Average Wavelet Coefficient-Based Detection of Chaos in Oscillatory Circuits

Vesna Rubežić, Igor Djurović, Ervin Sejdić

Abstract— **The wavelet transform-based detection of chaos in the oscillatory circuits has been proposed. The proposed detection is developed by using the specific measure, obtained by averaging wavelet coefficients. This measure exhibits various values for chaotic and periodic states. The proposed detection is applied to signals from autonomous systems such as the Chua's oscillatory circuit, the Lorenz chaotic system and non-autonomous systems such as the Duffing oscillator. In addition, the detection is applied to sequences obtained from the logistic map. The results are compared to those obtained with a detrended fluctuation analysis and a time-frequency signal analysis based on detectors of chaotic states. The proposed technique is robust to the noise influence, having smaller calculation complexity with respect to the state-of-the-art techniques.**

Index Terms—**Average wavelet coefficients, Chaos detection, Oscillatory circuits, Wavelet transform.**

I. Introduction

OSCILLATORY electrical circuits belong to a large group of nonlinear systems exhibiting different types of dynamic behavior (Drazin, 1992), (Sprott, 2003), (Kennedy, 1993), (Liu et al., 2010), (Chen et al., 2012), (Chen et al., 2014). The diverse dynamics of these systems complicate their analysis; however they facilitate different applications. Detection of dynamic changes and transitions in signals, as well as estimation of the current state of the system, are attractive research subjects.

Since the classical methods for detection of chaos in dynamical systems (Wolf et al., 1985), (Rosenstein et al., 1993) (Grassberger and Procaccia, 1983a) (Grassberger and Procaccia, 1983b) are computationally complex, sensitive to noise, and generally unsuitable for short data series, numerous alternatives have been developed. Chaos titration, permutation entropy procedures and recurrence plots have been proposed in (Poon and Barahona, 2001), (Bandt and Pompe, 2002), (Gao, 1999). The time-frequency (TF) signal analysis detectors of chaotic state are proposed in (Rubežić et al., 2006), (Djurović and Rubežić, 2007), (Djurović and Rubežić, 2008). These algorithms properly differentiate chaos and noise for environments of moderate noise levels. The scale and scaling exponents have been shown as excellent tools for analysis of signals coming from various real-life systems (Costa et al., 2005). A chaos detector for oscillators based on detrended fluctuation analysis (DFA) has been developed in (Djurović et al., 2013). This method was simple for implementation, with reasonable computational complexity and good accuracy for noisy environments. In (Gottwald and Melbourne, 2004), (Gottwald and Melbourne, 2009) a test for chaos from time series that works effectively for longer series in the presence of low levels of measurement noise is introduced. Test for chaos formulated in (Eyebe Fouda et al., 2013) is applicable only to discrete-time systems. Recently, an efficient algorithm for automatic chaotic modes detection in real-time and in the presence of noise is proposed in (Carlos M. N. Velosa, 2015). The key idea behind technique proposed in (Carlos M. N. Velosa, 2015) is that a single component of a chaotic trajectory tends to exhibit an infinite number of local maxima at different time instants. Using an auxiliary system acting as a denoiser and resorting to simple mathematical operation, it is established a parameter that characterizes the type of motion based on a specific threshold. Algorithm is effective even for a very small observation window of only 50 points, but it requires extension of the proposed concept to discrete time systems.

All of the above methods have their advantages and disadvantages. It is important to find alternative ways to analyze the chaotic systems' outputs, as more analysis tools mean more opportunities for appropriate estimation of chaotic state outputs. In this sense, we present a new method based on the wavelet transform (WT) for detection of current states of chaotic oscillators, based on a series of data. The proposed method has high accuracy for emphatic noise environment, which is the main advantage over existing methods.

The WT is a particularly popular time-scale signal analysis tool used in numerous fields of science ranging from quantum physics to cosmology (Askar et al., 1996), (Michtchenko and Nesvorny, 1996). There are many applications of wavelets to

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nonlinear dynamics (Eke et al., 2002), (Schaefer et al., 2014), (Chandre et al., 2003), (Kaur et al., 2014). Time-frequency/scale analysis using wavelets has been applied to dissipative chaotic systems (Duffing oscillator (Permann and Hamilton, 1992), Morse oscillator (Permann and Hamilton, 1994)). In this paper, the WT is applied to design the chaos detector for oscillatory circuits. A wavelet analysis represents a windowing technique with variable-sized regions, allowing an equally precise characterization of low and high frequency dynamics in a signal. The wavelet analysis breaks up the signal into shifted and scaled versions of the mother wavelet. In fact, the WT works as a "mathematical microscope" on a specific part of a signal to extract local structures and singularities (Mallat, 1999), (Daubechies, 1992). This makes the WT ideal tool for handling non-stationary and transient signals, as well as for fractal-type structures (Eke et al., 2002); while periodic and quasiperiodic trajectories have no significant difference between the windowed (short-time) Fourier (used for chaos detection in (Rubežić et al., 2006), (Djurović and Rubežić, 2007), (Djurović and Rubežić, 2008)) and the WT. Since the WT is a time-scale representation, it is quite natural to attempt to connect it to the scaling exponent estimation or other tools for scale analysis which are required for the detection of chaos in the oscillating systems.

In this paper, a specific measure is calculated by using average wavelet coefficients (AWC) (Simonsen et al., 1998) and fractal properties of chaotic signal (Eke et al., 2002). This measure exhibits different values for periodic and chaotic states. Current state of systems is estimated without a priori knowledge of the structure and parameters of the oscillators. Simulation examples are done for well-examined Chua's oscillator and other chaotic oscillator circuits but this method is applicable for chaos detection in other nonlinear systems. Detection of the chaotic state of signals from circuits was performed in a simpler manner with respect to the existing approaches (Poon and Barahona, 2001), (Bandt and Pompe, 2002), (Gao, 1999). The obtained detector exhibits better accuracy than the relevant techniques from (Rubežić et al., 2006), (Djurović et al., 2013) for noisy environments.

