

# COMPRESSIVE ASYNCHRONOUS DECOMPOSITION OF HEART SOUNDS

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## ABSTRACT

Because of the need to control power consumption, in many biomedical applications asynchronous processing of the data is more appropriate. In this paper, we present a scale-based decomposition algorithm for analog signals similar to the wavelet decomposition. Our procedure uses asynchronous sigma delta modulators (ASDMs) to represent the amplitude of a signal using the zero-crossing times of a binary signal. Changing the zero-crossing times into random sequences of pulse widths, it can be shown to be equivalent to an optimal level-crossing sampler using local averages as the quantization levels. Applying the generation of multi-level signals from the output of ASDMs for different scale parameters we are able to obtain a decomposer that in a few stages provides a close representation of the signal. To illustrate the performance of the proposed decomposition, we consider its application to the representation of heart sounds.

**Index Terms**— Asynchronous signal processing, continuous time digital signal processing, time-encoding, level-crossing sampling.

## 1. INTRODUCTION

Conventional digital signal processors operate synchronously in time. At each sampling time the amplitude is approximated by a binary code. Such synchronous processing suffers from quantization error and possible frequency aliasing. Furthermore, sample values in synchronous systems are collected independent of whether the value is significant or not. Differently, in level-crossing (LC) sampling [1] samples values are collected only when a specified quantization level is attained, giving a non-uniform sampling in time. The advantage of LC is that only samples are collected when there is significant information in the signal — which is the case of many biological signals. A drawback of LC sampling, however, is that a set of quantization levels needs to be specified *a-priori* and that the sampling times and the corresponding amplitudes must be kept. Furthermore, only a multilevel reconstruction is possible. However, the LC sampling is not hampered by aliasing or quantization error, and can be processed in continuous-time [2].

A time-encoding method using asynchronous sigma delta modulators [5] can be shown to be equivalent to an LC sampler with quantization levels set as local estimates of the signal average. The ASDM [3, 4, 5], a non-linear feedback system, generates a binary sequence of pulses with widths proportional to the local average of the signal, and controlled by a scale parameter of the ASDM. Thus different from the LC sampler, only the zero-crossing times of the output of the ASDM are needed to obtain a multilevel approximation of the signal, and the original analog signal can be reconstructed by interpolation using prolate spheroidal wave functions [6].

In this paper, we consider a signal decomposition of analog signals using a scale parameter. This procedure is similar to the wavelet decomposition. The binary output of an ASDM provides the local average of the signal based on the properties of a time-encoder. A multi-level decomposition of a signal is obtained by cascading modules consisting of an ASDM followed by an averager and a low-pass filter. This approach allows us to obtain a compressed signal representation. The proposed procedure shares the greedy characteristics of the compressive sensing.

As a practical example, we consider the decomposition of heart sounds. Cardiovascular disease remains the leading cause of death worldwide despite numerous advances in monitoring and early detection of the disease. Fortunately, clinical experience has shown that heart sounds can be an effective tool to non-invasively diagnose some of the heart failures, since they provide clinicians with valuable diagnostic and prognostic information concerning the heart valves and hemodynamics. Heart auscultation is an important technique allowing the detection of abnormal heart behavior before it can be detected using other techniques such as the ECG [7]. However, continuous monitoring of heart sounds can pose severe constraints on data acquisition and processing systems in telemedicine. One approach to alleviate these issues is to efficiently compress the signal [8, 9]. In this paper, we consider the suitability of the proposed decomposition for efficient compression of these signals.

## 2. ASYNCHRONOUS SIGMA DELTA MODULATOR (ASDM) TIME-ENCODING

The asynchronous sigma delta modulator (ASDM), Fig. 1, is a nonlinear feedback system consisting of an integrator and a Schmitt trigger [3, 5]. The ASDM maps the amplitude information of a bounded input signal  $x(t)$  into a time sequence  $t_k$ , or the zero crossings of the binary output of the ASDM.

For a bounded input  $x(t)$  ( $|x(t)| < c$ ) the desired output of the ASDM is a binary signal  $z(t)$  with values of  $+b$  or  $-b$ , and zero-crossing times related to the amplitude of  $x(t)$ . The bias  $\pm b$  is chosen bigger than the bound of  $x(t)$  to obtain increasing/decreasing function of time when integrated. When the output of the integrator reaches a predefined values  $\pm\delta$  the output  $z(t)$  is triggered to the opposite state. How fast the triggering occurs is related to the value of  $\kappa$ , which connects with the maximum frequency of the input signal. The information provided by the amplitude of  $x(t)$  is also given by the zero-crossing times of the binary output signal  $z(t)$ . The zero-crossing times as well as the design parameters  $\kappa, \delta, b$  (strictly positive real numbers) depend on the nature of the signal.

