

# AugerBot Calculations

Quit

Trial 1: 9/19 - 9/26

Modified Francisco Calculations : 10/18 (Do not use!)

Plotting  $F_x$  to find  $U$  which Balances Forces:  
11/1

Plotting  $F_x$  to find  $U$  which Balances Forces 2 :  
11/9

Inside Equation for Thrust: 11/16

Backtracking: 11/27

Francisco' s with Chen' s Coefficients: 11/30  
(CORRECT ONES)

Quit;

## Francisco With Chen' s Coefficients

```

In[1]:= (*Parameters from paper*)
r = 5/1000; w = 10.4; n = 1;

(*LP poppy Fourier coefficients*)
A00 = 0.051; A10 = 0.047; B11 = 0.053; B01 = 0.083;
Bn11 = 0.020; C11 = -0.026; C01 = 0.057; Cn11 = 0; D10 = 0.025;

In[3]:=  $\beta = (\text{Pi}/2) - \phi$ ; (* $\phi$  is symbolic, radians*)
 $\alpha_z = Bn11 * \text{Sin}[2 * \text{Pi} * (-\beta / \text{Pi})] + A00 * \text{Cos}[2 * \text{Pi} * \theta] +$ 
 $B01 * \text{Sin}[2 * \text{Pi} * \theta] + A10 * \text{Cos}[2 * \text{Pi} * (\beta / \text{Pi})] + B11 * \text{Sin}[2 * \text{Pi} * (\beta / \text{Pi})];$ 
(*Vertical stress per unit depth, N/m^3*)
 $\alpha_x = Cn11 * \text{Cos}[2 * \text{Pi} * (-\beta / \text{Pi})] + C01 * \text{Cos}[2 * \text{Pi} * \theta] +$ 
 $D10 * \text{Sin}[2 * \text{Pi} * (\beta / \text{Pi})] + C11 * \text{Cos}[2 * \text{Pi} * (\beta / \text{Pi})];$ 
(*Horizontal stress per unit depth, N/m^3*)

(*Friction coefficients, expressed in terms of  $\phi$ *)
d = 0.05; (*Depth robot buried, 50mm*)
Cn =  $\alpha_x * d$ ; (*N/m^2*)
Ct =  $\alpha_z * d$ ;

Cn /.  $\phi \rightarrow 0$ 
Ct /.  $\phi \rightarrow 0$ 

Out[9]= 0.00415
Out[10]= 0.0002

In[11]:= (*Eq 2 - 10/21/17 | Parametrized f Integrated only wrt  $d\theta$ *)
FranChen[u_] :=  $(2 * \text{Pi} * n * r / \text{Cos}[\phi]) *$ 
 $((Cn - Ct) * r * w * \text{Sin}[\phi] * \text{Cos}[\phi] - u * (Ct * \text{Sin}[\phi]^2 + Cn * \text{Cos}[\phi]^2)) /$ 
 $\text{Sqrt}[u^2 + (r * w)^2];$ 

```

## Calculating U/Rw when $F_x = 0$ for Many $\phi$ Cases

```

In[12]:=  $\phi = 10 * \text{Pi} / 180 // N$ ; (*Local inclination, radians*)
 $\phi\text{store} = \{\}$ ;
 $u\text{store} = \{\}$ ;
 $F\text{store} = \{\}$ ;
 $F\text{Maxstore} = \{\}$ ; (* $F_x$  in when  $u = 0.001$ *)

```

```

In[17]:= (*Finding U intercepts*)
While[ $\phi < 90 * \text{Pi} / 180$ ,
  (*Print statements*)
  (*Print["Let  $\phi = "$ ,  $\phi * 180 / \text{Pi}$ , " deg"];*)
  (*Print@
    Plot[{FranChen[u], 0}, {u, 0, 0.2}, PlotLabel -> "Inner Fx vs. Helix Velocity U/Rw",
      AxesLabel -> {"U (m/s)", "Fx (N/m^2)"}, PlotRange -> All];*)

  (*Finding U intercept: Newton-Raphson Method*)
  guess = 0.001; (*Reset initial guess*)
  grad = D[FranChen[u], u];
  While[FranChen[guess] >  $10^{-6}$ , (*Keep iterating until FxIn  $\approx 0$ *)
    gradEval = grad /. u -> guess; (*Find FxIn'(guess)*)
    guess = guess - FranChen[guess] / gradEval (* $u_{i+1} = u_i - \text{FxIn}(u_i) / \text{FxIn}'(u_i)$ *)
  ];
  uint = guess; (*U/Rw intercept found*)

  (*Storing data in arrays*)
   $\phi$ store = Join[ $\phi$ store, { $\phi$ }]; (*Storing  $\phi$  in Radians*)
  FMaxstore = Join[FMaxstore, {FranChen[ $10^{-3}$ ]}]; (*FxIn(0.001)*)
  ustore = Join[ustore, {uint}]; (*U found when FxIn <  $10^{-6}$ *)
  Fstore = Join[Fstore, {FranChen[uint]}]; (*FxIn val at u-intercept*)

   $\phi = \phi + (1 * \text{Pi} / 180)$  (*Increment by 1 deg*)
]

