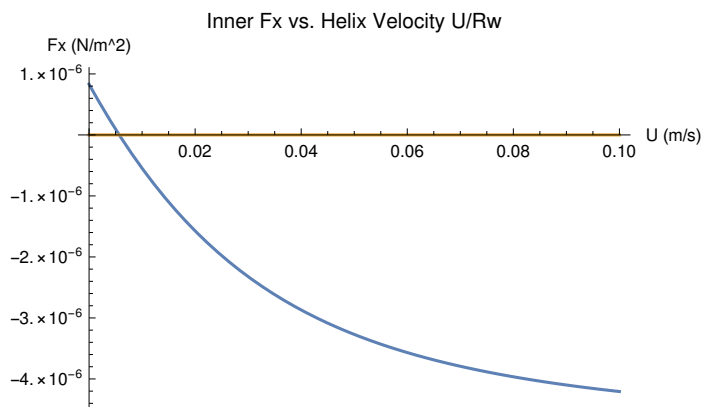
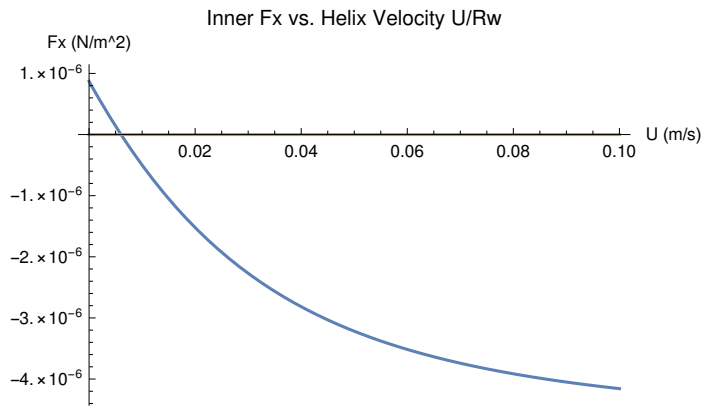


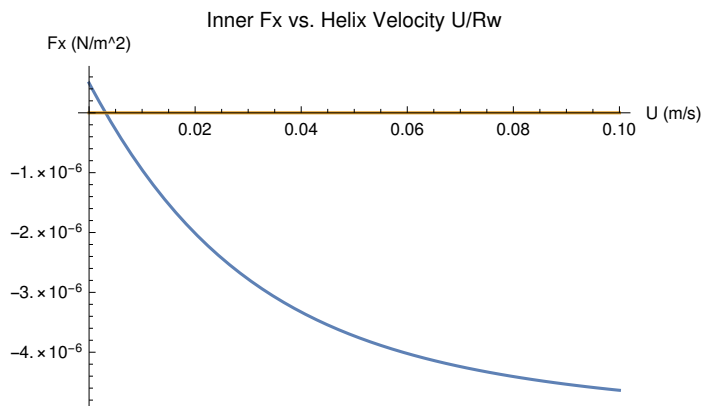
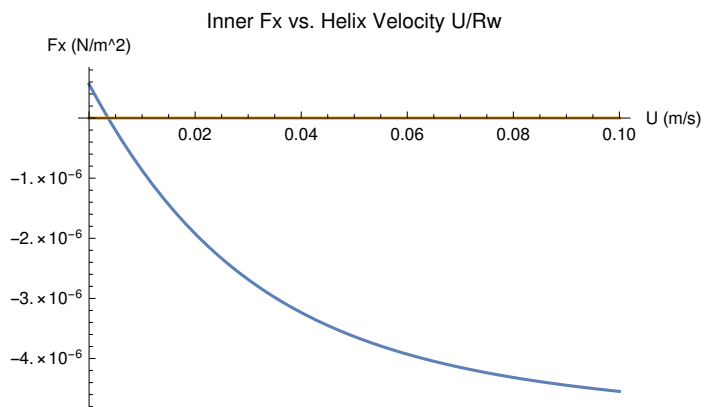
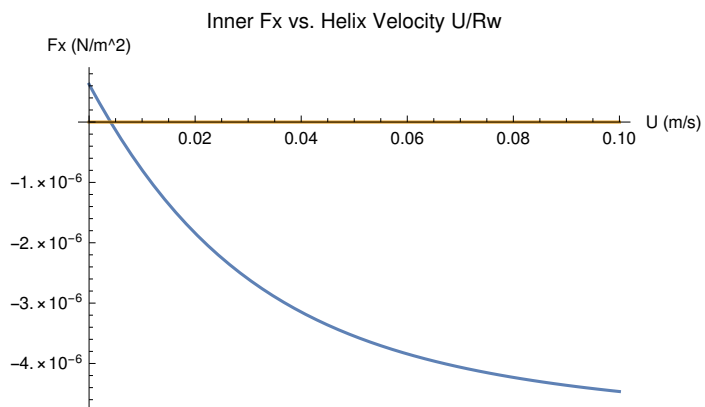
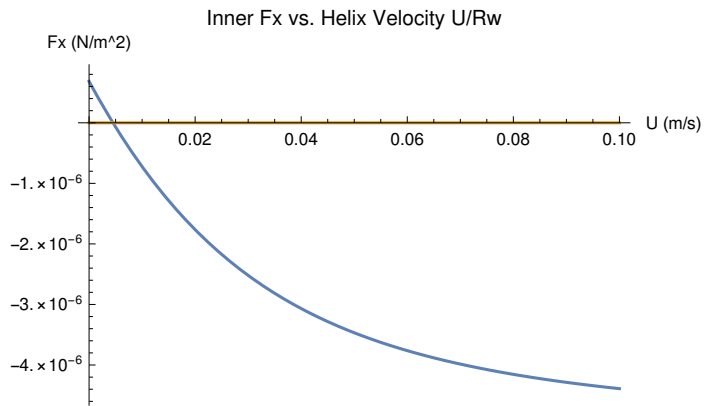


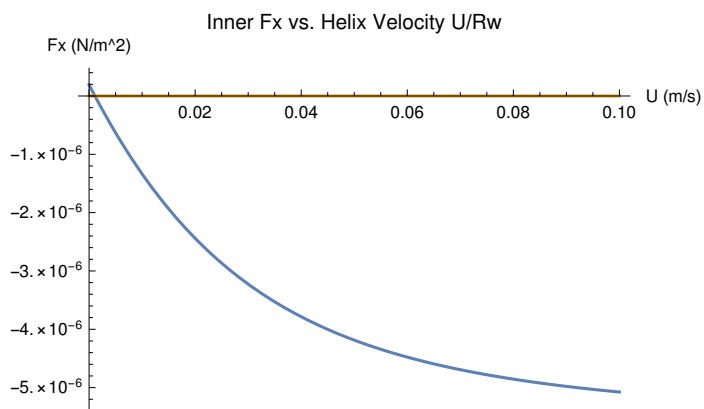
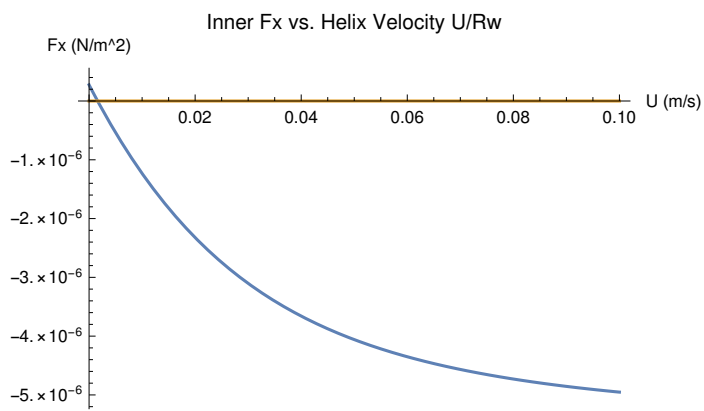
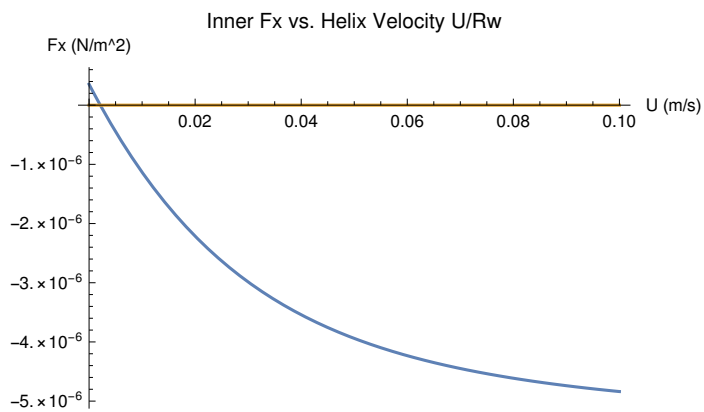
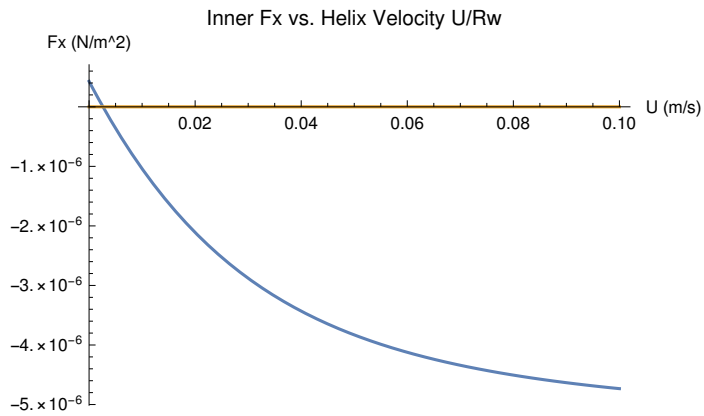


