UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Exam in: Introduction to robotics (INF 4380)

Day of exam: 28th of may at 14:30

Exam hours: 14:30 – 18:30 (4 hours)

This examination paper consists of: 5 pages + 3 pages appendix.
Appendices: Rules & Formulas INF4380

Permitted materials:

a. Mark W. Spong, Seth Hutchinson, M. Vidyasagar: *Robot Modeling and Control*, 2005. Wiley. ISBN: 978-0-471-649908.

b. Karl Rottmann, Matematisk Formelsamling (all editions)

c. Approved calculator

Make sure that your copy of this examination paper is complete before answering.

The exam can be answered in either Norwegian or English.

Exercise 1 (15%)

We want to use the 3DOF crustcrawler setup with 45 degree angle offset at the tip (similar to the one we used in the last assignment), and now we want to draw a circle somewhere on a slightly curved surface (se figure 1). The whole circle will be within the curved surface.

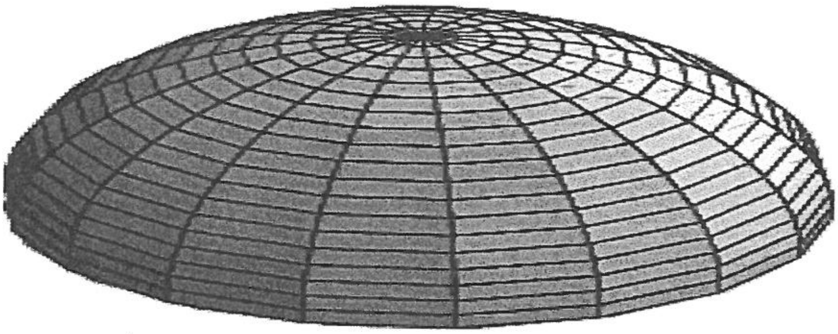


Figure 1. Curved surface

Assume the whole hemisphere is well within the workspace of the robot, and stationary.
You can also consider the friction between the pen and the surface to be zero, although the friction is high enough for the pen to leave a trace wherever it touches the hemisphere.

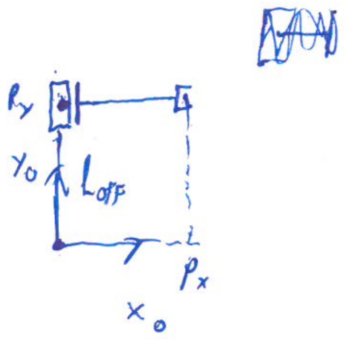
a) (5%) Briefly explain why we could use a force sensor to draw the circle on the hemisphere. Would you then need to model the hemisphere? What kind of controller would you use, and why?

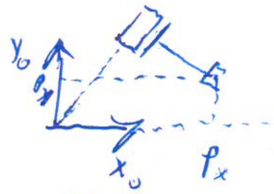
b) (5%) Assume we now have a 3DOF wrist with a gripper attached to the tip of the robot arm. The pen used to draw circles is now held in a penholder in front of the robot. Before the robot can draw circles it now needs to pick up the pen. We will use a camera mounted to the tip of the robot to help control the robot to perform this task.
What method of visual servo control would you use to help pick up the pen? Describe the concept of the method.

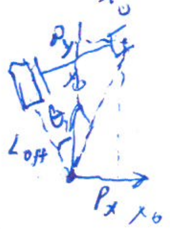
c) (5%) Working further with this robot you are tasked with using the Robot Operating System (ROS). Here, messages are known to be language agnostic. What does it mean? If you were asked to code in a custom ROS message, which would include the robot name as well as starting and finishing points in a two-dimensional map, how would the .msg file look?

