

1.

i. Derive the formula for number of comparisons for bubble sort:

Given that n is equal to the length of the array passed to bubble sort, the outer loop would iterate n times. Given that the inner loop is to iterate from $[0, n - i - 1)$ and that a comparison is done each time the inner loop is iterated through, the number of comparisons can be given by the series:

$$(n - 1) + (n - 2) + \dots + 2 + 1$$

According to what was said by Gauss, this series can be represented as follows:

$$\frac{[(n-1) + (n-2) + \dots + 2 + 1] + [1 + 2 + \dots + (n-2) + (n-1)]}{2}$$

Which can then be simplified and written as:

$$\frac{(n-1+1) + (n-2+2) + \dots + (2 + n-2) + (1+n-1)}{2}$$

Each term in the above series in the numerator equals n