EXERCISE 1

a)

$$U_a(s) = \left(\theta^d(s) - \theta(s)\right)\left(C_1 + sC_2\right)$$

$$U_b(s) = C_1\left(\theta^d(s) - \theta(s)\right) - sC_2\theta(s)$$

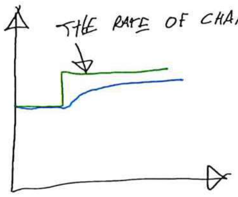
WORKING FURTHER WITH $U_a(s)$:

$$U_a(s) = \left(\theta^d(s) - \theta(s)\right)\left(C_1 + sC_2\right)$$

$$U_a(s) = C_1\left(\theta^d(s) - \theta(s)\right) + sC_2\left(\theta^d(s) - \theta(s)\right)$$

$$U_a(s) = C_1\left(\theta^d(s) - \theta(s)\right) + \cancel{sC_2\theta^d(s)} - sC_2\theta(s)$$

WHEN WORKING WITH A SETPOINT TRACKING SYSTEM, WE ARE TRACKING A CONSTANT/STEP REFERENCE COMMAND. THE DERIVATIVE GAIN WORKS BY LOOKING AT THE RATE OF CHANGE IN THE ERROR, THERFORE THIS TERM IS IRELLEVANT/WILL BE 0.



