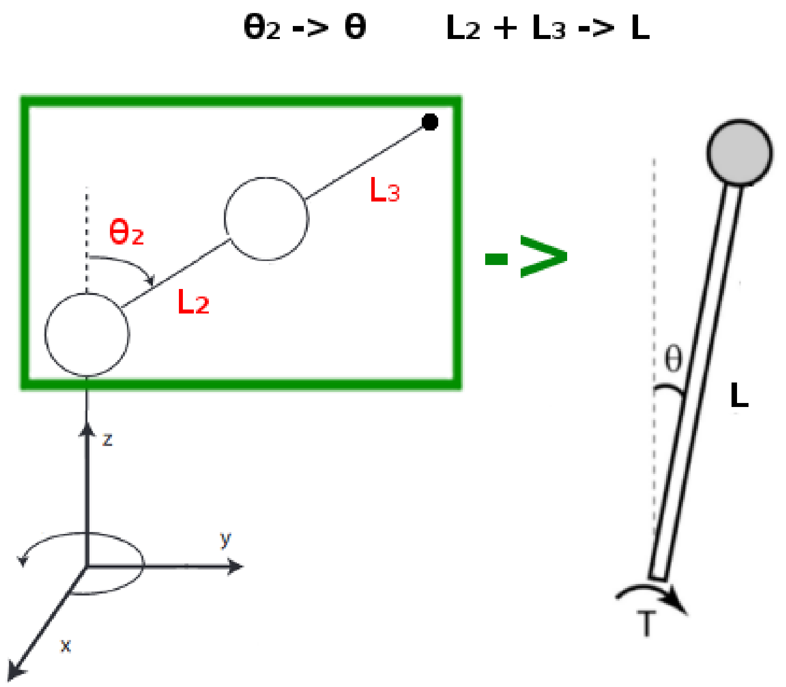
# IN3140/IN4140 Assignment 4 Task 1A Supplementary Notes (Spring 2019)

In the previous assignments, we have worked with the three-link CrustCrawler model. This assignment introduces a different model - the inverted pendulum model, in order to explore control of a single joint (the one that used to be joint 2 before). The variable mapping is roughly as follows:




You can see that we completely disregard a part of the old model (base joint and its link). We are now left with one composite link, which used to be links 2 and 3. The length of this link is denoted as L. Only one joint is now active (former joint 2), and it can rotate the link by angle $\theta$ (former $\theta_2$).

We assume that the whole new model has mass **m** (in terms of the old variables, it would be equal to $\mathbf{m_2 + m_3}$). **This mass is spread the same way as before, and inertia is still present**. Having understood the inverted pendulum model, we can begin solving the task.

**a) Make the equation of motion for Joint 2 independent (of other joints).**
There are two approaches here: you can either actually reduce the $\tau_2$ expression from as-

signment 3 or build a new dynamical model from scratch.

**If you obtained the equation from assignment 3**, you can complete this task easily. We know that base joint and its link are removed, we also know that former joint 3 is now inactive. Thus, we need to set some of the variables to zero. Those are: $\theta_1$, $\dot{\theta}_1$, $\ddot{\theta}_1$, $\theta_3$, $\dot{\theta}_3$, $\ddot{\theta}_3$.

This procedure should get rid of centrifugal ($\dot{\theta}_i\dot{\theta}_j$) and coriolis ($\dot{\theta}_i^{\,2}$) force terms in the equation. These forces represent joint interactions - they are exactly what we do **not** want anymore. However, it is possible that after setting the unused variables to zero, you still have terms like $\dot{\theta}_2^2$. Such terms seem to denote centrifugal force from joint 2 on itself - **they probably should not have been in the equation to begin with, and you need to remove them as well**. Once you have done that, you can move on to the next subtask.

**In case you did not obtain the equation from assignment 3**, the task will require a little more work - you will have to derive the dynamics for the new model. The potential energy of the system is easy to calculate, if we assume the new mass center is at the tip:

$$P = mgh = mgL\cos(\theta)$$

while the kinetic energy is not:

$$K_{general} = \frac{mv^2}{2} + \frac{\mathcal{I}\vec{\omega}^2}{2}$$

where, in this case,

$$\frac{mv^2}{2} = \frac{m(\omega r)^2}{2} = \frac{m\dot{\theta}^2L^2}{2}$$

$$\frac{\mathcal{I}\vec{\omega}^2}{2} \Rightarrow \frac{\vec{\omega}^T\mathcal{I}\vec{\omega}}{2} = \frac{\vec{\omega}^T(R_0^1)\mathbf{I}(R_0^1)^T\vec{\omega}}{2}$$

The expressions above can be a bit confusing. In the first part of the equation (the translational component), we define linear velocity $\mathbf{v}$ in terms of angular velocity $\omega$ - this is from high school physics. Since our joint is revolute, we can simply state that $\omega = \dot{\theta}$.

The second part of the equation (the rotational component) is much more complicated. Here, we need to perform matrix multiplication. Because of this, we should redefine angular velocity from scalar $\omega = \dot{\theta}$ to vector $\vec{\omega} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta} \end{bmatrix}$. The vector has to be assembled this way in order to follow DH rules (joint rotation is around z-axis).

To get the moment of inertia $\mathcal{I}$ right, we assume that we need to rotate our inertia tensor $\mathbf{I}$. For this part, we choose rotation matrix $R_1^0$ (rotation from our discarded base frame to the frame of the old joint 2) - reuse your forward kinematics here. Finally, our new link is a composite body, made from the old links 2 and 3, meaning that $\mathbf{I} = I_2 + I_3$ (probably). The terms $I_2$ and $I_3$ are the inertia tensors from the last assignment.

With all the definitions and assumptions in mind, we can get the full expression for kinetic energy:

$$K = \frac{m\dot{\theta}^2 L^2}{2} + \frac{\begin{bmatrix} 0 & 0 & \dot{\theta} \end{bmatrix} (R_1^0)(I_2 + I_3)(R_1^0)^T \begin{bmatrix} 0 \\ 0 \\ \dot{\theta} \end{bmatrix}}{2}$$

If you wish to define the inertia symbolically, use these notations:

$$I_2 = \begin{bmatrix} I_{2,xx} & 0 & 0 \\ 0 & I_{2,yy} & 0 \\ 0 & 0 & I_{2,zz} \end{bmatrix} \quad I_3 = \begin{bmatrix} I_{3,xx} & 0 & 0 \\ 0 & I_{3,yy} & 0 \\ 0 & 0 & I_{3,zz} \end{bmatrix}$$

Once you are done with the calculations, you can assemble the Lagrangian:

$$\mathcal{L} = K - P$$

Now, you can complete this subtask by finding the joint's torque:

$$\tau_2 = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\theta}} - \frac{\partial \mathcal{L}}{\partial \theta}$$