

AugerBot Calculations

Quit

Trial 1: 9/19 - 9/26

Modified Francisco Calculations : 10/18 (Do not use!)

Plotting F_x to find U which Balances Forces:
11/1

Plotting F_x to find U which Balances Forces 2 :
11/9

Inside Equation for Thrust: 11/16

Backtracking: 11/27

Francisco' s with Chen' s Coefficients: 11/30
(CORRECT ONES)

Quit;

Francisco With Chen' s Coefficients

```

In[1]:= (*Parameters from paper*)
r = 5/1000; w = 10.4; n = 3.5;

(*LP poppy Fourier coefficients*)
A00 = 0.051; A10 = 0.047; B11 = 0.053; B01 = 0.083;
Bn11 = 0.020; C11 = -0.026; C01 = 0.057; Cn11 = 0; D10 = 0.025;

In[3]:=  $\beta = (\text{Pi}/2) - \phi$ ; (* $\phi$  is symbolic, radians*)
 $\alpha_z = Bn11 * \text{Sin}[2 * \text{Pi} * (-\beta / \text{Pi})] + A00 * \text{Cos}[2 * \text{Pi} * \theta] +$ 
 $B01 * \text{Sin}[2 * \text{Pi} * \theta] + A10 * \text{Cos}[2 * \text{Pi} * (\beta / \text{Pi})] + B11 * \text{Sin}[2 * \text{Pi} * (\beta / \text{Pi})];$ 
(*Vertical stress per unit depth, N/m^3*)
 $\alpha_x = Cn11 * \text{Cos}[2 * \text{Pi} * (-\beta / \text{Pi})] + C01 * \text{Cos}[2 * \text{Pi} * \theta] +$ 
 $D10 * \text{Sin}[2 * \text{Pi} * (\beta / \text{Pi})] + C11 * \text{Cos}[2 * \text{Pi} * (\beta / \text{Pi})];$ 
(*Horizontal stress per unit depth, N/m^3*)

(*Friction coefficients, expressed in terms of  $\phi$ *)
d = 0.05; (*Depth robot buried, 50mm*)
Cn =  $\alpha_x * d$ ; (*N/m^2*)
Ct =  $\alpha_z * d$ ;

Cn /.  $\phi \rightarrow 0$ 
Ct /.  $\phi \rightarrow 0$ 

Out[9]= 0.00415
Out[10]= 0.0002

In[11]:= (*Eq 2 - 10/21/17 | Parametrized f Integrated only wrt  $d\theta$ *)
FranChen[u_] :=  $(2 * \text{Pi} * n * r / \text{Cos}[\phi]) *$ 
 $((Cn - Ct) * r * w * \text{Sin}[\phi] * \text{Cos}[\phi] - u * (Ct * \text{Sin}[\phi]^2 + Cn * \text{Cos}[\phi]^2)) /$ 
 $\text{Sqrt}[u^2 + (r * w)^2];$ 

```

Calculating U/Rw when $F_x = 0$ for Many ϕ Cases

```

In[12]:=  $\phi = 10 * \text{Pi} / 180 // N$ ; (*Local inclination, radians*)
 $\phi\text{store} = \{\}$ ;
 $u\text{store} = \{\}$ ;
 $F\text{store} = \{\}$ ;
 $F\text{Maxstore} = \{\}$ ; (* $F_x$  in when  $u = 0.001$ *)

