# Google Colab Code 1 Notes

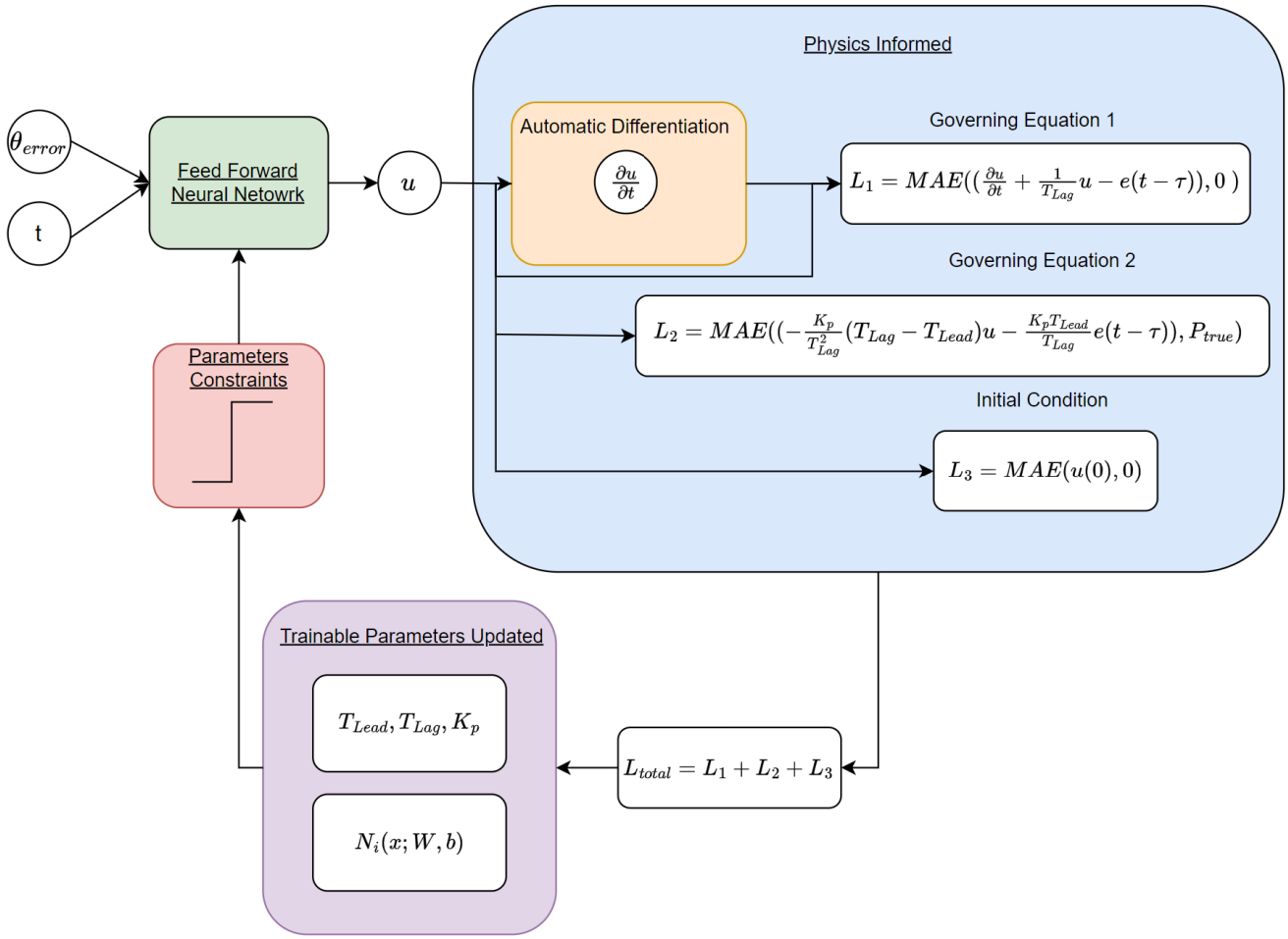
Code 1 Link:
https://colab.research.google.com/drive/1YemTI3xDJqHXdpAdvv4pxHG36A9k8WDh?usp=sharing

Intro: In this code the pilot model equations are taken from Thomas's thesis, make sure to take them from this bc the ones from the Mandal thesis are off by negative 1. These equations are for the pilot in which to keep in mind, the time delay $\tau$ is implicit and thus we are modeling the data as is without predicting the time delay -> aka we are only estimating $T_{Lag}, T_{Lead}, K_p$ . These equations are also without the Pade approximation thus $x$ is a 1x1.

$$\dot{x}(t) = -\frac{1}{T_{Lag}}x(t) + e(t-\tau)$$

$$p(t) = -\frac{K_p}{T_{Lag}^2}(T_{Lag} - T_{Lead})x(t) - \frac{K_p T_{Lead}}{T_{Lag}}e(t-\tau)$$

Architecture:




The feed forward Neural Neural network block is used to denote the Neural Network used in this work. The architecture of the network used is 2 hidden layers of 64 and 128 neurons respectively with Tanh() activation functions, Adam optimizer of learning rate of 0.0001, and 1001 epochs. Note for this formulation, the pilot parameters are defined as trainable parameters in the training loop, i.e. they are updated through each epoch just like the Neural Network weights and biases by the Adam optimizer. The automatic differentiation block allows the network to go from $x$ to $\dot{x}$ and this is done by Pytorch's automatic differentiation function torch.autograd(). Lastly the parameters constraints block was implemented to be able to saturate the McRuer model parameters within certain reasonable bounds as the training occured to reasonable min and max values as consistant with the literature.

Problem Formulation: If the standard relationship between the Neural Network input and output is,
$y = N_i(x; W, b)$ the relationship for the iPINN is $y = N_i(x; W, b, \theta)$ where $\theta$ can be denoted as any trainable parameters. For this setup the trainable parameters are the McRuer Pilot model parameters, $T_{Lead}, T_{Lag}, K_p$. The loss function for the iPINN will update the weights and biases of the Neural Network along with the pilot parameters.

The governing equations for the McRuer Pilot model are,

$\dot{x}(t) = -\frac{1}{T_{Lag}}x(t) + e(t-\tau)$

$p(t) = -\frac{K_p}{T_{Lag}^2}(T_{Lag} - T_{Lead})x(t) - \frac{K_p T_{Lead}}{T_{Lag}}e(t-\tau))$

$x(t=0) = 0$

The iPINN will utilize these governing equations as part of its loss function. The loss function, $L$, can be defined as,

$L_{total} = L_{Data} + L_{ODE} + L_{I.C.}$

Where $L_{Data}$ will be denoted as the residual of the pilot output, $L_{ODE}$ will be denoted as the residual of the differential equation and $L_{I.C.}$ will be denoted as the residual of the initial condition. The residuals can be calculated as,

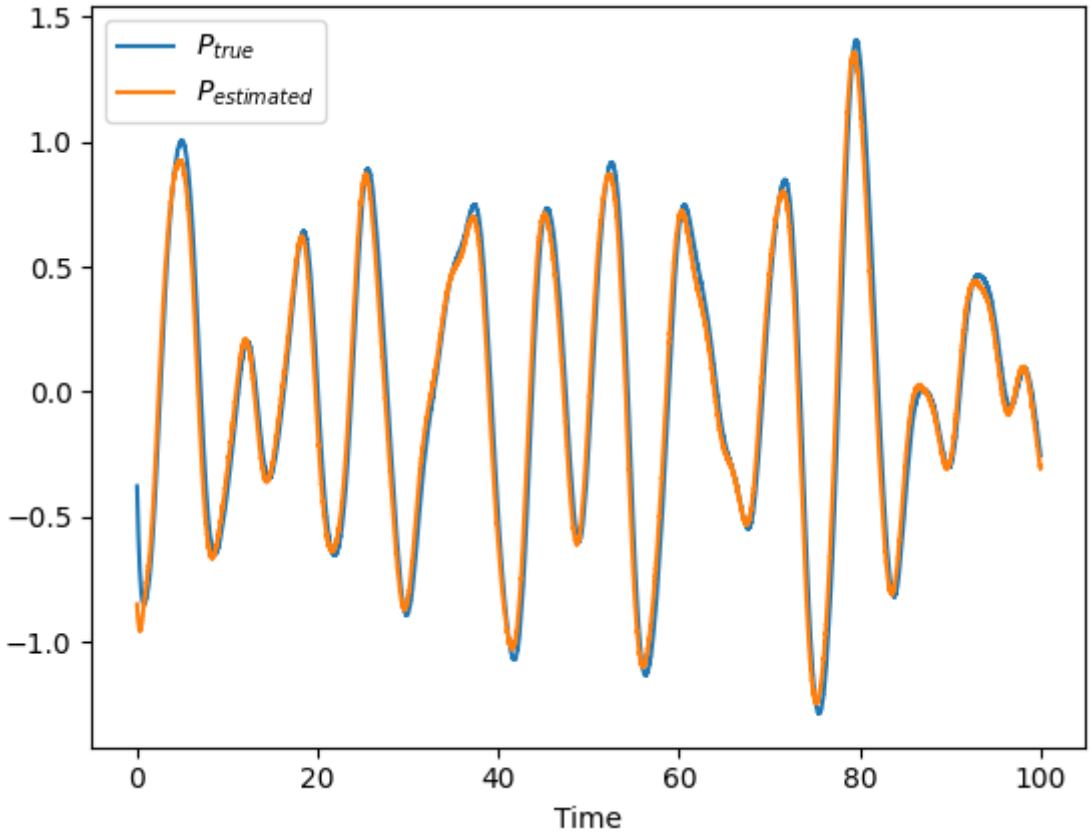
$L_{Data} = MAE((-\frac{K_p}{T_{Lag}^2}(T_{Lag} - T_{Lead})u - \frac{K_p T_{Lead}}{T_{Lag}}e(t-\tau)), P_{true})$