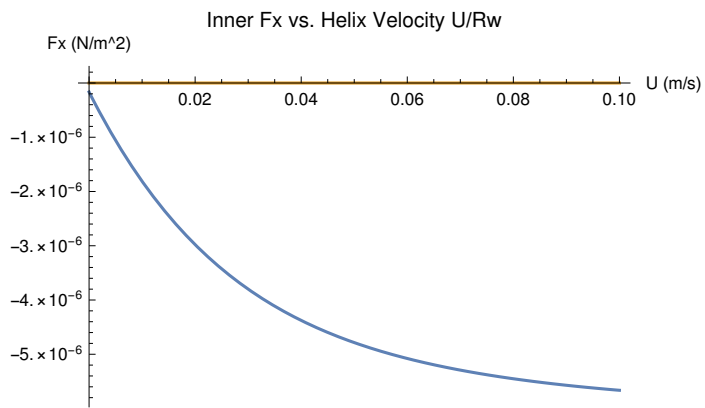
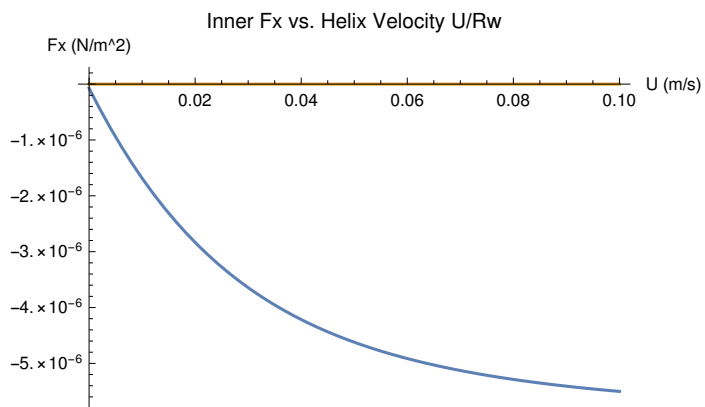
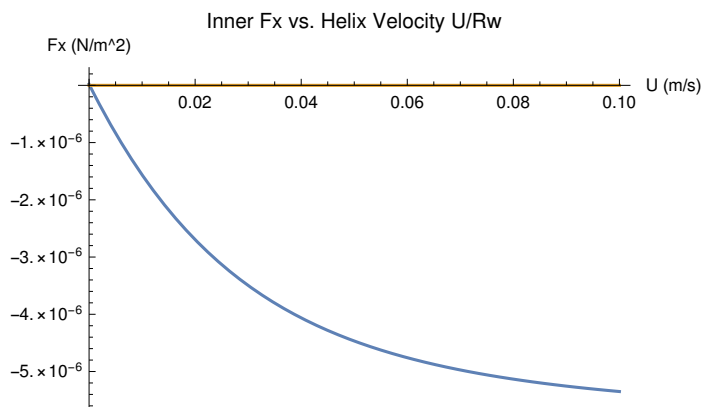
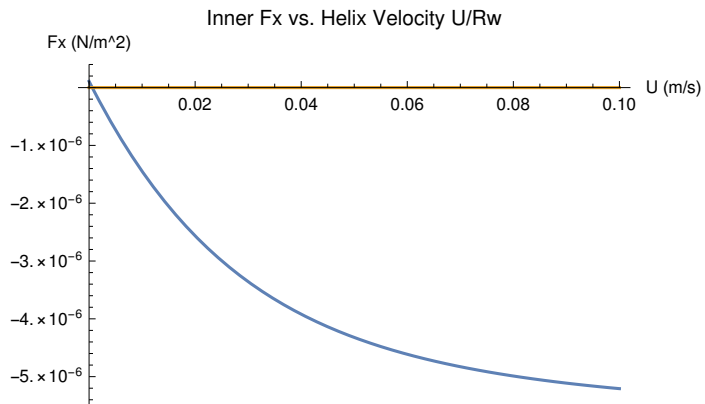


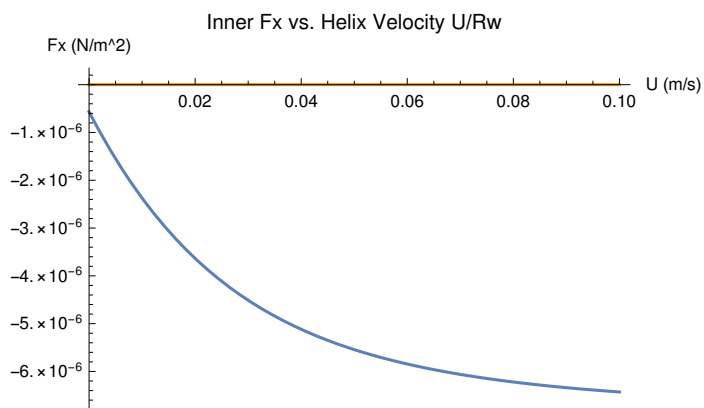
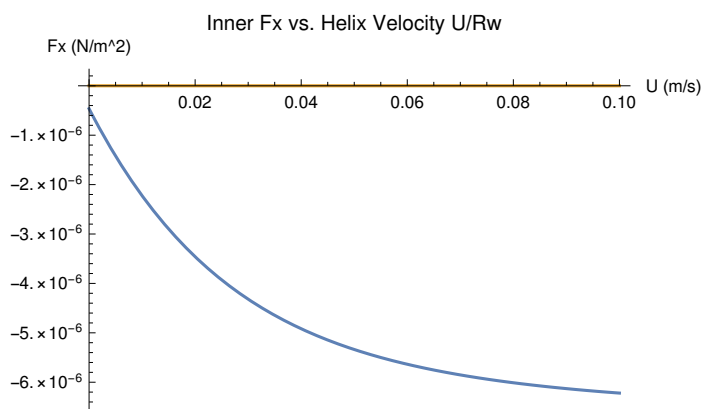
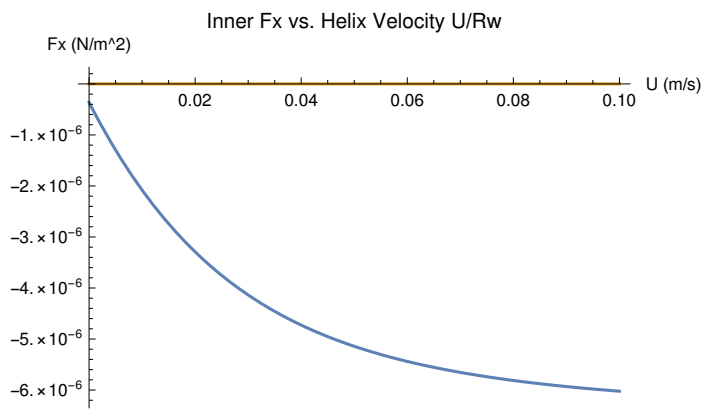
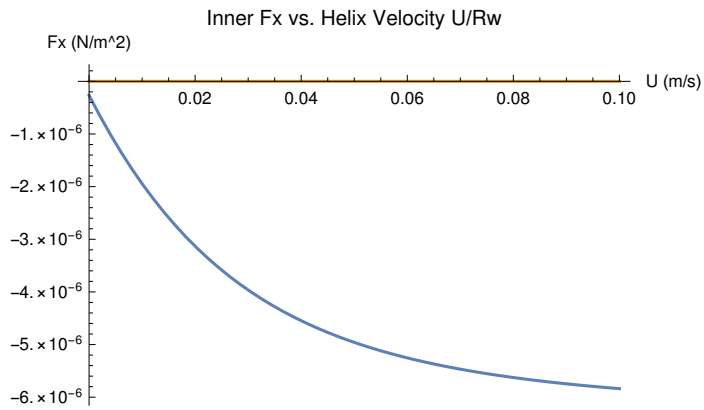


