Homework 1
MATH/STAT 4540/5540 Spr 2022 Time Series

Due date: Friday, Jan 28, before midnight, on Canvas/Gradescope **Instructor**: Prof. Becker
Theme: Intro to stationary processes

Instructions Collaboration with your fellow students is OK and in fact recommended, although direct copying is not allowed. The internet is allowed for basic tasks (e.g., looking up definitions on wikipedia) but it is not permissible to search for proofs or to *post* requests for help on forums such as `http://math.stackexchange.com/` or to look at solution manuals. Please write down the names of the students that you worked with.

An arbitrary subset of these questions will be graded.

Reading You are responsible for chapters 1 in our textbook (Brockwell and Davis, "Intro to Time Series and Forecasting", 3rd ed).

Problem 1: For this problem, **do not use the internet at all**. Give one explicit example of a stochastic process $\{X_t : t \in \mathbb{Z}\}$ that is

a) weakly stationary but not strictly stationary.

b) strictly stationary but not weakly stationary.

Problem 2: Consider the continuous-time process

$$X(t) = U \sin t + V \cos t, \qquad t \in \mathbb{R}$$

where U and V are independent random variables, each with mean zero and variance 1.

a) What do the realizations of this process look like? Sketch three different sample paths on $[0, 2\pi]$, and explain observed behavior.

b) Is $\{X(t) : t \in \mathbb{R}\}$ a weakly stationary process? Justify your answer. *Hint: you can use trig identities you find in a book or on the internet.*

c) (Optional, more difficult) Is $\{X(t) : t \in \mathbb{R}\}$ a strictly stationary process? Justify your answer.

Problem 3: **Pseudo AR(1) model** Suppose that $Y_t = \phi \cdot Y_{t-1} + Z_t$, for $t \in \mathbb{N} \stackrel{\text{def}}{=} \{0, 1, 2, \ldots\}$, where:

- $Y_0 = 0$
- $|\phi| < 1$
- $\{Z_t\} \sim \text{WN}(0, \sigma^2)$
- $\forall t \in \mathbb{N}$, Z_t is uncorrelated with Y_s for all $s < t$

a) Explain why this is not a *standard* AR(1) model

b) Determine $\mathbb{E}[Y_t]$ for each $t \in \mathbb{N}$.

c) Determine $\text{Var}[Y_t]$ for every $t \in \mathbb{N}$.

d) Is $\{Y_t\}$ weakly stationary? Explain!

e) (**Graduate students only**) Determine $\text{Cov}(Y_{t-h}, Y_t)$ for each $0 \le h \le t$.

Problem 4: (**Graduate students only**) Suppose $\{X(t) : t \in \mathbb{Z}\}$ is a sequence of iid random variables with mean zero and variance σ^2. Define the weighted partial sum process

$$S(n) = \sum_{t=1}^{n} w_t X(t)$$

for some real-valued weights $w_1, w_2, \ldots$. Under what conditions on $\{w_t\}$ will $\{S(n)\}$ be weakly stationary?