```

```

In[17]:= (*Finding U intercepts*)
While[ $\phi < 90 * \text{Pi} / 180$ ,
  (*Print statements*)
  (*Print["Let  $\phi = "$ ,  $\phi * 180 / \text{Pi}$ , " deg"];*)
  (*Print@
    Plot[{FranChen[u], 0}, {u, 0, 0.2}, PlotLabel -> "Inner Fx vs. Helix Velocity U/Rw",
      AxesLabel -> {"U (m/s)", "Fx (N/m^2)"}, PlotRange -> All];*)

  (*Finding U intercept: Newton-Raphson Method*)
  guess = 0.001; (*Reset initial guess*)
  grad = D[FranChen[u], u];
  While[FranChen[guess] >  $10^{-6}$ , (*Keep iterating until FxIn  $\approx 0$ *)
    gradEval = grad /. u -> guess; (*Find FxIn'(guess)*)
    guess = guess - FranChen[guess] / gradEval (* $u_{i+1} = u_i - \text{FxIn}(u_i) / \text{FxIn}'(u_i)$ *)
  ];
  uint = guess; (*U/Rw intercept found*)

  (*Storing data in arrays*)
   $\phi$ store = Join[ $\phi$ store, { $\phi$ }]; (*Storing  $\phi$  in Radians*)
  FMaxstore = Join[FMaxstore, {FranChen[ $10^{-3}$ ]}]; (*FxIn(0.001)*)
  ustore = Join[ustore, {uint}]; (*U found when FxIn <  $10^{-6}$ *)
  Fstore = Join[Fstore, {FranChen[uint]}]; (*FxIn val at u-intercept*)

   $\phi = \phi + (1 * \text{Pi} / 180)$  (*Increment by 1 deg*)
]

```

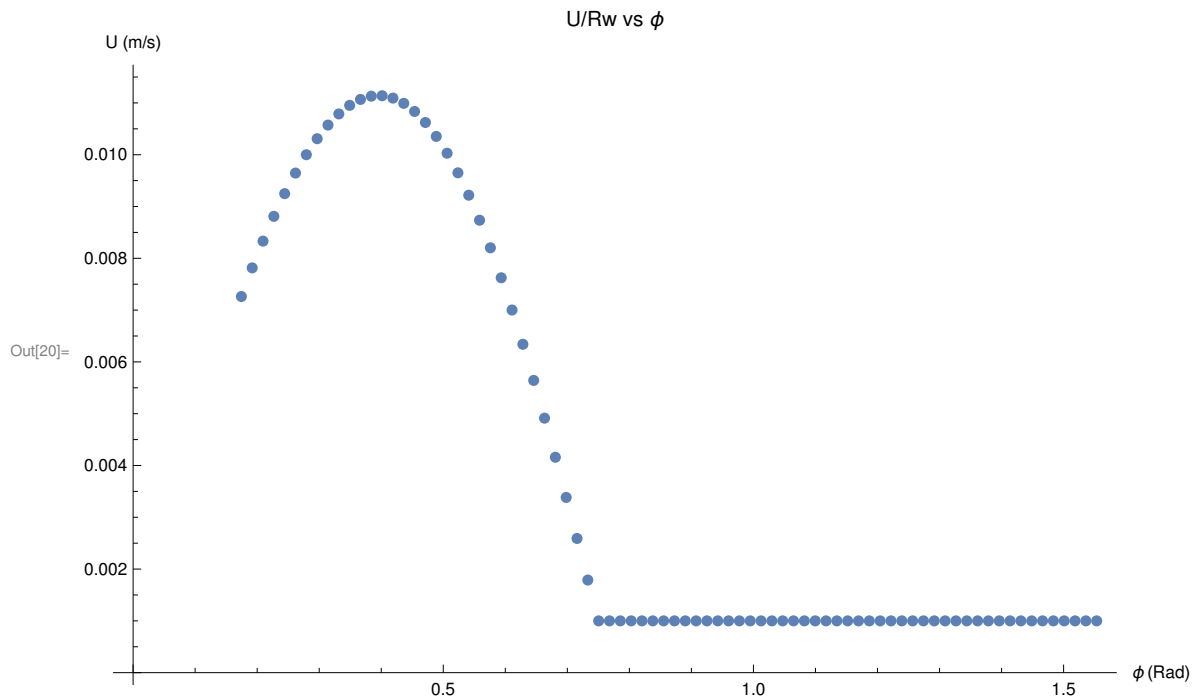
Plotting Francisco - Chen Results

```
In[18]:=  $\phi$ FCcol = { $\phi$ store}T; uFCcol = {ustore}T;
```

```
(*Helix Translation Speed vs. Local Incline Angle  $\phi$ *)
```

```
dataPlot = Join[ $\phi$ FCcol, uFCcol, 2];
```

```
ListPlot[dataPlot, PlotLabel → "U/Rw vs  $\phi$ ",  
  AxesLabel → {" $\phi$  (Rad)", "U (m/s)"}, ImageSize → Large]
```



Using Chen's Fourier coefficients does result in a maximum speed!!! Only phi values between 0 and 45 ° produce net forward thrust results!