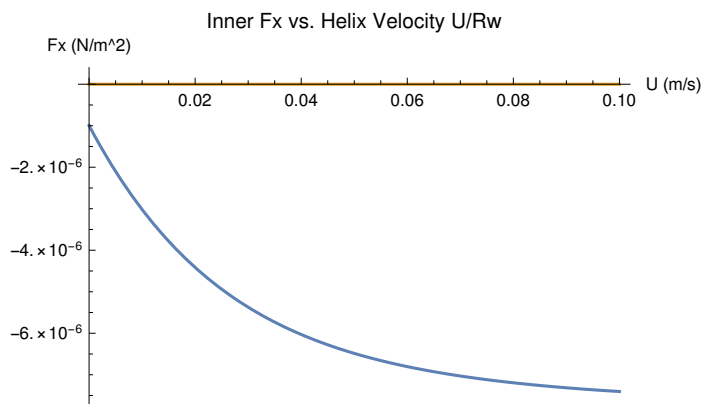
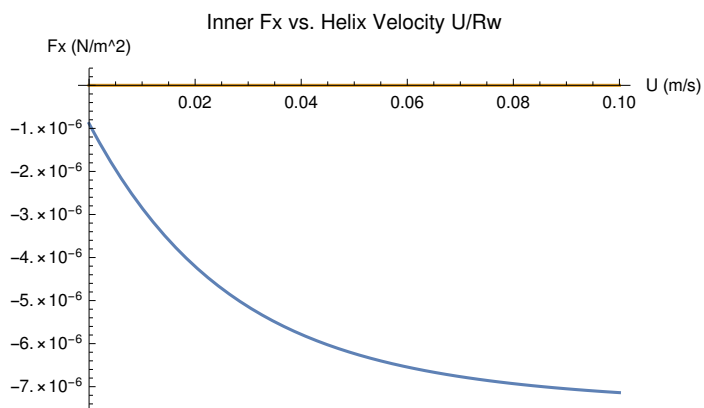
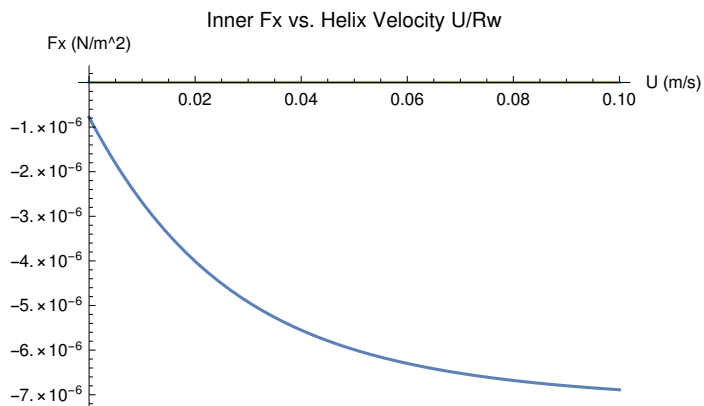
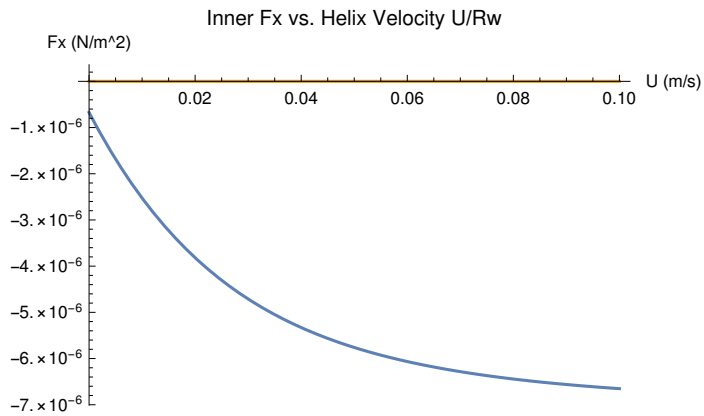


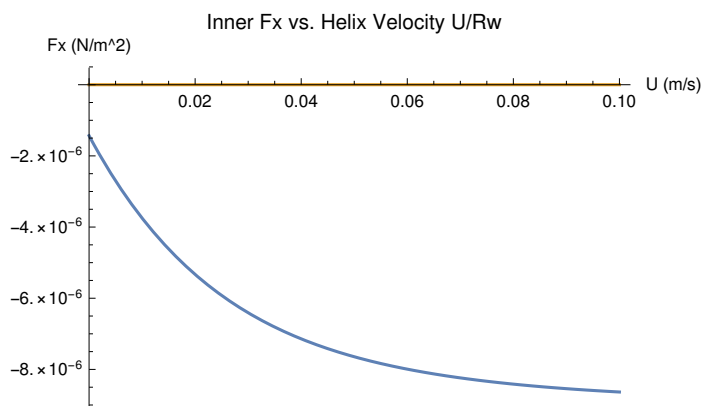
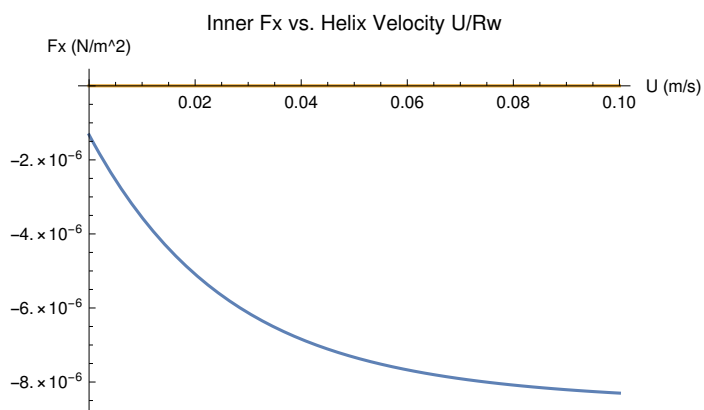
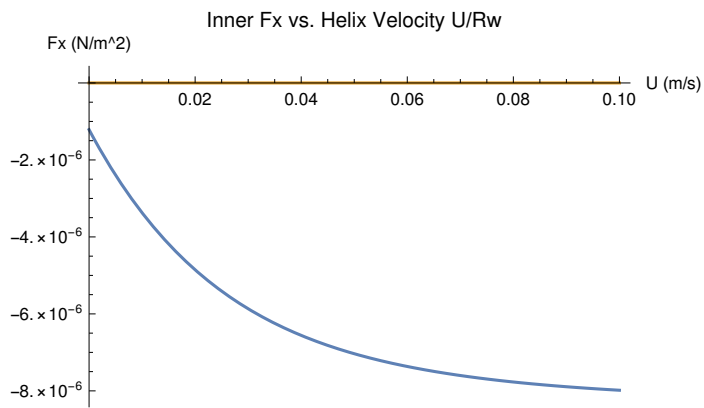
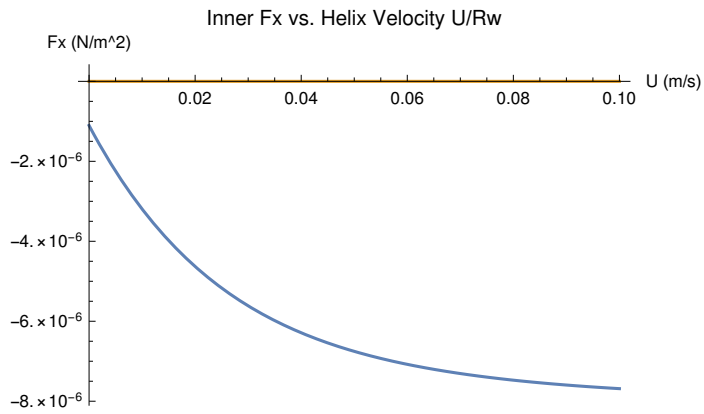


