2. Stationarity and Autocorrelation

Tuesday, January 11, 2022 10:14 PM

See §1.4 and §2 Brockwell + Davis

Stationarity

Def If $\{X_t\}$ (alternate notation: $\{X(t)\}$) is a time series with $\mathbb{E} X_t^2 < \infty \ \forall t$, then define

the mean function to be $\mu_X(t) := \mathbb{E} X_t$

or just $\mu(t)$ is X is clear from context

and the covariance function $\gamma_X(s,t) = \text{Cov}[X_s, X_t]$

We want time series that change in time, but often we don't want statistics to change in time

Def A t.s. $\{X_t\}$ is weakly stationary (aka wide-sense stationary)

if ① $\mu(t) = \mu$, i.e., a constant mean (no trends allowed)

② $\gamma(t+h, t) = \gamma(s+h, s)$ for all t, s

... only depends on relative difference

so $= \gamma(0+h, 0)$

and we just write $\boxed{\gamma(h)}$ "autocovariance function" ACVF

Remark So what is "non" weak stationarity?

"Strict stationarity" means $\forall n, \forall h,$

$$(X_1, \ldots, X_n) \sim (X_{1+h}, \ldots, X_{n+h})$$

have same distribution

in contrast, weak stationarity means the distributions only need to have the same 1st two moments ... and often that's all we need

From now on, we say "stationary" to mean "weakly stationary"

Def If $\{X_t\}$ is stationary,

h is called the "lag"

the ACVF is $\gamma_X(h) = \gamma_X(0, h) = \text{Cov}[X_0, X_h]$

and the autocorrelation function

ACF is $\rho_X(h) = \dfrac{\gamma_X(h)}{\gamma_X(0)} = Cor[X_0, X_h]$

$\gamma_X(0) = Var[X_0] = Var[X_t]$

i.e.,

ACF is ACVF normalized to value 1 at lag 0

also unitless

ACF, ACVF are basically the same info

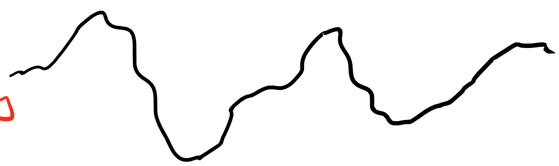
Recall: $Cov[X,Y] = \mathbb{E}[X \cdot Y] - \mathbb{E}[X] \cdot \mathbb{E}[Y]$

$Cor[X,Y] = \dfrac{Cov[X,Y]}{\sqrt{Var[X] \cdot Var[Y]}}$ aka Pearson correlation

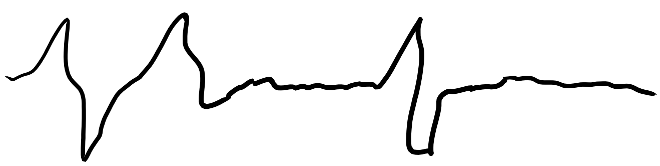
Ex (graphically)

Stationary means $\{X_t\}$ acts similarly in every window

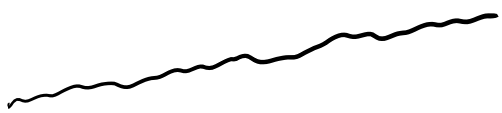
stationary



not stationary



not stationary



See PDF for examples of stationary t.s. and their ACF

ACFsAndSims