Testing Auger Model with Correct Chen Coefficients

Parameters

For Helix

```
In[1]:= (*Current param: R = 1.8cm, n = 3.5*)
```

```
R = 0.018; (*Screw radius, m*)
```

```
n = 3.5; (*Number of helix turns*)
```

For Material

```
In[3]:= (*LP poppy Fourier coefficients*)
A00 = 0.051; A10 = 0.047; B11 = 0.053; B01 = 0.083;
Bn11 = 0.020; C11 = -0.026; C01 = 0.057; Cn11 = 0; D10 = 0.025;

β = (Pi/2) - φ; (*φ is symbolic, radians*)
αZ = Bn11 * Sin[2 * Pi * (-β/Pi)] + A00 * Cos[2 * Pi * 0] +
    B01 * Sin[2 * Pi * 0] + A10 * Cos[2 * Pi * (β/Pi)] + B11 * Sin[2 * Pi * (β/Pi)];
(*Vertical stress per unit depth, N/m^3*)
αX = Cn11 * Cos[2 * Pi * (-β/Pi)] + C01 * Cos[2 * Pi * 0] +
    D10 * Sin[2 * Pi * (β/Pi)] + C11 * Cos[2 * Pi * (β/Pi)];
(*Horizontal stress per unit depth, N/m^3*)

d = 0.05; (*Depth robot buried, m*)

(*Friction coefficients, expressed in terms of φ*)
Cn = αX * d; (*N/m^2*)
Ct = αZ * d;
```

For Motor

```
In[10]:= w = 2 * 1000 * (2 * Pi) / 3584; (*Angular velocity with 12V source, rad/s*)
    ■ (2 ticks/ms)*(1000 ms/s)*(2*Pi rad/rev)*(1 rev/3584 ticks)
```

Horizontal Thrust Inner Equation

```
In[11]:= FxIn[U_] := (2 * Pi * n / Cos[φ]) *
    ((Cn - Ct) * w * Sin[φ] * Cos[φ]) * (((R / (2 * w^2)) * Sqrt[(R * w)^2 + U^2]) +
    ((U^2 / (2 * w^3)) * (Log[U] - Log[R * w + Sqrt[(R * w)^2 + U^2]]))) -
    (U * (Ct * Sin[φ]^2 + Cn * Cos[φ]^2) * (Sqrt[(R * w)^2 + U^2] - U) / w^2);

φ input must be in radians
```