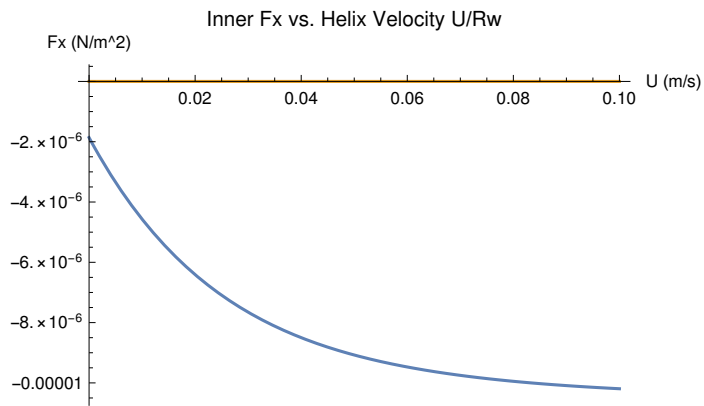
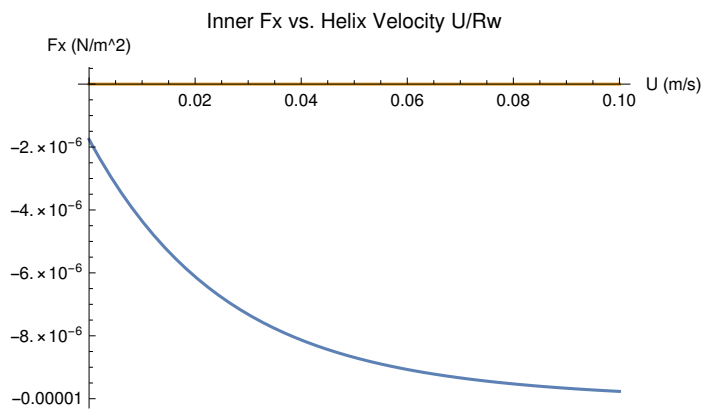
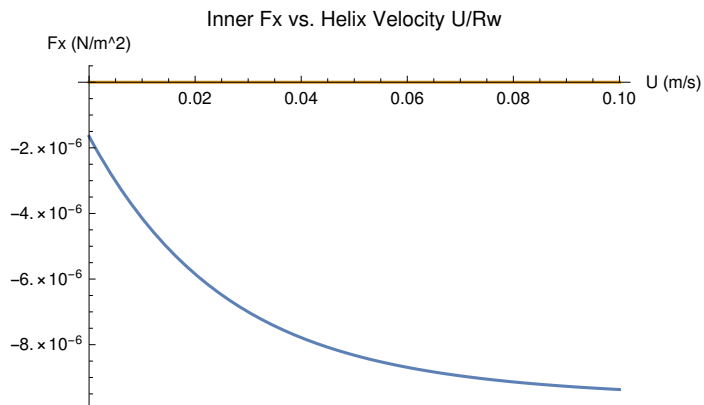
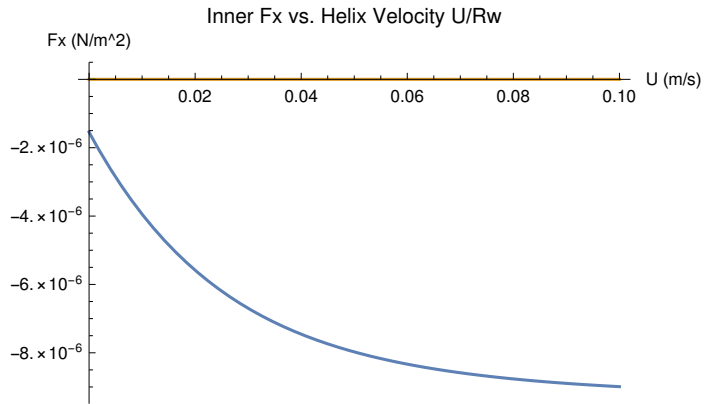




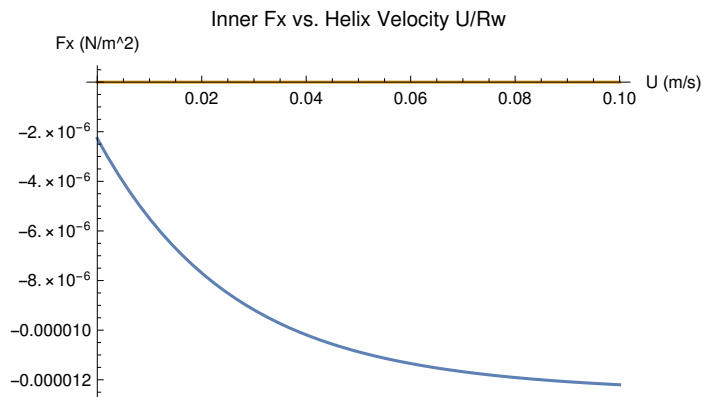
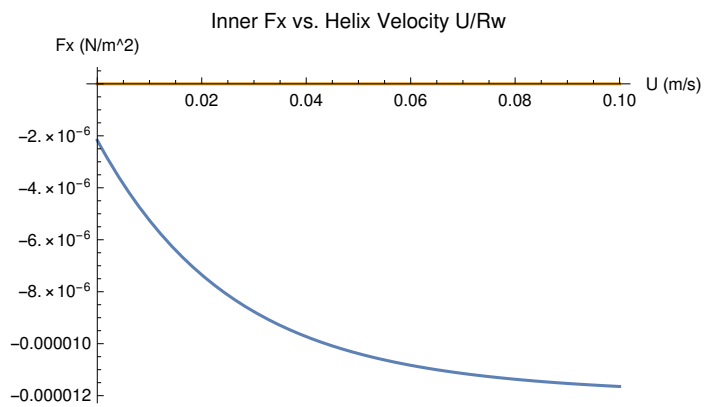
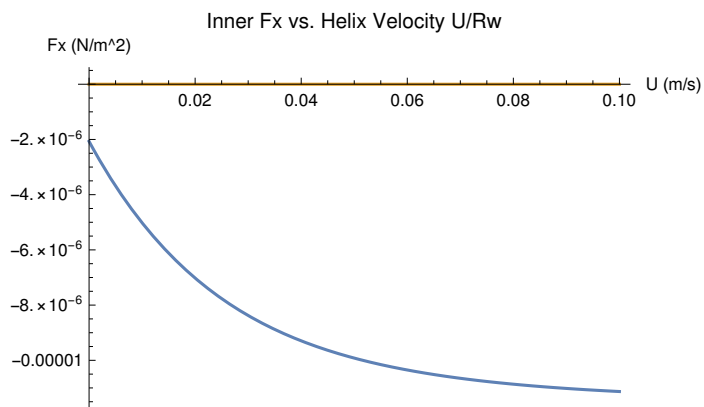
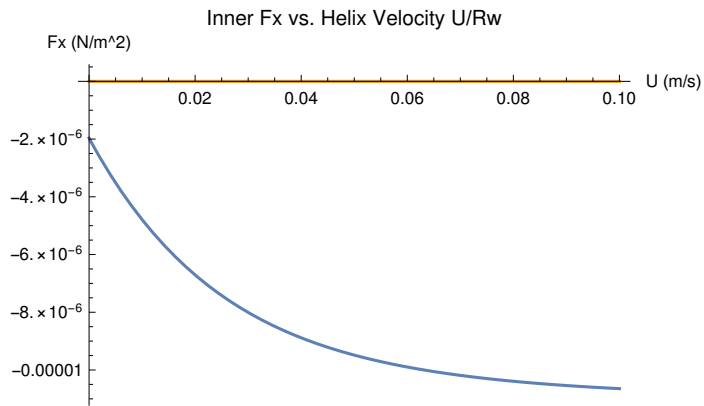