The paper is organized as follows. The WT is presented in brief in Section II. In Section III WT and AWC of a chaotic process are derived. In Section IV, an algorithm based on the AWC is described. This algorithm is tested on outputs from nonlinear oscillatory circuits (Section V). Comparison of the proposed technique with the state-of-the-art approaches for noisy environments is given in Section VI. Concluding comments are given in Section VII.

II. Wavelet Transformation - in brief

The continuous WT (CWT) is a tool used for signal decomposition into different signal components where each component is analyzed with a resolution that matches its scale. The basis for the WT is a wavelet function. Wavelet functions are families of functions satisfying prescribed conditions, such as continuity, zero mean amplitude, and finite or near finite duration. The translated (shifted) and scaled versions of the mother wavelet function $\Psi(t)$ can be defined as:

$$\Psi_{a,b}(t) = \Psi\left(\left(t-b\right)/a\right), \tag{1}$$

where the scale parameter is $a > 0$, and the translation parameter is $-\infty < b < \infty$.

The continuous WT of a given function $x(t)$ is defined as:

$$W[x](a,b) = a^{-1/2}\int_{-\infty}^{\infty} \Psi_{a,b}^{*}(t)\, x(t)\mathrm{d}t\,, \tag{2}$$

where $*$ denotes the complex conjugate.

The WT maps the signal into a two-dimensional time-scale domain (coordinate *a* corresponds to scale while *b* corresponds to time or shift). Here, attention will be concentrated only to the scale parameter. Averaging the wavelet coefficients is performed to obtain the representative amplitude for a given scale. This average is denoted as (Simonsen et al., 1998):

$$W[x](a) = \left\langle \left| W[x](a,b) \right| \right\rangle_b \,, \tag{3}$$

where $\langle \cdot \rangle_b$ is the standard arithmetic mean value operator with respect to variable *b*, while $W[x](a)$ is the average wavelet coefficient - AWC. Averaging the absolute values of wavelet coefficients produces a kind of "wavelet energy" function for an individual scale (Simonsen et al., 1998).

More details about the WT can be found in (Mallat, 1999), (Daubechies, 1992).

III. Wavelet Transformation of chaotic process

The purpose of this section is to prove that the AWC are related to scaling exponent. The scaling exponents are different for chaotic and periodic signals i.e., they can be used in detection of chaos. Since the proposed detector of chaos uses self-affine properties of chaotic signals we are briefly reviewing it in this section. For more details, see (Eke et al., 2002).

The main feature of self-affine processes is the scaling property. Consider function $h(x)$. Self-affinity is defined through statistical invariance under the transformation:

$$\begin{aligned} x &\to \lambda x, \\ h &\to \lambda^{H} h. \end{aligned} \tag{4}$$

Here H is the Hurst exponent used for description of fractal properties in (Eke et al., 2002) instead of fractal dimension. An alternative way of expressing this invariance is

$$h(x) \square \lambda^{-H} h(\lambda x), \tag{5}$$

where the symbol $\square$ means statistical equality.

The scaling property of self-affine process (5) in the wavelet domain can be written $W\left[h(x)\right](a,b) \square W\left[\lambda^{-H} h(\lambda x)\right](a,b)$. Now it follows:

$$\begin{aligned} & W\left[h(x)\right](a,b) \square W\left[\lambda^{-H} h(\lambda x)\right](a,b) = \\ & = a^{-1/2} \int_{-\infty}^{\infty} \lambda^{-H} h(\lambda x) \Psi^{*}\left((x-b)/a\right) \mathrm{d}x = \\ & = \lambda^{(-1/2-H)} (\lambda a)^{-1/2} \int_{-\infty}^{\infty} h(x') \Psi^{*}\left((x'-\lambda b)/\lambda a\right) \mathrm{d}x' = \\ & = \lambda^{(-1/2-H)} W\left[h(x)\right](\lambda a, \lambda b). \end{aligned} \tag{6}$$

Thus, the scaling relation in wavelet domain is obtained as

$$W[h](\lambda a, \lambda b) \square \lambda^{1/2+H} W[h](a,b). \tag{7}$$

Therefore, an isotropic rescaling with factor λ in the wavelet domain of a self-affine function leads to the same rescaling the wavelet amplitude of the original domain with a factor $\lambda^{1/2+H}$. Obviously the scale is important feature of the self-affine functions while translation is not of our interest. Averaging (3) scaling relation (7) yields

$$W[h](\lambda a) \square \lambda^{1/2+H} W[h](a). \tag{8}$$

Fractal methods are diverse, but their approaches have one thing in common: they fit to the data pairs of log *feature* versus log *scale* for finding the scaling exponent, from the regression slope. In the detection algorithm that is proposed in the next section, $W[h](a)$ is given with respect to scale a in a log-log plot. The linear dependence implies a self-affine behavior of chaotic data. The slope of this straight line is $1/2+H$ and based on that it the Hurst exponent can be estimated (Mallat, 1999). This function is not a straight line for periodic signals. Our goal is to use this difference between periodic and chaotic signals for detection of systems state (periodic or chaotic).

IV. THE DETECTOR BASED ON THE AVERAGE WAVELET COEFFICIENTS

In this section, the chaos detector based on the specific measure obtained from the AWC is summarized.

1. The original signal $x(t)$, $0 \le t \le t_s$ is divided into overlapping blocks $x_k(t)$ of length T (that are centered in the considered instant):

$$x_k(t) = x(t)\, w(t - k\tau),\ k = 0,1,\ldots,n. \tag{9}$$

where $w(t) = 1$ for $0 \le t \le T$ and 0 otherwise and $n = \left[(t_s - T)/\tau\right]$, where [] signifies whole parts of the number, while τ is the time shift.

