Task 1 Transformation

The task is to find the homogenous transformation that relates the base frame {B} with respect to the task frame {T}. We can relate the transformation matrix as below

$$T_B^W \times T_T^B = T_T^W$$

then the objective is to find the transformation matrix T_T^B so we can formulate the matrix

$$T_T^B = (T_B^W)^{-1} \times T_T^W \tag{1}$$

$$T_T^W = \begin{bmatrix} R_T^W & t_T^W \\ 0 & 1 \end{bmatrix}$$

$$R_T^W = R_z(180).R_x(180)$$

$$R_T^W = \begin{bmatrix} C_\theta & -S_\theta & 0 \\ S_\theta & C_\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_\theta & -S_\theta \\ 0 & S_\theta & C_\theta \end{bmatrix}$$

$$R_T^W = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$R_T^W = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$t_T^W = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$t_T^W = \begin{bmatrix} 1000 \\ 400 \\ 900 \end{bmatrix}$$

$$T_T^W = \begin{bmatrix} -1 & 0 & 0 & 1000 \\ 0 & 1 & 0 & 400 \\ 0 & 0 & -1 & 900 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_B^W = \begin{bmatrix} R_B^W & t_B^W \\ 0 & 1 \end{bmatrix}$$

$$R_B^W = R_x(-180).R_z(90)$$

$$R_B^W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_\theta & -S_\theta \\ 0 & S_\theta & C_\theta \end{bmatrix} \begin{bmatrix} C_\theta & -S_\theta & 0 \\ S_\theta & C_\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_B^W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_B^W = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$t_B^W = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$t_B^W = \begin{bmatrix} 250 \\ 650 \\ 1000 \end{bmatrix}$$

$$T_T^W = \begin{bmatrix} 0 & -1 & 0 & 250 \\ -1 & 0 & 0 & 650 \\ 0 & 0 & -1 & 1000 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

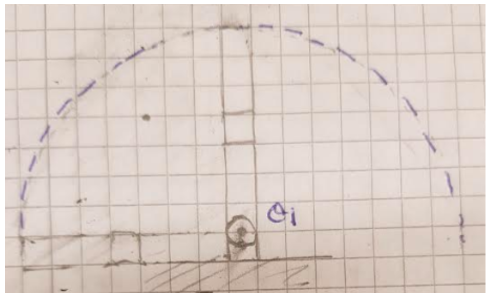
From the equation (1) we get

$$T_T^B = (T_B^W)^{-1} \times T_T^W$$

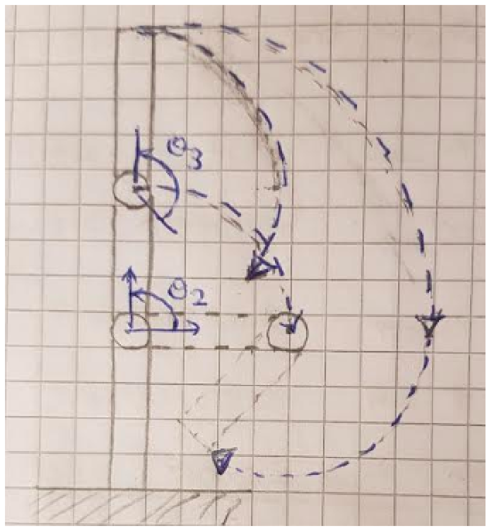
$$T_T^B = \begin{bmatrix} 0 & -1 & 0 & 250 \\ -1 & 0 & 0 & 650 \\ 0 & 0 & -1 & 1000 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} -1 & 0 & 0 & 1000 \\ 0 & 1 & 0 & 400 \\ 0 & 0 & -1 & 900 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_T^B = \begin{bmatrix} 0 & -1 & 0 & 250 \\ 1 & 0 & 0 & -750 \\ 0 & 0 & 1 & 100 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

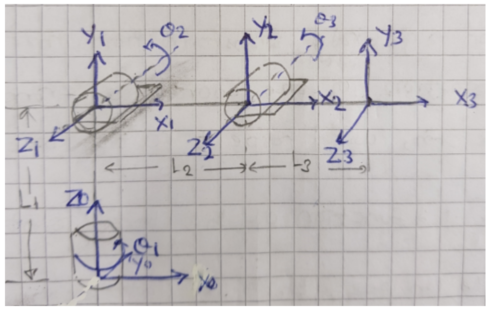
Task 2. a) A sketch about the workplace of a robot. 3D drawing from top view is show as below