```

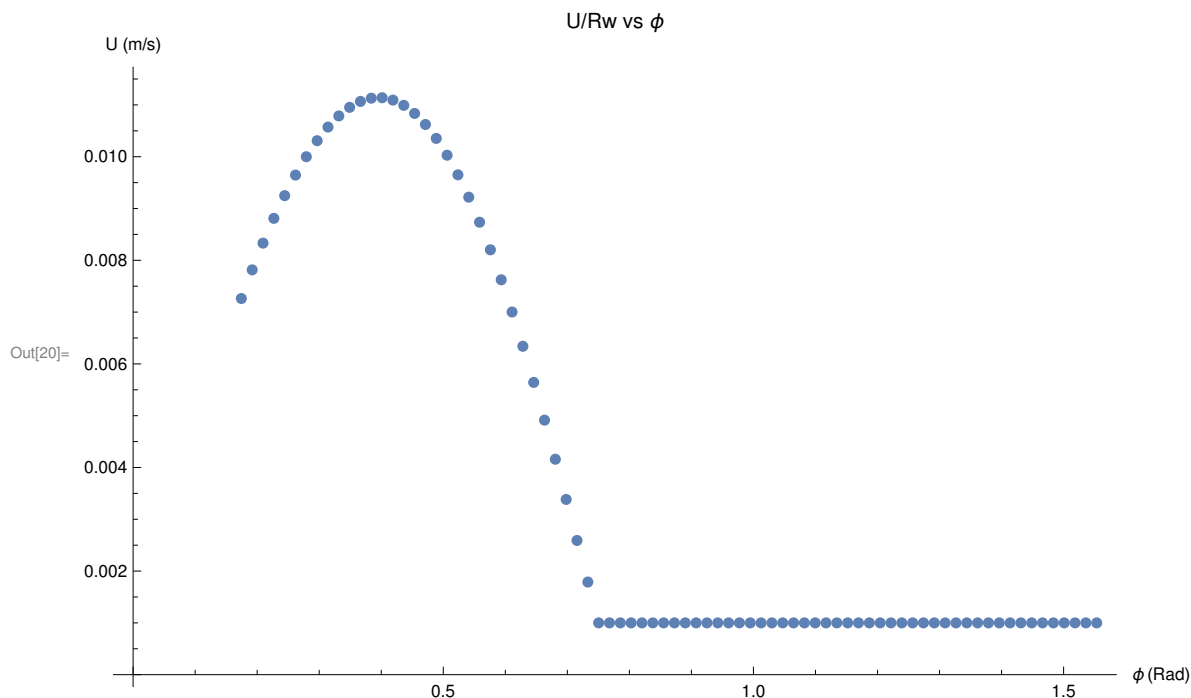
## Plotting Francisco - Chen Results

```
In[18]:=  $\phi$ FCcol = { $\phi$ store}T; uFCcol = {ustore}T;
```

```
(*Helix Translation Speed vs. Local Incline Angle  $\phi$ *)
```

```
dataPlot = Join[ $\phi$ FCcol, uFCcol, 2];
```

```
ListPlot[dataPlot, PlotLabel → "U/Rw vs  $\phi$ ",  
  AxesLabel → {" $\phi$  (Rad)", "U (m/s)"}, ImageSize → Large]
```



Using Chen's Fourier coefficients does result in a maximum speed!!! Only phi values between 0 and 45 ° produce net forward thrust results!

