Quadratic majorization

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Setup

Hessian $H_{p \times p}$

gradient at $\beta = 0$: $g_{0_{p \times 1}}$

penalty parameter λ

starting value $\beta_0 \neq 0$

Problem

minimize $g_0^T \beta + \frac{1}{2} \beta^T H \beta + \lambda ||\beta||_2$

Note for $x_0, x_1 > 0$,

$$\sqrt{x_1} \geq \sqrt{x_0} + \frac{x_1 - x_0}{2\sqrt{x_0}} = \frac{x_1}{\sqrt{x_0}} + \text{constant}$$

So minimizing $g_0^T \beta + \frac{1}{2} \beta^T H \beta + \frac{\lambda}{2} \cdot \frac{||\beta||_2^2}{||\beta_0||_2}$ drives the original problem toward a better value.

Thus we can solve

$$g_0 + \left(H + \frac{\lambda}{||\beta_0||_2} I \right) \beta = 0$$

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$$\beta_{new} = - \left(H + \frac{\lambda}{||\beta_0||_2} I \right)^{-1} g_0$$

Multinomial bound

$$Q(\eta) \geq const. + -l'(\eta_0) \cdot (\eta - \eta_0) + \frac{\tau}{2} (\eta - \eta_0)^T (\eta - \eta_0)$$

$$= const. + -(Y - \mu)^T X (\beta - \beta_0) + \frac{\tau}{2} (\beta - \beta_0)^T X^T X (\beta - \beta_0)$$

$$= const. + ()$$