2. The time series mean is computed as:

$$\bar{x}_k = (1/T) \int_{k\tau}^{T+k\tau} x_k(t) \mathrm{d}t,\ k = 0,1,\ldots,n. \tag{10}$$

An integrated time series $y_k(t)$ is obtained as:

$$y_k(t) = \int_0^t \left(x_k(t) - \bar{x}_k\right) \mathrm{d}t,\ \ k\tau \le t \le k\tau + T,\ k = 0,1,\ldots,n. \tag{11}$$

3. The continuous WT of a signal $y_k(t)$ is calculated according to (2).

4. Then, the AWC is calculated according to (3).

5. Function $W_l = f(a_l)$ where $\log\left(W[y](a)\right) = W_l$ and $\log(a) = a_l$ is interpolated with a linear function. Denote the obtained linear function as $y_l(a_l)$.

6. Fluctuation, i.e., the difference between $f(a_l)$ and $y_l(a_l)$ is calculated as:

$$z_l(a_l) = f(a_l) - y_l(a_l)\ . \tag{12}$$

7. The root mean square value (RMS) of fluctuation $z_l(a_l)$ is obtained as:

$$F_{t_k} = \sqrt{\left(1/\left(a_{l\max} - a_{l\min}\right)\right) \int_{a_{l\min}}^{a_{l\max}} z_l^2(a_l) \mathrm{d}a_l}\ . \tag{13}$$

Specifically, it is expected that the deviation $W_l(a_l)$ from the linear function $y_l(a_l)$ is small for the chaotic signal but large for the periodic signal. This deviation (the RMS of the difference between $f\left(a_l\right)$ and its linear approximation $y_l\left(a_l\right)$) from the linear function was used in detector of the chaotic/periodic states. It exhibits a small value for the chaotic signal, and a large value for the periodic signal (see Section III).

8. F_{t_k} is measure $m\left(t_k\right)$ for the time instant $t_k = k\tau + T/2, \quad k = 0,1,\ldots,n$.

9. The decision on the system state is made by comparing the function $m(t_k)$ with the detection threshold C:

$$\begin{aligned} m\left(t_k\right) &\le C \quad \text{Chaotic regime} \\ m\left(t_k\right) &> C \quad \text{Periodic regime.} \end{aligned} \tag{14}$$

The proposed algorithm enables chaos detection from a single time series, it does not require knowledge of systems and does not depend of nature of system. It is not limited only to oscillatory circuits but it can be applied on time series from any chaotic systems (discrete time or continuous time). In addition, it is not limited by a system dimension. In order to detect chaos considered is interval (window) of relatively small number of samples (in our experiments only 60). This keeps low detector complexity (the most complex operation is the WT evaluation). Therefore, it is suitable for real-time detection with delay that is about half of the window width.

The proposed algorithm is different from the previous techniques. In (Permann and Hamilton, 1992), function $W_l=f(a_l)$ (where $W_l = \log\left(W[y](a)\right)$ and $a_l = \log\left(a\right)$) is a linear function for the considered systems, which is then used to estimate the Hurst exponent. However, for systems considered here, function $W_l=f(a_l)$ is not linear for the periodic regime, and it is necessary to perform steps 5 to 9 where the RMS of the fluctuation is calculated. Furthermore, there is a significant difference between this approach for evaluation of the specific measure and the technique from (Djurović et al., 2013). Namely, the signal analysis is not performed on the signal series in (Djurović et al., 2013), but on the series representing intervals between signal extreme values. In this way the signal is significantly downsampled, limiting the ability to detect the narrow intervals of chaotic or periodic states. However, this problem can be avoided with the proposed algorithm, since the measure is obtained directly from the signal. Additionally, calculating the proposed measure is much simpler than in (Rubežić et al., 2006). Accuracy of the algorithm from (Rubežić et al., 2006) depends on several parameters, the most notably on the applied window width used in the TF representation. However, the proposed detector depends only on the selection of the mother wavelet and length of the block T. In our experiments, we have found that there is no significant difference related to the choice of the wavelet function. Accuracy of the considered detectors depends mainly on the selected threshold.

V. Proposed Detector for Chaotic Oscillators

There are numerous chaotic oscillatory circuits. Among them are autonomous systems such as the Chua's, Lorenz, and Rossler chaotic oscillators, as well as non-autonomous systems such as the Duffing oscillator. All these are commonly considered examples of chaotic systems. These circuits, together with appropriate chaos detectors, are analyzed in our papers (Rubežić et al., 2006), (Djurović and Rubežić, 2007), (Djurović and Rubežić, 2008), (Djurović et al., 2013). By changing one of these circuit parameters, they can pass through various modes of transition to chaos such as period-doubling, torus breakdown, subcritical and intermittency routes (Sprott, 2003), (Kennedy, 1993). Some of these regimes (period-doubling, torus breakdown) are characterized by long intervals of chaos. However, a signal is virtually periodic in intermittent transition to chaos, except for some irregular bursts. With changes to the parameter, these bursts appear more frequently and the average time between two consecutive bursts is shorter.

Here, the Chua's, Lorenz, and Duffing chaotic oscillators are considered with the same set of parameters as in (Rubežić et al., 2006) and (Djurović et al., 2013).