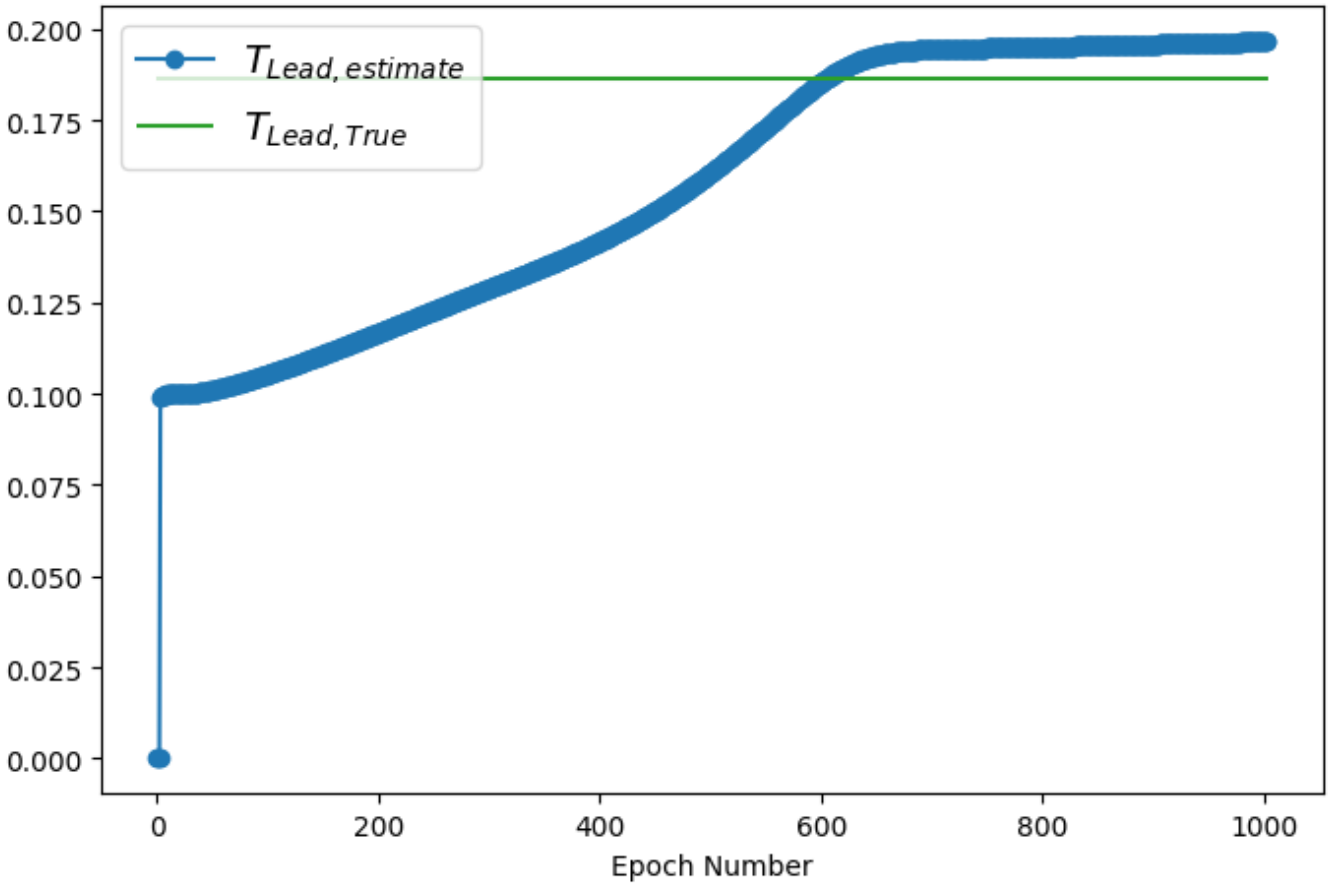
$L_{ODE} = MAE((-\frac{K_p}{T_{Lag}^2}(T_{Lag} - T_{Lead})u - \frac{K_p T_{Lead}}{T_{Lag}}e(t-\tau)), P_{true})$

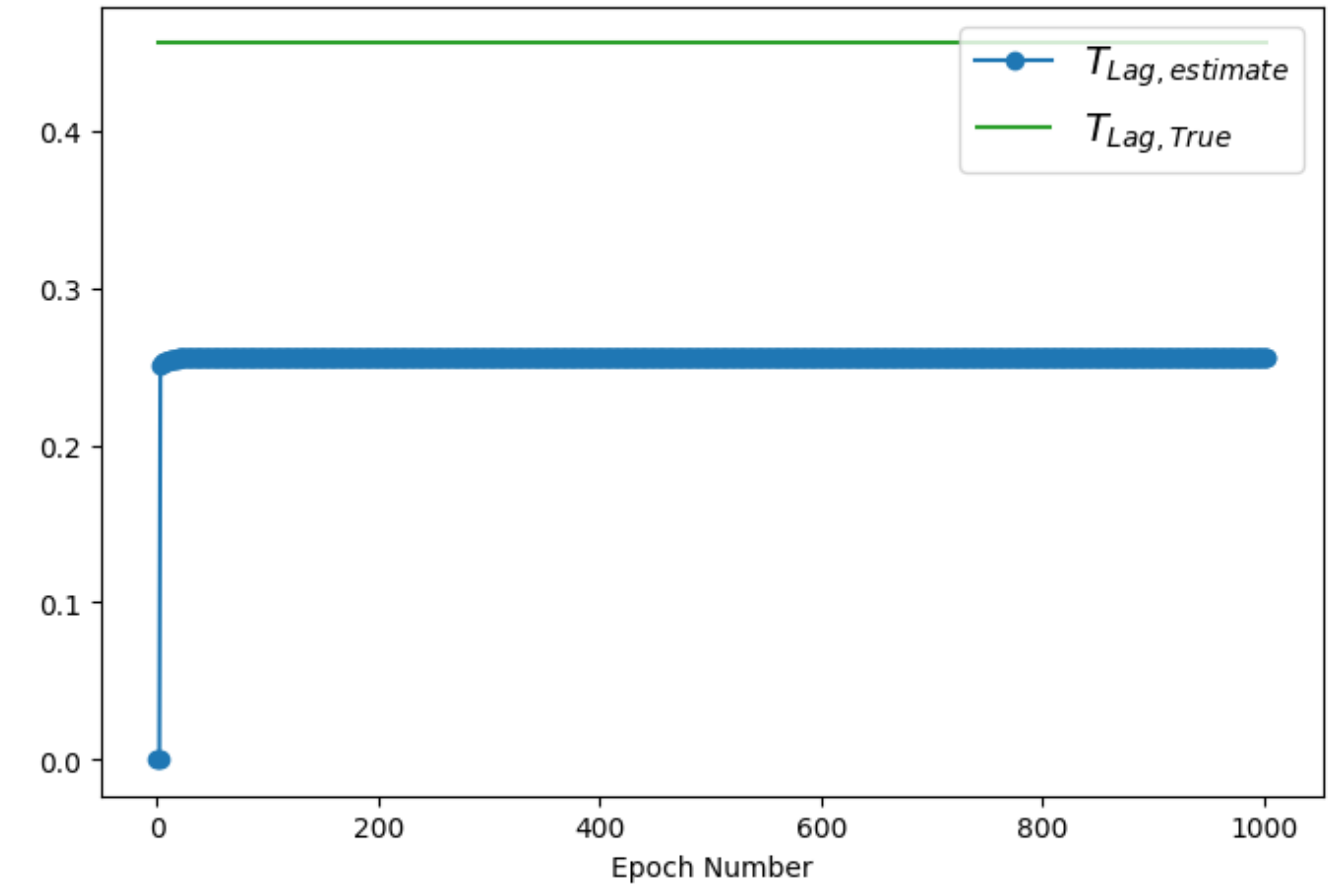
$L_{I.C.} = |u(t=0) - 0|$

Where MAE stands for the mean absolute error.

Results: From training and testing on 1 case




$T_{Lead, estimate}$
$T_{Lead, True}$
0.200
0.175
0.150
0.125
0.100
0.075
0.050
0.025
0.000
0
200
400
600
800
1000
Epoch Number


$T_{Lag, estimate}$
$T_{Lag, True}$
0.4
0.3
0.2
0.1
0.0
0
200
400
600
800
1000
Epoch Number

