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The Development of Geometry in Ancient Greece

Geometry, from the Greek "geometrein," means "earth measuring." Though the word is Greek, we know geometry was studied by civilizations as old as ancient Egypt and Babylon. The Egyptians, for example, had discovered a formula for the volume of a pyramidal frustrum, and the Israelites had approximated pi to be about three (1 Kings 7:23). However, at this time, geometry was predominantly studied for practical purposes. But the ancient Greeks studied geometry for its own sake. They pioneered a more scientific study of geometry which focused on abstract principles and arriving at conclusions or formulas by logical deduction rather than measurement or trial and error. In this paper, we will learn about a few of the key mathematicians who helped geometry significantly develop, namely Thales, Pythagoras, Euclid, Archimedes, and Hypatia.

The first great geometer that this paper will cover is Thales of Miletus, who, according to most manuscripts, is credited with beginning Greek geometry in the 600's BC. He introduced proofs in geometry and proposed axioms such as the following: A circle is bisected by any of its diameters, the base angles of an isosceles triangle are equal, opposite angles are equal, a triangle inscribed in a semi-circle has a right angle on the arc of the semi-circle, and two triangles with one congruent side and two congruent angles are themselves congruent.

Thales' pupil, Pythagoras, known for the famous "Pythagorean Theorem," made significant developments in geometry as well. He is credited for the following discoveries: the sum of a triangle's interior angles equals two right angles, the sum of a triangle's exterior angles equals four right angles, the sum of an *n*-sided polygon's interior angles equals $2n$ - 4 right angles, the sum of a polygon's exterior angles equals four right angles, and finally, the Pythagorean Theorem, which proved that the squared length of a right triangle's hypotenuse is the sum of the squared length of the two other sides.

Another significant geometer, Euclid, continuing along the pattern of geometric proofs by logical reasoning, published his famous thirteen volume series entitled, "The Elements." In this work, Euclid set out to propose and prove geometric statements without any methods of empirical measurement. The Elements begins with listing out axioms which were statements Euclid claimed to be self-evident, intuitive truths in geometry.

Here are some of the axioms: Things which are equal to the same thing are equal to one another, if equals are added to equals, the wholes are equal, if equals are subtracted from equals, the remainders are equal, things which coincide with one another are equal to one another, and the whole is greater than the part. In addition to axioms, Euclid also defined some geometric terms: A point is that which has no part, a line is a breadthless length, the ends of a line are points, a straight line is a line which lies evenly with the points on itself, and a surface is that which has length and breadth only. Next, Euclid assumed five postulates: A straight line can be drawn from any point to any point, an infinite line can be extended from any finite line, a circle may be described with a center and a radius, all right angles equal one another, and two straight lines will eventually meet if one can draw a straight line through them that results in the interior angles summing to less than two right angles.

On the foundation of these axioms, definitions, and postulates, Euclid began proving geometric propositions: To construct an equilateral triangle on a given finite straight line, to place a straight line equal to a given straight line with one end at a given point, to bisect a given angle, to bisect a finite straight line, to draw a straight line through a given point parallel to a given straight line, etc. Though modern mathematicians are unsatisfied with Euclid's lack of rigor in defining terms and his geometry being limited only to "Euclidean planes," his Elements are frequently taught in schools, and his methods of logical deduction from self-evident truths were an important contribution to geometry and all of mathematics.

Euclid's Elements even made an impact on President Abraham Lincoln, whose law partner, Billy Herndon, said this: "He studied and nearly mastered the six-books of Euclid since he was a member of Congress. He began a course of rigid mental discipline with the intent to improve his faculties, especially his powers of logic and language." One may wonder if Euclid's first axiom of equality impacted Lincoln's view on the moral cruelty of slavery.

While Euclid is famous for his propositions, the next geometer, Archimedes, is famous for his formulas. Archimedes devised many formulas for calculating area and volume, including the volume of a sphere being equal to two thirds the volume of a cylinder of the sphere's height and radius. Archimedes is also famous for his approximation of pi. By inscribing a circle in an n-sided polygon, and inscribing a smaller n-sided polygon inside the circle, he was able to find the areas of the n-sided polygons, and showed that the value of the circle's area was between the polygons' areas. He then increased the value of n to make the area of the polygons closer and closer to the circle's area, since a circle is said to have infinite sides. The idea of using an upper and lower bound in this way allowed Archimedes to approximate pi between 223/71 and 22/7 (or between about 3.1408 and 3.1429).

The last geometer we will cover is Hypatia. Hypatia, influenced by Euclid's Elements, began studying mathematics at an early age. She eventually became a leading mathematician in her time, and is regarded as the first female to make significant contributions to mathematics.

Some of her main contributions to geometry lie in her edits of Apollonius' work in conic sections.

Each of these geometers made significant contributions to the geometry we have today. The Pythagorean Theorem is used commonly across the world. Geometer David Hilbert designed his own set of axioms for non-Euclidean geometry based on the work of Euclid. Archimedes' 22/7 is used as a practical approximation of pi. Based on the works of Archimedes and others, geometer Eratosthenes was able to approximate the earth's circumference to extreme accuracy, epitomizing the etymology of the term geometry. Even some of the most abstract discoveries by these mathematicians have found practical applications in technology today, which would have been no surprise to Pythagors, who is known to have said, "There is geometry in the humming of the strings, there is music in the spheres."

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