## Testing Auger Model with Correct Chen Coefficients

### Parameters

#### For Helix

```
In[1]:= (*Current param: R = 1.8cm, n = 3.5*)
```

```
R = 0.018; (*Screw radius, m*)
```

```
n = 1; (*Number of helix turns*)
```

## For Material

```
In[3]:= (*LP poppy Fourier coefficients*)
A00 = 0.051; A10 = 0.047; B11 = 0.053; B01 = 0.083;
Bn11 = 0.020; C11 = -0.026; C01 = 0.057; Cn11 = 0; D10 = 0.025;

β = (Pi/2) - φ; (*φ is symbolic, radians*)
αZ = Bn11 * Sin[2 * Pi * (-β/Pi)] + A00 * Cos[2 * Pi * 0] +
    B01 * Sin[2 * Pi * 0] + A10 * Cos[2 * Pi * (β/Pi)] + B11 * Sin[2 * Pi * (β/Pi)];
(*Vertical stress per unit depth, N/m^3*)
αX = Cn11 * Cos[2 * Pi * (-β/Pi)] + C01 * Cos[2 * Pi * 0] +
    D10 * Sin[2 * Pi * (β/Pi)] + C11 * Cos[2 * Pi * (β/Pi)];
(*Horizontal stress per unit depth, N/m^3*)

d = 0.05; (*Depth robot buried, m*)

(*Friction coefficients, expressed in terms of φ*)
Cn = αX * d; (*N/m^2*)
Ct = αZ * d;
```

## For Motor

```
In[10]:= w = 2 * 1000 * (2 * Pi) / 3584; (*Angular velocity with 12V source, rad/s*)
    ■ (2 ticks/ms)*(1000 ms/s)*(2*Pi rad/rev)*(1 rev/3584 ticks)
```

## Horizontal Thrust Inner Equation

```
In[11]:= FxIn[U_] := (2 * Pi * n / Cos[φ]) *
    ((Cn - Ct) * w * Sin[φ] * Cos[φ]) * (((R / (2 * w^2)) * Sqrt[(R * w)^2 + U^2]) +
    ((U^2 / (2 * w^3)) * (Log[U] - Log[R * w + Sqrt[(R * w)^2 + U^2]]))) -
    (U * (Ct * Sin[φ]^2 + Cn * Cos[φ]^2) * (Sqrt[(R * w)^2 + U^2] - U) / w^2);

