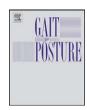
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An empirical examination of detrended fluctuation analysis for gait data

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ARTICLE INFO

Article history: Received 20 June 2009 Received in revised form 2 December 2009 Accepted 10 December 2009

Keywords: Stride interval dynamics Detrended fluctuation analysis Scaling exponent Estimation Sample size Box size fitting range

ABSTRACT

Stride interval series exhibit statistical persistence, and detrended fluctuation analysis (DFA) is a routinely employed technique for describing this behavior. However, the implementation of DFA to gait data varies considerably between studies. We empirically examine two practical aspects of DFA which significantly affect the analysis outcome: the box size range and the stride interval series length. We conduct an analysis of their effect using stride intervals from 16 able-bodied adults, for overground walking, treadmill walking while holding a handrail, and treadmill walking without using a handrail. Our goal is to provide general guidelines for these two choices, with the aim of standardizing the application of DFA and facilitating inter-study comparisons. Based on the results of our analysis, we propose the use of box sizes from 16 to N/9, where N is the number of stride intervals. Moreover, for differentiating between normal and pathological walking with reasonable accuracy, we recommend a minimum of 600 stride intervals.

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1. Introduction

The analysis of stride interval dynamics has become an integral part of quantitative gait research [1]. In their seminal paper, Hausdorff et al. [2] showed that stride intervals exhibit statistical persistence, similarly to many other physiological phenomena such as heartbeat [3], brain [4], and muscle [5] activity. To quantify this behavior researchers use the scaling exponent α , which measures the rate of decay of a series' autocorrelation. Interestingly, studies have revealed that α is a useful metric for distinguishing between healthy and pathological walking due to amyotrophic lateral sclerosis, Parkinson's disease, and Huntington's disease [6,7]. Moreover, α has been associated with walking stability [8], and has been found to be influenced by gait speed [9,10] and gait maturation [11].

Detrended fluctuation analysis (DFA) is a common technique for estimating α in the presence of non-stationarities [12], and has become the de facto standard for analyzing statistical persistence in gait data. Briefly, DFA fits a power law to the series' average fluctuations, F(n), across different scales (box sizes) n, and estimates α as the slope of $\log F(n)$ versus $\log n$. Despite DFA's popularity in gait research, the procedure is not yet standardized.

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Two underlying implementation aspects, namely the range of box sizes and the number of stride intervals used for estimating α , vary considerably between studies. Both decisions have a significant impact on α , and the subsequent inference and conclusions. However, there is no methodological consensus around these two choices, and this is exactly the problem we address.

First, we consider the range of box sizes. Given exact knowledge of the average fluctuations, there should be a strict linear relationship between $\log F(n)$ and $\log n$. In practice, however, F(n) is estimated, which leads to deviations from linearity even for simulated data [13,14]. Additionally, these deviations occur primarily for smaller and larger box sizes [3,15]. In order to avoid these effects, DFA is commonly performed on a restricted range of box sizes, n_1 to n_m , where the linear relationship is most stable. Various ranges have been used in the literature, from $[n_1=4,n_m=N/4]$ [9] (N being the number of stride intervals), to $[n_1=6,n_m=600]$ with $N \approx 3000$ [16], and $[n_1=10,n_m=20]$ with $N \approx 400$ [6,11]. Moreover, simulation experiments [15] suggest $n_m=N/10$, with n_1 depending on N and α .

Second, we consider the minimum number of stride intervals. As with every statistical procedure, more data yields more accurate results. There are, however, practical limitations to the sample size. For example, participants should not walk for prolonged periods of time because of fatigue, which can alter the properties of the series and the walking conditions under study. Moreover, appending several series from independent sessions to form a longer one is not a valid alternative, because this practice leads to artificial changes in α [17]. DFA should be performed on data from a single, continuous walking session, so it is reasonable to look for a sufficient sample size for obtaining reliable results. Once again,

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various sample sizes have been used in the literature, ranging from $N \approx 300$ for subjects with amyotrophic lateral sclerosis [7], to $N \approx 3000$ for healthy adults [16].

