

INSTITUTO FEDERAL DE EDUCAÇÃO, CIÊNCIA E TECNOLOGIA DE SÃO
PAULO – CAMPUS CUBATÃO

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Turma: CTII 317

TAREFA BÁSICA – SISTEMAS LINEARES – REGRA DE CRAMER – ESCALONAMENTO (Gauss)

QUESTÕES

REGRA DE CRAMER

01. a)

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Tarefa Básica - Sistemas Lineares

Regra de Cramer

01. a) $\begin{cases} 2x - y = 2 \\ -x + 3y = -3 \end{cases}$

$D = \begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix}$
 $6 - 1 = 5$

$D_x = \begin{vmatrix} 2 & 2 \\ -1 & -3 \end{vmatrix}$
 $-6 - (-2) = -4$

$D_y = \begin{vmatrix} 2 & -1 \\ -3 & 3 \end{vmatrix}$
 $6 - 3 = 3$

$x = \frac{D_x}{D} = \frac{-4}{5}$ $y = \frac{D_y}{D} = \frac{3}{5}$

$V = \left\{ \begin{pmatrix} 3 \\ 5 \end{pmatrix}, \begin{pmatrix} -4 \\ 5 \end{pmatrix} \right\}$

b)

$$c) \begin{cases} 3x - y + z = 1 \\ 2x + 3z = -1 \\ 4x + y - 2z = 7 \end{cases}$$

$$D = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 0 & 3 \\ 4 & 1 & -2 \end{vmatrix}$$

$$0 - 12 + 2 = -10$$

$$4 + 9 + 0 = 13$$

$$-10 - 13 = -23$$

$$D_x = \begin{vmatrix} 1 & -1 & 1 \\ -1 & 0 & 3 \\ 7 & 1 & -2 \end{vmatrix}$$

$$0 - 22 + 1 = -22$$

$$-2 + 3 + 0 = 1$$

$$D_x = -22 - 1 = -23$$

$$D_y = \begin{vmatrix} 3 & 1 & 1 \\ 2 & -1 & 3 \\ 4 & 7 & -2 \end{vmatrix}$$

$$6 + 12 + 14 = 32$$

$$-4 + 63 - 4 = 55$$

$$32 - 55 = -23$$

$$D_z = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 0 & -1 \\ 4 & 1 & 7 \end{vmatrix}$$

$$0 + 4 + 2 = 6$$

$$-14 - 3 + 0 = -17$$

$$6 - (-17) = 23$$

$$\star$$

$$Dz = \begin{array}{ccc|cc} 3 & -1 & 1 & 3 & -1 \\ 2 & 0 & -1 & 2 & 0 \\ 4 & 1 & 7 & 4 & 1 \end{array}$$

$$0 + 4 + 2 = 6$$

$$-14 - 3 + 0 = -17$$

$$6 - (-17) = 23$$

$$x = \cancel{Dx} = \frac{+23}{+23} = 1$$

$$D \quad +23$$

$$y = \frac{Dy}{D} = \frac{+23}{+23} = 1$$

$$z = \frac{Dz}{D} = \frac{23}{-23} = -1$$

$$V = \{(1, 1, -1)\}$$

02.

$$\begin{cases} 3x + 4y - z = 1 \\ 4x + 5y + 2z = 12 \\ x - 2y + 3z = 8 \end{cases}$$

$$D = \begin{vmatrix} 3 & 4 & -1 \\ 4 & 5 & 2 \\ 1 & -2 & 3 \end{vmatrix} = \begin{vmatrix} 3 & 4 & -1 \\ 4 & 5 & 2 \\ 1 & -2 & 3 \end{vmatrix}$$

$$45 + 8 + 8 = 61$$

$$\star 48 - 12 - 5 = 31$$

$$61 - 31 = 30$$

$$D_y = \begin{vmatrix} 3 & 1 & -1 \\ 4 & 12 & 2 \\ 1 & 8 & 3 \end{vmatrix} = \begin{vmatrix} 3 & 1 & -1 \\ 4 & 12 & 2 \\ 1 & 8 & 3 \end{vmatrix}$$

$$108 + 2 - 32 = 78$$

$$12 + 48 - 12 = 48$$

$$78 - 48 = 30$$

$$y = \frac{D_y}{D} = \frac{30}{30} = 1$$

$$D = 30$$

Resposta: Letra A.

03.

