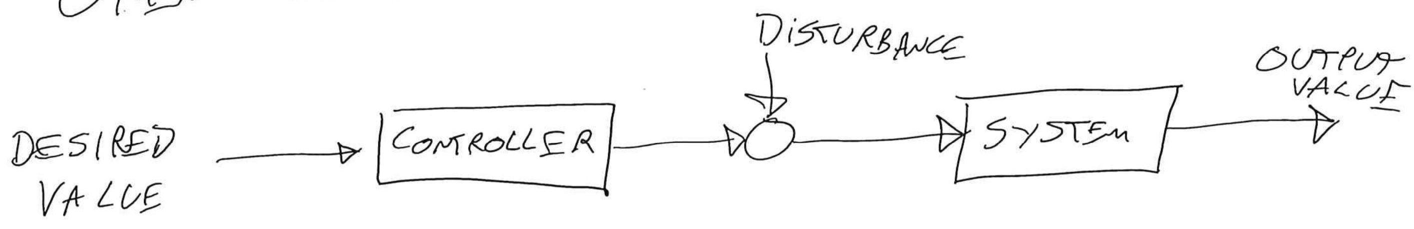
EXERCISE 1

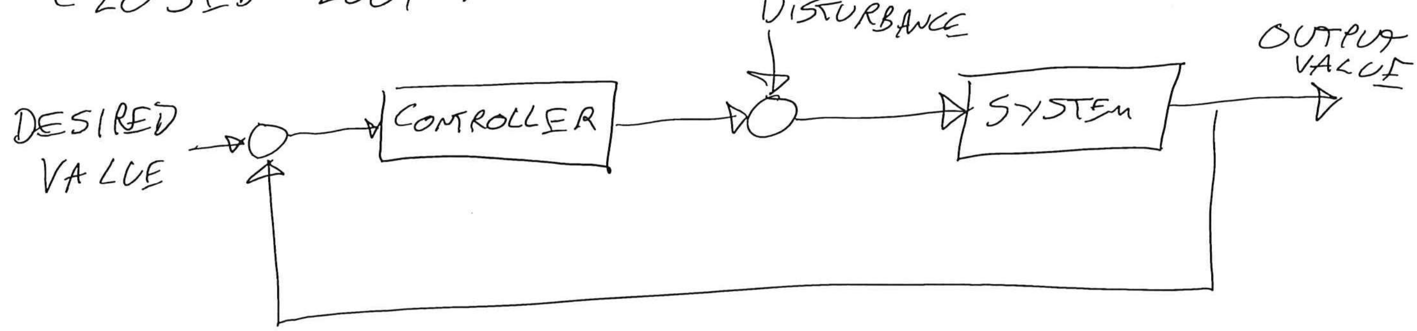
a)

THE DIFFERENCE BETWEEN CLOSED AND OPEN LOOP SYSTEMS IS THE FEEDBACK.

OPEN LOOP:



CLOSED LOOP:



THE BENEFIT OF A CLOSED LOOP SYSTEM IS THE FEEDBACK PART, WHICH ENABLES YOU TO KNOW WHAT THE ACTUAL OUTPUT VALUE IS. THIS CAN BE USED TO COMPENSATE FOR WRONG MODELLING OF THE CONTROLLER OR THE SYSTEM.

b)

THE LAPLACE TRANSFORM CONVERTS INTEGRATION IN THE TIME DOMAIN TO DERIVATION BY S, AND DERIVATION IN THE TIME DOMAIN TO MULTIPLICATION IN LAPLACE DOMAIN, MAKING A UNLINEAR DIFFERENTIAL EQUATION TO A LINEAR EQUATION. THESE DIFFERENTIAL EQUATIONS USUALLY COMES FROM THE DYNAMICS OF THE SYSTEM.

c)

ROS GIVES YOU A LOT OF MODULES/PACKAGES ETC ALREADY MADE, SO YOU DON'T HAVE TO "REINVENT THE WHEEL". ROS IS ALSO VERY TRANSPARENT BECAUSE OF THE MODULAR SYSTEM, CONSISTING OF A MASTER, TALKING TO NODES. IT IS THEREFORE EASY TO SUBSCRIBE TO THE DATA SENT (SUBSCRIBE TO TOPICS).