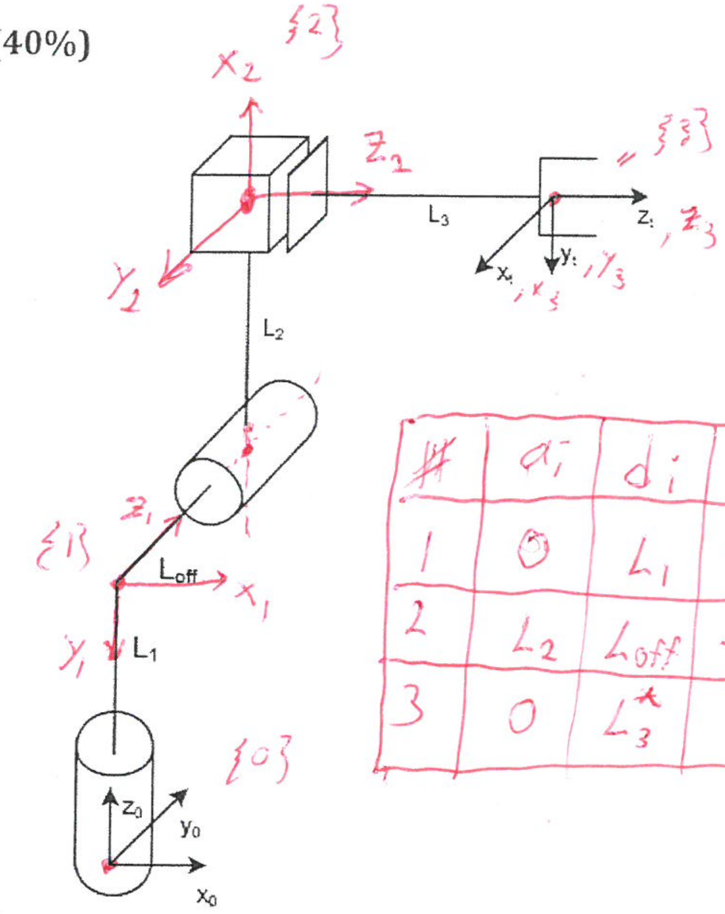
$L_2 S_2$ $\quad L_3^* \cdot C_2 \Rightarrow L_2 S_2 + L_3^* C_2 = X \Rightarrow$

Exercise 2 (40%)









#	a_i	d_i	α_i	θ_i
1	0	L_1	-90	θ_1^*
2	L_2	L_{off}	-90	$\theta_2^* - 90$
3	0	L_3^*	0	90

Figure 2: Robot

The given 3DOF robot in figure 2 comprises a rotational joint 1 with vertical axis of rotation (about Z_0), a rotational joint 2 with an offset L_{off} initially parallel to Y_0 and with an axis of rotation perpendicular to joint 1 and initially with axis of rotation about Y_0 whereas joint 3 is prismatic working on an axis perpendicular to the two first axis of rotations and initially in the direction of X_0.

√ a) (5%) Assign coordinate frames on the robot in figure 2 using the Denavit-Hartenberg convention. Write the Denavit-Hartenberg parameters in a table.

√ b) (5%) Derive the forward kinematics for the robot from the base coordinate system to the tool coordinate system at the tip of the robot.

√ c) (10%) Derive the Jacobian

√ d) (10%) Derive the Inverse kinematics for the robot

? e) (10%) Compute the robots singularities. Draw all the different singularities. Explain all of the different singularities.

Exercise 3 (20%)