03.
$$\begin{cases} x + 2y + z = 1 \\ 3x + y - 11z = -2 \\ 2x + 3y - z = 1 \end{cases}$$

$$D = \begin{vmatrix} 1 & 2 & 1 & 1 & 2 \\ 3 & 1 & -11 & 3 & 1 \\ 2 & 3 & -1 & 2 & 3 \end{vmatrix}$$

$$-1 - 44 + 9 = -36$$

$$-6 - 33 + 2 = -37$$

$$-36 \neq (-37) = 1$$

$$D_x = \begin{vmatrix} 1 & 2 & 1 & 1 & 2 \\ -2 & 1 & -11 & -2 & 1 \\ 1 & 3 & -1 & 1 & 3 \end{vmatrix}$$

*

$$-1 - 22 - 6 = -29$$

$$4 - 33 + 1 = -28$$

$$-29 \neq (-28) = -1$$

$$D_y = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 3 & -2 & -11 & 3 & -2 \\ 2 & 1 & -1 & 2 & 1 \end{vmatrix}$$

$$2 - 22 + 3 = -17$$

$$-3 - 11 - 4 = -18$$

$$Dz = \begin{vmatrix} 1 & 2 & 1 & 1 & 2 \\ 3 & 1 & -2 & 3 & 1 \\ 2 & 3 & 1 & 2 & 3 \end{vmatrix}$$

$$1 - 8 + 9 = 2$$

$$6 - 6 + 2 = 2$$

$$2 - 2 = 0$$

$$\star X = Dx = \frac{-1}{1} = -1 \quad \left\{ \begin{array}{l} Dz = Dx = 1 = 1 \\ D = 1 \end{array} \right.$$

$$a + w + c = -1 + 0 + 1$$

$$a + w + c = 0$$

Resposta: Letra C.

04.

$$04. \begin{cases} x + 2y - 3z = 29 \\ x + 3y + 2z = 4 \\ x - y - 2z = 8 \end{cases}$$

$$D = \begin{vmatrix} 1 & 2 & -3 & 1 & 2 \\ 1 & 3 & 2 & 1 & 3 \\ 1 & -1 & -2 & 1 & -1 \end{vmatrix}$$

$$-6 + 4 + 3 = 1$$

$$-4 - 2 - 9 = -15$$

$$1 - (-15) = 16$$

$$Dx = \begin{vmatrix} 29 & 2 & 3 \\ 4 & 3 & 2 \\ 8 & -1 & 2 \end{vmatrix} \begin{vmatrix} 29 & 2 \\ 4 & 3 \\ 8 & -1 \end{vmatrix}$$

$$-574 + 32 + 12 = -550$$

$$-16 - 58 + 72 = -2$$

$$-550 - (-2) = -548$$

$$Dy = \begin{vmatrix} 1 & 29 & 3 \\ 1 & 4 & 2 \\ 1 & 8 & -2 \end{vmatrix} \begin{vmatrix} 1 & 29 \\ 1 & 4 \\ 1 & 8 \end{vmatrix}$$

$$-8 + 58 + 24 = 74$$

$$-58 + 16 + 12 = -30$$

$$* 74 - (-30) = 104$$

$$Dz = \begin{vmatrix} 1 & 2 & 29 \\ 1 & 3 & 4 \\ 1 & -1 & 8 \end{vmatrix} \begin{vmatrix} 1 & 2 \\ 1 & 3 \\ 1 & -1 \end{vmatrix}$$

$$24 + 8 - 29 = 3$$

$$16 - 4 + 87 = 99$$

$$3 - 99 = -96$$

$$x = Dx = \frac{-548}{-19} = 28.84$$

$$y = Dy = \frac{104}{16} = 6.5 = \frac{13}{2}$$

$$z = Dz = \frac{-96}{16} = -6$$

$$x + y + z = -19 + 13 + 6$$

$$x + y + z = -19 + 19$$

$$x + y + z = 0$$

Resposta: Letra A.

05.

$$05. \begin{cases} 2x + y = 5 \\ 2y + z = 3 \\ 3x + 2y + z = 7 \end{cases}$$

$$D = \begin{array}{ccc|cc} 2 & 1 & 0 & 2 & 1 \\ 0 & 2 & 1 & 0 & 2 \\ 3 & 2 & 1 & 3 & 2 \end{array}$$

$$4 + 3 + 0 = 7$$

$$0 + 4 + 0 = 4$$

$$4 - 4 = 0$$

$$Dx = \begin{array}{ccc|cc} 5 & 1 & 0 & 5 & 1 \\ 3 & 2 & 1 & 3 & 2 \\ 7 & 2 & 1 & 7 & 2 \end{array}$$

$$10 + 7 + 0 = 17$$

$$3 + 10 + 0 = 13$$

$$17 - 13 = 4$$

$$Dy = \begin{array}{ccc|cc} 2 & 5 & 0 & 2 & 5 \\ 0 & 3 & 1 & 0 & 3 \\ 3 & 7 & 1 & 3 & 7 \end{array}$$