First, we considered the period doubling route to chaos in the Chua's oscillator with parameters similar to that of (Rubežić et al., 2006) and (Djurović et al., 2013). For the WT in all the examples, the Daubachies' order 12 wavelet is used (Daubechies, 1992). Similar results were obtained by applying other wavelets. Fig. 1a) and c) show the periodic signal from the Chua's oscillator and the coefficients of its continuous WT, respectively. Fig. 1e) depicts function $W_l = f(a_l)$, where $W_l = \log\left(W[y]\left(a\right)\right)$ and $a_l = \log\left(a\right)$ (blue line). It can be seen that it is of oscillatory nature and cannot be simply described. Fig. 1b) and d) show the chaotic signal from Chua's circuit and the coefficients of its continuous WT, respectively. Fig. 1f) depicts $W_l = f(a_l)$ for chaotic region. For chaotic signal it is almost a straight line that can be approximated by linear functions, so the fluctuation error is small. The main conclusion influencing development of the proposed detector is that the linear function on the log-log plot corresponds to chaotic regime while the function different of the straight line indicates periodic regime. Large RMS occurs when function from Fig. 1e) is approximated by the linear function (green line). This deviation from the linear function leads to the idea that the RMS value, which is derived from the given dependence and its linear approximation, can be used for the chaos detection measure, as described in previous section. This difference (fluctuation) has a small value for the chaotic signal and a large value for the periodic signal, which is important feature to distinguish between these two states.

Proposed measure for chaotic signal has small value. Namely, in all our experiments (for different systems and different routes to chaos) the value of measure for chaotic signal is smaller than 0.01. For periodic signal measure has bigger values (different for different periodic regimes). Difference between the measure in the chaotic and periodic regime was large enough to set a safe threshold between these regimes. In experiments we concluded that in noisy environment the value of measure decreases, which will be described later. Because of that we decided to set the threshold on the lowest allowed value and it is the maximal value we have found in the chaotic regime. This choice enables reliable detection in noisy environment. Therefore, based on the numerous sets of experiments for the considered routes to chaos and the considered circuits, a threshold was selected as 0.01.

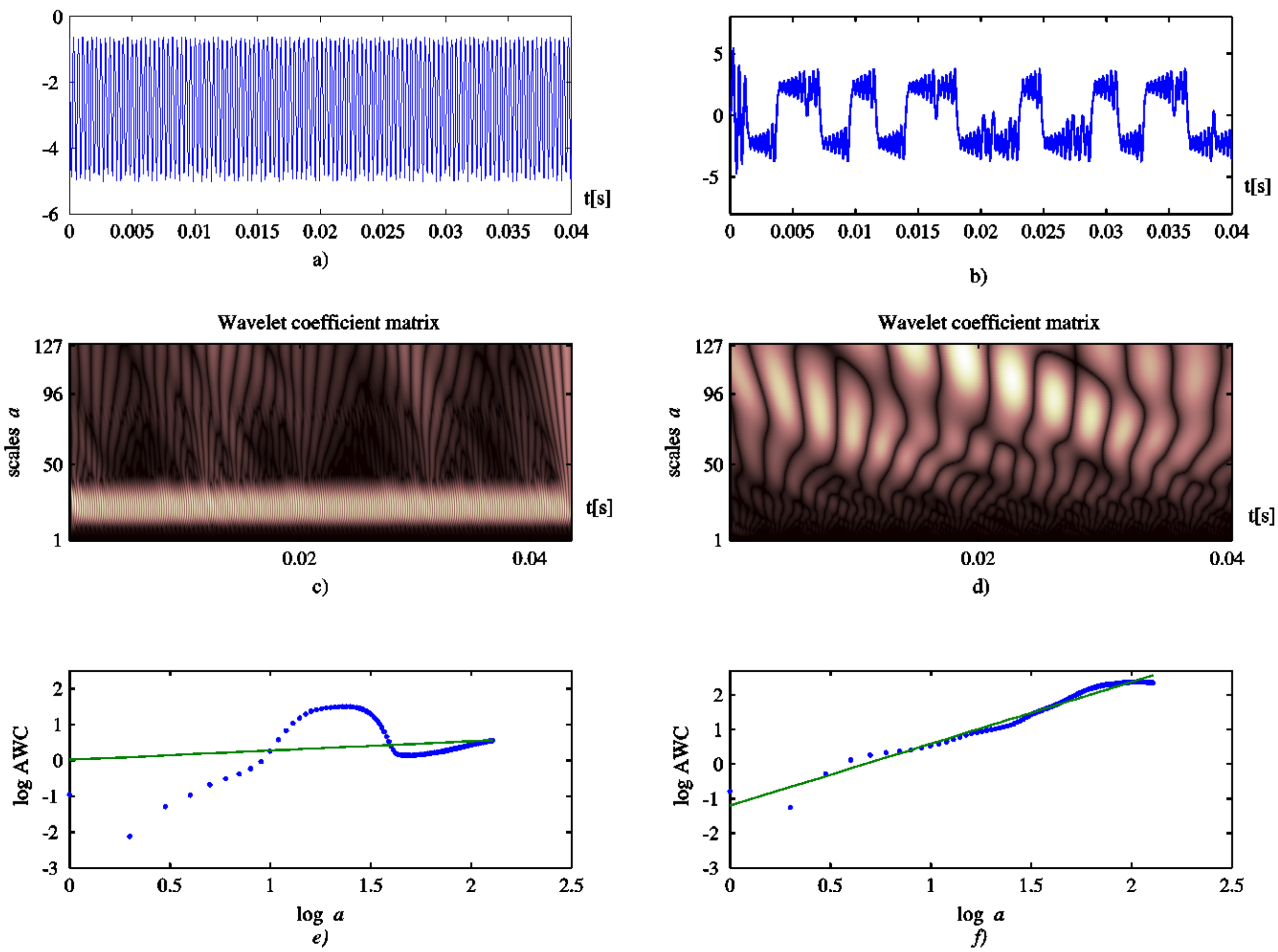

Fig. 1. Steps of analysis by the average wavelet coefficient method. a) periodical signal; b) chaotic signal; c), d) wavelet coefficient matrix; e), f) Blue line – function $\log AWC = \log(W[y](a)) = W_l = f(a_l) = f(\log a)$ versus $\log a$, green line – its linear approximation.

