UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Exam in: Introduction to robotics (INF3480)

Day of exam: 28th of may at 14:30

Exam hours: 14:30 – 18:30 (4 hours)

This examination paper consists of: 4 pages + 3 pages appendix.
Appendices: Rules & Formulas INF3480

Permitted materials:

- **a.** Mark W. Spong, Seth Hutchinson, M. Vidyasagar: *Robot Modeling and Control*, 2005. Wiley. ISBN: 978-0-471-649908.
- **b.** Karl Rottmann, Matematisk Formelsamling (all editions)
- **c.** Approved calculator

Make sure that your copy of this examination paper is complete before answering.

The exam can be answered in either Norwegian or English.

Exercise 1 (15%)

a) (5%) In the context of Robot Operating System (ROS) and MoveIt!, there were 5 types of motion planning kinematic constraints discussed. Please list all 5 types together with a brief explanation about each one.

b) (5%) Robot Operating System (ROS) messages are known to be language agnostic. What does it mean? If you were asked to code in a custom ROS message, which would include the robot name as well as starting and finishing points in a two-dimensional map, how would the .msg file look?

c) (5%) One of the main advantages of Robot Operating System (ROS) is modular design. What is it and why is it beneficial when designing a robotic system with many hardware components?

Exercise 2 (40%)

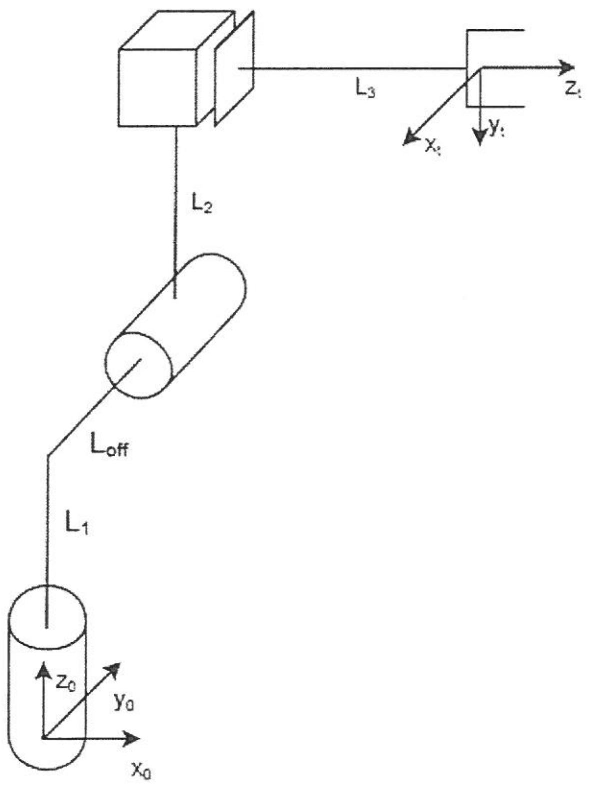

Figure 1: Robot

The given 3DOF robot in figure 2 comprises a rotational joint 1 with vertical axis of rotation (about Z_0), a rotational joint 2 with an offset L_{off} initially parallel to Y_0 and with an axis of rotation perpendicular to joint 1 and initially with axis of rotation about Y_0 whereas joint 3 is prismatic working on an axis perpendicular to the two first axes of rotation and initially in the direction of X_0.

a) (5%) Assign coordinate frames on the robot in figure 1 using the Denavit-Hartenberg convention. Write the Denavit-Hartenberg parameters in a table.
b) (5%) Derive the forward kinematics for the robot from the base coordinate system to the tool coordinate system at the tip of the robot.
c) (10%) Derive the Jacobian
d) (10%) Derive the Inverse kinematics for the robot
e) (10%) Compute the robots singularities. Draw all the different singularities. Explain all of the different singularities.