MENTION 3 OF THESE:

- Motion Planning providing fast and good quality paths and kinematic constraints
- Fast and flexible collision checking
- Integrated Kinematics
- Integrated Perception for Environment Representation
- Standardised Interfaces to Controllers
- Execution and Monitoring
- Kinematic Analysis
- Simulated Robots

d)

The "reality gap" is a problem within ER where the optimization algorithm is able to "over fit" (or exploit features) of a simulation that are not present in the real-world. We can see this effect in the difference in performance between simulation and real-world.

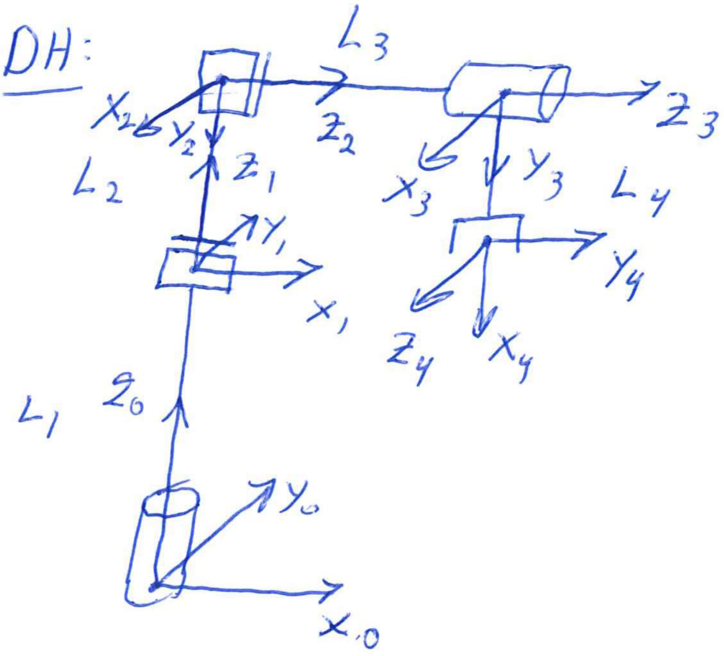
The course described three ways of dealing with the "reality gap".

1. Increase simulation fidelity. We can optimize our simulator to more accurately reflect the real-world. This can be done through better modelling or automatic tuning.
2. Do not accept solutions that behave badly. This can be done through encouraging slow movement, adding noise to the simulation or avoid solutions that perform badly in the real-world (we can spot-test some solutions during evolution to try and avoid controllers that are not performing as they should).
3. Adapt controller online in the real-world. We can accept the reality gap problem during simulation and later optimize the controller in the real-world through reinforcement learning or other means.

Fasit Examen i INF 3480/4380, Vår 2017

Ex. 2

a) DH:



i	a_i	d_i	α_i	θ_i
1	0	L_1	0	θ_1^*
2	0	L_2^*	-90	-90
3	0	L_3^*	0	0
4	L_4	0	90	θ_4^*

$(\Delta\theta_4 + 90)$

$L_2^* = L_2 + \Delta L_2$

$L_3^* = L_3 + \Delta L_3$

$\theta_4^* = \Delta\theta_4 + 90$

b) Forward Kinematics:

$$A_1 = \begin{bmatrix} C_1 & -S_1 C(0) & S_1 S(0) & 0 \cdot C_1 \\ S_1 & C_1 C(0) & -C_1 S(0) & 0 \cdot S_1 \\ 0 & S(0) & C(0) & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} C_{-90} & -S_{-90} C_{-90} & S_{-90} S_{-90} & 0 \cdot C_{-90} \\ S_{-90} & C_{-90} C_{-90} & -C_{-90} S_{-90} & 0 \cdot S_{-90} \\ 0 & S_{-90} & C_{-90} & L_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & L_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} C(0) & -S(0) C(0) & S(0) S(0) & 0 \\ S(0) & C(0) C(0) & -C(0) S(0) & 0 \\ 0 & S(0) & C(0) & L_3^* \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_3^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} C_4 & -S_4 C_{90} & S_4 S_{90} & L_4 C_4 \\ S_4 & C_4 C_{90} & -C_4 S_{90} & L_4 S_4 \\ 0 & S_{90} & C_{90} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C_4 & 0 & S_4 & L_4 C_4 \\ S_4 & 0 & C_4 & L_4 S_4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_1^0 = A_1 = \begin{array}{c} \begin{array}{cccc} & & z_1 & \end{array} \\ \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \cdot \begin{array}{c} A_2 \\ \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & L_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$T_2^0 = A_1 \cdot A_2 = \begin{array}{c} \begin{array}{cccc} & & z_2 & \end{array} \\ \begin{bmatrix} S_1 & 0 & C_1 & 0 \\ -C_1 & 0 & S_1 & 0 \\ 0 & -1 & 0 & L_2^* + L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \cdot \begin{array}{c} A_3 \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_3^* \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$T_3^0 = A_1 \cdot A_2 \cdot A_3 = \begin{array}{c} \begin{array}{cccc} & & z_3 & \end{array} \\ \begin{bmatrix} S_1 & 0 & C_1 & C_1 L_3^* \\ -C_1 & 0 & S_1 & S_1 L_3^* \\ 0 & -1 & 0 & L_2^* + L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ \begin{array}{cccc} & & & O_3 \end{array} \end{array} \cdot \begin{array}{c} A_4 \\ \begin{bmatrix} C_4 & 0 & S_4 & L_4 C_4 \\ S_4 & 0 & C_4 & L_4 S_4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$T_4^0 = A_1 \cdot A_2 \cdot A_3 \cdot A_4 = \begin{array}{c} \begin{bmatrix} S_1 C_4 & C_1 & S_1 S_4 & S_1 C_4 L_4 + C_1 L_3^* \\ -C_1 C_4 & S_1 & -C_1 S_4 & -C_1 C_4 L_4 + S_1 L_3^* \\ -S_4 & 0 & -C_4 & L_1 + L_2^* - S_4 L_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ \begin{array}{cccc} & & & O_4 \end{array} \end{array}$$

⇒ The forward kinematics of the system.

c) The Jacobian:

$$J = \begin{bmatrix} J_v \\ \hline J_w \end{bmatrix} = \begin{array}{cccc} \text{link 1} & \text{link 2} & \text{link 3} & \text{link 4} \\ \left[\begin{array}{c|c|c|c} Z_0 \times \underbrace{(O_4 - O_0)}_{O_4} & Z_1 & Z_2 & Z_3 \times (O_4 - O_3) \\ \hline Z_0 & 0 & 0 & Z_3 \end{array}\right] \end{array}$$

$$Z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = Z_1 \quad ; \quad Z_2 = \begin{bmatrix} C_1 \\ S_1 \\ 0 \end{bmatrix} = Z_3$$

$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} ; \quad O_4 - O_3 = \begin{bmatrix} S_1 C_4 L_4 \\ -C_1 C_4 L_4 \\ -S_4 L_4 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$O_3 = \begin{bmatrix} C_1 L_3^* \\ S_1 L_3^* \\ L_1 + L_2^* \end{bmatrix} ; \quad O_4 = \begin{bmatrix} S_1 C_4 L_4 + C_1 L_3^* \\ -C_1 C_4 L_4 + S_1 L_3^* \\ L_1 + L_2^* - S_4 L_4 \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$$

$Z_0 \times O_4$

$$\begin{array}{ccc|cc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ 0 & 0 & 1 & 0 & 0 \\ A & B & C & A & B \end{array}$$

$$\Rightarrow \underline{-B\,\vec{i} + A\,\vec{j} + 0 \cdot \vec{k}}$$

$Z_3 \times (O_4 - O_3)$

$$\begin{array}{ccccc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ C_1 & S_1 & 0 & C_1 & S_1 \\ a & b & c & a & b \end{array}$$

$$\Rightarrow \underline{\underline{(S_1 \cdot c)\,\vec{i} - (C_1 \cdot c)\,\vec{j} + (C_1 b - S_1 \cdot a)\,\vec{k}}}$$

$$J = \left[\begin{array}{c|c|c|c} +C_1 C_4 L_4 - S_1 L_3^* & 0 & C_1 & -S_1 S_4 L_4 \\ S_1 C_4 L_4 + C_1 L_3^* & 0 & S_1 & +C_1 S_4 L_4 \\ 0 & 1 & 0 & (-C_1^2 C_4 L_4 - S_1^2 C_4 L_4) \\ \hline 0 & 0 & 0 & C_1 \\ 0 & 0 & 0 & S_1 \\ 1 & 0 & 0 & 0 \end{array}\right]$$

$$(-C_1^2 C_4 L_4 - S_1^2 C_4 L_4) \Rightarrow \quad S_1^2 + C_1^2 = 1 \Downarrow \quad \underline{\underline{-C_4 L_4}}$$