In Fig. 2 a), b), c), the responses of the proposed detector, of the detector from (Djurović et al., 2013), and of the detector from (Rubežić et al., 2006) for the period doubling route to chaos for Chua's oscillatory circuit, are shown respectively. We consider Chua's circuit with the following set of parameters: $L = 18mH$, $C_1 = 10nF$, $C_2 = 100nF$, $G_a = -757.576\mu S$, $G_b = -409.091\mu S$, $E = 1V$, $R_0 = 12.5\Omega$ with G that linearly increases from $G = 530\mu S$ to $G = 565\mu S$ and after that decreases toward initial value. The initial conditions are $(v_1, v_2, i_3) = (0, 0.4590, 0.0006)$ and the sampling interval is $\Delta t = 12.63\mu s$. The time-domain representation of considered signal is shown in Fig. 2d). In order to visualize periodic regions (especially periodic windows) and chaotic regions the short-time Fourier transform (STFT) of the considered signal is shown in Fig. 2 e). The STFT is defined as (Boashash, 2016):

$$STFT(t, f) = \int_{-\infty}^{\infty} x(t+\tau) w(\tau) e^{-j2\pi f\tau} d\tau, \tag{15}$$

where $w(\tau)$ is the window function, $w(\tau) = 0$ for $|\tau| > T/2$ and T is window width. In simulations the voltage $v_1(t)$ is considered.

Detector response $m(t)$ (Fig. 2 a)) is larger for the periodic than for the chaotic regime. If $m(t)$ is above the selected threshold, it can be concluded that the fluctuation is large and that the state of circuit is periodic. If the measure is below the threshold, the considered signal is chaotic. The results obtained in our simulations agree with the theoretical considerations and experimental results reported in (Sprott, 2003), (Kennedy, 1993), (Rubežić et al., 2006), (Djurović et al., 2013). The obtained detection accuracy is satisfactory for longer chaotic or periodic regions, but also in the area of narrow periodic windows. Namely, all narrow periodic windows that can be seen in the time-frequency representation in the chaotic region were detected, which was not the case for the algorithm from (Djurović et al., 2013). In Fig. 2 some of periodic windows are shown by arrows. Changing the parameters of the algorithm, such as reducing the length of the block T (duration of the segment can be very short), may have increased the accuracy, which could not be done in the algorithm from (Djurović et al., 2013) because DFA requires a relatively wide window to produce accurate results. Similar results could be obtained for other signals from this circuit ($v_2(t)$ and $i_3(t)$) and other routes to chaos. Then intermittency and torus breakdown routes to chaos in the Chua's oscillatory circuits are considered. Considered signals in time-domain are shown in Fig. 3 a) and b). In Fig 3 c) and d) the proposed detector responses are shown. Theoretical knowledge related to behavior in this circuit and routes to chaos is wide and it is the simplest way to compare obtained results with related theory and other techniques.