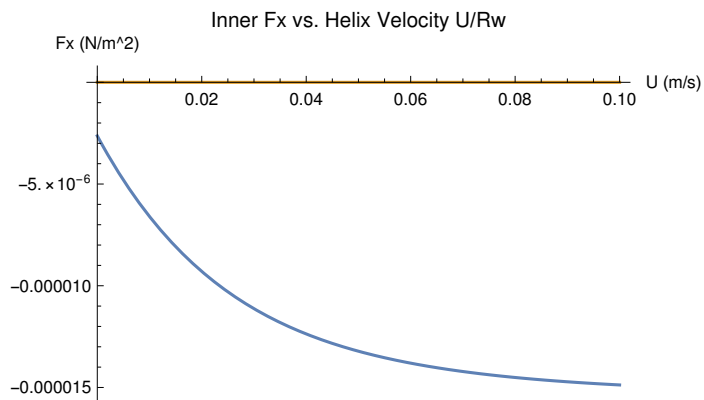
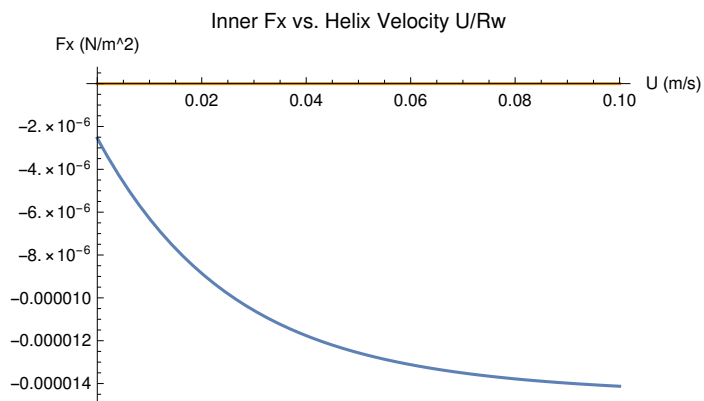
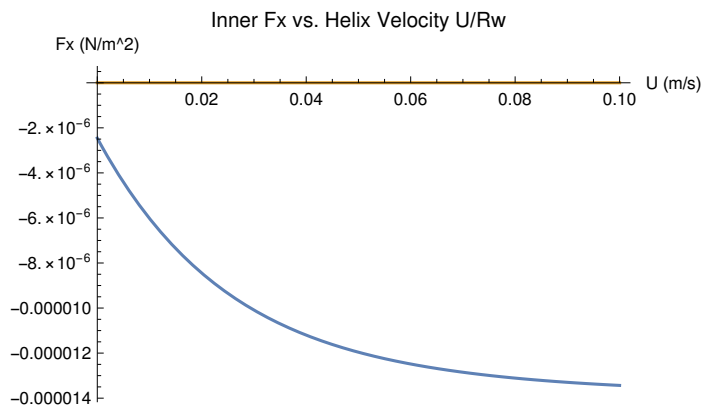
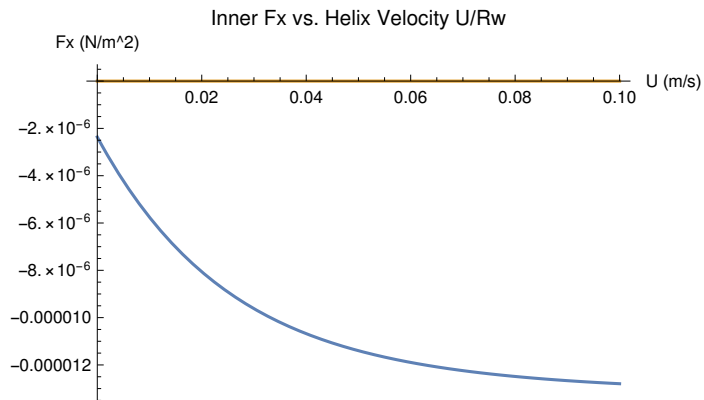


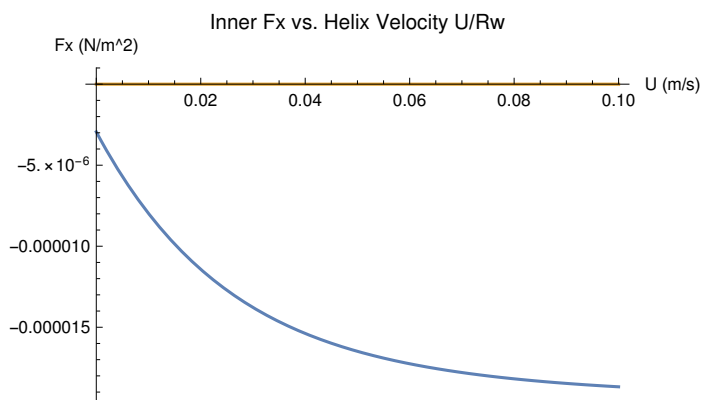
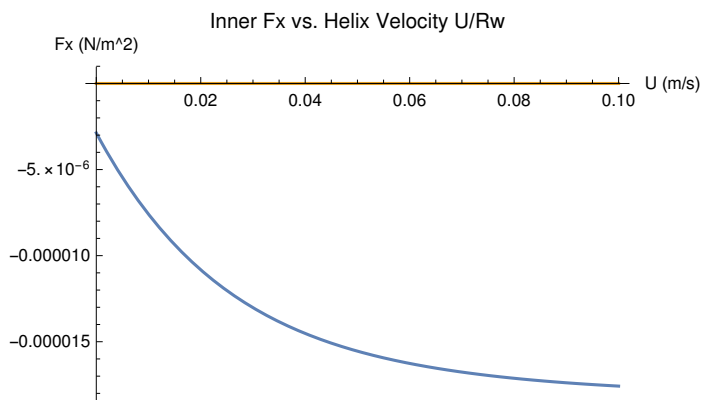
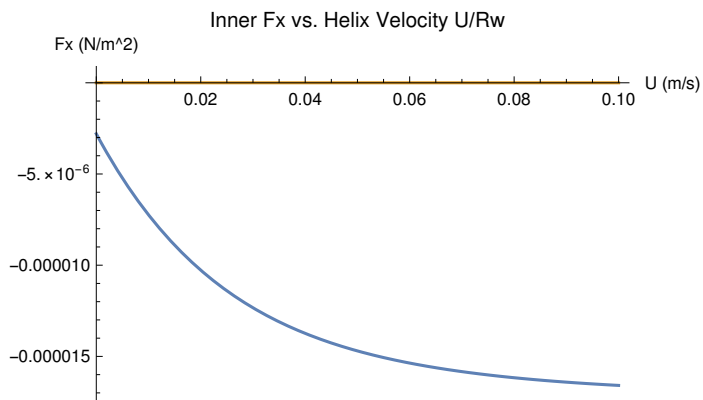
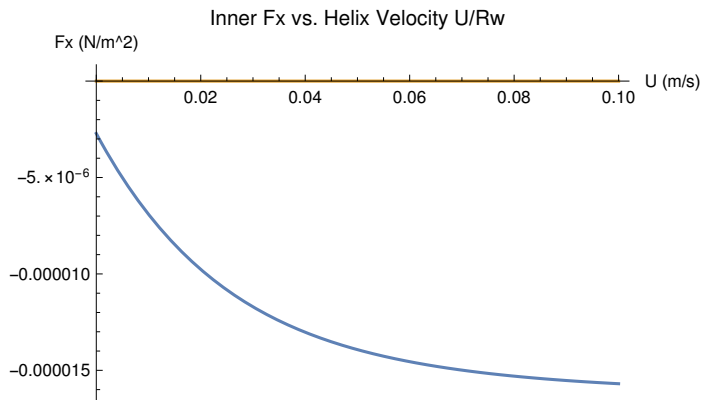


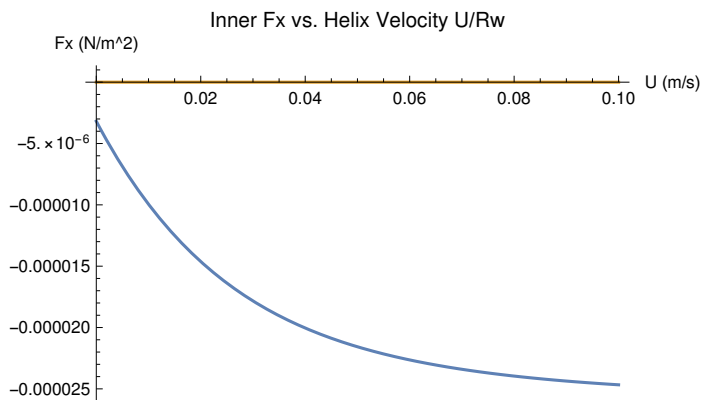
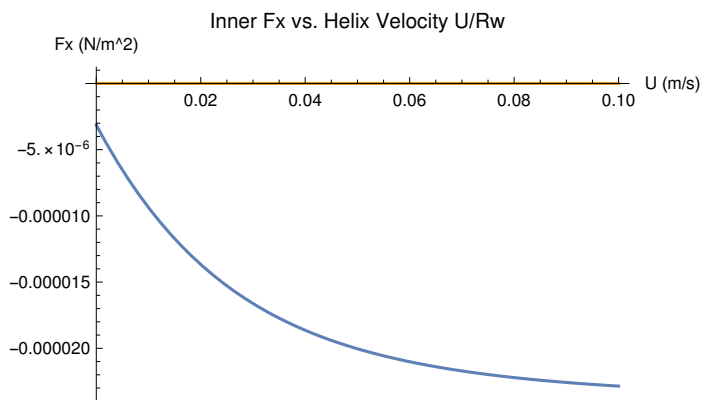
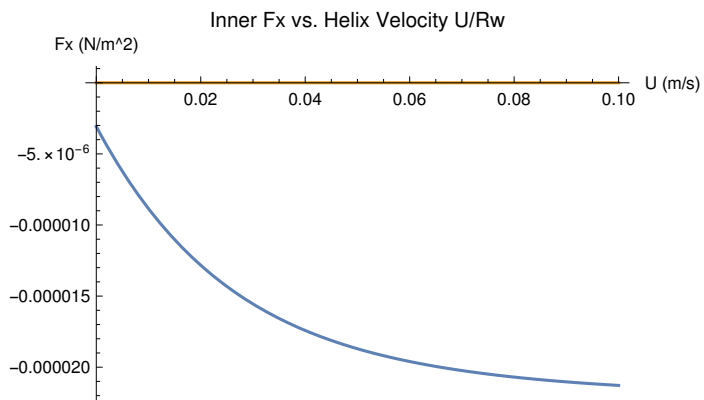
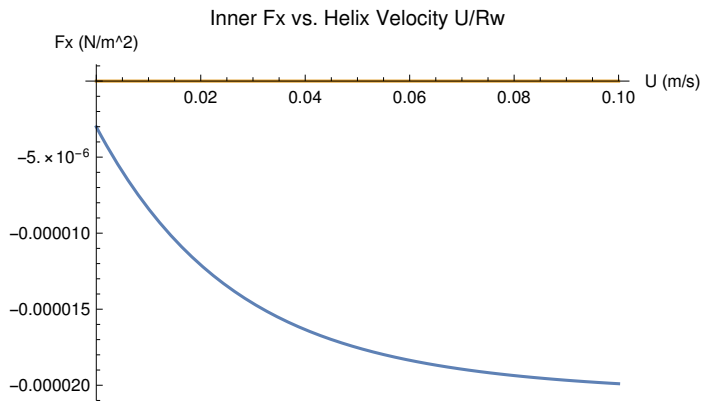