The variation in box size ranges and numbers of stride intervals should not be surprising, given that these decisions are typically made on an ad hoc or subjective basis. Our goal is to quantify the effect of these choices on α , and provide rigorous guidelines for selecting them. For this we use stride intervals from healthy adults for three different walking conditions, namely overground walking, and treadmill walking with and without holding a handrail. Healthy adults are frequently used as controls in gait research, and we examine different walking conditions because they can affect gait dynamics [18,19]. Ultimately, we expect our guidelines to make the application of DFA to gait data more consistent, and enable inter-study comparisons.

2. Methods

2.1. Participants

Sixteen able-bodied, right-foot dominant (self-reported) participants (6 males) with normal or corrected-to-normal vision were recruited from the Bloorview Research Institute, Bloorview Kids Rehab, as a convenience sample. Participants with previous or existing neurological pathologies, injuries or illnesses likely to affect their gait were excluded. Mean age was 23.3 (SD 3.3) years, mean height was 1.68 (SD 0.08) m, and mean mass was 59.9 (SD 11.5) kg. The study was approved by the Research Ethics Board of the Bloorview Kids Rehab Centre, Toronto, Canada. All participants provided informed written consent prior to participation.

2.2. Protocol design

Each of the 16 participants walked for two 15 min sessions on different days, and under three different conditions: overground walking (OW), treadmill walking while holding a front handrail (TW-R), and treadmill walking without holding a handrail (TW-NR). In total, 32 sessions were recorded for each condition (two per participant). For each participant the sequence of walking conditions was randomized, with an option to rest between them. Participants walked on a rectangular path (walkway width = 2.43 m, perimeter length = 84.87 m) in a hallway without obstructions or interference. A motorized treadmill (Q55, Quinton Cardiology Systems) with front handrails (height = 1.02 m) was used for treadmill walking.

Participants walked at a self-selected comfortable speed via instruction to "walk at a comfortable pace". The speed of the treadmill was incrementally increased or decreased to determine the participant's comfortable walking speed. Participants were given a 2 min practice period before each session.

2.3. Assessment of gait dynamics

An ultra-thin, force-sensitive resistor (FSR; #406, Interlink Electronics) was taped below the heel of the participant's right shoe. Each time the FSR made contact with the ground, a change in voltage occurred. These voltages were directly captured by a CompactFlash data acquisition card (CF-6004, National Instruments) and stored in a small, lightweight (8.1 cm \times 1.9 cm \times 12.1 cm; 0.195 kg) personal digital assistant (#X11-15454, Hewlett-Packard) carried in a fanny pack. The wiring was clipped to clothing along the lateral side of the right leg and the fanny pack was centered at the front of the participant's waist to ensure the instrumentation did not interfere with natural gait. The footswitch signals were sampled at 200 Hz and subsequently uploaded to a PC for analysis. The stride interval time series was extracted using an algorithm that locates the initial time of heel strikes [20]. To minimize 'start-up' and 'end' effects, the first and last 30 s of the signals were excluded.

2.4. Detrended fluctuation analysis

For completeness, we describe the DFA procedure in detail and illustrate its steps in Fig. 1. In order to estimate the scaling exponent α of a time series $\{x(i)\}_{i=1}^{N}$, we take the following steps [3]:

- 1. Integrate the series' deviations from its mean, \vec{x} , yielding $y_i = \sum_{j=1}^i [x(j) \vec{x}]$, for $i = 1, \dots, N$.
- 2. For a given box size n, divide $\{y(i)\}_{i=1}^N$ into $\lfloor N/n \rfloor$ non-overlapping boxes of length n, where $\lfloor \cdot \rfloor$ is the greatest integer function.
- 3. Fit a least squares line to the data within each box. The sequence of fitted lines constitutes the trend series $\{y_n(i)\}_{i=1}^{N_n}$, where $N_n = n \lfloor N/n \rfloor$ is the total number of data points falling within the boxes.
- 4. Calculate the average fluctuation, F(n), of the integrated series y around the trend series y_n . Explicitly, $F(n) = \sqrt{\frac{1}{N_n} \sum_{i=1}^{N_n} |y(i) y_n(i)|^2}$.
- 5. Repeat steps 2–4 for different box sizes n_1, \ldots, n_m , to get a range of fluctuations $F(n_1), \ldots, F(n_m)$.
- 6. Plot $\log F(n_i)$ versus $\log n_i$, for $i=1,\ldots,m$. If there is an obvious linear relationship between them, the least squares fitted slope provides an estimate of the scaling exponent α .

Since we estimate the slope in log-space, it is preferable to have equidistant box sizes in log-space. Practically, this means we increase n_i relative to n_{i-1} by a fixed factor $f = 2^{1/8}$, following Peng et al. [12].

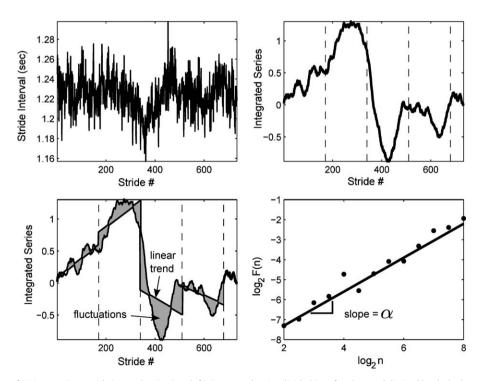


Fig. 1. Graphical illustration of DFA steps. Raw stride interval series (top left), integrated series divided into four boxes, delimited by dashed vertical lines (top right), fitted linear trends within each box, with shaded area representing fluctuations (bottom left), and log-log plot of average fluctuations F(n) versus box sizes P(n) (bottom right). The estimated scaling exponent P(n) is the slope of the least squares fitted line.

2.5. Box size fitting range

We want to identify a range of box sizes that provides stable estimates of α . To accomplish this, we sequentially remove box sizes which cause significant shifts in the fitted slope. Conceptually, if all log-fluctuations lay on a straight line, then removing one would not alter the slope. Conversely, if a log-fluctuation deviates from the line, then removing it would lead to a considerable change in α . This is particularly true for points at the boundaries of the fitting range, because they exert more influence on the slope, also referred to as leverage in statistics [21].

For quantifying the effect of each box size on α we use an established regression diagnostic measure known as *DFBETAS* [21]. Assume a sequence of box sizes, $[n_1,\ldots,n_m]$, and their corresponding average fluctuations $F(n_1),\ldots,F(n_m)$. Let α be the fitted slope using all points, $\alpha_{(i)}$ be the fitted slope excluding the pair $(n_i,F(n_i))$, and $\operatorname{std}(\alpha_{(i)})$ be the estimated standard deviation of $\alpha_{(i)}$, for $i=1,\ldots,m$. The latter is given by the standard deviation of the regression model's slope at step 6 of DFA. The influence of each box size n_i is measured by

$$DFBETAS(i) = \frac{\alpha - \alpha_{(i)}}{\mathsf{std}(\alpha_{(i)})}$$

Essentially, *DFBETAS* is the standardized difference in the scaling exponent before and after removing the *i*th box size. Under standard linear regression assumptions, the *DFBETAS* are distributed as $t(m-2)/\sqrt{m}$, where t(m-2) is the *t*-distribution with m-2 degrees of freedom and m is the number of box sizes [21]. Even though the regression assumptions are not strictly applicable to DFA (e.g. mean fluctuations are not independent), we still use the result as an approximation. We identify box sizes leading to significant changes in α by checking whether or not their absolute *DFBETAS* are above a specific cutoff value C. We set the cutoff value to $C = t_{.975}(m-2)/\sqrt{m}$, at the usual 5% significance level.