Exercise 3 (20%)

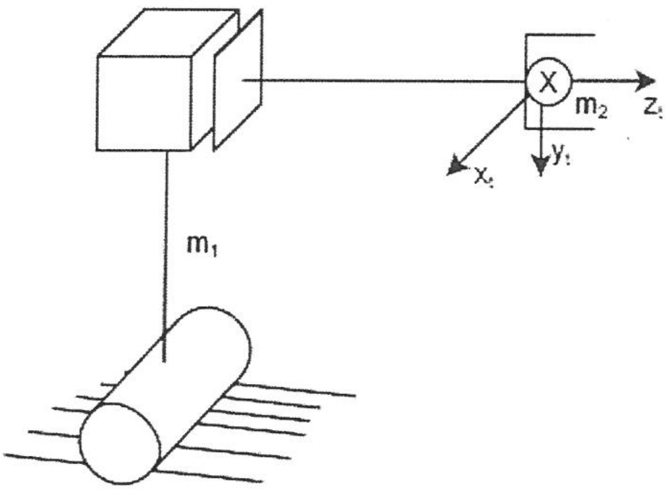

Figure 2: Simplified robot

Figure 2 shows a planar robot with two degrees of freedom. This is a simplification of the robot in exercise2, where joint 1, link 1 and the offset L_{off} is not in use. Assume that link 1 has a length L_2 and an equally distributed mass m_1 with the inertia tensor: $I_1 = \begin{bmatrix} I_{1xx} & 0 & 0 \\ 0 & I_{1yy} & 0 \\ 0 & 0 & I_{1zz} \end{bmatrix}$, and that link 2 has a length L_3 and has a point mass m_2 located at the tip of the robot.

a) (10%) Find the Lagrangian $\mathcal{L}$ of the robotic system in Figure 2.
b) (10%) Derive the dynamic equations for the robot using the Euler-Lagrange formulation. Formulate the Euler-Lagrange equations on the form $D(q) \cdot \ddot{q} + C(q;\dot{q}) \cdot \dot{q} + G(q) = \tau$

Exercise 4 (25%)

We have the system $J\ddot{\theta}(t) + B\dot{\theta}(t) + K\theta(t) = \tau$. When we use the Laplace transform on the system we get $Js^2\theta(s) + Bs\theta(s) + K\theta(s) = \tau$.

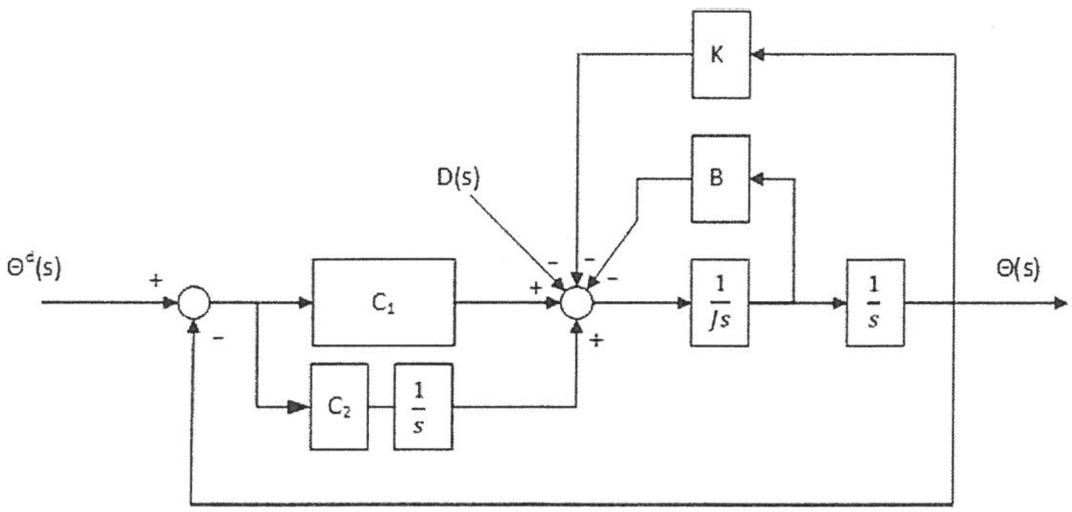

Figure 3: Control system

a) (2.5%) Figure 3 shows a set-point tracking control system in the s domain. What is the name of the controller used here? What properties does it provide to the system?