EXERCISE 2

d)

FROM FORWARD KINEMATICS:

$$P_x = S_1 C_4 L_4 + C_1 L_3^*$$

$$P_y = -C_1 C_4 L_4 + S_1 L_3^*$$

$$P_z = L_1 + L_2^* - S_4 L_4$$

$$\Downarrow$$

4

$$\boxed{L_2^* = P_z - L_1 + S_4 L_4}$$

$$\boxed{L_3^* = \frac{P_y - C_1 C_4 L_4}{S_1}}$$

SUMMING & SQUARING GIVES:

$$P_x^2 = S_1^2 C_4^2 L_4^2 + 2 S_1 C_4 L_4 C_1 L_3^* + C_1^2 L_3^{*2}$$

$$P_y^2 = C_1^2 C_4^2 L_4^2 - 2 C_1 C_4 L_4 S_1 L_3^* + S_1^2 L_3^{*2}$$

$$P_x^2 + P_y^2 = \cancel{S_1^2} C_4^2 L_4^2 + \cancel{2 S_1 C_4 L_4 C_1 L_3^*} + \cancel{C_1^2} L_3^{*2} + \cancel{C_1^2} C_4^2 L_4^2 - \cancel{2 C_1 C_4 L_4 S_1 L_3^*} + \cancel{S_1^2} L_3^{*2}$$

$$P_x^2 + P_y^2 = C_4^2 L_4^2 + L_3^{*2}$$

$$C_4^2 = \frac{P_x^2 + P_y^2 - L_3^{*2}}{L_4^2}$$

$$C_4^2 = \frac{Px^2 + Py^2 - L_3^{*}}{L_4^2}$$

$$C_4 = \sqrt{\frac{Px^2 + Py^2 - L_3^{*2}}{L_4^2}}$$

$$c^2 + s^2 = 1$$

$$s^2 = 1 - c^2$$

$$s = \sqrt{1 - c^2}$$

(3)

$$\boxed{\theta_4 = \text{ATAN2}\left(\frac{\sqrt{1 - \sqrt{\frac{Px^2 + Py^2 - L_3^{*2}}{L_4^2}}}}{\sqrt{\frac{Px^2 + Py^2 - L_3^{*2}}{L_4^2}}}\right)}$$

↑ + 7%

↓ + 3%

$$T_4^0 = \begin{bmatrix} R_{11} & R_{12} & R_{13} & Px \\ R_{21} & R_{22} & R_{23} & Py \\ R_{31} & R_{32} & R_{33} & Pz \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^0 = T_1^0 \, T_2^1 \, T_3^2 \, T_4^3$$

IDENTITY MATRIX

$$(T_1^0)^{-1} T_4^0 = \cancel{(T_1^0)^{-1} T_1^0} \, T_2^1 \, T_3^2 \, T_4^3$$

$$(T_1)^{-1}\, T_4^0 = T_2^1\, T_3^2\, T_4^3$$

$$(A_1)^{-1}\, T_4^0 = A_2 A_3 A_4$$

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[ r11*cos(T1) + r21*sin(T1), r12*cos(T1) + r22*sin(T1), r13*cos(T1) + r23*sin(T1), Px*cos(T1) + Py*sin(T1)]
[ r21*cos(T1) - r11*sin(T1), r22*cos(T1) - r12*sin(T1), r23*cos(T1) - r13*sin(T1), Py*cos(T1) - Px*sin(T1)]
[                       r31,                       r32,                       r33,                  Pz - L1]
[                         0,                         0,                         0,                        1]
```

$=$