In summary, we propose the following procedure for identifying a stable fitting range for DFA:

- 1. Beginning with a wide range of box sizes, calculate the DFBETAS of each one.
- If the absolute DFBETAS value of either the smallest or largest box size is outside the cutoff, C, remove these box sizes and recalculate the DFBETAS of the remaining ones.
- 3. Repeat the previous step until the absolute *DFBETAS* of the outermost box sizes lie below the cutoff.

This procedure leads to a stable estimate of α in the sense that it is not overly reliant on small or large box sizes. Since we check recursively for influential points, the procedure is susceptible to multiple testing [22], meaning that the overall significance level can be higher than the one set for individual comparisons. For our purposes, however, this is not a concern, since it can only lead to a more

conservative range. Provided there are sufficient points for fitting α , using a smaller range should not adversely affect the results.

The proposed procedure identifies a stable fitting range for each series separately. It would be convenient, however, to have a single range for series of the same conditions. The advantages of a common range are twofold. First, researchers do not have to repeat the procedure for every series, and second and most important, a single range facilitates comparisons between subjects and studies. To arrive at this general range, we propose using box sizes that fall within the majority (95%) of the individual ranges. Using this shorter, general range does not aggravate the estimation of α , provided there are sufficient points for fitting the slope.

2.6. Number of strides

There is statistical uncertainty associated with every scaling exponent α derived from DFA, which decreases with the length of the series. This uncertainty can be estimated by the standard deviation of the slope in DFA, and we examine how it relates to the series length. For simple linear regression, the standard deviation of the slope decreases at an exponential rate of $N^{-0.5}$ with the number of data N [23]. However, this result does not hold for DFA because the distribution of the regression data, $(\log n, \log F(n))$, violates two invariance assumptions. First, the standard deviation of the dependent variable, $\log F(n)$, decreases with the series length, because mean fluctuations are also estimated from the series. Second, the range of the independent variable, $\log n$, usually increases with the series length. For these reasons, we asses the relationship between the series length and α 's standard deviation empirically, using a similar exponential model.

To estimate the behavior of α 's standard deviation we apply DFA to subsets of increasing length from each series. We start with a minimum subset of 256 stride intervals, and expand it by 10 stride intervals at a time until we reach the full length of the series. The box size range used at each step is based on the method outlined in Section 2.5. For each series and each subset length, we estimate the corresponding standard deviation of α . We then average the standard deviations at each subset length across all series. Finally, we plot the average standard deviation of α versus the subset length, and fit a power law describing their relationship.

3. Results

We implement the procedure in Section 2.5, starting with a wide range of $[n_1 = 4, n_m = 256]$, and sequentially pruning the outermost box sizes according to their *DFBETAS*. The identified stable fitting ranges are presented in Fig. 2. Each horizontal line

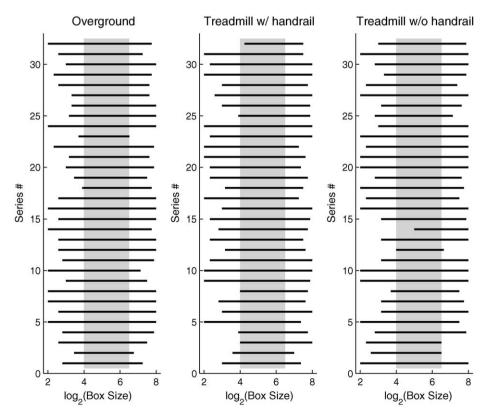


Fig. 2. Plots of stable box size fitting ranges as identified by the procedure in Section 2.5, for three different walking conditions. Horizontal lines delimit the fitting range for each series in \log_2 -space. Shaded area represents the general fitting range $[n_1 = 16, n_m = 91]$, which is contained in 95% of the individual ranges.