φ input must be in radians
```

## Calculating U/Rw when Fx = 0 for Many φ Cases

```
In[43]:= φ = 10 * Pi / 180 // N; (*Local inclination, radians*)
φstore = {};
Ustore = {};
Fstore = {};
FMaxstore = {}; (*FxIn when u = 0.001*)
```

```

In[48]:= While[ $\phi < 90 * \text{Pi} / 180$ ,
  (*Print statements*)
  (*Print["Let  $\phi = ", \phi * 180 / \text{Pi}$ , " deg"];*)
  (*Print@Plot[{FxIn[U], 0}, {U, 0, 0.1}, PlotLabel->"Inner Fx vs. Helix Velocity U/Rw",
    AxesLabel->{"U (m/s)", "Fx (N/m^2)"}, PlotRange->All];*)

  (*Finding U intercept: Newton-Raphson Method*)
  guess =  $10^{-6}$ ; (*Reset initial guess*)
  grad = D[FxIn[U], U];

  (*Print[FxIn[guess]];*)

  While[FxIn[guess] >  $10^{-12}$ , (*Keep iterating until FxIn  $\approx 0$ *)
    gradEval = grad /. U -> guess; (*Find FxIn'(guess)*)
    guess = guess - FxIn[guess] / gradEval (* $u_{i+1} = u_i - \text{FxIn}(u_i) / \text{FxIn}'(u_i)$ *)
  ];
  Uint = guess; (*U intercept found*)

  (*Storing data in arrays*)
   $\phi$ store = Join[ $\phi$ store, { $\phi$ }]; (*Storing  $\phi$  in Radians*)
  FMaxstore = Join[FMaxstore, {FxIn[ $10^{-3}$ ]}]; (*FxIn(0.001)*)
  Ustore = Join[Ustore, {Uint}]; (*U found when FxIn <  $10^{-6}$ *)
  Fstore = Join[Fstore, {FxIn[Uint]}]; (*FxIn val at U-intercept*)

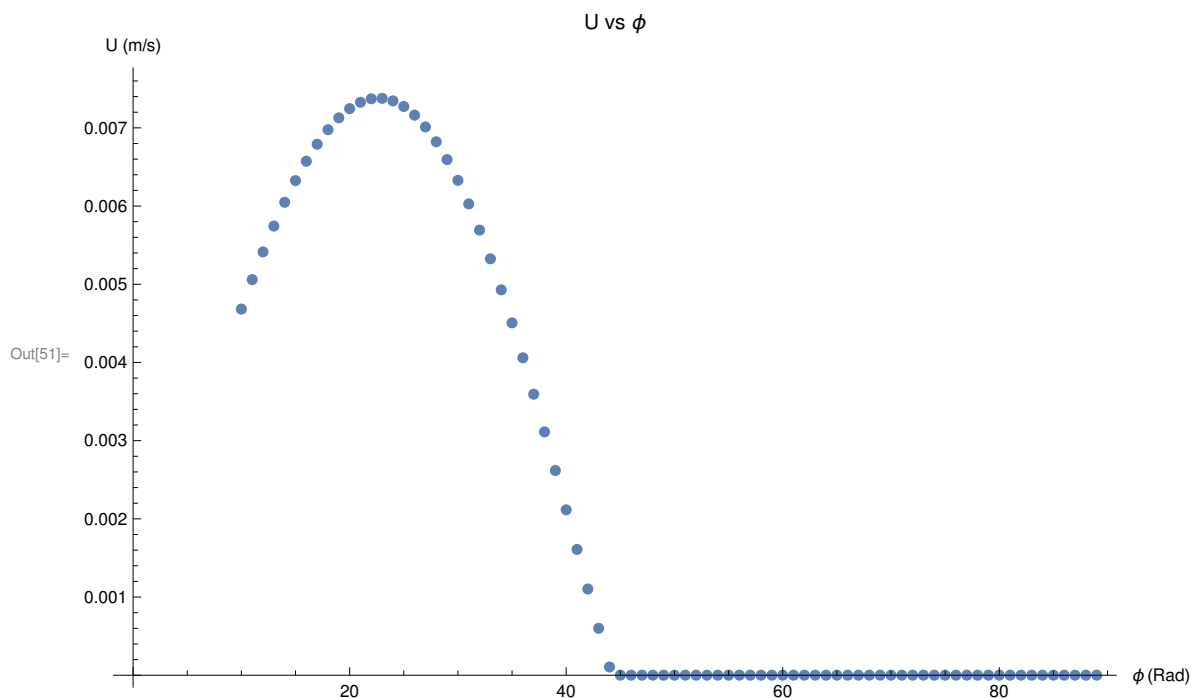
   $\phi = \phi + (1 * \text{Pi} / 180)$  (*Increment by 1 deg*)
]