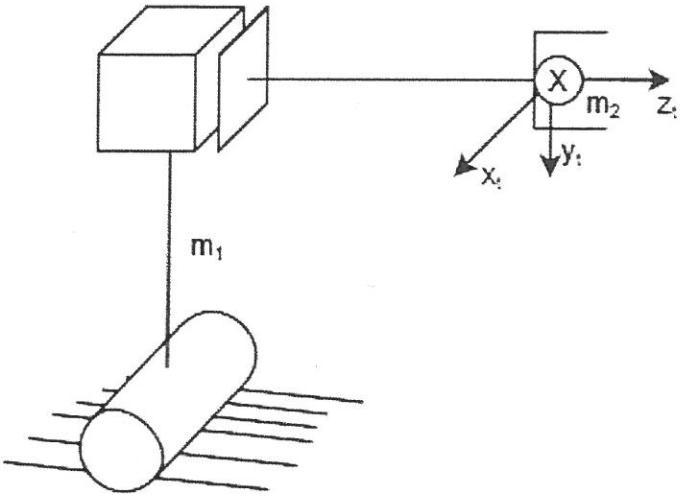


Figure 3: Simplified robot

Figure 3 shows a planar robot with two degrees of freedom. This is a simplification of the robot in exercise2, where joint 1, link 1 and the offset L_{off} is not in use. Assume that link 1 has a length L_2 and an equally distributed mass m_1 with the inertia tensor: $I_1 = \begin{bmatrix} I_{1xx} & 0 & 0 \\ 0 & I_{1yy} & 0 \\ 0 & 0 & I_{1zz} \end{bmatrix}$, and that link 2 has a length L_3 and has a point mass m_2 located at the tip of the robot.

a) (10%) Find the Lagrangian $\mathcal{L}$ of the robotic system in Figure 3.
b) (10%) Derive the dynamic equations for the robot using the Euler-Lagrange formulation. Formulate the Euler-Lagrange equations of the form $D(q) \cdot \ddot{q} + C(q;\dot{q}) \cdot \dot{q} + G(q) = \tau$

Exercise 4 (25%)

We have the system $J\ddot{\theta}(t) + B\dot{\theta}(t) + K\theta(t) = \tau$. When we use the Laplace transform on the system we get $Js^2\theta(s) + Bs\theta(s) + K\theta(s) = \tau$.

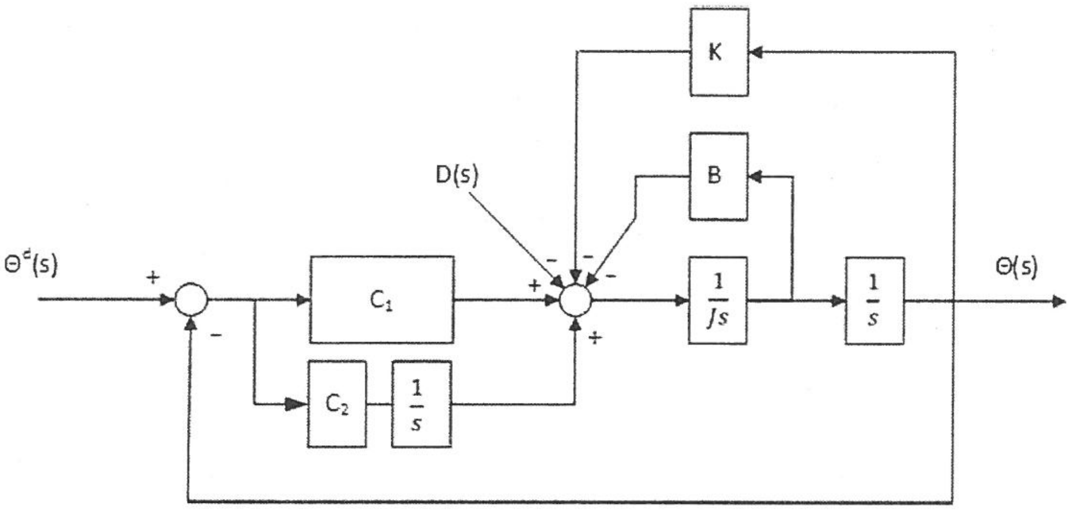


Figure 4: Control system

a) (2.5%) Figure 4 shows a set-point tracking control system in the s domain. What is the name of the controller used here? What properties does it provide to the system?

b) (2.5%) The system with the controller in Figure 4 have oscillations, how can we remove the oscillations while keeping the other system properties? What is the name of this new controller?

c) (5%) Find the closed loop transfer function between the input value ($\Theta^d(s)$ - desired angle) and output value ($\Theta(s)$ - actual/measured angle) for the system with this new improved controller.

d) (5%) Use the final value theorem to calculate the steady state error for the closed loop control system with this new improved controller, when both the desired angle $\Theta^d(s)$ and the disturbance D(s) are "step inputs". Comment on the result.