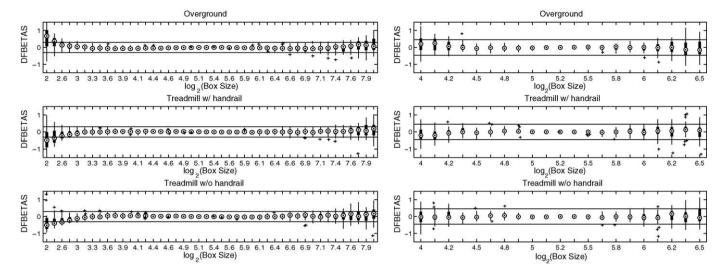


Fig. 3. Box-plots of the distribution of *DFBETAS* values at different box sizes, and for different walking conditions. The left panel contains box sizes in the initial fitting range $[n_1 = 4, n_m = 256]$, and the right panel contains box sizes in the proposed general range $[n_1 = 16, n_m = 91]$. Solid lines represent the 95% bounds for identifying influential box sizes.

delimits the range for a particular series and walking condition. There is some variability in the ranges, with n_1 ranging from 4 to 32, and n_m ranging from 91 to 256. However, even the smallest range contains 21 box sizes, which are sufficient for estimating α . These fitting ranges give robust estimates, in the sense that removing any boundary box size would not cause a significant change in α .

There is considerable overlap in the individual ranges in 2, so we calculate common ranges for each walking condition. The ranges $[n_1 = 13, n_m = 91], [n_1 = 16, n_m = 128], and$ $[n_1 = 16, n_m = 91]$ are contained in 95% of the individual ranges for OW, TW-R, and TW-NR, respectively. Since these are similar, we also provide the general range $[n_1 = 16, n_m = 91]$, which is contained in 95% of all the individual ranges. The latter comprises 21 box sizes, and is represented by the shaded regions in Fig. 2. We also assess the consistency of estimation duplicate sessions from each subject using the proposed range. For all three walking conditions, OW, TW-R, and TW-NR, paired t-tests failed to reject the null hypothesis of no systematic difference (P = 0.9114, 0.1631, and 0.5928, respectively), and the Bland-Altman coefficients of repeatability showed practically sufficient agreement between duplicate α 's (CR = 0.1047, 0.1175, and 0.1017, respectively). Thus, we propose this range as a general guideline for series with a similar length of N = 800. For series with substantially different lengths, we propose the size-adjusted range $[n_1 = 16, n_m = N/9]$, which essentially guarantees a minimum of nine boxes used in detrending for the maximum box size n_m .

We elucidate the advantages of the general range $[n_1 = 16, n_m =$ 91] through a comparison with the initial range $[n_1 = 4, n_m = 256]$. Fig. 3 presents box-plots, grouped by box size, of the DFBETAS values of all series for different walking conditions. Plots on the left comprise box sizes in the initial range, plots on the right comprise box sizes in the proposed range, and horizontal lines delimit the cutoff bounds for identifying influential box sizes. For the initial range, the mean DFBETAS of smaller boxes are outside the bounds, resulting in a systematic bias for α . Small boxes contain fewer points for fitting the trend, which can lead to over/underestimation of α , in agreement with crossover phenomena (slope shifts) reported in such cases [15]. The DFBETAS for larger box sizes in the initial range do not exhibit bias, but are more dispersed. Large box sizes provide sufficient data for fitting the trend, but average fluctuations around the trend become more variable, making the estimation of α less stable [14]. Both observations warrant the use of a restricted fitting range. For the shorter, proposed range, the majority of DFBETAS fall within the cutoff bounds, demonstrating the improved stability in estimating α .

We examine the effect of sample size on the standard deviation of α by applying DFA to increasing subsets from each series. Following our previous recommendation, we use the range $[n_1=16,n_m=N_s/9]$ at each particular subset length N_s . The solid lines in Fig. 4 present the average (across series) standard deviation of α

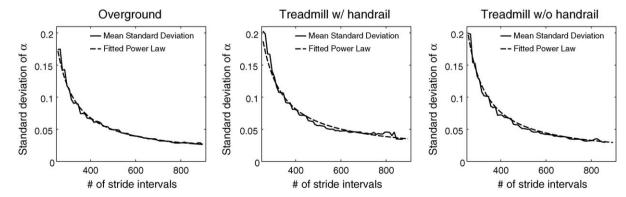


Fig. 4. Plots of standard deviation of α versus series length, for different walking conditions. Solid lines represent the average (across all series) standard deviation, and dashed lines represent the fitted power laws.

versus the number of stride intervals used in DFA, for different walking conditions.