b) (2.5%) The system with the controller in Figure 3 have oscillations, how can we remove the oscillations while keeping the other system properties? What is the name of this new controller?

c) (5%) Find the closed loop transfer function between the input value (Θ^d(s) - desired angle) and output value (Θ(s) - actual/measured angle) for the system with this new improved controller.

d) (5%) Use the final value theorem to calculate the steady state error for the closed loop control system with this new improved controller, when both the desired angle Θ^d(s) and the disturbance D(s) are "step inputs". Comment on the result.

e) (5%) Transform the following equation into the s domain using the Laplace transform; $m\ddot{x} + kx = u$. Find the transfer function of this system, assuming that u is input and x is output. Draw a block diagram of the system.

f) (2.5%) How would you examine the stability of the control system, and what is required to get a stable system? Find the poles and zeros of the system in 4e) when the spring constant k=8 and the mass of the system m=2 and show them in a plot. What do we call a system like this?

g) (2.5%) What is a Root locus plot? Explain what it tells us and show it graphically.

Rules & Formulas INF3480/INF4380

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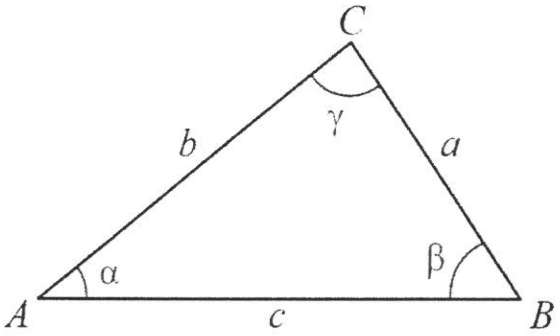
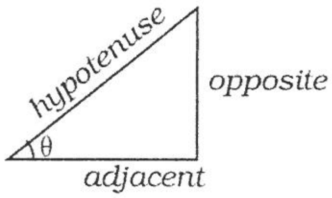




$\sin\theta = \frac{o}{h}$

$\cos\theta = \frac{a}{h}$

$\tan\theta = \frac{o}{a}$

$$c^2 = a^2 + b^2 - 2ab\ \cos\gamma$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\text{radians} = \text{degrees} \times \frac{\pi}{180}$$

$$\cos^2\theta + \sin^2\theta = 1$$

$$\sin^2 u = \frac{1-\cos(2u)}{2}$$
$$\cos^2 u = \frac{1+\cos(2u)}{2}$$
$$\tan^2 u = \frac{1-\cos(2u)}{1+\cos(2u)}$$

$$\sin(-u) = -\sin u \qquad \cos(-u) = \cos u$$

$$\begin{aligned}\cos 2\theta &= \cos^2\theta - \sin^2\theta\\ &= 2\cos^2\theta - 1\\ &= 1 - 2\sin^2\theta\\ &= \frac{1-\tan^2\theta}{1+\tan^2\theta}\end{aligned}$$