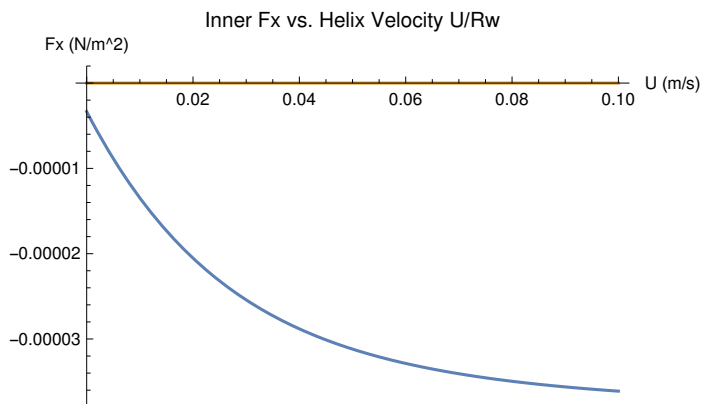
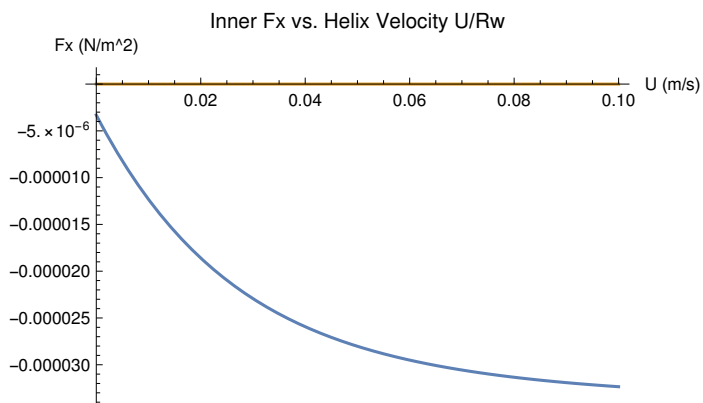
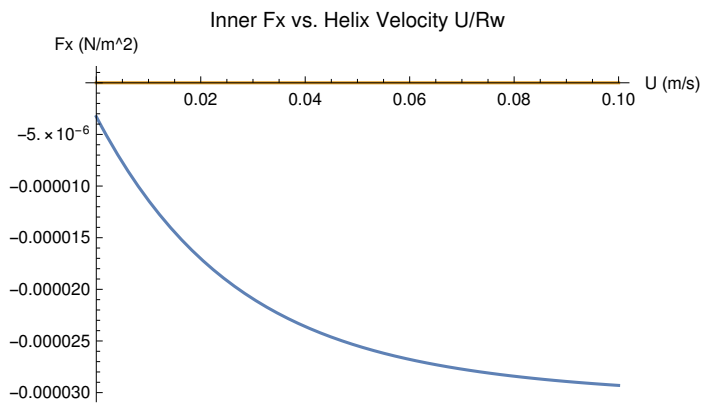
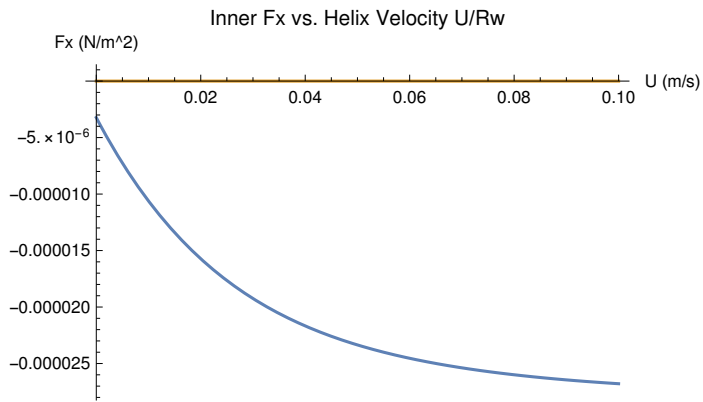


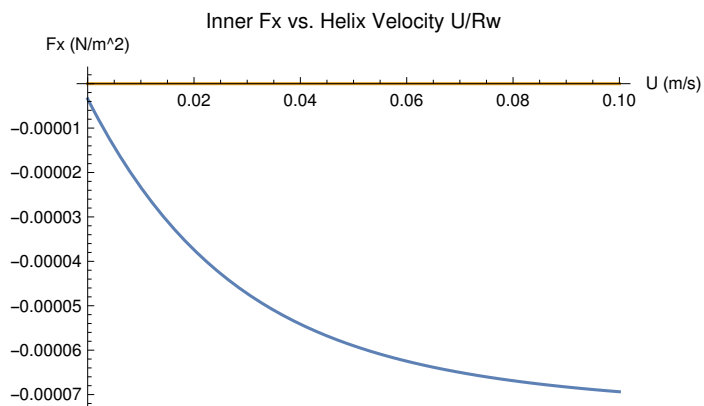
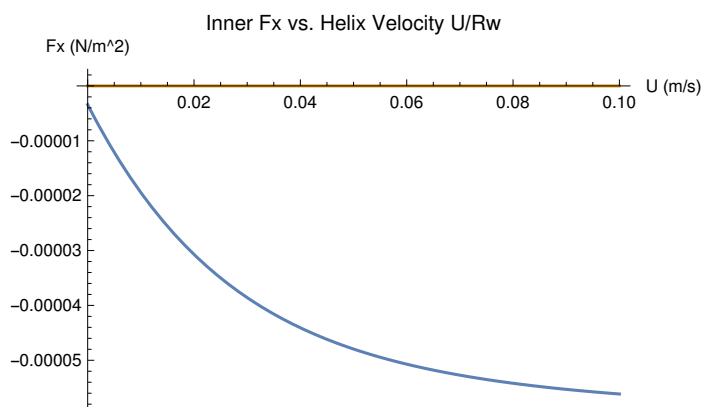
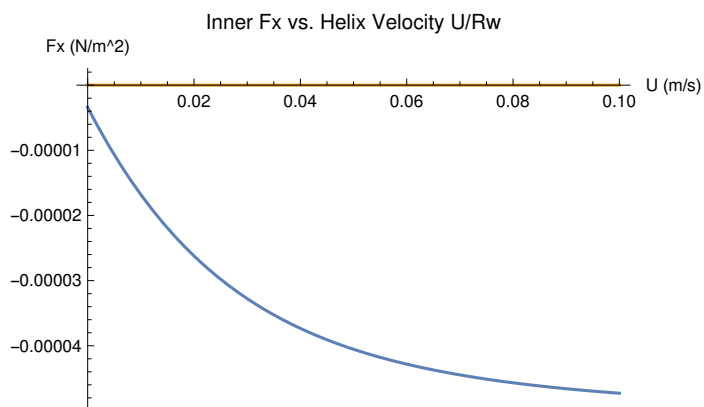
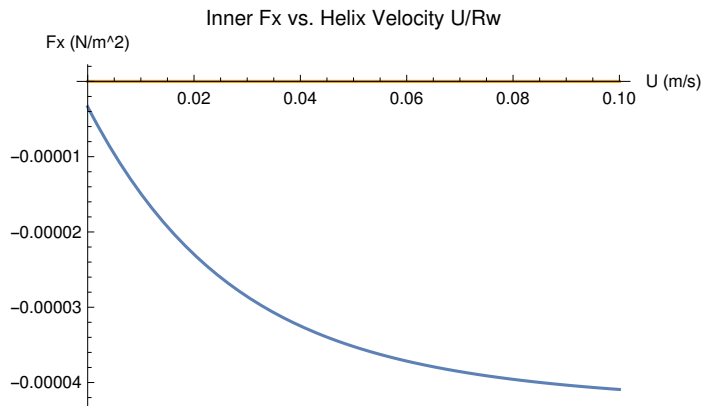