```
[          0, 1,          0,                  L3d]
[ -cos(T4), 0, -sin(T4),          -L4*cos(T4)]
[ -sin(T4), 0,  cos(T4), L2d - L4*sin(T4)]
[          0, 0,          0,                    1]
```

$$\Downarrow$$

$$P_x C_1 + P_y S_1 = L_3^*$$

$$\boxed{P_y C_1 - P_x S_1 = -L_4 C_4}$$

$$P_z - L_1 = L_2^* - L_4 S_4$$

$$C_4 = \frac{P_x S_1 - P_y C_1}{L_4}$$

FROM OVER

$$\boxed{L_3^* = \frac{P_y - C_1 C_4 L_4}{S_1}}$$

$$L_3^* = \frac{P_y - C_1\left(\frac{P_x S_1 - P_y C_1}{\cancel{L_4}}\right)\cancel{L_4}}{S_1}$$

$$L_3^* = \frac{P_y - P_x S_1 C_1 - P_y C_1^2}{S_1}$$

2

$$\boxed{L_3^* = \frac{P_y(1 - C_1^2) - P_x S_1 C_1}{S_1}}$$

EXERCISE 2

e)

YO CAN FIND THE SINGULARITIES BY TAKING THE DETERMINANT OF J_V.

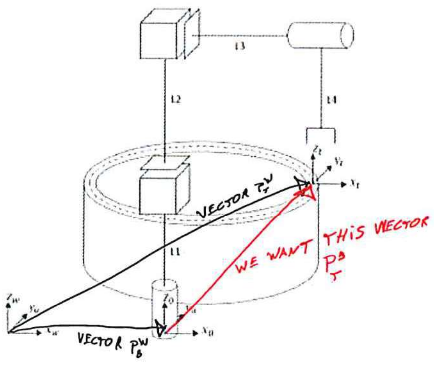
NOT REQUIRED TO ANSWER: HERE WE HAVE A 3x4 MATRIX OF J_V. TO GET IT SQUARE (FOR TAKING THE DERMINANT), YOU CAN TAKE $J_V^T J_V$, AND THEN TAKE THE DETERMINANT.

AT JOINT SPACE SINGULARITY, INFINITE INVERSE KINEMATICS SOLUTIONS MAY EXIST, ALSO SMALL CARTESIAN MOTIONS MAY REQUIRE INFINITE JOINT VELOCITIES, CAUSING A PROBLEM (UNWANTED VELOCITY, TORQUE OR FORCE). WE OFTEN GET JOINT SPACE SINGULARITY WHEN WE GET ALIGNMENT OF THE ROBOT AXES IN SPACE. F.EKS. ROTATION OF ONE CAN BE CANCELLED BY COUNTER ROTATION OF THE OTHER. ALSO CERTAIN KINEMATIC ALIGNMENTS SPESIFIC TO EACH MANIPULATOR CAN CAUSE THESE.

WORKSPACE SINGULARITIES HAPPENS WHEN THE ROBOT IS FULLY EXSTENDED (AND THE END EFFECTOR IS AT IT'S OUTER-MOST PLACE IN SPACE). HERE THE ROBOT LOOSES ONE OR MORE DEGREES OF FREEDOM (RESTRICTED MOVEMENT/MOBILITY IS REDUCED), OR THE ROBOT CANNOT MOVE IN ANY DIRECTION/GETS STUCK.

EXERCISE 2

F)



(WEE NEED THE POINT OF THE TASK COORDINATE FRAME EXPRESSED IN THE BASE COORDINATE FRAME, IN ORDER TO USE THE INVERSE KINEMATIC EQUATIONS FOR CALCULATING THE JOINT CONFIGURATION)

THE CALCULATIONS OVER THE GREEN LINE IS NOT REQUIRED, BUT GOOD FOR SHOWING UNDERSTANDING OF THE TRANSFORMATIONS

$$T_t^B = T_W^B \, T_t^W$$

$$T_t^B = \left(T_B^W\right)^{-1} T_t^W$$

$$T_B^W = \begin{bmatrix} 1 & 0 & 0 & x_B \\ 0 & 1 & 0 & y_B \\ 0 & 0 & 1 & z_B \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\left(T_B^W\right)^{-1} = \begin{bmatrix} 1 & 0 & 0 & -x_B \\ 0 & 1 & 0 & -y_B \\ 0 & 0 & 1 & -z_B \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_t^W = \begin{bmatrix} 1 & 0 & 0 & x_t \\ 0 & 1 & 0 & y_t \\ 0 & 0 & 1 & z_t \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & x_B \\ 0 & 1 & 0 & y_B \\ 0 & 0 & 1 & z_B \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & x_t \\ 0 & 1 & 0 & y_t \\ 0 & 0 & 1 & z_t \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_t^B = \begin{bmatrix} 1 & 0 & 0 & x_t+x_B \\ 0 & 1 & 0 & y_t+y_B \\ 0 & 0 & 1 & z_t+z_B \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

