Homework 4
APPM 4490/5490 Theory of Machine Learning, Spring 2024

Due date: Friday, Feb 16 '24, before 11 AM, via paper or via Gradescope **Instructor**: Prof. Becker
Revision date: 2/12/2024

Theme: VC dimension

Instructions Collaboration with your fellow students is OK and in fact recommended, although direct copying is not allowed. The internet is allowed for basic tasks (e.g., looking up definitions on wikipedia) but it is not permissible to search for proofs or to *post* requests for help on forums such as `http://math.stackexchange.com/` or to look at solution manuals. Please write down the names of the students that you worked with.

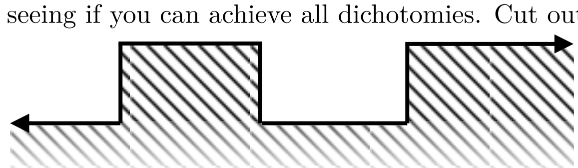
An arbitrary subset of these questions will be graded.

Reading You are responsible for reading chapter 6 of "Understanding Machine Learning" by Shai Shalev-Shwartz and Shai Ben-David (2014, Cambridge University Press). Note: you can buy the book for about $45 on Amazon (or less for an e-book), and the authors host a free PDF copy on their website (but note that this PDF has different page numbers).

Note For VC dimension calculations, you should have a convincing argument, but you don't need to spend a very long time writing out a rigorous proof (e.g., when appropriate, a "proof by pictures" or a "I checked all possibilities and it works" may suffice). When in doubt, ask the instructor at office hours.

Problem 1: Exercise 3.15 in Mohri et al: let $I_\alpha = [\alpha, \alpha+1] \cup [\alpha+2, \infty)$, and the classifier $h_\alpha(x) = 1_{I_\alpha}(x)$ the indicator function associated with this set. Let $\mathcal{H}$ be the set of all such classifiers h_α. What is the VC dimension of $\mathcal{H}$?

Hint: For fun (and because it is helpful), for a set C, you can check if it is shattered by this $\mathcal{H}$ by sliding a template over the points and seeing if you can achieve all dichotomies. Cut out a paper template in the shape of the h, i.e.,



Problem 2: Exercise 6.2a in Shalev-Shwartz and Ben-David: Let the domain $\mathcal{X}$ be a finite set with $|\mathcal{X}| = n$, and fix any integer $k \in [0, n]$. Let $\mathcal{H}$ be the set of all binary functions on $\mathcal{X}$ that give an output of 1 for exactly k of the inputs. What is the VC dimension of $\mathcal{H}$?

Note: Exercise 6.2b is not required, but if you want to do it for extra practice, define $\mathcal{H}$ to be the set of all binary functions that gives an output of 1 for at most k of the inputs (in the textbook, ignore the confusing "or" statement in the definition of $\mathcal{H}$).