Integral $\uparrow$ Derivative $\downarrow$

$\sin\theta$
$\cos\theta$
$-\sin\theta$
$-\cos\theta$
$\sin\theta$

$$\sin(u+v) = \sin u\cos v + \cos u\sin v$$

$$\cos(u+v) = \cos u\cos v - \sin u\sin v$$

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u\tan v}$$

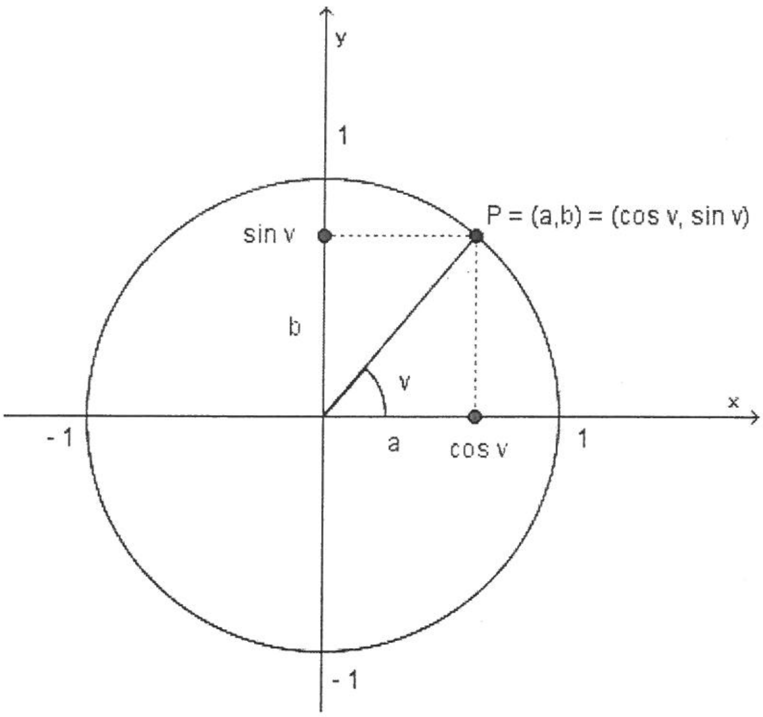
$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}} \qquad \cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\sin(u-v) = \sin u\cos v - \cos u\sin v$$

$$\cos(u-v) = \cos u\cos v + \sin u\sin v$$

$$\tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u\tan v}$$

$$\begin{aligned}\sin 2\theta &= 2\sin\theta\cos\theta\\ &= \frac{2\tan\theta}{1+\tan^2\theta}\end{aligned}$$



$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\operatorname{atan2}(y,x) = \begin{cases} \arctan(\frac{y}{x}) & \text{if } x > 0, \\ \arctan(\frac{y}{x}) + \pi & \text{if } x < 0 \text{ and } y \geq 0, \\ \arctan(\frac{y}{x}) - \pi & \text{if } x < 0 \text{ and } y < 0, \\ +\frac{\pi}{2} & \text{if } x = 0 \text{ and } y > 0, \\ -\frac{\pi}{2} & \text{if } x = 0 \text{ and } y < 0, \\ \text{undefined} & \text{if } x = 0 \text{ and } y = 0. \end{cases}$$

Deg	**0**	**30**	**45**	**60**	**90**
Rad	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
Sin	0	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$
Cos	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	0
Tan	0	$\sqrt{3}^{-1}$	$\sqrt{3}^{0}$	$\sqrt{3}^{1}$	**Not defined**

$$A = [a, b, c] \qquad B = [d, e, f]$$

$$\begin{array}{ccccc} x & y & z & x & y \\ a & b & c & a & b \\ d & e & f & d & e \end{array}$$

$$A \times B = [(bf - ce), (cd - af), (ae - bd)]$$

Consider the matrices

$$A = \begin{bmatrix} -1 & 0 \\ 2 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$$

Multiplying gives

$$AB = \begin{bmatrix} -1 & -2 \\ 11 & 4 \end{bmatrix}, \quad BA = \begin{bmatrix} 3 & 6 \\ -3 & 0 \end{bmatrix}$$

Thus, $AB \neq BA$.

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} \times \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \end{bmatrix}$$

$$3 \times 2 \quad \times \quad 2 \times 5 \quad = \quad 3 \times 5$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}$$

$$A \qquad\qquad B \qquad\qquad C$$

A, B and C are square metrices of size N x N
a, b, c and d are submatrices of A, of size N/2 x N/2
e, f, g and h are submatrices of B, of size N/2 x N/2

Time domain	Laplace domain	Time domain	Laplace domain
$x(t)$	$x(s) = L\{x(t)\} = \int_0^\infty e^{-st} x(t) dt$	$x(t-\alpha)H(t-\alpha)$	$e^{-as}x(s)$
$\dot{x}(t)$	$sx(s) - x(0)$	$e^{-at}x(t)$	$x(s+a)$
$\ddot{x}(t)$	$s^2x(s) - sx(0) - \dot{x}(0)$	$x(at)$	$\frac{1}{a}x\left(\frac{s}{a}\right)$
Ct	$\frac{C}{s^2}$	$C\delta(t)$	C
step	$\frac{1}{s}$		
$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$		
$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$		