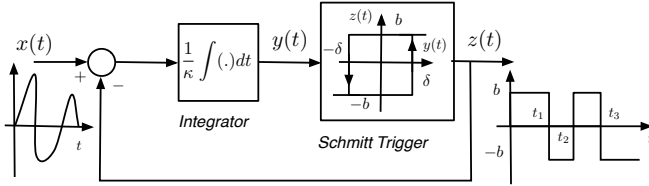


Fig. 1. Asynchronous sigma delta modulator

It can be shown [5] that for an increasing sequence  $\{t_k\}$ , the following integral equation can be obtained

$$\int_{t_k}^{t_{k+1}} x(\tau) d\tau = (-1)^k [-b(t_{k+1} - t_k) + 2\kappa\delta] \quad (1)$$

According to (1), the input  $x(t)$  is related to the zero-crossings  $\{t_k\}$  and the parameters  $b, \delta$  and  $\kappa$  of the ASDM. A multi-level approximation for  $x(t)$  can be obtained by relating the width of the pulses in  $z(t)$  with local averages of the signal. An analog interpolation is obtained by first approximating the integral using the trapezoidal rule and then assuming the prolate spheroidal wave functions as the basis functions for  $x(t)$  [6].

A parameter critical in our decomposition is the scale parameter  $\kappa$ . To obtain bounds for it, we use  $|x(t)| \leq c$  and  $b > c$ , so that

$$-c(t_{k+1} - t_k) \leq \int_{t_k}^{t_{k+1}} x(\tau) d\tau \leq c(t_{k+1} - t_k)$$

By using the above constraint in (1), the bound for  $\kappa$  can be obtained as

$$\frac{(b-c)(t_{k+1} - t_k)}{2\delta} < \kappa < \frac{(b+c)(t_{k+1} - t_k)}{2\delta} \quad (2)$$

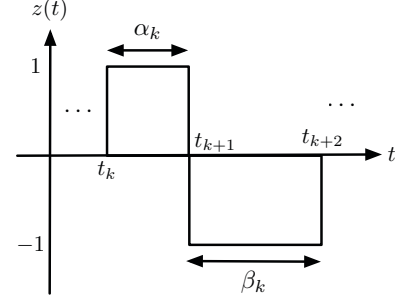


Fig. 2. The parameters  $\alpha_k$  and  $\beta_k$  in  $z(t)$ .

In the case of non-uniform sampling, a sufficient condition for reconstruction of band-limited signals is that the maximum of  $\{t_{k+1} - t_k\}$  should be less the sampling period  $T_s$ . In such a case letting  $\delta = 0.5$ ,  $b = 1$  and  $b = c + \varepsilon$ , for a positive value  $\varepsilon \rightarrow 0$ , the relationship of  $\kappa$  with the maximum frequency  $f_{max}$  of the signal is obtained from the upper bound of 2:

$$\kappa \leq (2c + \varepsilon)T_s \leq \frac{1 - 0.5\varepsilon}{f_{max}} \approx \frac{1}{f_{max}}. \quad (3)$$

## 3. ASYNCHRONOUS DECOMPOSITION

Assuming  $\delta = 0.5$  and  $b = 1$ , and evaluating eqn. (1) from  $t_k$  to  $t_{k+2}$ , we have

$$\int_{t_k}^{t_{k+2}} x(\tau) d\tau = \underbrace{(t_{k+2} - t_{k+1})}_{\beta_k} - \underbrace{(t_{k+1} - t_k)}_{\alpha_k}$$

where  $\alpha_k$  and  $\beta_k$  are defined as in Fig. 2 and  $t_k < t_{k+1} < t_{k+2}$ . If we then let  $T_k = \beta_k + \alpha_k$ , then the local average

$$\bar{x}_k = \frac{1}{T_k} \int_{t_k}^{t_{k+2}} x(\tau) d\tau = \frac{\beta_k - \alpha_k}{\beta_k + \alpha_k} \quad (4)$$

