6a Big-O notation

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$x \to \infty$ case

"Big O"

$f = O(g)$ means asymptotically, f grows no faster than g

$f = o(g)$ means asymptotically, f grows slower than $\alpha \cdot g$ $\forall \alpha \in \mathbb{R}^+$

"little o"

Formally: f, g positive real-valued functions ($f, g : \mathbb{R} \to \mathbb{R}^+$)

$f = O(g)$ means $\exists x_0 \geq 0, \exists \alpha \geq 0$ st $\forall x > x_0$, $f(x) \leq \alpha g(x)$

$f = o(g)$ means $\forall \alpha > 0$, $\exists x_0$ st $\forall x > x_0$, $f(x) \leq \alpha g(x)$

So... $\underbrace{f = o(g)}_{\text{stronger}} \Rightarrow f = O(g)$

$f = \Omega(g)$ means $g = O(f)$

$f = \omega(g)$ means $g = o(f)$

$f = \Theta(g)$ means $f = O(g)$ and $g = O(f)$

$f \sim g$ is an even stronger version of this

$f = \tilde{O}(g)$ means $\exists k \in \mathbb{N}$ st. $f = O(g \log^k(g))$

This is all setup for "computer science" where we ask about, say, $\underbrace{\text{runtime}}_{\text{"}f(x)\text{"}}$ as a function of $\underbrace{\text{problem size}}_{\text{"}x\text{"}}$, $x \to \infty$

Ex: $n = O(n^2)$

$\log(n) = O(n)$, $n^2 \log(n) = O(n^3)$

$= \tilde{O}(n^2)$

$x \to 0$ case

But analogous notation exists for "analysis" where "x" gets small

i.e. $f(h)$, $h \to 0$

Ex: $h^2 = O(h)$ (as $h \to 0$)

NOT TRUE in $h \to \infty$ case