$K_{p, estimate}$
0.40
0.35

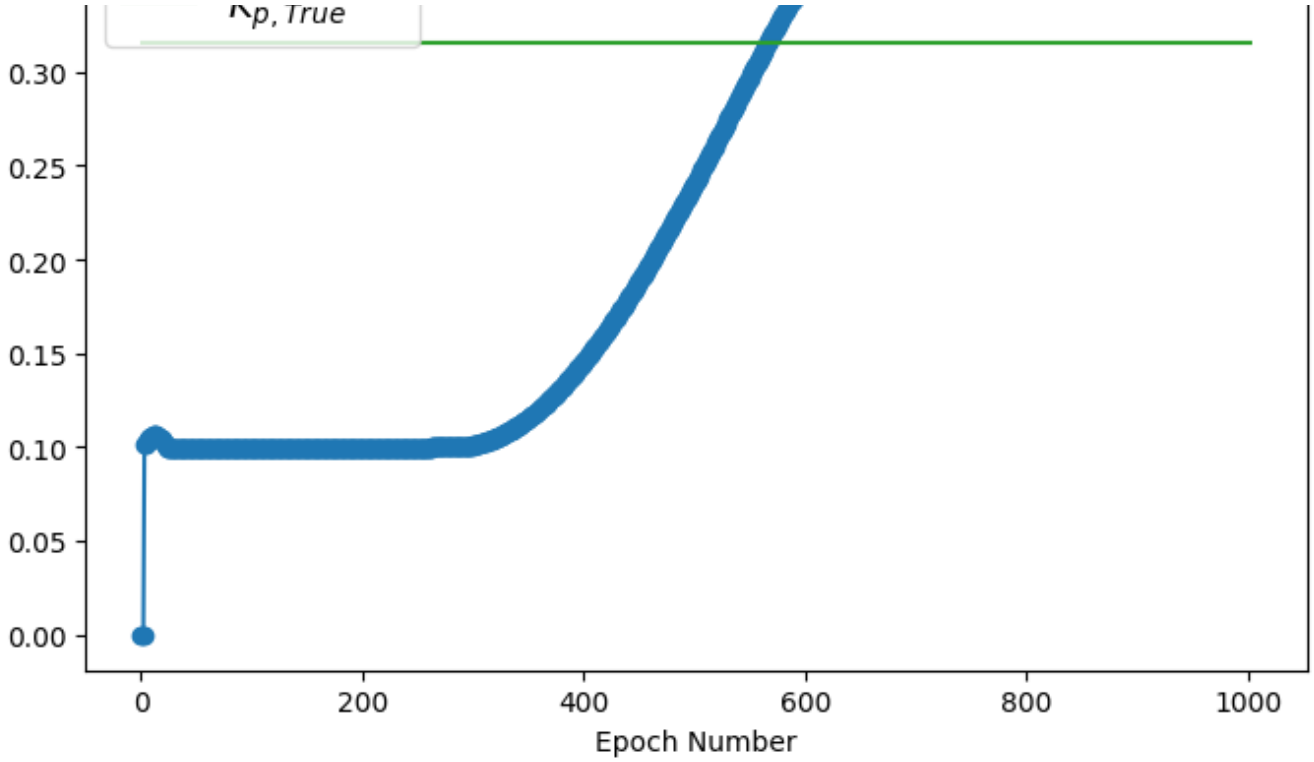
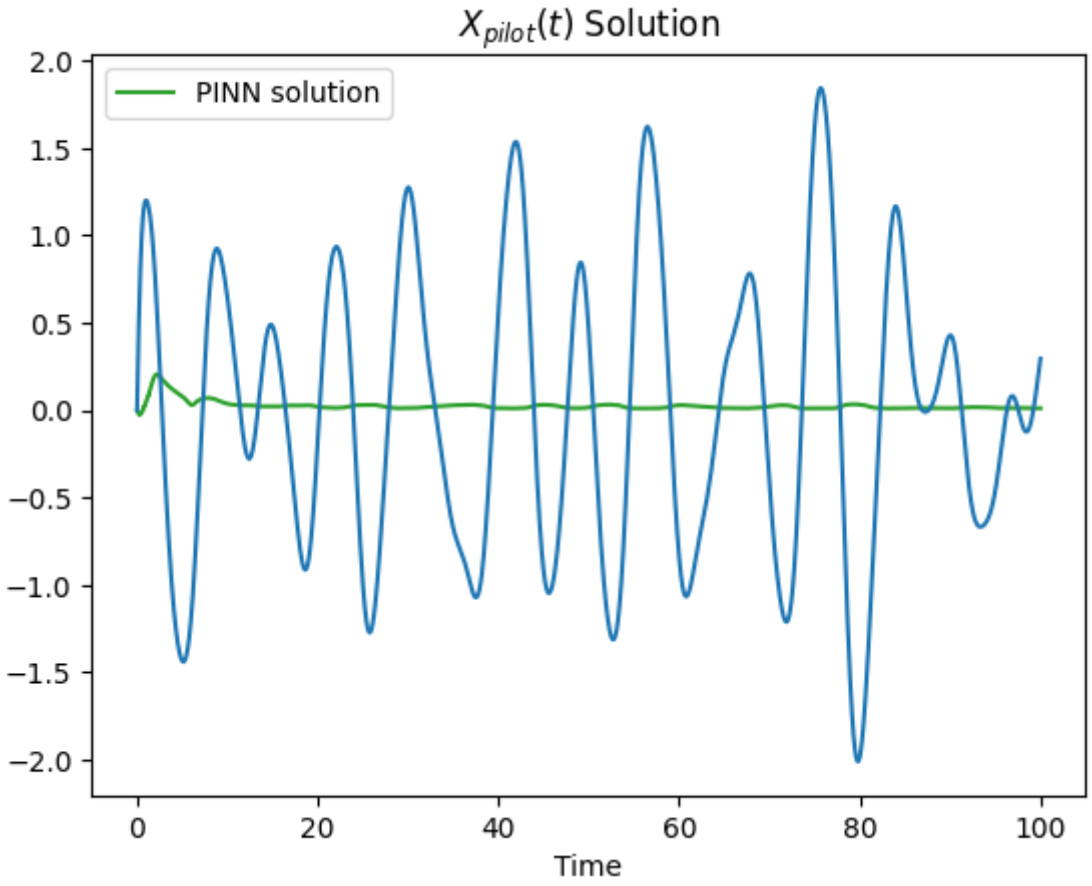



Table 1: Analysis Results from iPINN training

| | $p(t)$ | $T_{Lead}$ | $T_{Lag}$ | $K_p$ |
|---|---|---|---|---|
| $MAE$ | 0.0916 | 0.0103 | 0.1993 | 0.0629 |
| % difference | 6.69 | 5.52 | 4.377 | 19.97 |




Analysis: With a lack of a forcing function in the loss function for the parameters to converge to their true values, e.g. $L_4 = MAE(\hat{K}_p, K_{p_{true}})$ the Network simply just learns a combination of the parameters that results in a minimized loss function. The result of the parameters constraints can be seen in the parameters training figures where the parameters are initialized at $1 * 10^{-4}$ but after the first epoch they jump to the lower bound of the constraint and train from there. Even though the parameters may have trouble converging to the true values the overall response of the system, the pilot output does converge very well. Looking at weighting each part of the loss function can also be explored as well.

The drawbacks for this method is that now the pilot parameters are a part of the NN thus if we were to use what looks like a well trained NN on a "different pilot" aka different parameters used in simulation we would find that the pilot output will not estimate well bc it is using the previously learned parameters. So we can only say the NN learned the behavior of this 1 pilot. Lastly its interesting that the pilot output can match but the $x(t)$ does not match at all.

The true parameters for this run are:
$T_{Lead} = 0.1862$
$T_{Lag} = 0.4533$
$\tau = 0.0.3152$
The parameters were clamped for this way such that,
$0.1 < T_{Lead} < 3$

$0.25 < T_{Lag} < 0.65$

$0.1 < K_P < 0.5$

We constrain the parameters consistent with the literature's approximate values and to allow the neural network to learn values that are realistic (see below).

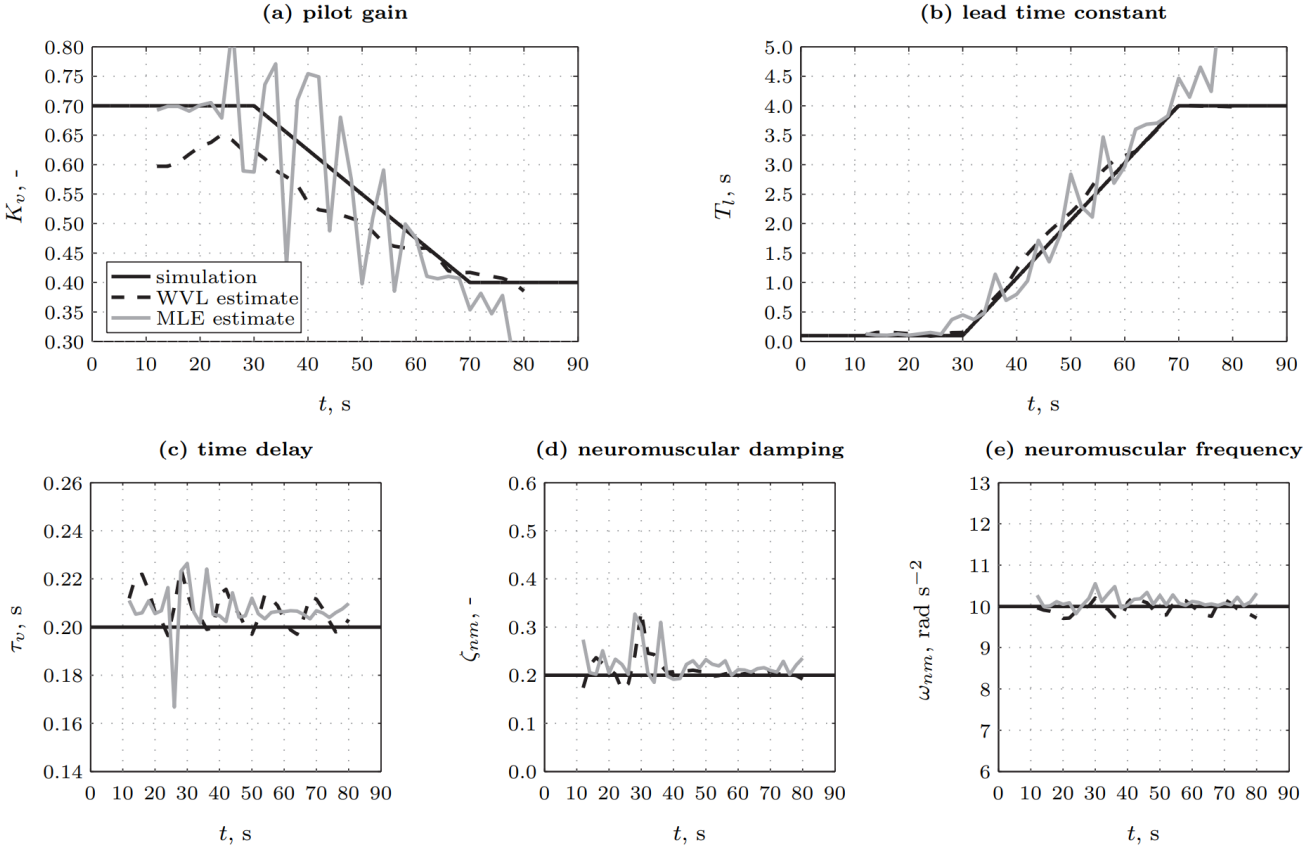



Figure 14: Estimated time-varying pilot model parameters from wavelet and MLE methods ($P_n$=0).