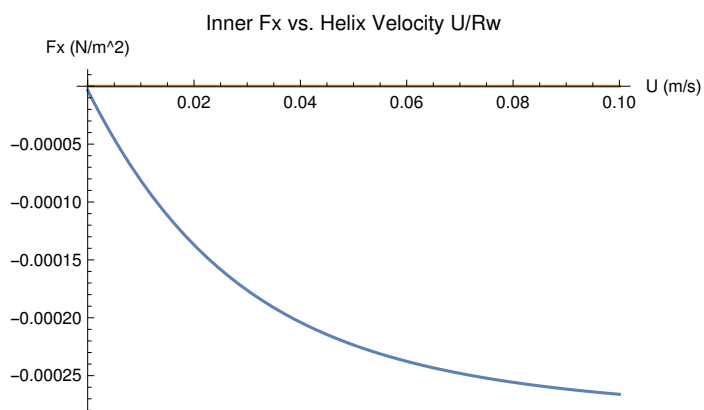
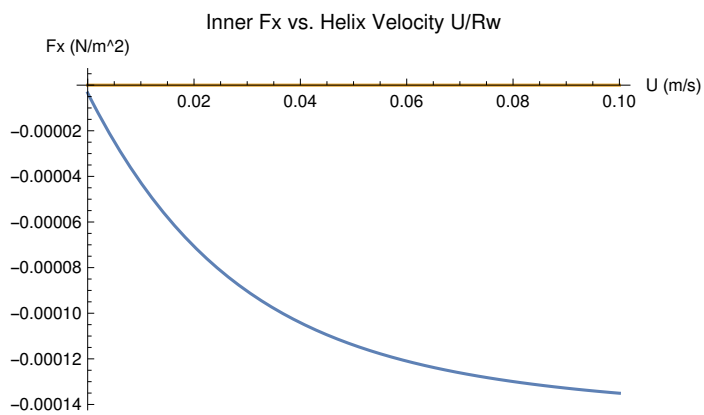
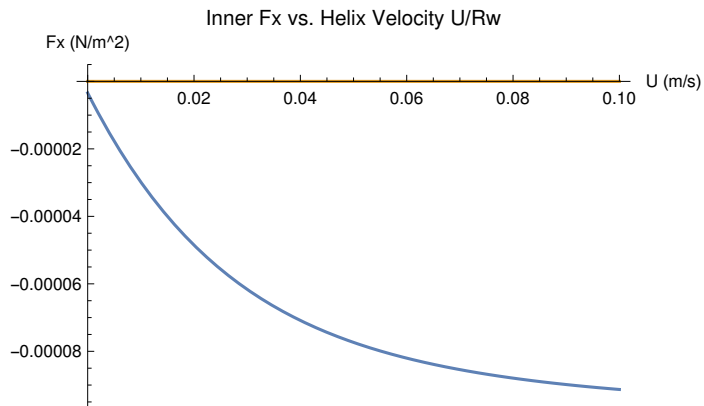












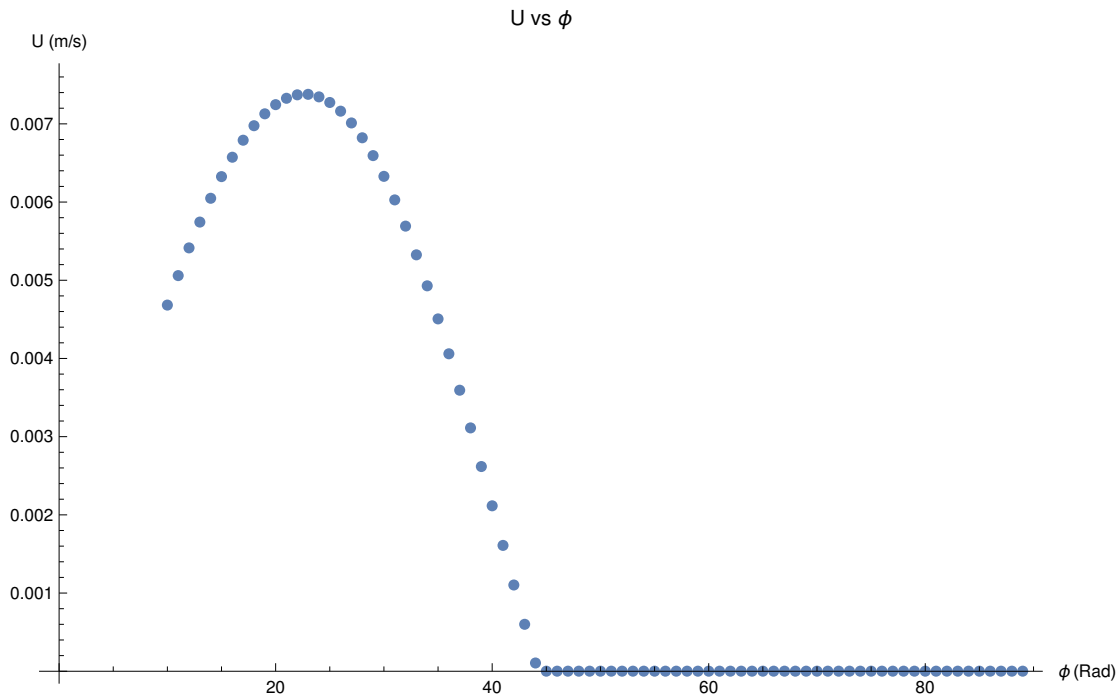
## Analysis

```

 $\phi\text{col} = \{\phi\text{store}\}^T$ ;
 $U\text{col} = \{U\text{store}\}^T$ ;
 $F\text{col} = \{F\text{store}\}^T$ ;
 $F\text{maxcol} = \{F\text{Maxstore}\}^T$ ;

(*Helix Translation Speed vs. Local Incline Angle  $\phi$ *)
dataPlot = Join[ $\phi\text{col} * 180 / \text{Pi}$ ,  $U\text{col}$ , 2];
ListPlot[dataPlot, PlotLabel → "U vs  $\phi$ ",
  AxesLabel → {" $\phi$  (Rad)", "U (m/s)"}, ImageSize → Large]

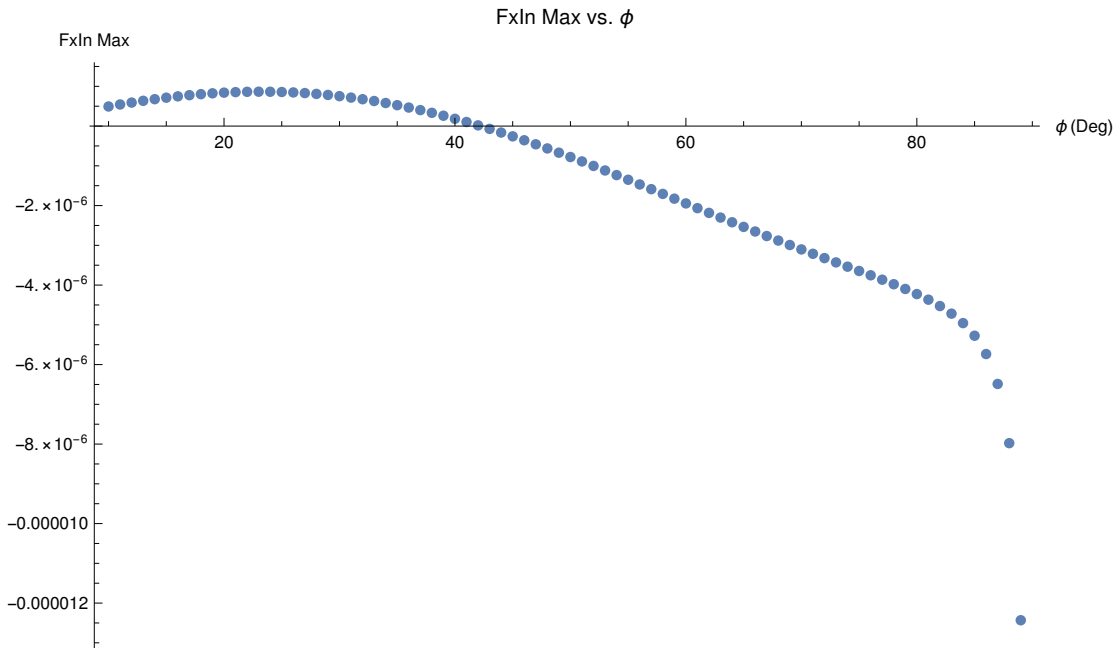
```