e) (5%) Transform the following equation into the s domain using the Laplace transform; $m\ddot{x} + kx = u$. Find the transfer function of this system, assuming that u is input and x is output. Draw a block diagram of the system.

f) (2.5%) How would you examine the stability of the control system, and what is required to get a stable system? Find the poles and zeros of the system in 4e) when the spring constant k=8 and the mass of the system m=2 and show them in a plot. What do we call a system like this?

g) (2.5%) What is a Root locus plot? Explain what it tells us and show it graphically.

Rules & Formulas INF3480/INF4380

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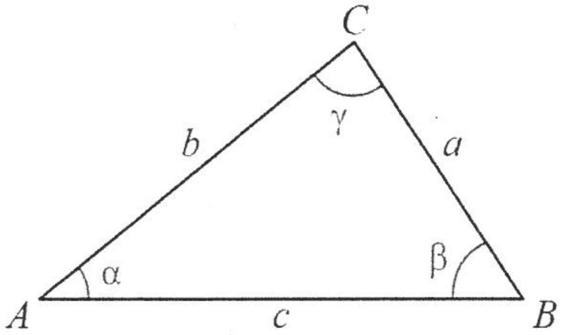
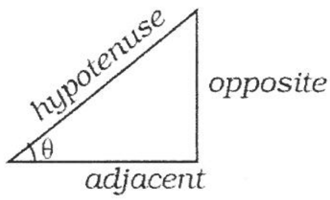




$\sin\theta = \frac{o}{h}$

$\cos\theta = \frac{a}{h}$

$\tan\theta = \frac{o}{a}$

$$c^2 = a^2 + b^2 - 2ab\ \cos\gamma$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\text{radians} = \text{degrees} \times \frac{\pi}{180}$$

$$\cos^2\theta + \sin^2\theta = 1$$

$$\sin^2 u = \frac{1-\cos(2u)}{2}$$

$$\cos^2 u = \frac{1+\cos(2u)}{2}$$

$$\tan^2 u = \frac{1-\cos(2u)}{1+\cos(2u)}$$

$$\sin(-u) = -\sin u \qquad \cos(-u) = \cos u$$

$$\begin{aligned}\cos 2\theta &= \cos^2\theta - \sin^2\theta \\ &= 2\cos^2\theta - 1 \\ &= 1 - 2\sin^2\theta \\ &= \frac{1-\tan^2\theta}{1+\tan^2\theta}\end{aligned}$$

Integral ↑

$\sin\theta$

$\cos\theta$

$-\sin\theta$

$-\cos\theta$

$\sin\theta$

Derivative ↓

$$\sin(u+v) = \sin u\cos v + \cos u\sin v$$

$$\cos(u+v) = \cos u\cos v - \sin u\sin v$$

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u\tan v}$$

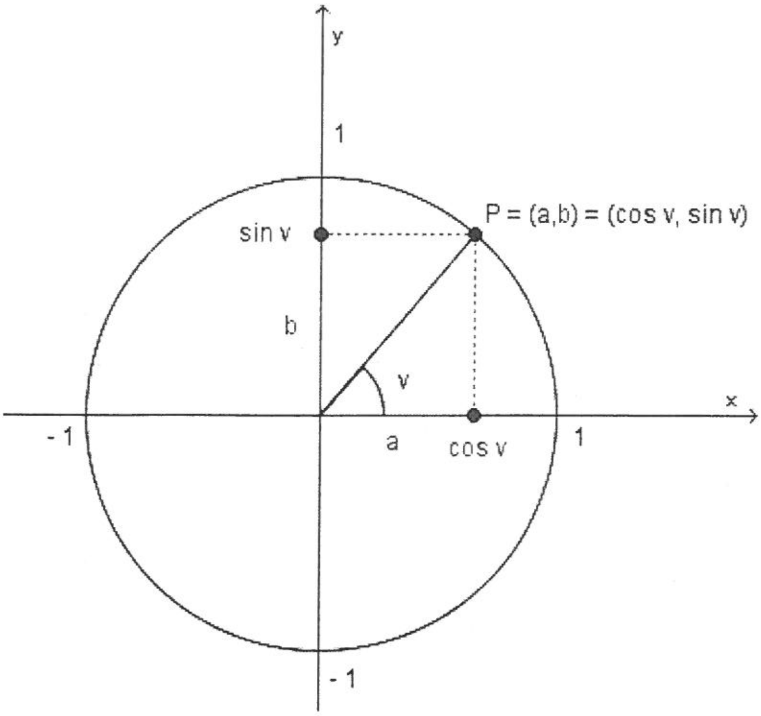
$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}} \qquad \cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\sin(u-v) = \sin u\cos v - \cos u\sin v$$