Calculating U/Rw when Fx = 0 for Many φ Cases

```
In[12]:= φ = 10 * Pi / 180 // N; (*Local inclination, radians*)
φstore = {};
Ustore = {};
Fstore = {};
FMaxstore = {}; (*FxIn when u = 0.001*)
```

```

In[17]:= While[ $\phi < 90 * \text{Pi} / 180$ ,
  (*Print statements*)
  (*Print["Let  $\phi = "$ ,  $\phi * 180 / \text{Pi}$ , " deg"];*)
  (*Print@Plot[{FxFIn[U], 0}, {U, 0, 0.1}, PlotLabel -> "Inner Fx vs. Helix Velocity U/Rw",
    AxesLabel -> {"U (m/s)", "Fx (N/m^2)"}, PlotRange -> All];*)

  (*Finding U intercept: Newton-Raphson Method*)
  guess = 0.001; (*Reset initial guess*)
  grad = D[FxFIn[U], U];
  While[FxFIn[guess] >  $10^{-6}$ , (*Keep iterating until FxFIn  $\approx 0$ *)
    gradEval = grad /. U -> guess; (*Find FxFIn'(guess)*)
    guess = guess - FxFIn[guess] / gradEval (* $u_{i+1} = u_i - \text{FxFIn}(u_i) / \text{FxFIn}'(u_i)$ *)
  ];
  Uint = guess; (*U intercept found*)

  (*Storing data in arrays*)
   $\phi$ store = Join[ $\phi$ store, { $\phi$ }]; (*Storing  $\phi$  in Radians*)
  FMaxstore = Join[FMaxstore, {FxFIn[ $10^{-3}$ ]}]; (*FxFIn(0.001)*)
  Ustore = Join[Ustore, {Uint}]; (*U found when FxFIn <  $10^{-6}$ *)
  Fstore = Join[Fstore, {FxFIn[Uint]}]; (*FxFIn val at U-intercept*)

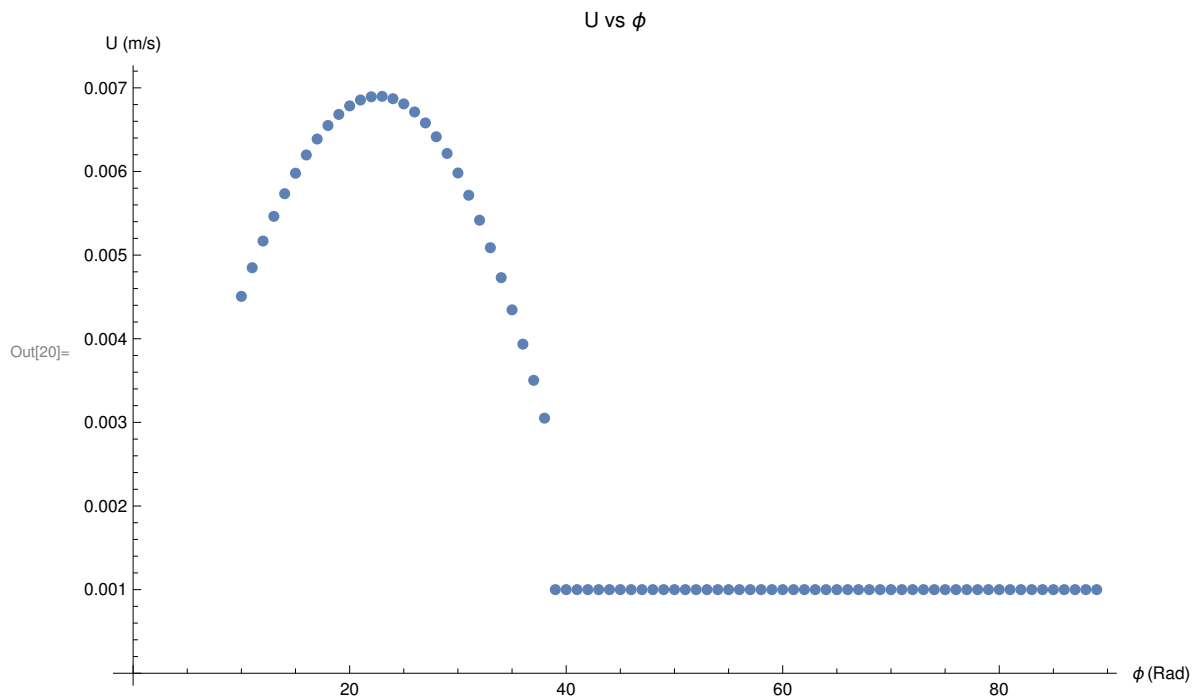
   $\phi = \phi + (1 * \text{Pi} / 180)$  (*Increment by 1 deg*)
]