```

## Analysis

```
In[49]:=  $\phi\text{col} = \{\phi\text{store}\}^T$ ;
 $U\text{col} = \{U\text{store}\}^T$ ;
 $F\text{col} = \{F\text{store}\}^T$ ;
 $F\text{maxcol} = \{F\text{Maxstore}\}^T$ ;

(*Helix Translation Speed vs. Local Incline Angle  $\phi$ *)
dataPlot = Join[ $\phi\text{col} * 180 / \text{Pi}$ , Ucol, 2];
ListPlot[dataPlot, PlotLabel -> "U vs  $\phi$ ",
  AxesLabel -> {" $\phi$  (Rad)", "U (m/s)"}, ImageSize -> Large]
```

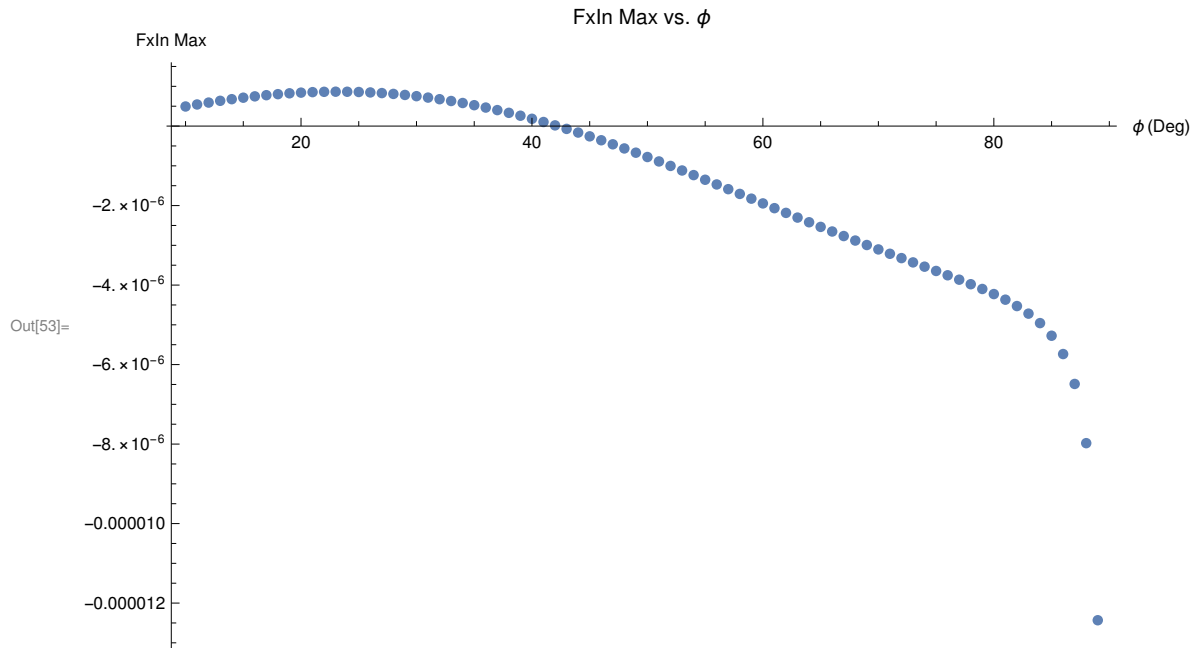


With Chen's coefficients, the dimensional  $F_x$  equation does yield a  $U$  maximum speed :) Same deal with Francisco's model, where net forward thrust only exists for  $0 < \phi < 40^\circ$

```

In[52]:= (*FxIn Max vs. Local Incline Angle  $\phi$ *)
dataMax = Join[ $\phi$ col * 180 / Pi, Fmaxcol, 2];
ListPlot[dataMax, PlotLabel -> "FxIn Max vs.  $\phi$ ",
  AxesLabel -> {" $\phi$  (Deg)", "FxIn Max"}, ImageSize -> Large, PlotRange -> Full]

```



```

In[54]:= (*Checking Data Values:  $\phi$  is left col, U/Rw is right col*)
dataTable = Join[ $\phi$ col * 180 / Pi, Ucol, Fcol, 2];
Grid[Join[{" $\phi$  (Deg)", "U", "FxIn[U]"}, dataTable, 1]]

```

$\phi$ (Deg)	U	FxIn[U]
10.	0.00468258	$1.8561 \times 10^{-15}$
11.	0.00505958	$3.17729 \times 10^{-15}$
12.	0.00541394	$5.0654 \times 10^{-15}$
13.	0.00574407	$7.59502 \times 10^{-15}$
14.	0.0060484	$1.07902 \times 10^{-14}$
15.	0.00632535	$1.46078 \times 10^{-14}$
16.	0.00657334	$1.89275 \times 10^{-14}$
17.	0.00679084	$2.35511 \times 10^{-14}$
18.	0.00697635	$2.82127 \times 10^{-14}$
19.	0.00712844	$3.25989 \times 10^{-14}$
20.	0.00724577	$3.63793 \times 10^{-14}$
21.	0.00732711	$3.92417 \times 10^{-14}$
22.	0.00737137	$4.09297 \times 10^{-14}$
23.	0.00737762	$4.12758 \times 10^{-14}$
24.	0.00734515	$4.0226 \times 10^{-14}$
25.	0.00727345	$3.78512 \times 10^{-14}$
26.	0.00716228	$3.4342 \times 10^{-14}$
27.	0.00701169	$2.99876 \times 10^{-14}$
28.	0.00682204	$2.51412 \times 10^{-14}$
29.	0.00659404	$2.0176 \times 10^{-14}$