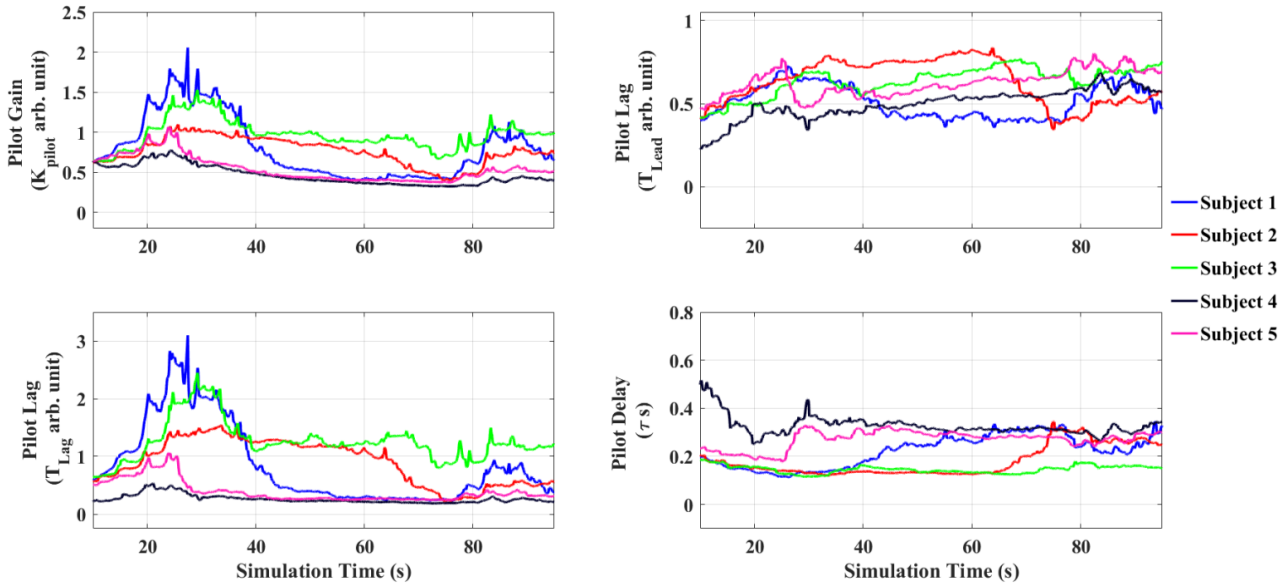



FIGURE 3.18: Comparison of estimated parameters for 5 different subjects for a single trial under nominal condition.

*Table 5.2* Time-invariant pilot model parameters used to produce the results shown in cases 1 and 3. These values were derived from Monte Carlo simulations and retrieved from similar studies of the McRuer model [5] [6].

| Parameter | Value |
|---|---|
| $K_p$ | 0.54 |
| $T_{Lead}$ | 0.32 |
| $T_{Lag}$ | 0.4 |
| $\tau$ | 0.25s |

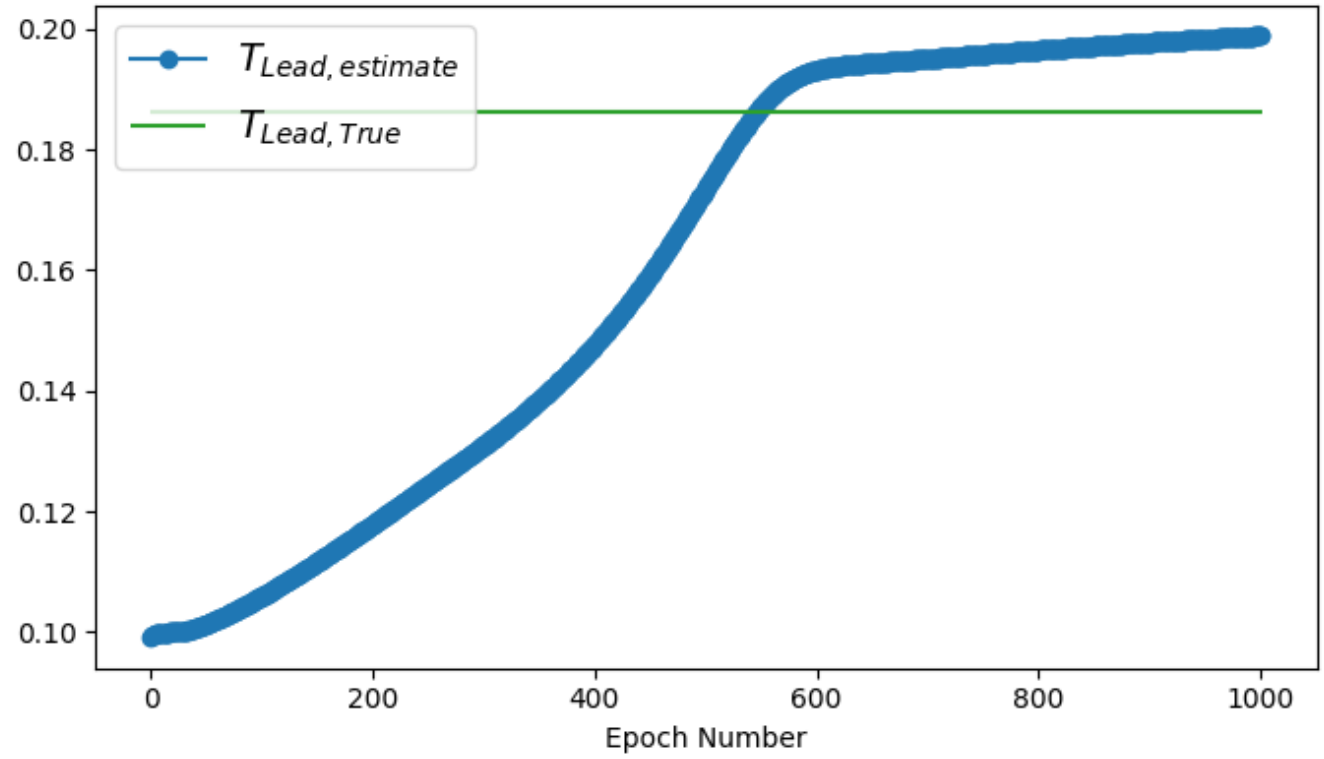
To see the estimation with a larger range of saturation I adjusted the bounds to:

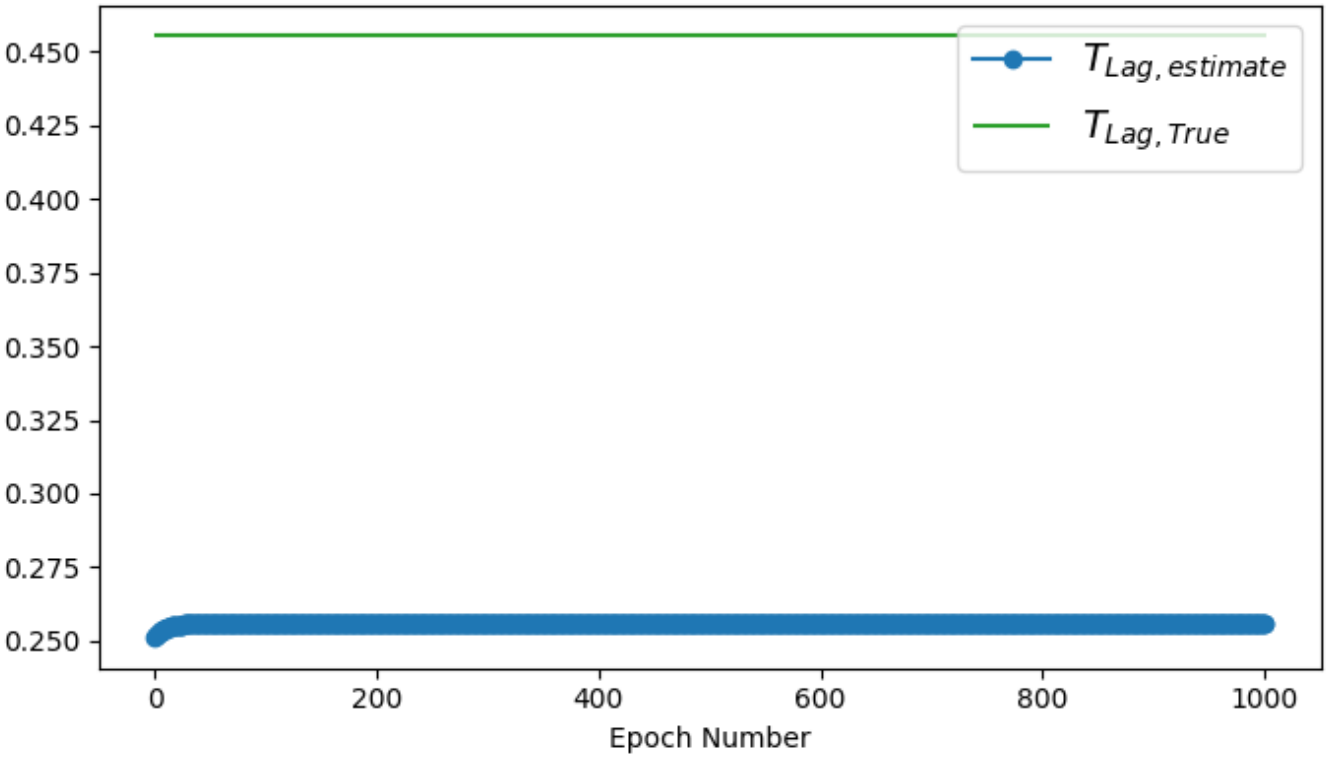
$0.1 < T_{Lead} < 3$

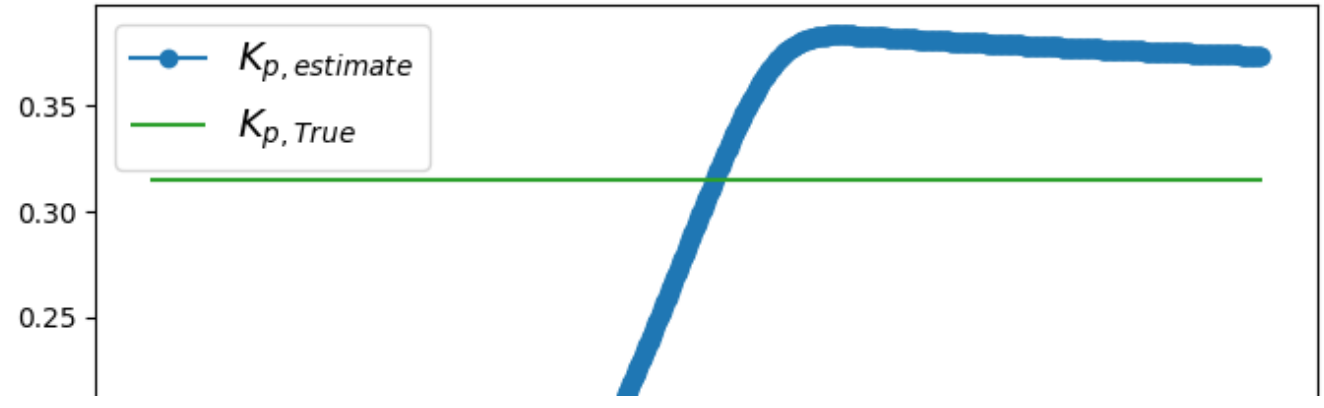
$0.25 < T_{Lag} < 3$

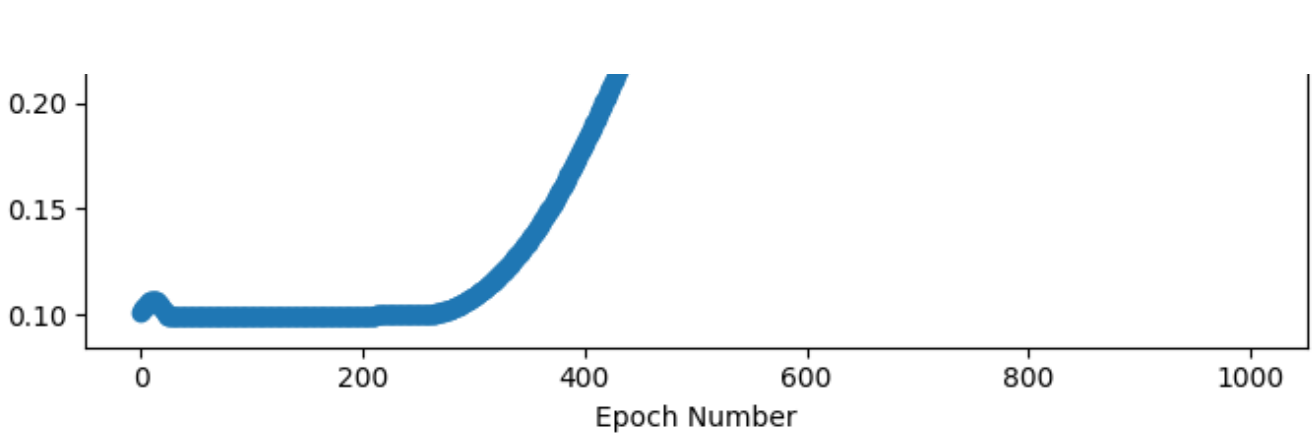
$0.1 < K_P < 2$

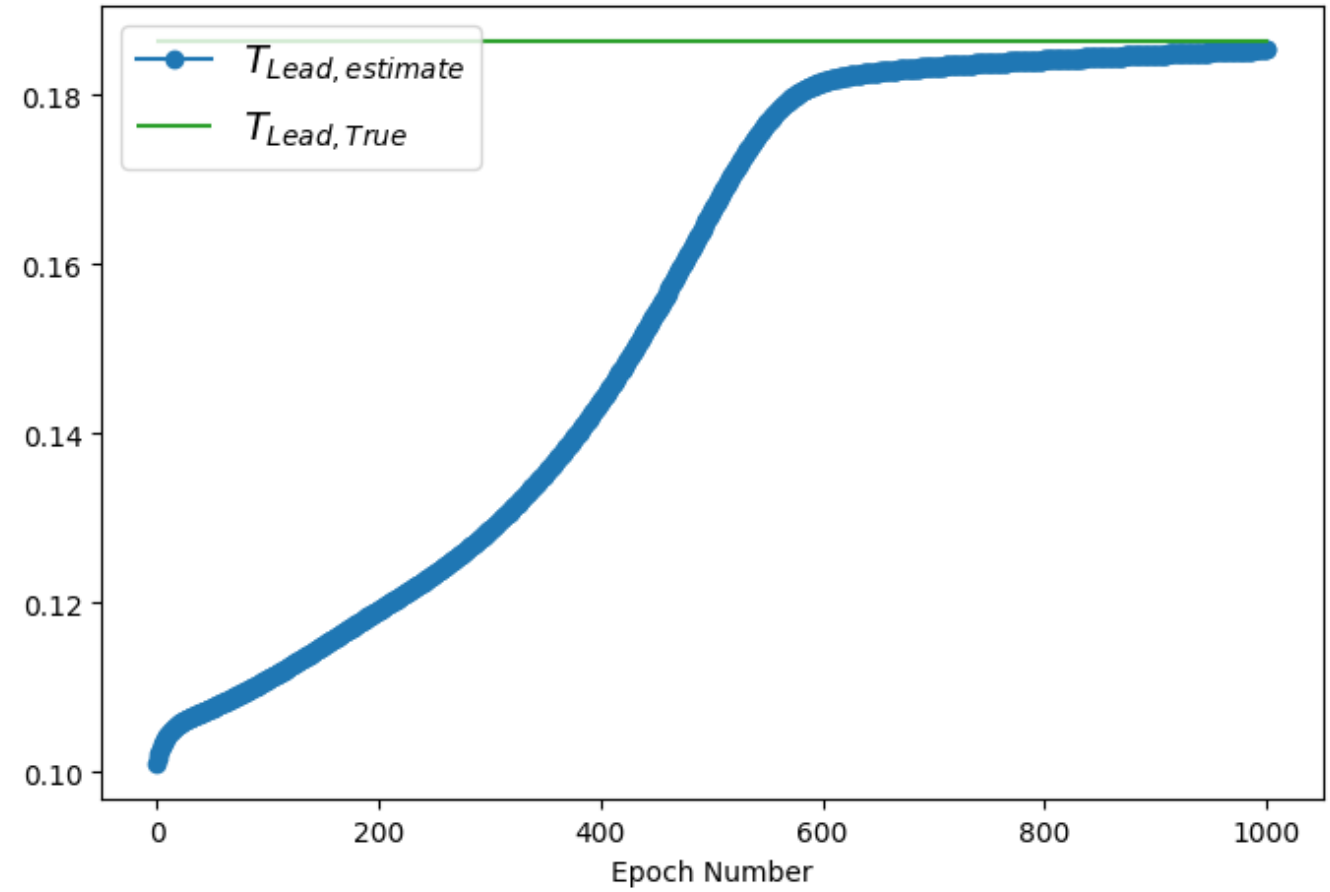
the Results can be seen below:

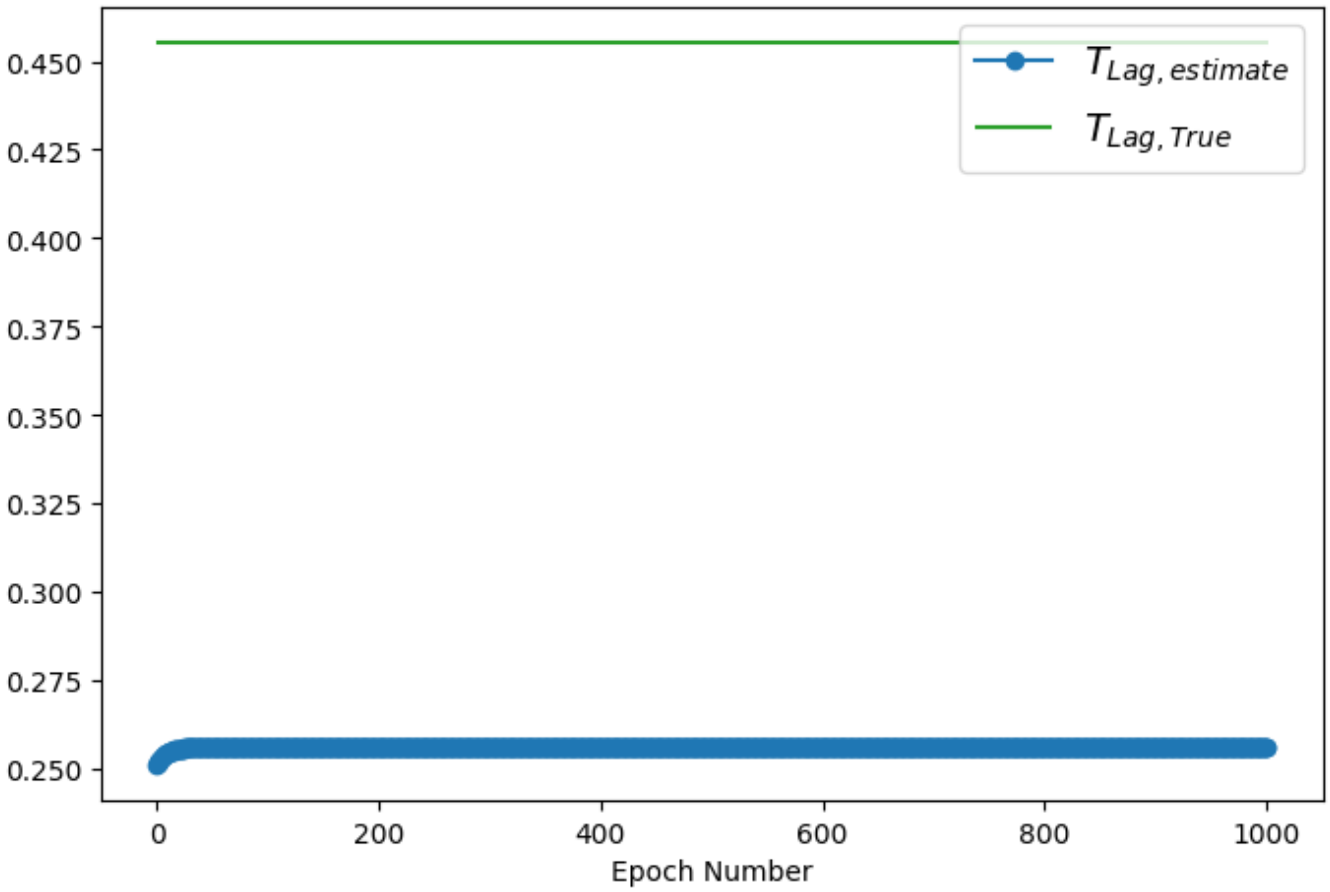

$T_{Lead, estimate}$
$T_{Lead, True}$
0.20
0.18
0.16
0.14
0.12
0.10
0
200
400
600
800
1000
Epoch Number


$T_{Lag, estimate}$
$T_{Lag, True}$
0.450
0.425
0.400
0.375
0.350
0.325
0.300
0.275
0.250
0
200
400
600
800
1000
Epoch Number

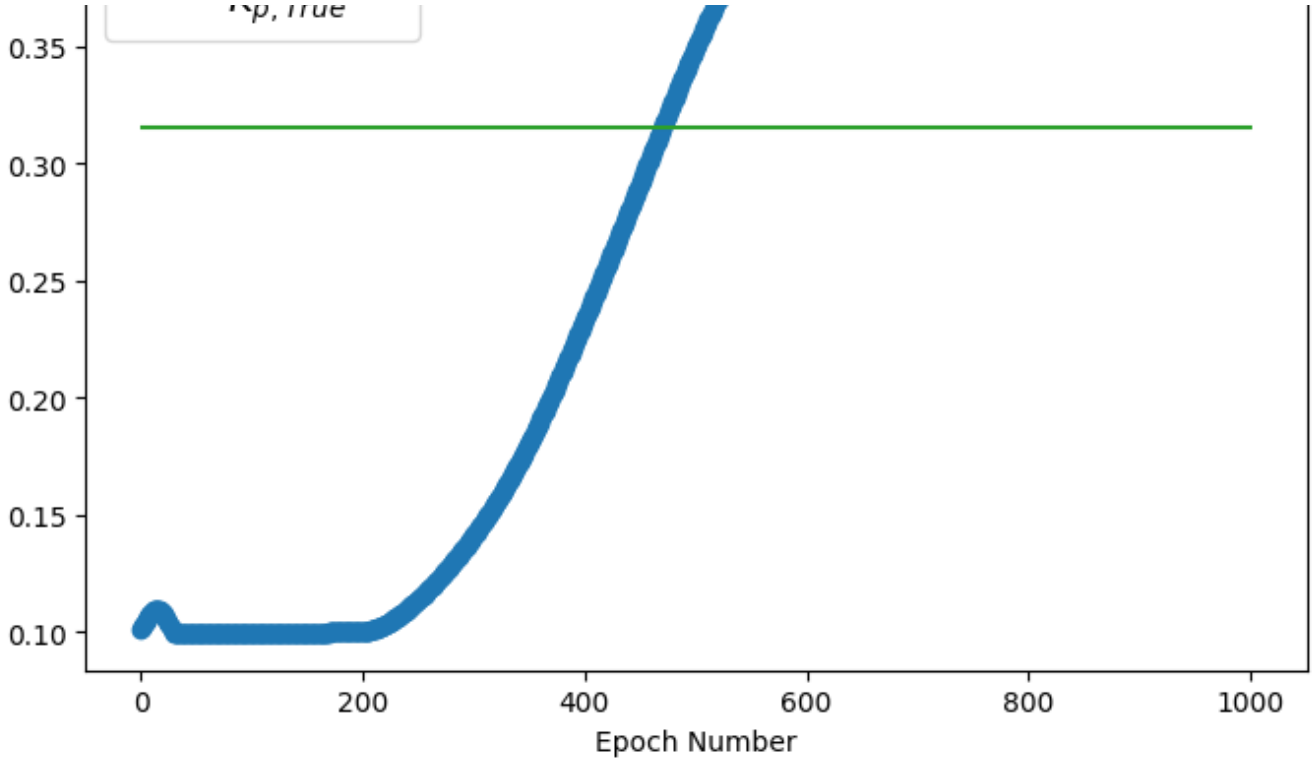

$K_{p, estimate}$
$K_{p, True}$
0.35
0.30
0.25

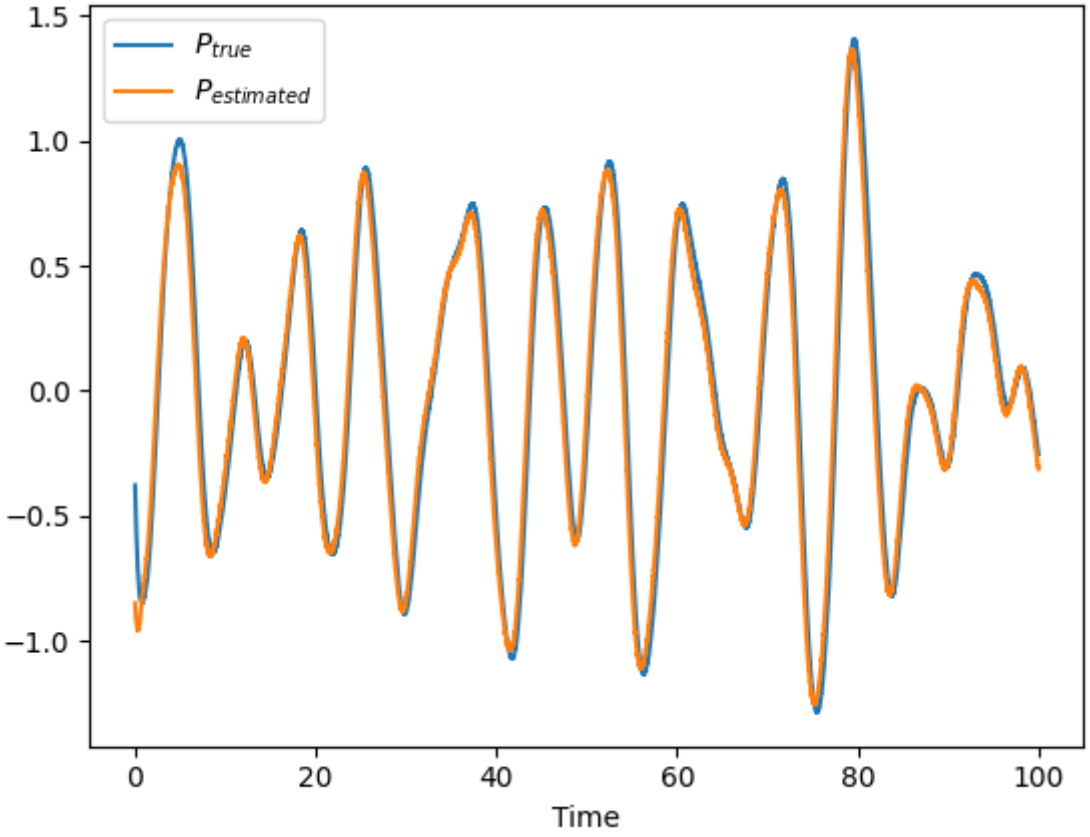

0.20
0.15
0.10
0
200
400
600
800
1000
Epoch Number

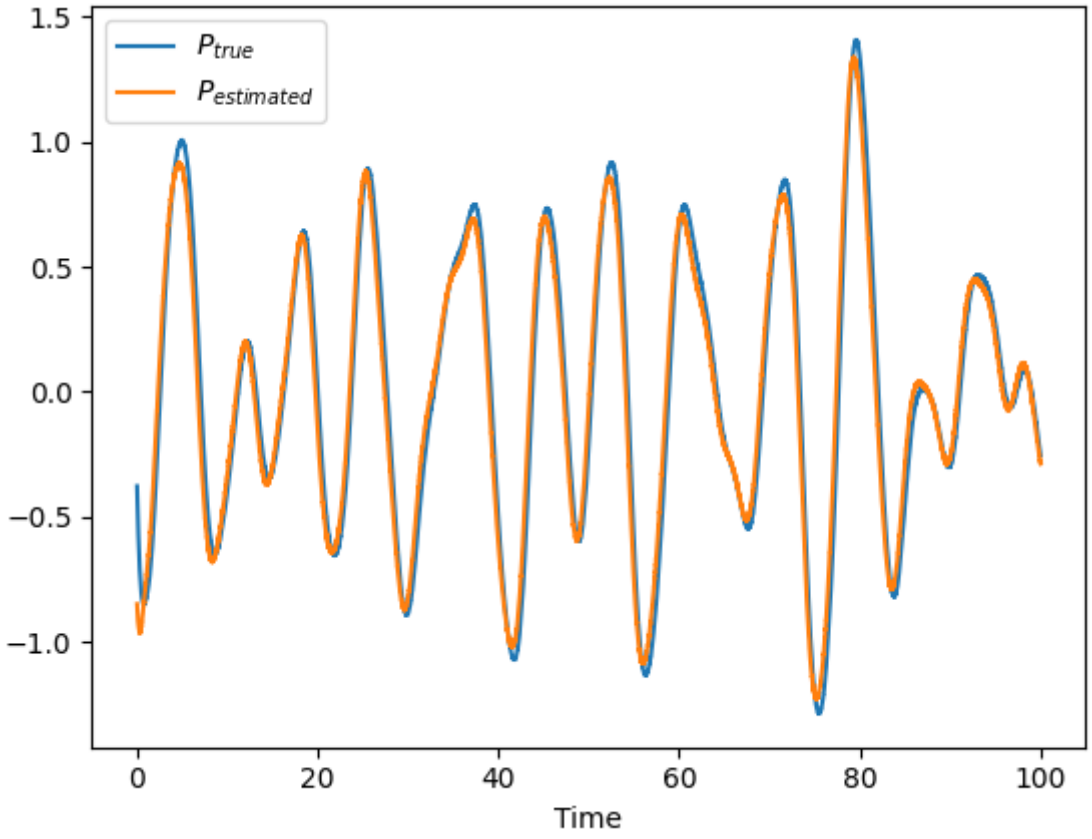

$T_{Lead, estimate}$
$T_{Lead, True}$
0.18
0.16
0.14
0.12
0.10
0
200
400
600
800
1000
Epoch Number