To describe the accuracy gain from using more data, we fit a power law to the average standard deviation of α . The relationships for each walking condition are

 $\begin{array}{l} \bullet \; \; \text{OW:} \; \; \text{std}(a_{N_s}) \approx 3.716 \times N_b^{-1.623} \\ \bullet \; \; \text{TW-R:} \; \; \text{std}(a_{N_s}) \approx 2.883 \times N_b^{-1.445} \\ \bullet \; \; \text{TW-NR:} \; \; \text{std}(a_{N_s}) \approx 4.429 \times N_b^{-1.645}, \end{array}$

where $N_b = 8(\log_2(N_s/9) - \log_2 16)$ measures the effective sample size, i.e. the number of $(\log n, \log F(n))$ pairs used in DFA. As shown by the dashed lines in Fig. 4, these laws provide a very good approximation to α 's standard deviation with respect to sample size $(R^2 > .97)$, at least for the sample sizes we examined.

4. Discussion

We discuss the application of our results to the analysis of gait data, focusing on two cases of interest: comparing α 's between control and test groups, and estimating α for individual subjects. We begin with the box sizes, for which we propose the general range [16, N/9], where N is the number of strides. Our recommendation is based on able-bodied adults because of its application to hypothesis testing. Since null hypotheses always presume identical gait dynamics for the control and test group, our proposed range is custom to the reference (i.e. control) population. For estimating individual α 's, subjects with pathological or otherwise dissimilar gait dynamics can have a different stable fitting range, in which case the procedure in section 2.5 can be repeated. However, for consistency and simplicity, we advocate the general use of the range [16, N/9].

We also describe the use of the empirical power laws. When comparing group α 's, the power of the hypothesis test relies on group sizes and variances. Moreover, each group's variance is the sum of inter-subject variance and estimation variance for α . Intersubject variance cannot be controlled, but estimation variance can be reduced with more stride intervals. Experimental design theory (e.g. see Cohen [24]) gives rules for selecting the number of subjects, based on group variance and the desired power. Using these rules and the empirical power laws, the researcher can select a convenient combination of number of subjects and strides. For example, if a test group cannot perform long walks, power can be improved by including more subjects in the group, and vice versa. For individual estimation, we want to evaluate α with certain accuracy. Studies have found differences in α of more than .1 between healthy and pathological gait [7]. Thus, a standard deviation of .05 is a safe prerequisite for estimating α with .1 error, i.e. within two standard deviations. To achieve this, the power laws commend sample sizes of N = 494,604, and 541, for OW, TW-R, and TW-NR, respectively. These numbers are based on able-bodied adults, but can be applied universally as conservative lower bounds. For subjects who can not complete that many strides at once, averaging their α 's from separate walking sessions can also decrease estimation errors. Overall, we propose at least 600 stride intervals for assessing individual subject's dynamics with reasonable confidence.

5. Conclusion

Using able-bodied, adult gait data, we have systematically investigated the choices of box size range and number of strides that yield stable and accurate estimates of α using detrended

fluctuation analysis. To obtain stable α estimates from series of length N, a box size range of [16,N/9] is recommended. Further, to estimate α with an accuracy of $\pm .1$, researchers should generally aim to capture upwards of 600 strides.

Acknowledgement

We thank the subjects for their participation, time and effort.

Conflict of interst statement

Sotirios Damouras and Ervin Sejdić are post-doctoral fellows in the Bloorview Research Institute. Tom Chau is the supervisor of Matthew Chang. There are no conflicts of interest between the coauthors of the paper.

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