```

Analysis

```
In[18]:=  $\phi\text{col} = \{\phi\text{store}\}^T$ ;
 $U\text{col} = \{U\text{store}\}^T$ ;
 $F\text{col} = \{F\text{store}\}^T$ ;
 $F\text{maxcol} = \{F\text{Maxstore}\}^T$ ;

(*Helix Translation Speed vs. Local Incline Angle  $\phi$ *)
dataPlot = Join[ $\phi\text{col} * 180 / \text{Pi}$ , Ucol, 2];
ListPlot[dataPlot, PlotLabel -> "U vs  $\phi$ ",
  AxesLabel -> {" $\phi$  (Rad)", "U (m/s)"}, ImageSize -> Large]
```

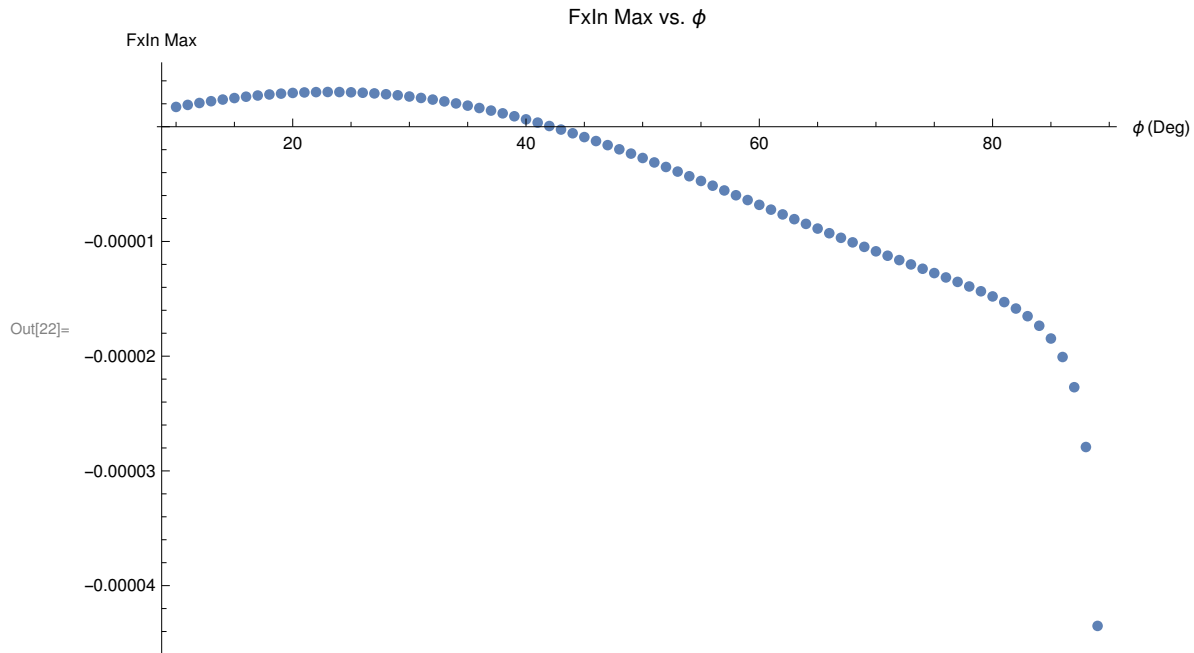


With Chen's coefficients, the dimensional F_x equation does yield a U maximum speed :) Same deal with Francisco's model, where net forward thrust only exists for $0 < \phi < 40^\circ$

```

In[21]:= (*FxIn Max vs. Local Incline Angle  $\phi$ *)
dataMax = Join[ $\phi$ col * 180 / Pi, Fmaxcol, 2];
ListPlot[dataMax, PlotLabel -> "FxIn Max vs.  $\phi$ ",
  AxesLabel -> {" $\phi$  (Deg)", "FxIn Max"}, ImageSize -> Large, PlotRange -> Full]

```



```

In[23]:= (*Checking Data Values:  $\phi$  is left col, U/Rw is right col*)
dataTable = Join[ $\phi$ col * 180 / Pi, Ucol, Fcol, 2];
Grid[Join[{" $\phi$  (Deg)", "U", "FxIn[U]"}, dataTable, 1]]