Thus,  $\bar{x}_k$  or the local average in  $[t_k, t_{k+2}]$  corresponds to the difference of the areas under two consecutive pulses in  $z(t)$  divided by the length of the two pulses. Using these connections between  $z(t)$  and the local averages, we can obtain a multi-level approximation of  $x(t)$ . This would be equivalent to using a level-crossing sampler with quantization levels  $\{\bar{x}_k\}$ . If we consider the  $\bar{x}_k$  the best linear estimator of the signal in  $[t_k, t_{k+2}]$  when no data is provided, the time-encoder can be thought of an optimal LC sampler—provided that the zero-time crossings  $\{t_k\}$  are available. This would require to process the signal first with an ASDM and then to use the obtained local averages as the quantization levels for the LC.

The proposed decomposition is shown in Fig. 3. It consists of cascading  $L$  modules, each having an ASDM, an averager, a low-pass filter and an adder. The number of modules,  $L$ , is determined by the scale parameters used to decompose the input signal. For a certain scale  $\kappa_i$ , the ASDM maps the input signal into a binary signal with sequences  $\{\alpha_k\}$  and

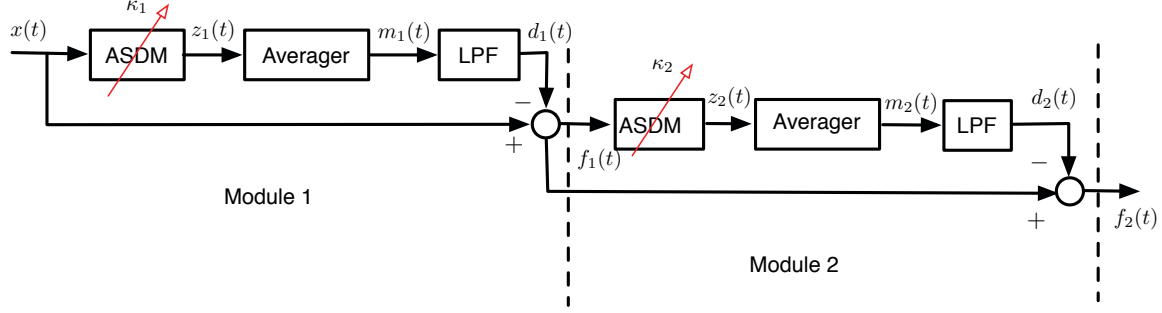


Fig. 3. Decomposer

$\{\beta_k\}$ , which the averager converts into local averages  $\{\bar{x}_k\}$ . The low-pass filter is used to smooth out the multi-level signal output so that there is no discontinuity inserted by the adder when the multi-level signal is subtracted from the input signal of the corresponding module. Each of these modules operates similarly but at a different scale.

We start with a scale factor  $\kappa_1$ , corresponding to a wide window and obtained by considering the maximum frequency present in  $x(t)$  as indicated in equation (3). The other scale are obtained according to

$$\kappa_\ell = \frac{\kappa_1}{2^{\ell-1}} \quad \ell = 2, \dots, L$$

for the  $\ell^{th}$  module. The input to the modules beyond the first one can be written sequentially as follows,

$$\begin{aligned} f_1(t) &= x(t) - d_1(t) \\ f_2(t) &= f_1(t) - d_2(t) = x(t) - d_1(t) - d_2(t) \\ &\vdots \\ f_j(t) &= x(t) - d_1(t) - d_2(t) \cdots - d_j(t) \\ &\vdots \\ f_L(t) &= x(t) - \sum_{l=1}^L d_l(t) \end{aligned} \quad (5)$$

where the  $d_i(t)$  are the outputs of the low-pass filters. We thus have the decomposition

$$x(t) = \sum_{\ell=1}^L d_\ell(t) + f_L(t) \quad (6)$$

where,  $f_L(t)$  can be thought as the error of the decomposition. This decomposition is analogous to the wavelet decomposition, but it is valid for continuous rather than discrete signals, and it uses scale rather than frequency to implement the decomposition.