3D drawing from side view is show as below



Task 2. b)A simple 3D illustration of the robot, showing the coordinate frames and the Denavit-Hartenberg parameters



A DH table is showing

No	Rz	Tz	Tx	Rx
1	theta1	L1	0	90
2	theta2	0	L2	0
3	theta3	0	L3	0

Table 1: DH table

First of all we need to have four frame on this manipulator, one frame for each joint and need to have a frame on the end effector. After that we need to go through DH rules.
The first rule is that Z axis has to be in the direction of the motion or the axis of rotation, meaning Z0 in the first joint has to be up or downwards. I choose upward, Z1 and Z2 has to be place in the axis. Z3 in the end-effector can be point in any direction, but for this case I choose it as the same direction as Z2.
Second rules, X-axis has to be perpendicular to its own Z-axis and Z-axis before. So for frame 0, I could make x into or out of the page, right or left. All of those is perpendicular to Zo-axis. In frame 1, I can make X1 either right or left, there are only two direction that could be perpendicular bot Z1 and Z0. I choose X1 on the right direction. In frame 2, X2 can go either right or left, up or down. I choose X2 as the same as X1, so to be X3 as well.
In rule 3,X-axis has to intersect hte Z-axis of the frame before. That rules does not apply for frame 0 becouse there is no axis before. But it apply frame 1. We can see that X1 intersect Z0 and X2 intersect Z1. In rule 4, Y axis has to follow right hand rules.
Lastly I have labled link length respectively.

Task 2. c) Calculating the forward kinematics for this robot.

$$H_3^0 = H_1^0 H_2^1 H_3^2 \tag{2}$$

$$H_1^0 = R_z T_z T_x R_x \tag{3}$$

$$H_1^0 = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^1 = R_z T_z T_x R_x \tag{4}$$

$$H_2^1 = \begin{bmatrix} C_2 & -S_2 & 0 & 0 \\ S_2 & C_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & L_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C_2 & -S_2 & 0 & C_2L_2 \\ S_2 & C_2 & 0 & S_2L_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3^2 = R_z T_z T_x R_x \tag{5}$$

$$H_3^2 = \begin{bmatrix} C_3 & -S_3 & 0 & 0 \\ S_3 & C_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & L_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C_3 & -S_3 & 0 & C_3L_3 \\ S_3 & C_3 & 0 & S_3L_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

From equation (2) we get

$$H_3^0 = H_1^0 H_2^1 H_3^2$$

$$H_3^0 = \begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_2 & -S_2 & 0 & C_2L_2 \\ S_2 & C_2 & 0 & S_2L_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_3 & -S_3 & 0 & C_3L_3 \\ S_3 & C_3 & 0 & S_3L_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3^0 = \begin{bmatrix} C_1C_2 & -C_1S_2 & S_1 & C_1C_2L_2 \\ S_1C_2 & -S_1S_2 & -C_1 & S_1C_2L_2 \\ S_2 & C_2 & 0 & S_2L_2 + L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_3 & -S_3 & 0 & C_3L_3 \\ S_3 & C_3 & 0 & S_3L_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3^0 = \begin{bmatrix} C_1C_2C_3 - C_1S_2S_3 & -C_1C_2S_3 - C_1S_2C_3 & S_1 & C_1C_2C_3L_3 - C_1S_2S_3L_3 + C_1C_2L_2 \\ S_1C_2C3 - S_1S_2S_3 & -S_1C_2S_3 - S_1S_2C_3 & -C_1 & S_1C_2C_3L_3 - S_1S_2S_3L_3 + S_1C_2L_2 \\ S_2C3 + C_2S_3 & -S_2S_3 + C_2C_3 & 0 & S_2C_3L_3 + C_2S_3L_3 + S_2L_2 + L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Task 3

Point p is located at the tip of the robot. Adjust the robot as displayed in assigment figure where $\phi_1 = 270°$, $\phi_2 = 60°$ and $\phi_3 = 45°$ and link length we have L_1=100.9mm,L_2=222.1mm and L_3=136.2mm respectively. We get the equation from task 2c and plug it this values into the equation.

$$H_3^0 = \begin{bmatrix} C_1C_2C_3 - C_1S_2S_3 & -C_1C_2S_3 - C_1S_2C_3 & S_1 & C_1C_2C_3L_3 - C_1S_2S_3L_3 + C_1C_2L_2 \\ S_1C_2C3 - S_1S_2S_3 & -S_1C_2S_3 - S_1S_2C_3 & -C_1 & S_1C_2C_3L_3 - S_1S_2S_3L_3 + S_1C_2L_2 \\ S_2C3 + C_2S_3 & -S_2S_3 + C_2C_3 & 0 & S_2C_3L_3 + C_2S_3L_3 + S_2L_2 + L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