```

ϕ (Deg)	U	FxIn[U]
10.	0.00450688	7.8351×10^{-8}
11.	0.00484907	9.34992×10^{-8}
12.	0.00516831	1.08693×10^{-7}
13.	0.0054637	1.23644×10^{-7}
14.	0.00573433	1.3808×10^{-7}
15.	0.00597925	1.5175×10^{-7}
16.	0.0061975	1.64422×10^{-7}
17.	0.00638811	1.75883×10^{-7}
18.	0.00655009	1.85942×10^{-7}
19.	0.0066825	1.94426×10^{-7}
20.	0.00678441	2.01188×10^{-7}
21.	0.00685494	2.06103×10^{-7}
22.	0.00689327	2.09069×10^{-7}
23.	0.00689869	2.10013×10^{-7}
24.	0.00687057	2.08885×10^{-7}
25.	0.00680842	2.0567×10^{-7}
26.	0.00671192	2.00381×10^{-7}
27.	0.00658089	1.93065×10^{-7}
28.	0.00641539	1.83806×10^{-7}

Out[24]=

29.	0.00621568	1.72725×10^{-7}
30.	0.00598224	1.59986×10^{-7}
31.	0.00571585	1.45793×10^{-7}
32.	0.00541752	1.304×10^{-7}
33.	0.00508859	1.14104×10^{-7}
34.	0.00473063	9.72539×10^{-8}
35.	0.00434554	8.02475×10^{-8}
36.	0.00393549	6.35323×10^{-8}
37.	0.00350289	4.76042×10^{-8}
38.	0.00305045	3.30051×10^{-8}
39.	0.001	9.11988×10^{-7}
40.	0.001	6.4262×10^{-7}
41.	0.001	3.59188×10^{-7}
42.	0.001	6.22345×10^{-8}
43.	0.001	-2.47663×10^{-7}
44.	0.001	-5.69891×10^{-7}
45.	0.001	-9.03807×10^{-7}
46.	0.001	-1.24874×10^{-6}
47.	0.001	-1.60398×10^{-6}
48.	0.001	-1.9688×10^{-6}
49.	0.001	-2.34247×10^{-6}
50.	0.001	-2.7242×10^{-6}
51.	0.001	-3.11322×10^{-6}
52.	0.001	-3.50872×10^{-6}
53.	0.001	-3.9099×10^{-6}
54.	0.001	-4.31595×10^{-6}
55.	0.001	-4.72605×10^{-6}
56.	0.001	-5.13937×10^{-6}
57.	0.001	-5.55512×10^{-6}
58.	0.001	-5.97248×10^{-6}
59.	0.001	-6.39069×10^{-6}
60.	0.001	-6.80897×10^{-6}
61.	0.001	-7.2266×10^{-6}
62.	0.001	-7.64286×10^{-6}
63.	0.001	-8.05712×10^{-6}
64.	0.001	-8.46877×10^{-6}
65.	0.001	-8.87726×10^{-6}
66.	0.001	-9.28214×10^{-6}
67.	0.001	-9.68305×10^{-6}
68.	0.001	-0.0000100797
69.	0.001	-0.0000104721
70.	0.001	-0.0000108601
71.	0.001	-0.0000112442
72.	0.001	-0.0000116248
73.	0.001	-0.0000120027
74.	0.001	-0.0000123794
75.	0.001	-0.0000127567
76.	0.001	-0.0000131372
77.	0.001	-0.0000135246

78.	0.001	-0.0000139241
79.	0.001	-0.0000143429
80.	0.001	-0.0000147915
81.	0.001	-0.0000152852
82.	0.001	-0.0000158473
83.	0.001	-0.0000165145
84.	0.001	-0.0000173485
85.	0.001	-0.0000184605
86.	0.001	-0.000020073
87.	0.001	-0.0000227059
88.	0.001	-0.0000279189
89.	0.001	-0.0000435114

$\phi = 23^\circ$ yields the highest value for U (m/s).