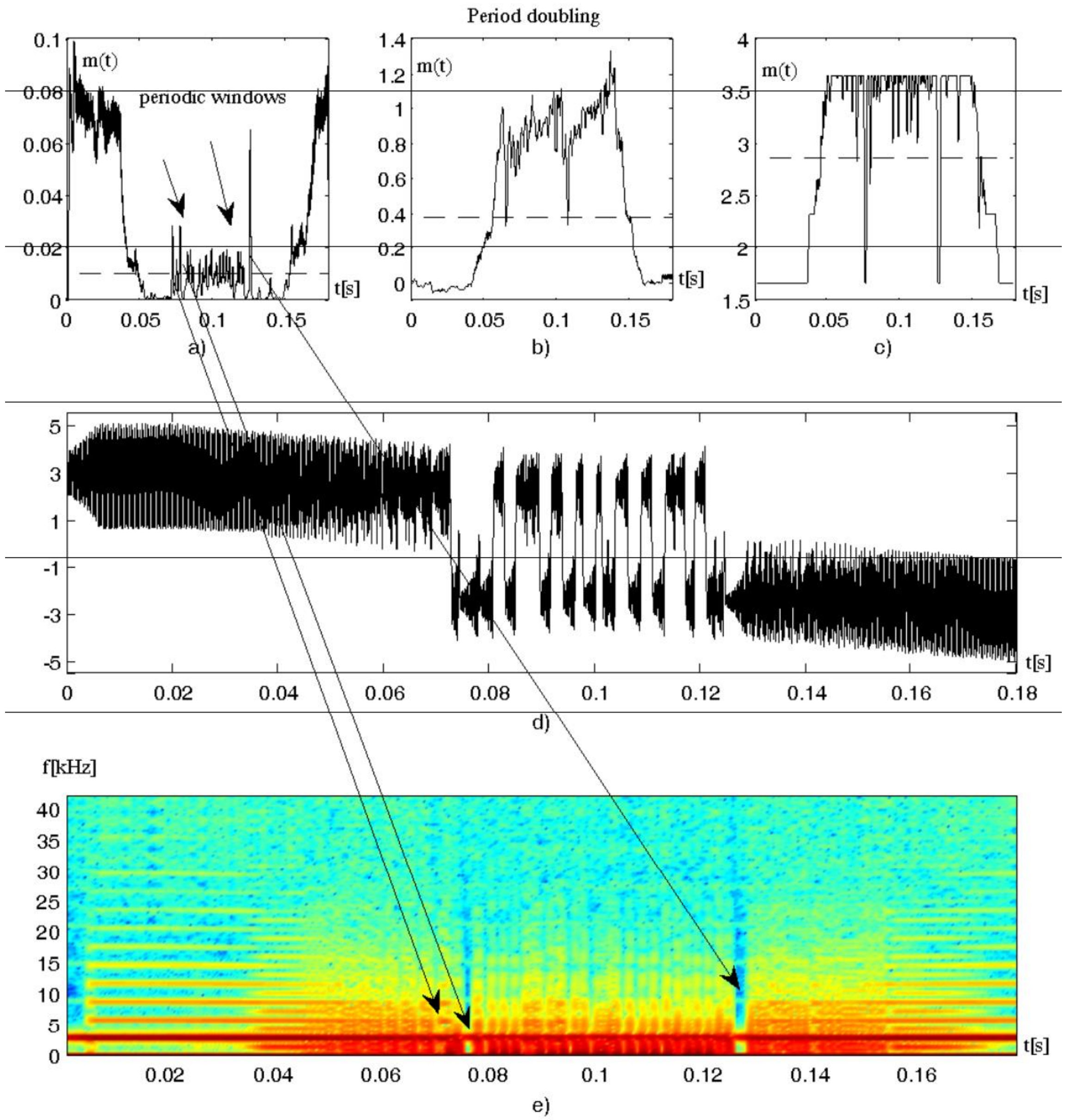

Fig. 2. Chua's oscillator – period-doubling route to chaos. a) the response of the proposed detector; b) detector response from (Djurović et al., 2013); c) detector response from (Rubežić et al., 2006); d) time-domain representation of the considered signal; e) time-frequency representation of the considered signal. Detection threshold - dashed line, solid line - detector response.

Performance of the proposed detector is also investigated for other chaotic systems, including the autonomous systems such as the Lorenz chaotic system, and non-autonomous systems such as the Duffing oscillator. In addition, we have applied the detector to the sequences obtained from the logistic map (with the same values of parameter as in (Rubežić et al., 2006)). Simulation results for these three systems are given in Fig. 4. The proposed procedure is not limited only on oscillatory circuits, but it can be applied on time series from any chaotic system. Chaotic regions (below the threshold) and periodic regions (above the threshold) detected by the proposed detector corresponded well to the theoretical expectations (Sprott, 2003), (Kennedy, 1993) and results from (Rubežić et al., 2006), (Djurović et al., 2013). Namely, the values of parameters of the considered systems for which their signals are periodic or chaotic can be found in (Sprott, 2003), (Kennedy, 1993). We can also detect periodic windows and chaotic signal by visually comparing the proposed detector with the time-frequency representation.

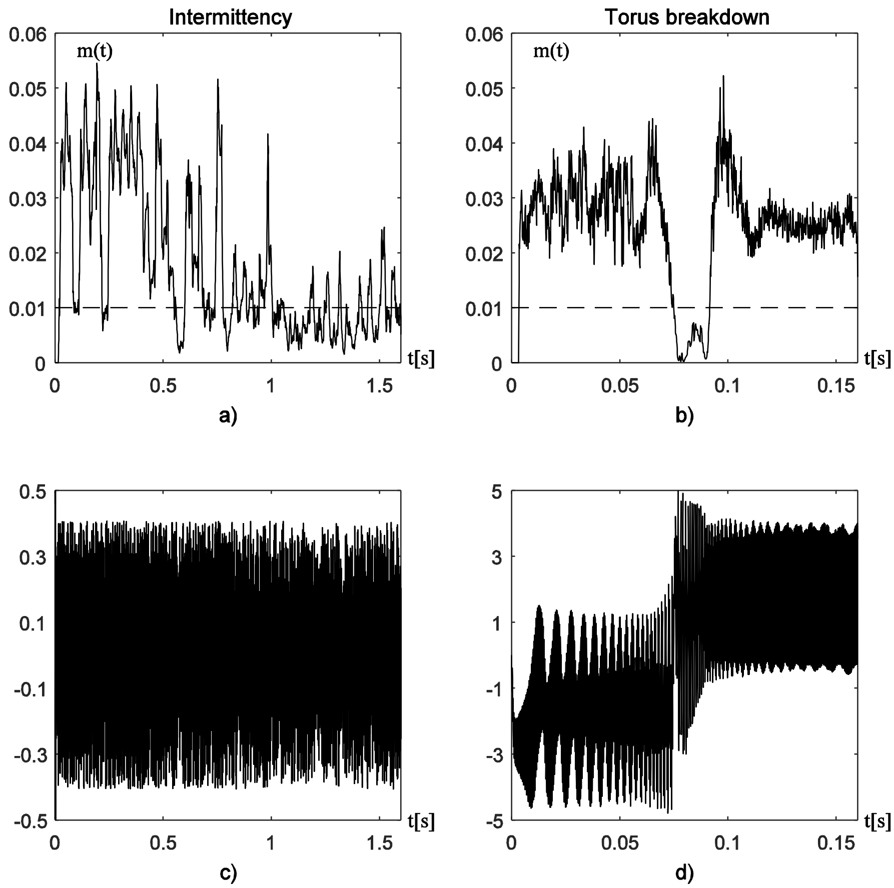

Fig.3. Chua's oscillator. Intermittency route to chaos: a) detector response; c) time-domain representation of considered signal; Torus breakdown route to chaos: b) detector response; d) time-domain representation of considered signal; Detection threshold - dashed line, solid line - detector response.