$$6 + 15 + 0 = 21$$

$$0 + 14 + 0 = 14$$

$$* \quad 21 - 14 = 7$$

$$b_2 = \begin{vmatrix} 2 & 1 & 5 & 2 & 1 \\ 0 & 2 & 3 & 0 & 2 \\ 3 & 2 & 7 & 3 & 2 \end{vmatrix}$$

$$28 + 9 + 0 = 37$$

$$0 + 12 + 30 = 32$$

$$37 - 32 = 5$$

$$x = \frac{\Delta x}{\Delta} = \frac{4}{3}$$

$$y = \frac{\Delta y}{\Delta} = \frac{7}{3}$$

$$z = \frac{\Delta z}{\Delta} = \frac{5}{3}$$

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$$v = \left\{ \begin{pmatrix} 4 & 7 & 5 \\ 3 & 3 & 3 \end{pmatrix} \right\}$$

Resposta: Letra D.

06.

$$06. \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ -1 \end{bmatrix}$$

matriz Resultante

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{pmatrix} x + 0 + 0 \\ 2x + y + 0 \\ -x + 2y + 2z \end{pmatrix}$$

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$$\begin{pmatrix} x + 0 + 0 \\ 2x + y + 0 \\ -x + 2y + 2z \end{pmatrix} = \begin{pmatrix} 3 \\ 7 \\ -1 \end{pmatrix} = \begin{cases} x = 3 \\ 2x + y = 7 \\ -x + 2y + 2z = -1 \end{cases}$$

Equações

$$x = 3$$

$$2 \cdot 3 + y = 7$$

$$\star 6 + y = 7$$

$$y = 7 - 6$$

$$y = 1$$

$$-3 + 2 \cdot 1 + 2 \cdot z = -1$$

$$-3 + 2 + 2 \cdot z = -1$$

$$-1 + 2z = -1$$

$$2 \cdot z = -1 + 1$$

$$2 \cdot z = 0$$

$$z = 0$$

$$2$$

$$z = 0$$

Resposta: Letra E.

ESCALONAMENTO

01.

Escalonamento

01.
$$\begin{cases} 2x - y - 3z = -5 \\ x + 3y - z = 11 \\ x - 5z = 3 \end{cases}$$

$$\begin{aligned} (-1) & \begin{pmatrix} 1 & 0 & -5 & 3 \\ 1 & 3 & -1 & 11 \\ 2 & -1 & -3 & -5 \end{pmatrix} \sim 1 \begin{pmatrix} x & y & z & \\ 0 & 3 & 4 & 8 \\ 0 & -1 & 7 & -11 \end{pmatrix} \sim \begin{pmatrix} x & y & z & \\ 0 & 0 & 25 & 25 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{aligned}$$

Equações

$$25z = -25$$
$$z = -1$$
$$25z = -25$$
$$z = -1$$

$$3y + 4z = 8$$
$$3y + 4(-1) = 8$$
$$3y - 4 = 8$$
$$3y = 8 + 4$$
$$3y = 12$$
$$y = \frac{12}{3}$$
$$y = 4$$

$$2x - y - 3z = -5$$
$$2x - 4 - 3(-1) = -5$$
$$2x - 4 + 3 = -5$$
$$2x - 1 = -5$$
$$2x = -5 + 1$$
$$2x = -4$$
$$x = \frac{-4}{2}$$
$$x = -2$$

$$V = \{(-2, 4, -1)\}$$

02.

02.
$$\begin{cases} x = 2y \\ 2y = 3z \\ x + y + z = 11 \end{cases} \rightarrow \begin{cases} x - 2y = 0 \\ 2x - 3z = 0 \\ x + y + z = 0 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & 2 & -3 & 0 \\ 1 & 1 & 1 & 11 \end{array} \right) \xrightarrow{(-1)} \left(\begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 1 & 1 & 1 & 11 \\ 0 & 2 & -3 & 0 \end{array} \right) \xrightarrow{(-1)} \left(\begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & 3 & 1 & 11 \\ 0 & 2 & -3 & 0 \end{array} \right) \sim$$

$$\left(\begin{array}{ccc|c} x & y & z & \\ 0 & 0 & -11 & -22 \end{array} \right)$$

Equações

$$\begin{aligned} -11z &= -22 \\ z &= +2 \\ +11 \\ z &= 2 \end{aligned} \quad \begin{cases} x = 2y \\ x = 2 \cdot 3 \\ x = 6 \end{cases}$$

$$\begin{aligned} 3y + 1z &= 11 \\ 3y + 1 \cdot 2 &= 11 \\ 3y + 2 &= 11 \\ 3y &= 11 - 2 \\ 3y &= 9 \\ y &= 3 \\ y &= 3 \end{aligned} \quad \begin{cases} x + 2y + 3z \\ 6 + 2 \cdot 3 + 3 \cdot 2 \\ 6 + 6 + 6 = 18 \end{cases}$$

Resposta = Letra B.