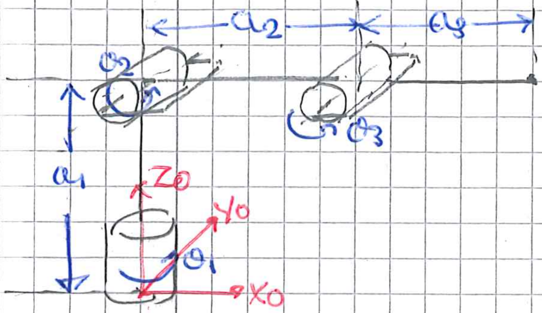
$$H_3^0 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ -0.484 & 0.875 & 0 & -176.98mm \\ 0.875 & 0.484 & 0 & 412.42mm \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Task 4.

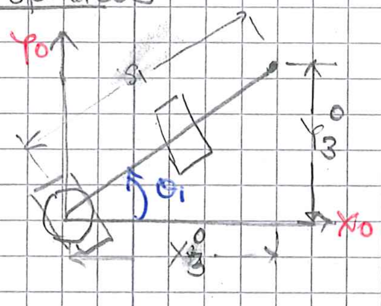
Inverse kinematic is basically finding the values of joint parameters in terms of the desired end-effector's positon and orientation.

In general, the inverse kinematics can be solved by various methods such as inverse transform, screw algebra, dual matrices, iterative, geometric approach, decoupling of position and orientation and geometric algebra. The two most common method are algebraic methods and geometric methods.

Here in task 4 is taking a method based geometric solution.

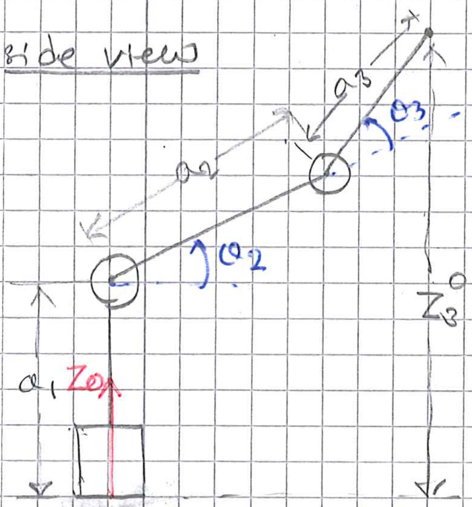


Top-view



$$\theta_1 = \tan^{-1} \frac{Y_3^0}{X_3^0} \quad \text{— (1)}$$

side view



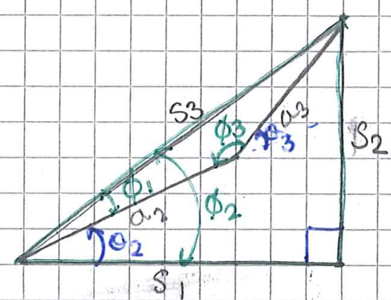


Fig: Albow down configuration

$$\theta_2 = \phi_2 - \phi_1 \text{ — (2)} ; \quad \phi_2 = \tan^{-1}\left(\frac{S_2}{S_1}\right) \text{ — (3)} ; \quad S_2 = Z_3^0 - a_1 \quad \text{— (3.1)}$$

$$S_1 = \sqrt{(X_3^0)^2 + (Y_3^0)^2} \quad \text{— (3.2)}$$

$$S_3 = \sqrt{S_1^2 + S_2^2} \quad \text{— (4.1)}$$

$$a_3^2 = a_2^2 + S_3^2 - 2 a_2 S_3 \cos \phi_1 \quad \text{— (4.2)}$$

$$\phi_1 = \cos^{-1}\left(\frac{a_2^2 + S_3^2 - a_3^2}{2 a_2 S_3}\right) \quad \text{— (4)}$$

$$\theta_3 = 180 - \phi_3 \text{ — (5)} ; \quad S_3^2 = a_2^2 + a_3^2 - 2 a_2 a_3 \cos \phi_3 \quad \text{— (5.2)}$$

$$\phi_3 = \cos^{-1}\left(\frac{a_2^2 + a_3^2 - S_3^2}{2 a_2 a_3}\right) \quad \text{— (5.1)}$$