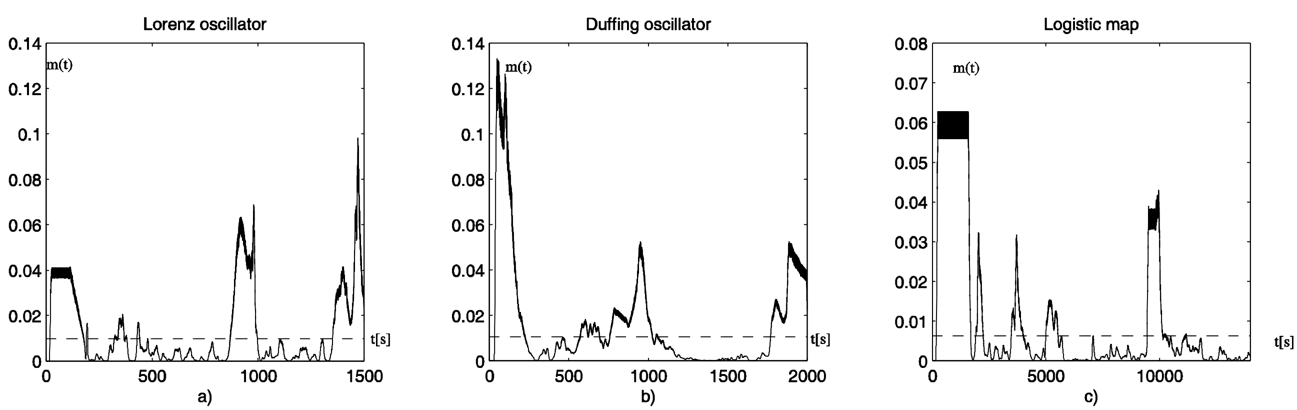

Fig. 4. Detector responses for other chaotic systems (Lorenz and Duffing oscillator and Logistic map). Detection threshold - dashed line, solid line - detector response.

VI. Noise influence

The main advantage of the proposed algorithm is robustness to the noise influence. Signals from the Chua's oscillator for a period-doubling route to chaos are typically corrupted by an additive Gaussian noise. Noise in the wavelet domain is spread over the entire time-scale plane. Averaging wavelet coefficients reduces the noise impact. However, the noise impact is noticed at scales with lower values of AWC. In other words, the noise raises AWC at these scales. In the example in Fig. 5 noise raises

AWC for $\log(W[y](a))$ below 0.5. For chaotic signal these are only lower scales, and for periodic both lower and higher (above $\log(a) = 1.6$). This leads to change of function $W_l = \log(W[y](a)) = f(a_l) = f(\log a)$ and its linear approximation.

However, the proposed measure remains almost the same for moderate level of noise. Therefore the noise does not significantly influence the proposed measure.

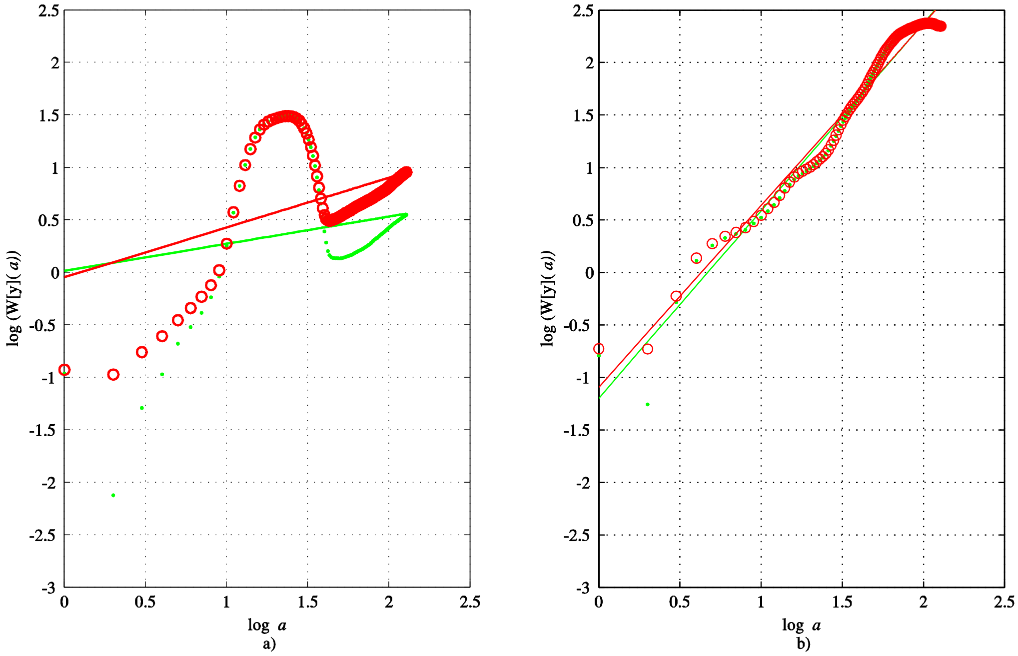

Fig. 5. Function $W_l = f(a_l)$ and its linear approximation. a) periodic, b) chaotic signal. Noiseless case (green line), SNR=5dB (red line).