In terms of compression, the decomposition of the input signal  $x(t)$  can be interpreted in two equivalent ways:

(i) Time-encoding — If the multilevel decomposition signals  $\{m_\ell(t)\}$  are inputted into ASDMs, for each of these

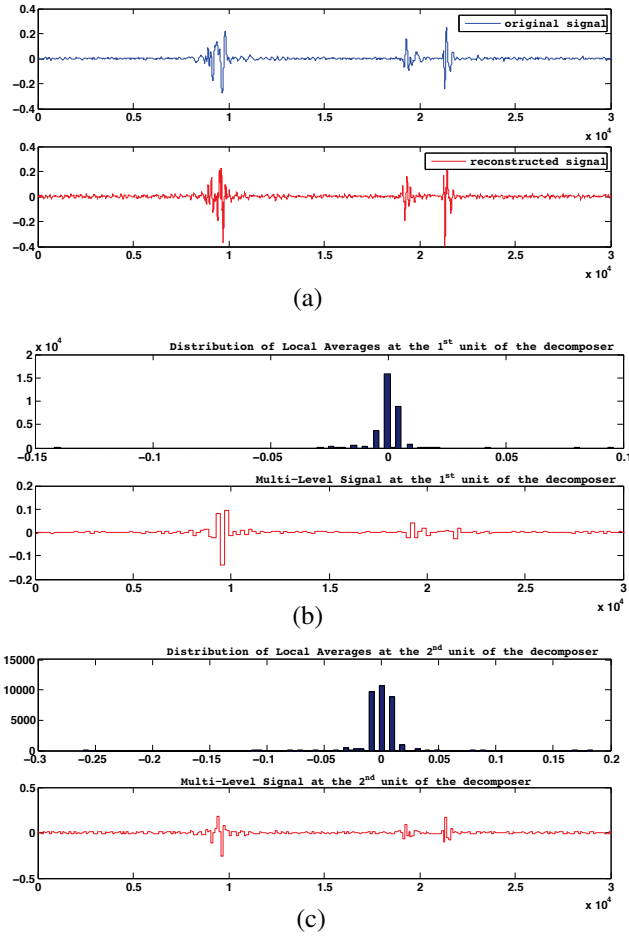
signals we will obtain binary signals  $\{z_\ell(t)\}$  with crossing times  $\{t_{\ell,k}\}$  that will permit us to reconstruct them. As before, these signals can then be low-pass filtered to obtain the  $\{d_\ell(t)\}$ , and approximate  $x(t)$  according to (6). The array  $\{t_{\ell,k}\}$ ,  $1 \leq \ell \leq L$ ,  $k \in K_\ell$ , where  $K_\ell$  corresponds to the number of pulses used in each decomposition stage, would provide a representation of the components and of the overall signal. This would use time-encoding [5].

(ii) Pulse-modulation — Each  $\{z_\ell(t)\}$ , the output of the ASDMs in the decomposition, provides random sequences  $\{\alpha_{\ell,k}, \beta_{\ell,k}\}$  from which we can compute sequences  $\{\bar{x}_{\ell,k}, T_{\ell,k}\}$  for  $1 \leq \ell \leq L$ ,  $k \in K$ , where  $K$  corresponds to the number of pulses used in each decomposition stage. For a non-deterministic signal  $\{\bar{x}_{\ell,k}\}$  are random, and as such their distributions will characterize  $d_\ell(t)$  as well as the signal  $x(t)$ . The  $\{\bar{x}_{\ell,k}, T_{\ell,k}\}$  provide the same compression as the one provided by  $\{\alpha_{\ell,k}, \beta_{\ell,k}\}$  and  $\{t_{\ell,k}\}$ . To obtain higher compression, we can consider the distribution of the  $\{\bar{x}_{\ell,k}\}$  and ignore values clustered around one of the averages.

#### 4. HEART SOUND COMPRESSIVE REPRESENTATION

Phonocardiograph recordings of actual heart sounds containing either the opening snap (OS) or the third heart sound (S3) in addition to first and second heart sounds were obtained from patients at St. Josephs Hospital in Toronto, Canada during clinical examinations that also included heart auscultation. The data acquisition system consisted of a PC fitted with a 16-bit acquisition board and of an analog recorder/player (Cambridge AVR-I). The heart sounds are sampled at 4000 Hz for 4096 samples (1.024 seconds in length) [10].