$$\cos(u-v) = \cos u\cos v + \sin u\sin v$$

$$\begin{aligned}\sin 2\theta &= 2\sin\theta\cos\theta \\ &= \frac{2\tan\theta}{1+\tan^2\theta}\end{aligned}$$

$$\tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u\tan v}$$



$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\operatorname{atan2}(y,x) = \begin{cases} \arctan(\frac{y}{x}) & \text{if } x > 0, \\ \arctan(\frac{y}{x}) + \pi & \text{if } x < 0 \text{ and } y \geq 0, \\ \arctan(\frac{y}{x}) - \pi & \text{if } x < 0 \text{ and } y < 0, \\ +\frac{\pi}{2} & \text{if } x = 0 \text{ and } y > 0, \\ -\frac{\pi}{2} & \text{if } x = 0 \text{ and } y < 0, \\ \text{undefined} & \text{if } x = 0 \text{ and } y = 0. \end{cases}$$

Deg	**0**	**30**	**45**	**60**	**90**
Rad	**0**	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
Sin	**0**	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$
Cos	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	**0**
Tan	**0**	$\sqrt{3}^{-1}$	$\sqrt{3}^{0}$	$\sqrt{3}^{1}$	**Not defined**

$A = [a, b, c] \qquad B = [d, e, f]$

$$\begin{array}{ccccc} x & y & z & x & y \\ a & b & c & a & b \\ d & e & f & d & e \end{array}$$

$A \times B = [(bf - ce), (cd - af), (ae - bd)]$

Consider the matrices

$$A = \begin{bmatrix} -1 & 0 \\ 2 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$$

Multiplying gives

$$AB = \begin{bmatrix} -1 & -2 \\ 11 & 4 \end{bmatrix}, \quad BA = \begin{bmatrix} 3 & 6 \\ -3 & 0 \end{bmatrix}$$

Thus, $AB \neq BA$.

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} \times \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \end{bmatrix}$$

$$3 \times 2 \quad \times \quad 2 \times 5 \quad = \quad 3 \times 5$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}$$

$$\quad A \qquad\qquad\qquad B \qquad\qquad\qquad C$$

A, B and C are square metrices of size N x N
a, b, c and d are submatrices of A, of size N/2 x N/2
e, f, g and h are submatrices of B, of size N/2 x N/2

Time domain	Laplace domain	Time domain	Laplace domain
$x(t)$	$x(s) = L\{x(t)\} = \int_0^\infty e^{-st} x(t) dt$	$x(t-\alpha)H(t-\alpha)$	$e^{-as} x(s)$
$\dot{x}(t)$	$sx(s) - x(0)$	$e^{-at} x(t)$	$x(s+a)$
$\ddot{x}(t)$	$s^2 x(s) - sx(0) - \dot{x}(0)$	$x(at)$	$\frac{1}{a} x\left(\frac{s}{a}\right)$
Ct	$\frac{C}{s^2}$	$C\delta(t)$	C
step	$\frac{1}{s}$		
$\cos(\omega t)$	$\frac{s}{s^2 + \omega^2}$		
$\sin(\omega t)$	$\frac{\omega}{s^2 + \omega^2}$		