$T_{Lag, estimate}$
$T_{Lag, True}$
0.450
0.425
0.400
0.375
0.350
0.325
0.300
0.275
0.250
0
200
400
600
800
1000
Epoch Number

$K_{p, estimate}$
0.40


0.35
0.30
0.25
0.20
0.15
0.10
0
200
400
600
800
1000
Epoch Number


$P_{true}$
$P_{estimated}$
1.5
1.0
0.5
0.0
−0.5
−1.0
0
20
40
60
80
100
Time

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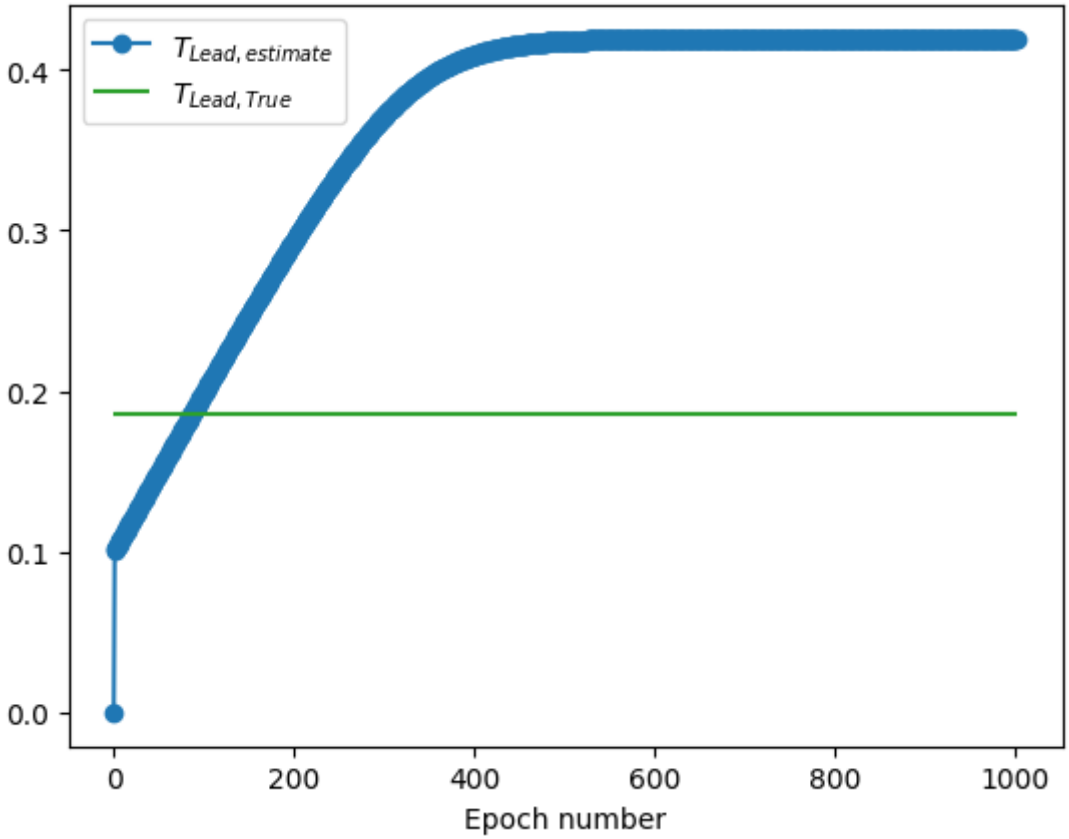
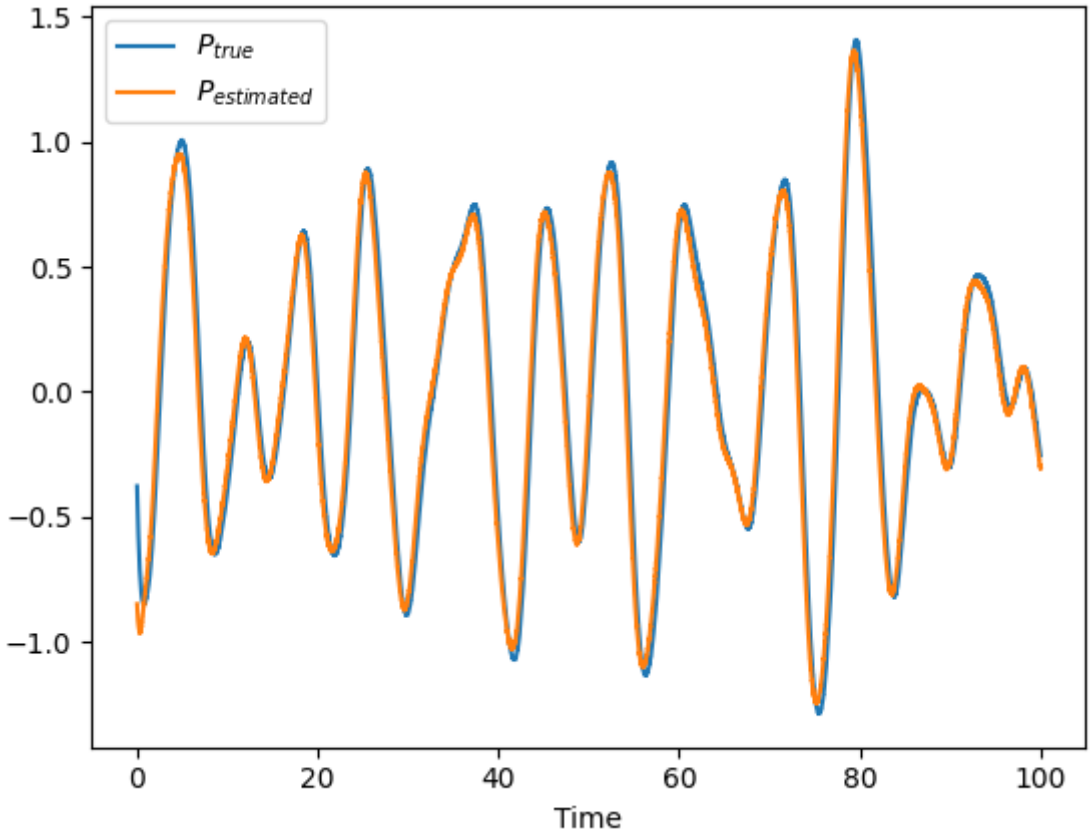
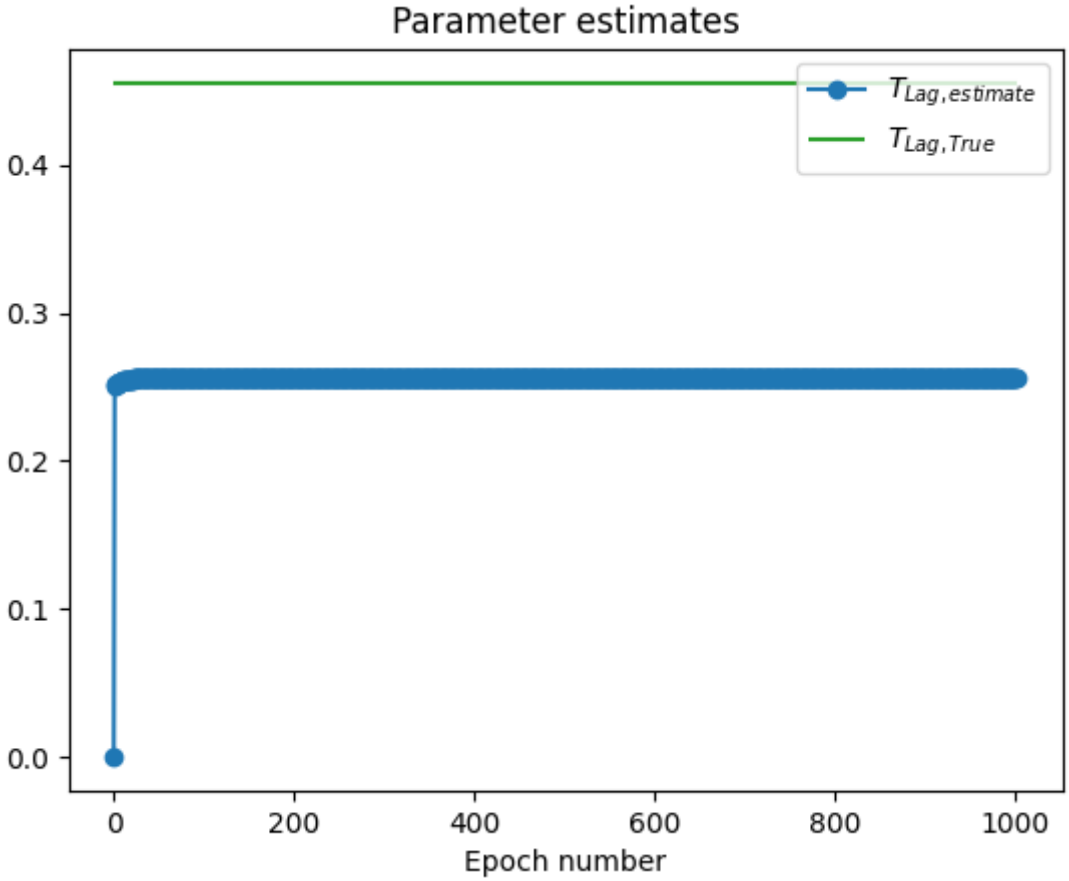
In an attempt to see the algorithm try instead of trying to estimate all 3 parameters at once and run into the issue of non uniqueness in the solution, I instead try and estimate each pilot model parameter 1 at a time (aka set 2 of them to the true value and let the third learn).