With Chen's coefficients, the dimensional  $F_x$  equation does yield a  $U$  maximum speed :) Same deal with Francisco's model, where net forward thrust only exists for  $0 < \phi < 40^\circ$



```
(*FxIn Max vs. Local Incline Angle  $\phi$ *)
dataMax = Join[ $\phi$ col * 180 / Pi, Fmaxcol, 2];
ListPlot[dataMax, PlotLabel -> "FxIn Max vs.  $\phi$ ",
  AxesLabel -> {" $\phi$  (Deg)", "FxIn Max"}, ImageSize -> Large, PlotRange -> Full]
```



```
(*Checking Data Values:  $\phi$  is left col, U/Rw is right col*)
dataTable = Join[ $\phi$ col * 180 / Pi, Ucol, Fcol, 2];
Grid[Join[{" $\phi$  (Deg)", "U", "FxIn[U]"}, dataTable, 1]]
```

$\phi$ (Deg)	U	FxIn[U]
10.	0.00468258	$1.8561 \times 10^{-15}$
11.	0.00505958	$3.17729 \times 10^{-15}$
12.	0.00541394	$5.0654 \times 10^{-15}$
13.	0.00574407	$7.59502 \times 10^{-15}$
14.	0.0060484	$1.07902 \times 10^{-14}$
15.	0.00632535	$1.46078 \times 10^{-14}$
16.	0.00657334	$1.89275 \times 10^{-14}$
17.	0.00679084	$2.35511 \times 10^{-14}$
18.	0.00697635	$2.82127 \times 10^{-14}$
19.	0.00712844	$3.25989 \times 10^{-14}$
20.	0.00724577	$3.63793 \times 10^{-14}$
21.	0.00732711	$3.92417 \times 10^{-14}$
22.	0.00737137	$4.09297 \times 10^{-14}$
23.	0.00737762	$4.12758 \times 10^{-14}$
24.	0.00734515	$4.0226 \times 10^{-14}$
25.	0.00727345	$3.78512 \times 10^{-14}$
26.	0.00716228	$3.4342 \times 10^{-14}$
27.	0.00701169	$2.99876 \times 10^{-14}$
28.	0.00682204	$2.51412 \times 10^{-14}$
29.	0.00659404	$2.0176 \times 10^{-14}$

30.	0.00632874	$1.5439 \times 10^{-14}$
31.	0.00602757	$1.12112 \times 10^{-14}$
32.	0.00569233	$7.67893 \times 10^{-15}$
33.	0.00532521	$4.92292 \times 10^{-15}$
34.	0.00492876	$2.92488 \times 10^{-15}$
35.	0.00450587	$1.58964 \times 10^{-15}$
36.	0.00405974	$7.76558 \times 10^{-16}$
37.	0.00359388	$3.32758 \times 10^{-16}$
38.	0.00311198	$1.20726 \times 10^{-16}$
39.	0.00261794	$3.51316 \times 10^{-17}$
40.	0.00211574	$7.50228 \times 10^{-18}$
41.	0.00160943	$9.99593 \times 10^{-19}$
42.	0.00110303	$8.16426 \times 10^{-13}$
43.	0.000600491	$8.02477 \times 10^{-14}$
44.	0.000105622	$8.52882 \times 10^{-17}$
45.	$\frac{1}{1\,000\,000}$	$-7.2164 \times 10^{-8}$
46.	$\frac{1}{1\,000\,000}$	$-1.66505 \times 10^{-7}$
47.	$\frac{1}{1\,000\,000}$	$-2.63455 \times 10^{-7}$
48.	$\frac{1}{1\,000\,000}$	$-3.62789 \times 10^{-7}$
49.	$\frac{1}{1\,000\,000}$	$-4.64271 \times 10^{-7}$
50.	$\frac{1}{1\,000\,000}$	$-5.67661 \times 10^{-7}$
51.	$\frac{1}{1\,000\,000}$	$-6.72712 \times 10^{-7}$
52.	$\frac{1}{1\,000\,000}$	$-7.79171 \times 10^{-7}$
53.	$\frac{1}{1\,000\,000}$	$-8.86782 \times 10^{-7}$
54.	$\frac{1}{1\,000\,000}$	$-9.95281 \times 10^{-7}$
55.	$\frac{1}{1\,000\,000}$	$-1.10441 \times 10^{-6}$
56.	$\frac{1}{1\,000\,000}$	$-1.21389 \times 10^{-6}$
57.	$\frac{1}{1\,000\,000}$	$-1.32345 \times 10^{-6}$
58.	$\frac{1}{1\,000\,000}$	$-1.43284 \times 10^{-6}$
59.	$\frac{1}{1\,000\,000}$	$-1.54176 \times 10^{-6}$
60.	$\frac{1}{1\,000\,000}$	$-1.64996 \times 10^{-6}$
61.	$\frac{1}{1\,000\,000}$	$-1.75716 \times 10^{-6}$
62.	$\frac{1}{1\,000\,000}$	$-1.8631 \times 10^{-6}$
63.	$\frac{1}{1\,000\,000}$	$-1.9675 \times 10^{-6}$
64.	$\frac{1}{1\,000\,000}$	$-2.07011 \times 10^{-6}$
65.	$\frac{1}{1\,000\,000}$	$-2.17066 \times 10^{-6}$
66.	$\frac{1}{1\,000\,000}$	$-2.2689 \times 10^{-6}$
67.	$\frac{1}{1\,000\,000}$	$-2.36458 \times 10^{-6}$

68.	$\frac{1}{1\,000\,000}$	$-2.45746 \times 10^{-6}$
69.	$\frac{1}{1\,000\,000}$	$-2.5473 \times 10^{-6}$
70.	$\frac{1}{1\,000\,000}$	$-2.63387 \times 10^{-6}$
71.	$\frac{1}{1\,000\,000}$	$-2.71694 \times 10^{-6}$
72.	$\frac{1}{1\,000\,000}$	$-2.79631 \times 10^{-6}$
73.	$\frac{1}{1\,000\,000}$	$-2.87177 \times 10^{-6}$
74.	$\frac{1}{1\,000\,000}$	$-2.94313 \times 10^{-6}$
75.	$\frac{1}{1\,000\,000}$	$-3.01019 \times 10^{-6}$
76.	$\frac{1}{1\,000\,000}$	$-3.0728 \times 10^{-6}$
77.	$\frac{1}{1\,000\,000}$	$-3.13078 \times 10^{-6}$
78.	$\frac{1}{1\,000\,000}$	$-3.18398 \times 10^{-6}$
79.	$\frac{1}{1\,000\,000}$	$-3.23228 \times 10^{-6}$
80.	$\frac{1}{1\,000\,000}$	$-3.27554 \times 10^{-6}$
81.	$\frac{1}{1\,000\,000}$	$-3.31365 \times 10^{-6}$
82.	$\frac{1}{1\,000\,000}$	$-3.34652 \times 10^{-6}$
83.	$\frac{1}{1\,000\,000}$	$-3.37407 \times 10^{-6}$
84.	$\frac{1}{1\,000\,000}$	$-3.39624 \times 10^{-6}$
85.	$\frac{1}{1\,000\,000}$	$-3.41301 \times 10^{-6}$
86.	$\frac{1}{1\,000\,000}$	$-3.42439 \times 10^{-6}$
87.	$\frac{1}{1\,000\,000}$	$-3.4305 \times 10^{-6}$
88.	$\frac{1}{1\,000\,000}$	$-3.43177 \times 10^{-6}$
89.	$\frac{1}{1\,000\,000}$	$-3.43047 \times 10^{-6}$

$\phi = 23^\circ$  yields the highest value for U (m/s).

$$\begin{aligned}
 \text{FxIn} = & (2 * \text{Pi} * n / \text{Cos}[\phi]) * \\
 & ((\text{Cn} - \text{Ct}) * w * \text{Sin}[\phi] * \text{Cos}[\phi]) * ((R / (2 * w^2)) * \text{Sqrt}[(R * w)^2 + U^2]) + \\
 & ((U^2 / (2 * w^3)) * (\text{Log}[U] - \text{Log}[R * w + \text{Sqrt}[(R * w)^2 + U^2]])) - \\
 & (U * (\text{Ct} * \text{Sin}[\phi]^2 + \text{Cn} * \text{Cos}[\phi]^2) * (\text{Sqrt}[(R * w)^2 + U^2] - U) / w^2) \\
 & - 3.6634 \times 10^{15} \\
 & \left( -0.000398577 U \left( -U + \sqrt{0.00398317 + U^2} \right) + 2.01457 \times 10^{-17} \left( 0.00073208 \sqrt{0.00398317 + U^2} + \right. \right. \\
 & \left. \left. \frac{1}{1953125 \pi^3} 702464 U^2 \left( \text{Log}[U] - \text{Log}[0.0631124 + \sqrt{0.00398317 + U^2}] \right) \right) \right)
 \end{aligned}$$