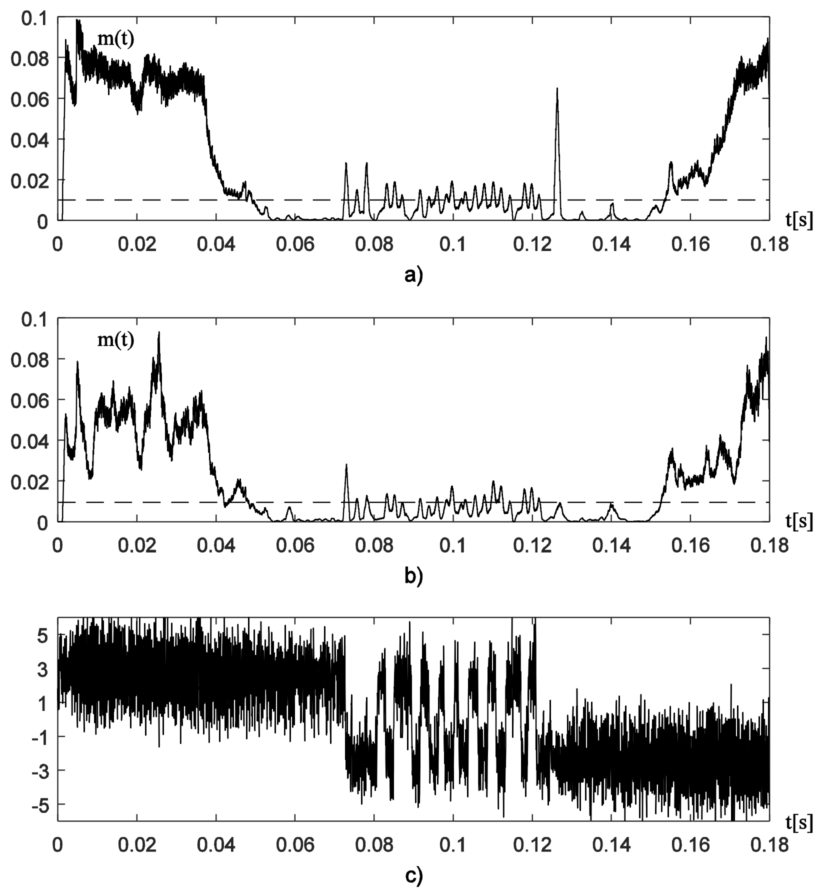

Fig. 6. Detector response for period doubling route to chaos in Chua's circuit. a) noiseless case, b) SNR=5dB, c) time-domain representation of considered noisy signal.

In Fig. 6 a) and b) detector responses are shown without noise and in noisy environment (SNR=5dB), respectively. In Fig 6 c) time-domain representation of considered noisy signal is shown. Detector recognized great number of periodic windows under conditions of intensive noise. Detector response for wide periodic region is lower (than in case without noise) but still above selected threshold.

In order to analyze accuracy of the proposed detector, we have compared its performance with the detectors from (Rubežić et al., 2006), (Djurović et al., 2013) for the signal to noise ratio in the range [0, 20]dB. The detection error is defined as the percentage of samples from the periodic regime which are incorrectly declared chaotic. In noisy environment the detector response decreases. It can lead to misidentification of samples from the periodic regime. In presence of noise the value of measure of chaotic signal that is already low becomes even lower and it is not possible to misidentify samples from the chaotic regime as a periodic for properly selected threshold (0.01 and above). As it can be seen in Fig. 7, the proposed detector is more accurate than those from (Rubežić et al., 2006), (Djurović et al., 2013).

Additionally, we have done a few experiments in noisy environment with different values of threshold. As it can be seen from Fig. 7, results are not significantly dependent on the threshold. It can be seen that detector performances for higher threshold (0.015) are almost the same as for the proposed threshold. However, smaller threshold (0.005) at the first glance slightly improves results in reducing probability of error in the periodic regime. However, it can causes misidentification of the chaotic regime so we do not recommend reduction of the threshold value below 0.01. Namely, recommended threshold is selected as maximal value of the detector response for chaotic regime and its reducing can produce errors in detection of chaos for high signal to noise ratio environments.

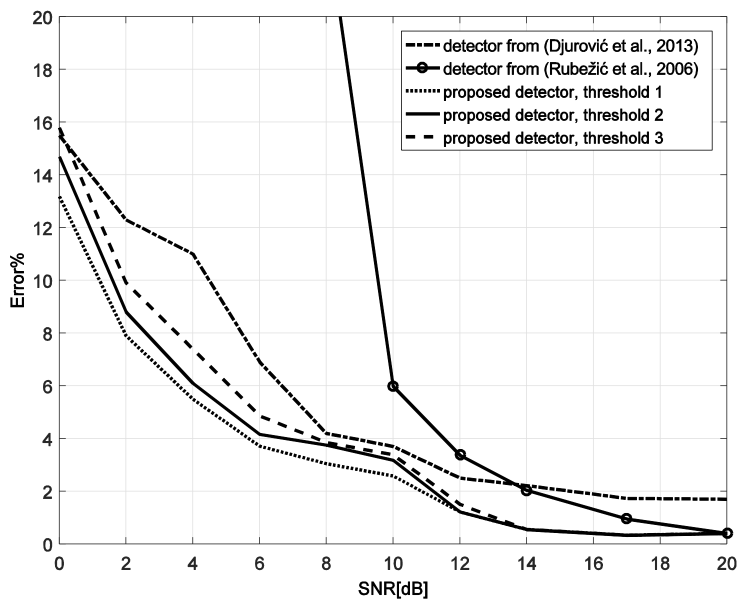

Fig. 7. Probability of error detection (the percentage of samples from the periodic regime which are incorrectly declared chaotic) with the proposed detector and with the algorithms from (Rubežić et al., 2006), (Djurović et al., 2013). The proposed detector: dotted line – threshold 1 (0.005), solid line – recommended threshold (0.01), dashed line – threshold 3 (0.015). Solid line with circles - the detector from (Rubežić et al., 2006). Dashed-dotted line - the detector from (Djurović et al., 2013).

VII. Conclusion

The AWC-based algorithm for detection of current states (periodic or chaotic) of oscillatory circuits is proposed. It enables chaos detection from a single time series, independent on the system nature and without apriory knowledge on the systems. Its effectiveness is demonstrated on the chaotic oscillatory circuits and other chaotic systems. With respect to the windowed (short-time) Fourier transform based detectors, this technique exhibits more accurate results in the noisy environment due to averaging of the WT coefficient values. Additionally, the proposed algorithm accurately detects short intervals of chaotic or periodic behavior in contrast to a DFA-based detector. In order to produce detection of chaos in the considered instant we consider signal in the narrow window with reasonable calculation complexity. Therefore, it is suitable for real-time detection with delay that is about half of the window width (about only 30 samples).

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