Out[55]=

30.	0.00632874	$1.5439 \times 10^{-14}$
31.	0.00602757	$1.12112 \times 10^{-14}$
32.	0.00569233	$7.67893 \times 10^{-15}$
33.	0.00532521	$4.92292 \times 10^{-15}$
34.	0.00492876	$2.92488 \times 10^{-15}$
35.	0.00450587	$1.58964 \times 10^{-15}$
36.	0.00405974	$7.76558 \times 10^{-16}$
37.	0.00359388	$3.32758 \times 10^{-16}$
38.	0.00311198	$1.20726 \times 10^{-16}$
39.	0.00261794	$3.51316 \times 10^{-17}$
40.	0.00211574	$7.50228 \times 10^{-18}$
41.	0.00160943	$9.99593 \times 10^{-19}$
42.	0.00110303	$8.16426 \times 10^{-13}$
43.	0.000600491	$8.02477 \times 10^{-14}$
44.	0.000105622	$8.52882 \times 10^{-17}$
45.	$\frac{1}{1\,000\,000}$	$-7.2164 \times 10^{-8}$
46.	$\frac{1}{1\,000\,000}$	$-1.66505 \times 10^{-7}$
47.	$\frac{1}{1\,000\,000}$	$-2.63455 \times 10^{-7}$
48.	$\frac{1}{1\,000\,000}$	$-3.62789 \times 10^{-7}$
49.	$\frac{1}{1\,000\,000}$	$-4.64271 \times 10^{-7}$
50.	$\frac{1}{1\,000\,000}$	$-5.67661 \times 10^{-7}$
51.	$\frac{1}{1\,000\,000}$	$-6.72712 \times 10^{-7}$
52.	$\frac{1}{1\,000\,000}$	$-7.79171 \times 10^{-7}$
53.	$\frac{1}{1\,000\,000}$	$-8.86782 \times 10^{-7}$
54.	$\frac{1}{1\,000\,000}$	$-9.95281 \times 10^{-7}$
55.	$\frac{1}{1\,000\,000}$	$-1.10441 \times 10^{-6}$
56.	$\frac{1}{1\,000\,000}$	$-1.21389 \times 10^{-6}$
57.	$\frac{1}{1\,000\,000}$	$-1.32345 \times 10^{-6}$
58.	$\frac{1}{1\,000\,000}$	$-1.43284 \times 10^{-6}$
59.	$\frac{1}{1\,000\,000}$	$-1.54176 \times 10^{-6}$
60.	$\frac{1}{1\,000\,000}$	$-1.64996 \times 10^{-6}$
61.	$\frac{1}{1\,000\,000}$	$-1.75716 \times 10^{-6}$
62.	$\frac{1}{1\,000\,000}$	$-1.8631 \times 10^{-6}$
63.	$\frac{1}{1\,000\,000}$	$-1.9675 \times 10^{-6}$
64.	$\frac{1}{1\,000\,000}$	$-2.07011 \times 10^{-6}$
65.	$\frac{1}{1\,000\,000}$	$-2.17066 \times 10^{-6}$
66.	$\frac{1}{1\,000\,000}$	$-2.2689 \times 10^{-6}$
67.	$\frac{1}{1\,000\,000}$	$-2.36458 \times 10^{-6}$

68.	$\frac{1}{1\,000\,000}$	$-2.45746 \times 10^{-6}$
69.	$\frac{1}{1\,000\,000}$	$-2.5473 \times 10^{-6}$
70.	$\frac{1}{1\,000\,000}$	$-2.63387 \times 10^{-6}$
71.	$\frac{1}{1\,000\,000}$	$-2.71694 \times 10^{-6}$
72.	$\frac{1}{1\,000\,000}$	$-2.79631 \times 10^{-6}$
73.	$\frac{1}{1\,000\,000}$	$-2.87177 \times 10^{-6}$
74.	$\frac{1}{1\,000\,000}$	$-2.94313 \times 10^{-6}$
75.	$\frac{1}{1\,000\,000}$	$-3.01019 \times 10^{-6}$
76.	$\frac{1}{1\,000\,000}$	$-3.0728 \times 10^{-6}$
77.	$\frac{1}{1\,000\,000}$	$-3.13078 \times 10^{-6}$
78.	$\frac{1}{1\,000\,000}$	$-3.18398 \times 10^{-6}$
79.	$\frac{1}{1\,000\,000}$	$-3.23228 \times 10^{-6}$
80.	$\frac{1}{1\,000\,000}$	$-3.27554 \times 10^{-6}$
81.	$\frac{1}{1\,000\,000}$	$-3.31365 \times 10^{-6}$
82.	$\frac{1}{1\,000\,000}$	$-3.34652 \times 10^{-6}$
83.	$\frac{1}{1\,000\,000}$	$-3.37407 \times 10^{-6}$
84.	$\frac{1}{1\,000\,000}$	$-3.39624 \times 10^{-6}$
85.	$\frac{1}{1\,000\,000}$	$-3.41301 \times 10^{-6}$
86.	$\frac{1}{1\,000\,000}$	$-3.42439 \times 10^{-6}$
87.	$\frac{1}{1\,000\,000}$	$-3.4305 \times 10^{-6}$
88.	$\frac{1}{1\,000\,000}$	$-3.43177 \times 10^{-6}$
89.	$\frac{1}{1\,000\,000}$	$-3.43047 \times 10^{-6}$

$\phi = 23^\circ$  yields the highest value for U (m/s).