03.

03.
$$\begin{cases} x + y + z = 0 \\ 2x - y - 2z = 1 \\ 6y + 3z = -12 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & -1 & -2 & 1 \\ 0 & 6 & 3 & -12 \end{array} \right) \sim \left(\begin{array}{ccc|c} x & y & z & \\ 0 & -3 & -4 & 1 \\ 0 & 6 & 3 & -12 \end{array} \right) \sim \left(\begin{array}{ccc|c} x & y & z & \\ 0 & 0 & -15 & -3 \end{array} \right)$$

$$\begin{aligned} -15z &= -30 \\ z &= +2 \\ +15 \\ z &= 2 \end{aligned}$$

Resposta: Letra D.

04.

$$04. \quad a + b + c = 68$$

$$b + 20 \cdot c = a$$

$$c + 20 \cdot a = 3 \cdot b$$

$$b + 1 \cdot c = a$$

$$5b + c = 5a$$

$$* \quad -5a + 5b + c = 0$$

$$c + 20a = 3b$$

$$5c + a = 15b$$

$$a - 15b + 5c = 0$$

$$a + b + c = 68$$

$$5a + 5b - c = 0$$

$$a - 15b + 5c = 0$$

$$\sim \begin{pmatrix} 1 & 1 & 1 & | & 68 \\ 5 & -5 & -1 & | & 0 \\ 1 & -15 & 5 & | & 0 \end{pmatrix} \xrightarrow{+6} \begin{pmatrix} a & b & c \\ 0 & -10 & -6 & | & -340 \\ 0 & -16 & 4 & | & -60 \end{pmatrix} \sim$$

$$\begin{pmatrix} a & b & c \\ 0 & 0 & -136 & | & -4760 \end{pmatrix}$$

Equação

$$-136c = -4760$$

$$c = -4760 : 136$$

$$c = 35$$

$$a + b + c = 68$$

$$a + 13 + 35 = 68$$

$$a = 68 - 48$$

$$a = 20$$

$$-10b - 6c = -340$$

$$-10b - 6 \cdot 35 = -340$$

$$-10b - 210 = -340$$

$$-10b = -340 + 210$$

$$-10b = -130$$

$$b = -130 : -10$$

$$b = 13$$

$$c = a$$

$$35 - 20 = 15$$



Resposta: Letra A.

05.

$$\begin{aligned} 05. \quad a &= 134,00 \\ w &= 115,00 \\ c &= 48,00 \end{aligned}$$

$$A = \begin{bmatrix} x & y & z \\ 0 & 3 & 4 \\ 1 & 0 & 5 \\ 2 & 1 & 0 \end{bmatrix} \cdot x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \begin{cases} 3y + 4z = 134 \\ x + 5z = 115 \\ 2x + z = 48 \end{cases} \quad *$$

$$\left(\begin{array}{ccc|c} 0 & 3 & 4 & 134 \\ 1 & 0 & 5 & 115 \\ 2 & 1 & 0 & 48 \end{array} \right) \xrightarrow{(-2)} \left(\begin{array}{ccc|c} 0 & 3 & 4 & 134 \\ 1 & 0 & 5 & 115 \\ 0 & 1 & -10 & 48 \end{array} \right) \xrightarrow{(-3)} \left(\begin{array}{ccc|c} a & w & c & \\ 0 & 1 & -10 & -182 \\ 0 & 3 & 4 & 134 \end{array} \right)$$

$$\sim \begin{pmatrix} a & w & c \\ 0 & 0 & 34 & 680 \end{pmatrix}$$

* Equações

$$\begin{aligned} 34c &= 680 \\ c &= 20 \\ 34 & \\ c &= 20 \end{aligned} \quad \begin{cases} a + 5c = 115 \\ a + 5 \cdot 20 = 115 \\ a = 115 - 100 \\ a = 15 \end{cases} \quad \begin{cases} w - 10c = -182 \\ w - 10 \cdot 20 = -182 \\ w = -182 + 200 \\ w = 18 \end{cases}$$

$$a + w + c$$

$$15 + 18 + 20 = 53$$

Resposta: Letra A.