1.

i. Derive the formula for number of comparisons for bubble sort:

Given that n is equal to the length of the array passed to bubble sort, the outer loop would iterate n times. Given that the inner loop is to iterate from [0, n - i -1) and that a comparison is done each time the inner loop is iterated through, the number of comparisons can be given by the series:

$$(n - 1) + (n - 2) + \dots + 2 + 1$$

According to what was said by Gauss, this series can be represented as follows:

$$\frac{[(n-1) + (n-2) + \dots + 2 + 1] + [1 + 2 + \dots + (n-2) + (n-1)]}{2}$$

Which can then be simplified and written as:

$$\frac{(n-1+1) + (n-2+2) + \dots + (2 + n-2) + (1+n-1)}{2}$$

Each term in the above series in the numerator equals n, and there are (n-1) terms in the series. So we can further simplify the above equation further to give the formula for the number of comparisons:

$$\frac{n(n-1)}{2}$$

ii. Derive the average-case number of swaps for bubble sort:

Given that the minimum number of swaps for bubble sort is 0 (if the array is already sorted), and the maximum number of swaps is $\frac{n(n-1)}{2}$ (if each comparison resulted in a swap). So the number of swaps for the average-case could be found by dividing the sum of every possible number of swaps by the total number of comparisons:

$$\frac{0 + 1 + \dots + \frac{n(n-1)}{2}}{\frac{n(n-1)}{2}}$$

From what was said by Gauss, since our numerator is a series we can write it as:

$$\frac{(0 + 1 + \dots + \frac{n(n-1)}{2}) + (\frac{n(n-1)}{2} + \dots + 1 + 0)}{2}$$

Which can be simplified as:

$$\frac{(0+\frac{n(n-1)}{2})+(1+\frac{n(n-1)}{2}-1)+\ldots+(\frac{n(n-1)}{2}-1+1)+(\frac{n(n-1)}{2}+0)}{2}$$

The term $\frac{n(n-1)}{2}$ is repeated $\frac{n(n-1)}{2}$ times it can be written as :

$$\frac{\frac{n(n-1)}{2}(\frac{n(n-1)}{2})}{2}$$

Then the can be reduced again to be:

$$\frac{[n(n-1)]^2}{8}$$

Now, replacing the previous the previous numerator with the one we just derived:

$$\frac{\frac{[n(n-1)]^2}{8}}{\frac{n(n-1)}{2}}$$

Which can be simplified to:

$$\frac{2[n(n-1)]^2}{8n(n-1)}$$

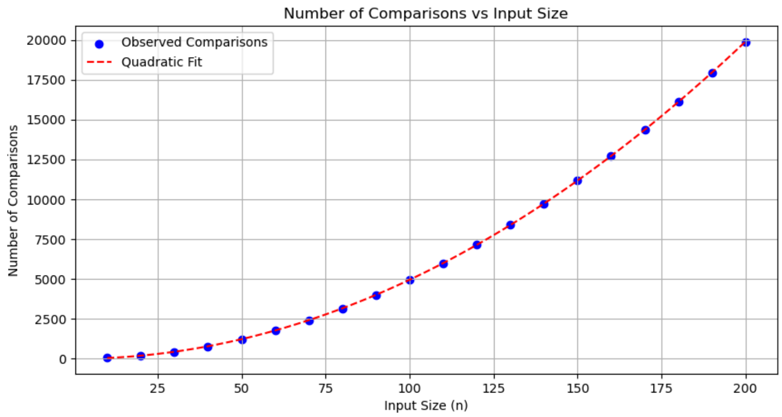
Then further reduced to get the derived equation for the average number of swaps to be:

$$\frac{n(n-1)}{4}$$

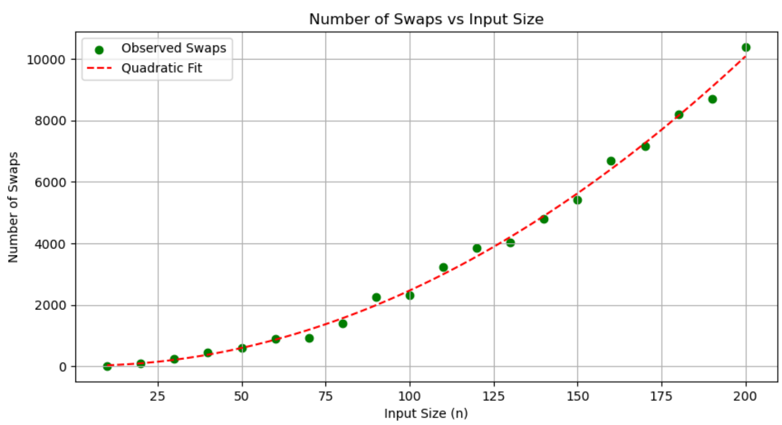
2. On a separate file

3. On a separate file

4.



The number of comparisons follows a quadratic trend, as the derived equation above for the number of comparisons was $O(n^2)$. The number of comparisons for each input size is the same no matter how many times the random array was generated, as the arrangement of the array values does not affect the number of comparisons. This is why it is expected that each data point of the comparisons lies on a quadratic fit.



The number of swaps also follows a quadratic trend as derived above for an average case. The further below the data point is than the quadratic fit, the closer that array was to a best case scenario. The further above the data point is than the quadratic fit, the closer the array was to a worst case scenario, where the number of swaps is equal to the number of comparisons. It is expected that the data points vary from the quadratic fit, as not every comparison leads to a swap, so each time the array was randomized, a different number of swaps was found. Overall both plots demonstrated that the number of comparisons and the average number of swaps had a quadratic relationship to the input size. This aligns with the derivation from part 1.