To illustrate the performance of the proposed decomposition, we consider two different heart sounds from patients. One contains the OS (Fig. 4) and the second contains the S3 (Fig. 5). Both of these signals required a decomposition using two modules. To make the signals approximate analog signals they are interpolated with a factor of 8. The reconstruction of the OS signal requires 517 local average values and the corresponding 517 zero crossing where these aver-



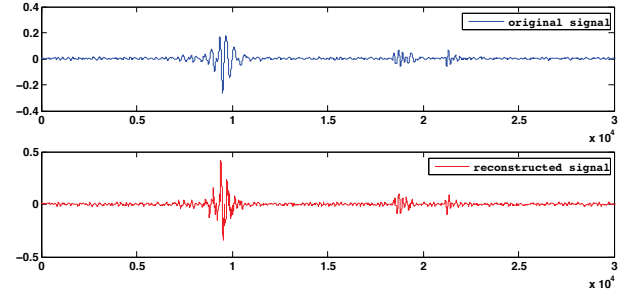
**Fig. 4.** (a) The original signal containing the S3 and the reconstructed signal; (b) Distribution and multi-level signal at the first module; (c) Distribution and multi-level signal at the second module.

ages occur and the two  $\kappa$  values used in the decomposition. This reconstruction gives a mean-square error of  $9.78 \times 10^{-4}$  and a compression ratio of 25.3%, when related to the original number of samples. Similar performance is obtained for the S3 signal. The distribution of the local averages is obtained via a histogram with 100 bins.

The results show that we can accurately reconstruct the heart sounds from the sparsely sampled recordings. A great amount of compression is achieved on both sparse data records.

## 5. CONCLUSION

In this paper, we proposed a novel asynchronous scale-based decomposition method for analog signals. The decomposer uses ASDMs to obtain binary signals representing the amplitude of the signal, from which we can obtain a multilevel representation based on local averages. To illustrate the com-



**Fig. 5.** The original signal containing the OS and the reconstructed signal. The mean-square error for this reconstruction is  $9.28 \times 10^{-4}$ .

pression ability of our procedure, we considered heart sound signals. Our procedure was able to compress these signals significantly.

## 6. REFERENCES

- [1] K. Guan and A. Singer, "A level-crossing sampling scheme for non-bandlimited signals," *IEEE Intl. Conf. Acoust., Speech and Sig. Proc. (ICASSP)*, vol. III, pp. 381-383, 2006.
- [2] Y. Tsividis, "Digital signal processing in continuous time: a possibility for avoiding aliasing and reducing quantization error," *IEEE Intl. Conf. on Acoust., Speech, and Sig. Proc. (ICASSP)*, vol. II, pp. 589-92, May 2004.
- [3] E. Roza, "Analog-to-digital conversion via duty-cycle modulation," *IEEE Trans. on Circuits and Systems II: Analog and Digital Signal Processing*, vol. 44, no. 11, pp. 907-914, Nov 1997.
- [4] E.V. Aksenov, Yu.M. Ljashenko, A.V. Plotnikov, D.A. Prilutskiy, S.V. Selishchev, E.V. Vetskiy, "Biomedical data acquisition systems based on sigma-delta analogue-to-digital converters," *Intl. Conf. of the IEEE Engineering in Medicine and Biology Society*, vol.4, pp. 3336-3337, 2001.
- [5] A. Lazar, and L. Toth, "Time encoding and perfect recovery of band-limited signals," *IEEE Intel. Conf. on Acous., Speech, and Sig. Proc. (ICASSP)*, VI- 709-12, Apr. 2003.
- [6] S. Senay, L.F. Chaparro, M. Sun, and R.J. Scabassi, "Adaptive level-crossing sampling and reconstruction," *18th European Signal Processing Conference*, pp. 1296-1300, Aug. 2010.
- [7] M. Obadiat, "Phonocardiogram signal analysis: Techniques and performance analysis," *Journal of Medical Engineering and Technology*, vol. 17, no. 6, pp. 221-227, Nov./Dec. 1993.
- [8] D. L. Donoho, "Compressed sensing," *IEEE Transactions on Information Theory*, vol. 52, no. 4, pp. 1289-1306, Apr. 2006.
- [9] E. Sejdić, A. Can, L. F. Chaparro, C. M. Steele, and T. Chau, "Compressive sampling of swallowing accelerometry signals using time-frequency dictionaries based on modulated discrete prolate spheroidal sequences," *EURASIP Journal on Advances in Signal Processing*, 2012, submitted.
- [10] E. Sejdić and J. Jiang, "Selective regional correlation for pattern recognition," *IEEE Transactions on Systems, Man and Cybernetics - Part A*, vol. 37, no. 1, pp. 82-93, Jan. 2007.