ONLY REQIRED TO WRITE THIS:

$$\Downarrow$$

$$P_t^B = (x_t + x_B,\ y_t + y_B,\ z_t + z_B)$$

THE X, Y AND Z VALUES OF THE POINT OVER WILL BE INSERTED INTO THE INVERSE KINEMATICS EQUATIONS, AND THEN THE JOINT CONFIGURATION WILL PUT THE TCP (TOOL CENTER POINT) IN (0,0,0) IN THE TARGET {t} COORDINATE FRAME.

EXERCISE 3

$$J = \left[\begin{array}{c|c|c|c} C_1C_4L_4 - S_1L_3^* & 0 & C_1 & -S_1S_4L_4 \\ S_1C_4L_4 + C_1L_3^* & 0 & S_1 & +C_1S_4L_4 \\ 0 & 1 & 0 & -C_4L_4 \\ \hline 0 & 0 & 0 & C_1 \\ 0 & 0 & 0 & S_1 \\ 1 & 0 & 0 & 0 \end{array}\right]$$

L_3^* BECOMES L_3, AND COLUMN 3 & 4 GOES AWAY.

$$J_{FORENKLET} = \left[\begin{array}{c|c} -S_1L_3 & 0 \\ +C_1L_3 & 0 \\ 0 & 1 \\ \hline 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{array}\right]$$

$$\mathcal{L} = K - P$$

VARIABEL

$$P = mgh \quad , \quad h = L_1 + L_2^* \quad , \quad L_2^* = L_2 + \Delta L_2$$

$$P = mg(L_1 + L_2^*)$$

$$K = \frac{1}{2} m V \cdot V^T = \frac{1}{2} m v^2$$

$$\xi = J_v(q)\dot{q}$$

3×2 $\qquad$ 2×1

$$\begin{array}{cc} 3\times 2 & 2\times 1 \end{array}$$

$$\begin{bmatrix} -S_1L_3 & 0 \\ +C_1L_3 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \Delta\dot{L}_2 \end{bmatrix} = \begin{bmatrix} -S_1L_3\dot{\theta}_1 \\ C_1L_3\dot{\theta}_1 \\ \Delta\dot{L}_2 \end{bmatrix} = \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}$$

$$V^2 = V_x^2 + V_y^2 + V_z^2$$

$$V^2 = S_1^2L_3^2\dot{\theta}_1^2 + C_1^2L_3^2\dot{\theta}_1^2 + \Delta\dot{L}_2^2$$

$$V^2 = L_3^2\dot{\theta}_1^2 + \Delta\dot{L}_2^2$$

$$K = \frac{mL_3^2\dot{\theta}_1^2}{2} + \frac{m\Delta\dot{L}_2^2}{2}$$

$$\mathcal{L} = \frac{mL_3^2\dot{\theta}_1^2}{2} + \frac{m\Delta\dot{L}_2^2}{2} - mgL_1 - mgL_2 - mg\Delta L_2$$

$$\tau_1 = \frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{\theta}_1} - \frac{\partial\mathcal{L}}{\partial\theta_1}$$

$$\frac{\partial\mathcal{L}}{\partial\theta_1} = 0$$

$$\frac{\partial\mathcal{L}}{\partial\dot{\theta}_1} = mL_3\dot{\theta}_1$$

$$\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{\theta}_1} = mL_3\ddot{\theta}_1$$

$$\tau_1 = mL_3\ddot{\theta}_1$$

$$\tau_2 = \frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\Delta\dot{L}_2} - \frac{\partial\mathcal{L}}{\partial\Delta L_2}$$

$$\frac{\partial\mathcal{L}}{\partial\Delta L_2} = -mg$$

$$\frac{\partial\mathcal{L}}{\partial\Delta\dot{L}_2} = m\Delta\dot{L}_2$$

$$\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\Delta\dot{L}_2} = m\Delta\ddot{L}_2$$

$$\tau_2 = m\Delta\ddot{L}_2 + mg$$

b)

NO $C(q,\dot{q})$ MATRIX

(INGEN SINGEL-DERIVERT, DERFOR INGEN $C(q,\dot{q})$ MATRISE)