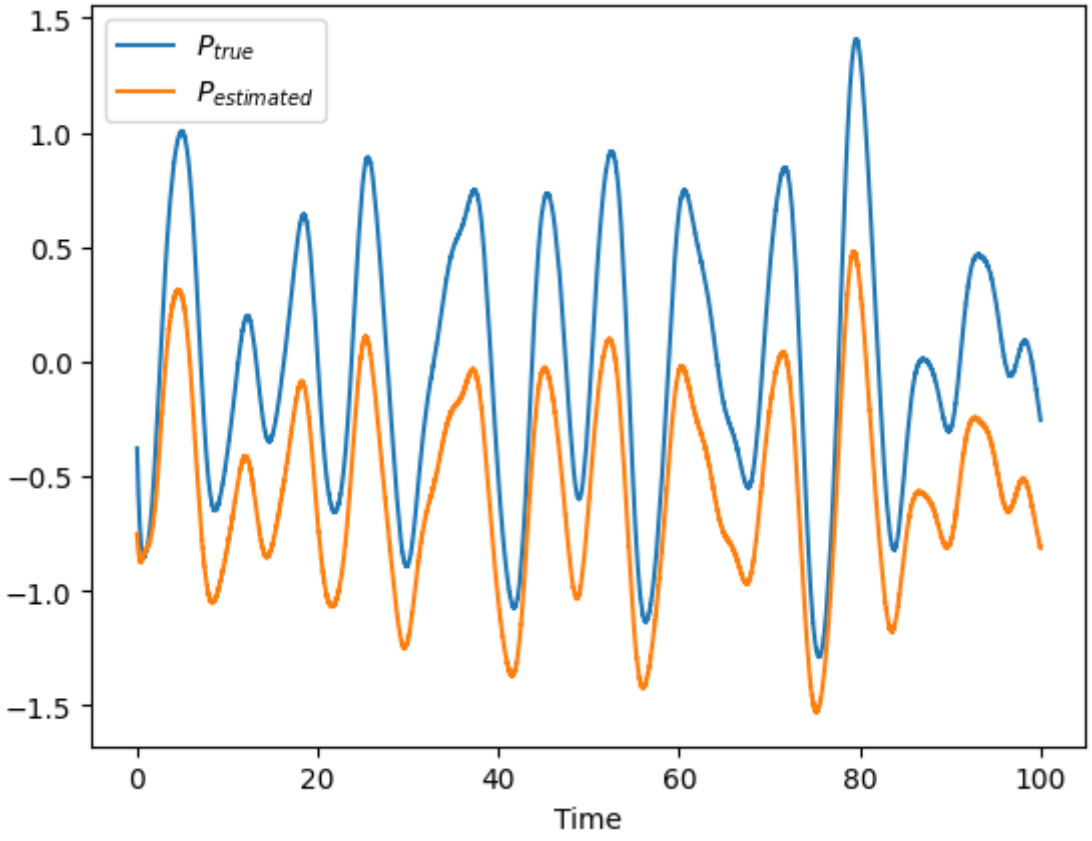
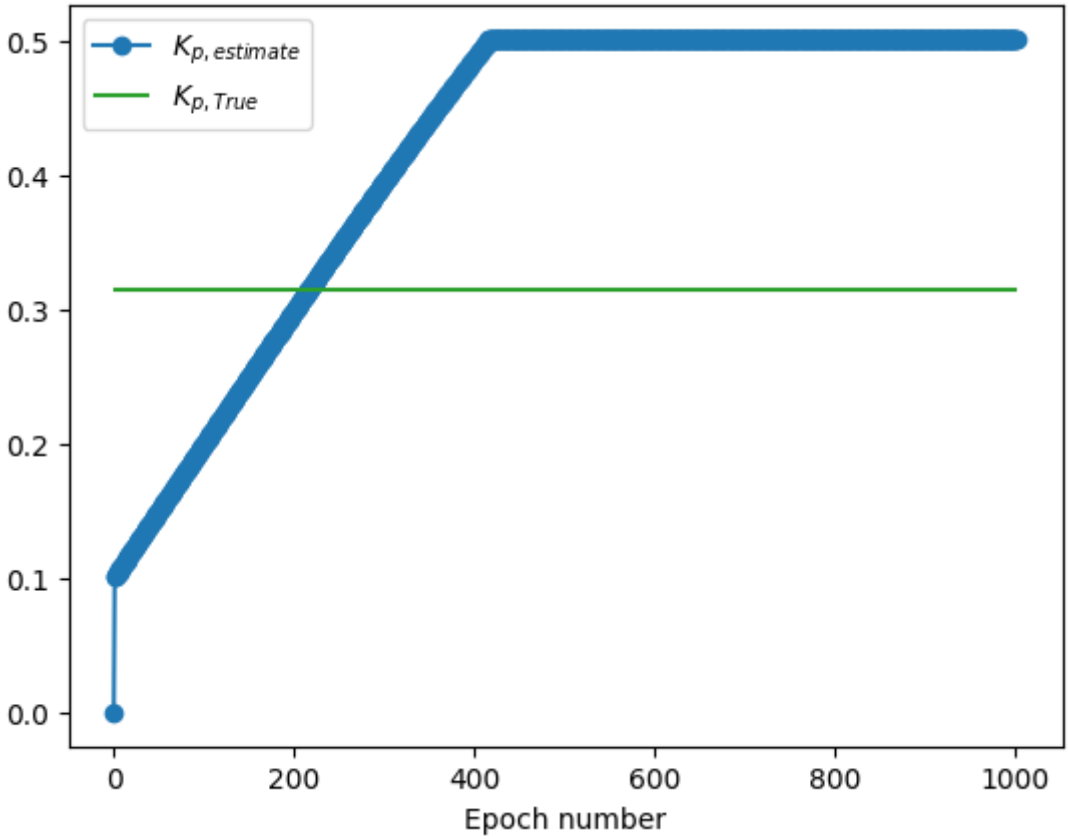
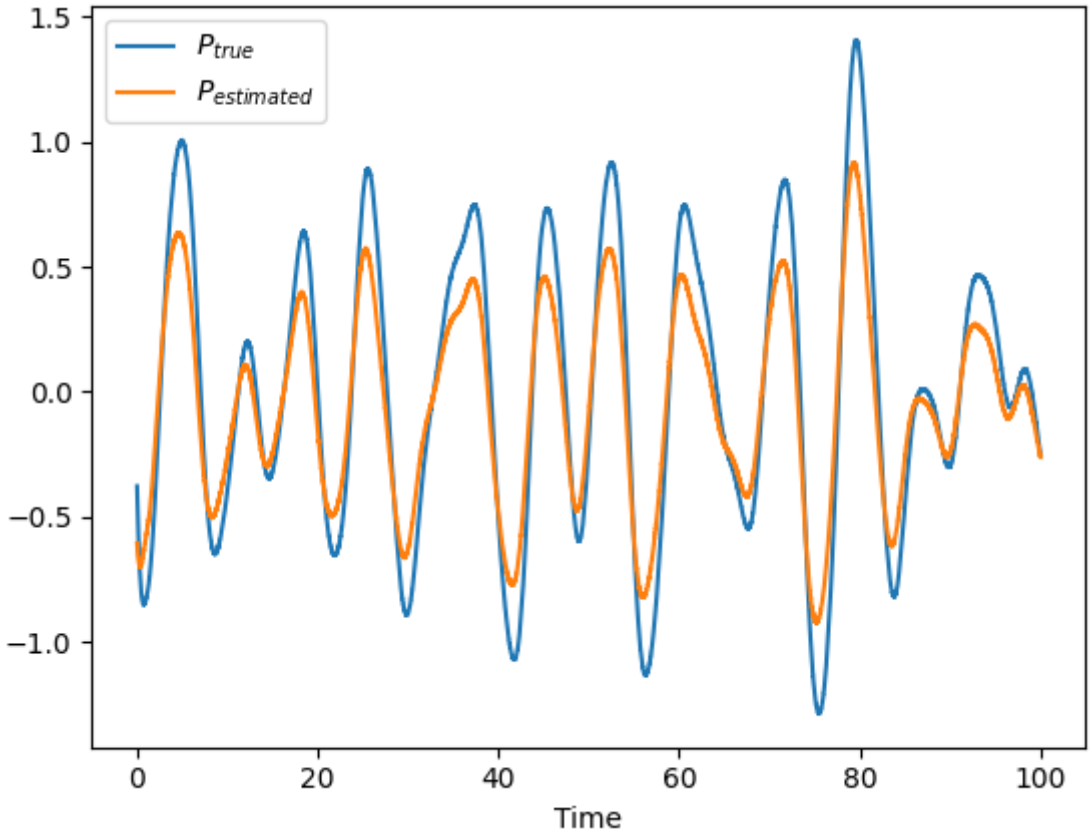
1-Holding $T_{Lag}, K_p$ as true and learning $T_{Lead}$



2-Learning $T_{Lag}$






3-Learning $K_p$