$$M \begin{bmatrix} mL_3 & 0 \\ 0 & m \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \Delta \ddot{L}_2 \end{bmatrix} + G \begin{bmatrix} 0 \\ mg \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

EXERCISE 4

a)

IT'S A PD CONTROLLER

b)

YOU CAN ADD AN INTEGRAL TERM.
THE NEW CONTROLLER WILL BE
A "PID" CONTROLLER.

TO GET A PID CONTROLLER,
JUST ADD $+\frac{C_3}{s}$ IN THE BOKS
WHERE C_1 IS.

$$\overbrace{C_1\left(\theta^d(s)-\theta(s)\right)}^{P} + \overbrace{\frac{C_3}{s}\left(\theta^d(s)-\theta(s)\right)}^{I} - D(s) - \overbrace{C_2 s\theta(s)}^{D} - Bs\theta(s) - K\theta(s) = Js^2\theta(s)$$

$$C_1\theta^d(s) - C_1\theta(s) + \frac{C_3}{s}\theta^d(s) - \frac{C_3}{s}\theta(s) - D(s) - C_2 s\theta(s) - Bs\theta(s) - K\theta(s) = Js^2\theta(s)$$

$$C_1\theta^d(s) + \frac{C_3}{s}\theta^d(s) = Js^2\theta(s) + C_1\theta(s) + C_2 s\theta(s) + Bs\theta(s) + K\theta(s) + D(s) + \frac{C_3}{s}\theta(s)$$

$$\theta^d(s)\left(C_1 + \frac{C_3}{s}\right) = \left(Js^2 + Bs + K + C_2 s + C_1 + \frac{C_3}{s}\right)\theta(s) + D(s)$$

$\theta^d(s)$ AND $D(s)$ IS STEP INPUT :

$$D(s) = \frac{D}{s}$$

$$\Theta^d(s) = \frac{\Omega^d}{s}$$

$$\frac{\Omega^d}{s}\left(C_1 + \frac{C_3}{s}\right) = \left(Js^2 + Bs + K + C_2 s + C_1 + \frac{C_3}{s}\right)\Theta(s) + \frac{D}{s}$$

$$\Theta(s) = \frac{\frac{\Omega^d C_1}{s} + \frac{\Omega^d C_3}{s^2} - \frac{D}{s}}{Js^2 + Bs + K + C_2 s + C_1 + \frac{C_3}{s}}$$

FINAL VALUE THEOREM:

$$\lim_{t \to \infty} \theta(t) = \lim_{s \to 0} s\Theta(s)$$

$$= \lim_{s \to 0} \frac{\cancel{s}\frac{\Omega^d C_1}{\cancel{s}} + \cancel{s}\frac{\Omega^d C_3}{s^{\cancel{2}}} - \cancel{s}\frac{D}{\cancel{s}}}{Js^2 + Bs + K + C_2 s + C_1 + \frac{C_3}{s}}$$

$$= \lim_{s \to 0} \frac{\Omega^d C_1 + \frac{\Omega^d C_3}{s} - D}{Js^2 + Bs + K + C_2 s + C_1 + \frac{C_3}{s}} \quad \begin{matrix} | \cdot s \\ | \cdot s \end{matrix}$$

$$= \lim_{s \to 0} \frac{s\Omega^d C_1 + \Omega^d C_3 - Ds}{Js^3 + Bs^2 + Ks + C_2 s^2 + C_1 s + C_3}$$

$$= \lim_{s \to 0} \frac{\cancel{s\Omega^d C_1} + \Omega^d C_3 - \cancel{D s}}{\cancel{J s^3} + \cancel{B s^2} + \cancel{K s} + \cancel{C_2 s^2} + \cancel{C_1 s} + C_3}$$

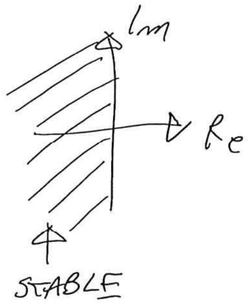
$$= \frac{\Omega^d C_3}{C_3}$$

$$= \underline{\underline{\Omega^d}}$$

BECAUSE OF THE "I" PART WE CAN SEE THAT WE END UP WITH GETTING THE DESIRED ANGLE (Ω^d IS THE STEP REFERENCE VERSION OF θd), SO WE HAVE NO STEADY STATE ERROR. THAT IS THE WHOLE MEANING OF THE INTEGRAL TERM, TO REMOVE STEADY STATE ERROR.

c)

THE STABILITY OF A CONTROL SYSTEM CAN BE INVESTIGATED BY FINDING THE POLES OF THE CHARACTERISTIC POLYNOM OF THE SYSTEM, AND SEE IF THEY ARE IN THE LEFT HALF PLANE:



d)

OUR DESIRED SYSTEM IS CALLED A "CRITICALLY DAMPED" SYSTEM. $\zeta = 1$