$\Downarrow$

$$U_a(s) = C_1\left(\theta^d(s) - \theta(s)\right) - sC_2\theta(s)$$

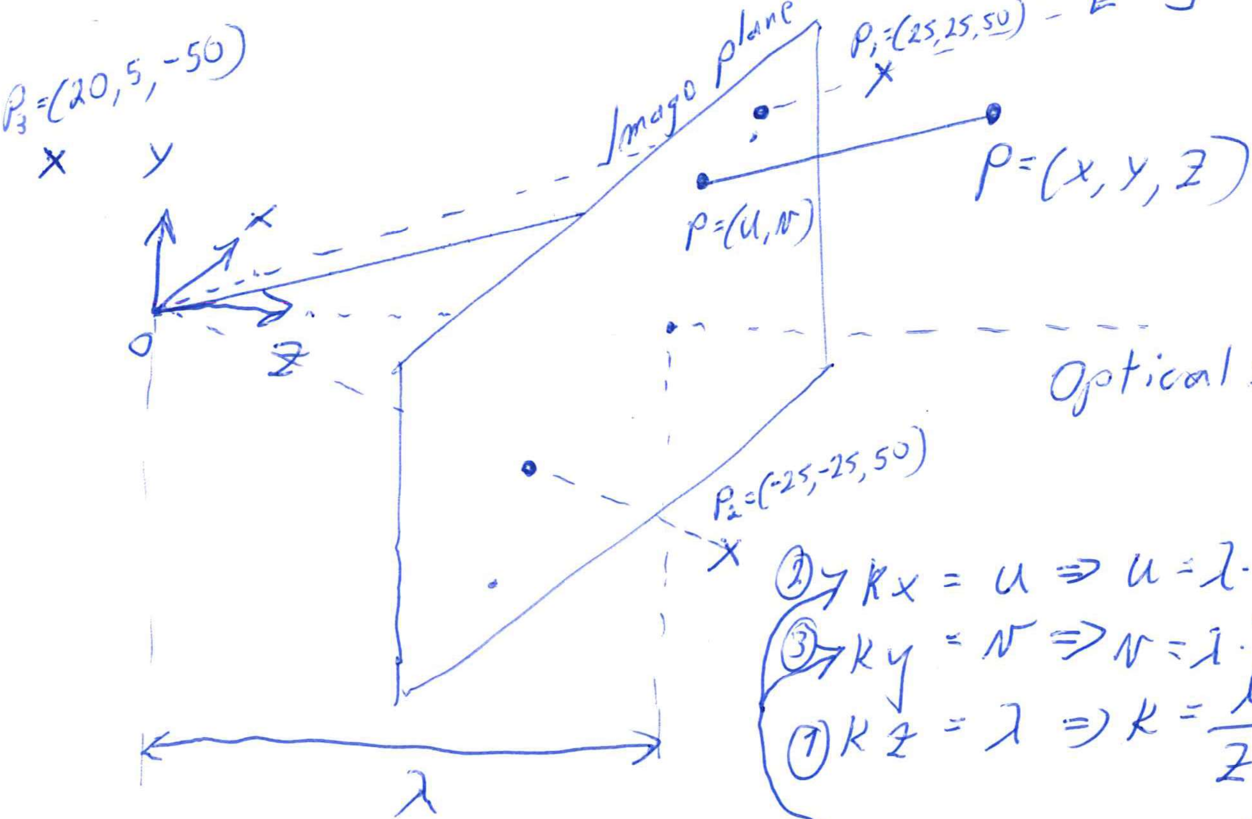
$$\parallel$$

$$U_b(s) = C_1\left(\theta^d(s) - \theta(s)\right) - sC_2\theta(s)$$

Fasit Ex. 1 Master:

a) Camera with focal length, $\lambda = 10$
What is the image plane coordinates given the following 3D points given in th camera frame?

Page 379 and 380 in book: $K \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} u \\ v \\ \lambda \end{bmatrix}$



(2) $kx = u \Rightarrow u = \lambda \cdot \frac{x}{z}$

(3) $ky = v \Rightarrow v = \lambda \cdot \frac{y}{z}$

(1) $kz = \lambda \Rightarrow k = \frac{\lambda}{z}$

(1) $u_1 = 10 \cdot \frac{25}{50} = 5 \qquad v_1 = 10 \cdot \frac{25}{50} = 5$

(2) $u_2 = -5 \; ; \; v_2 = -5$

(3) $u_3 = \text{NaN}$ → The point will not be imaged on the image plane due to a Z-koordinate lower than the focal length λ. The point has to be seen as through a "pin-hole" in the image plane loocking from the camera system.

Fasit master oppg 1 (p.381, 382)

c) - The extrinsic parameters (R, T) are, the position and orientation of the camera frame with respect to the world coordinate frame. Thus the extrinsic parameters are external to the camera and will change according to the set-up.

- The intrinsic parameters are camera dependent, and fixed for a particular camera and does not change when the camera moves. The intrinsic parameters is needed when mapping from 3D world coordinates ~~and~~ to pixel coordinates in the camera $(\lambda, s_x, o_r, s_y, o_c)$

- The process of finding the intrinsic and extrinsic parameters is called camera calibration.

d) ~~The i, j moment of~~ (Page 396-398, 401)
The moments of an object is used to summarise various aspects of the shape and size of objects in the image.
The i, j moment for the k^{th} object, denoted by $m_{ij}(k) = \sum_{r,c} r^i c^j I_k(r,c)$; order of moment i sum of $i+j$.

0-order : m_{00} is the number of pixels in the object

1-order : Is of interest when computing the centroid of an object. (center of mass)

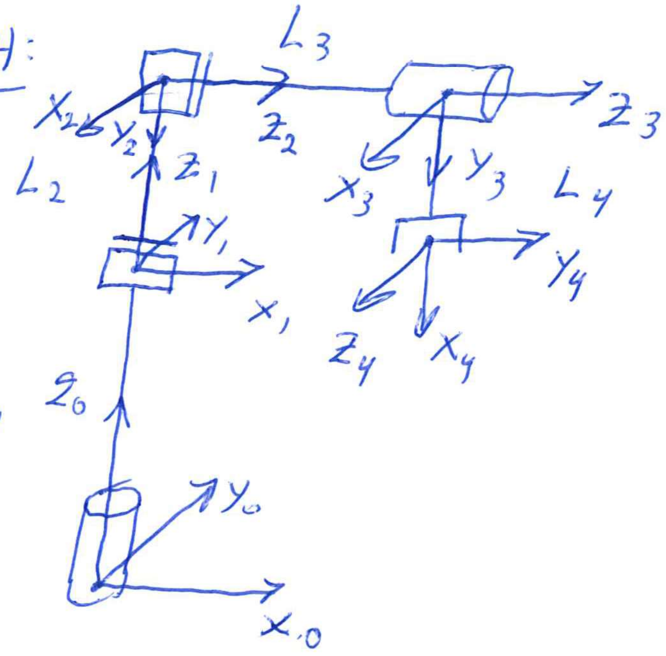
2-order : is relevant when computing the orientation of an object.

These parameters is used to define the position and orientation of the object seen from the camera, and is used in a visual servo system as points and orientation that the robot should move to. (Desired position and orientation)

Fasit Examen i INF 3480/4380, Vår 2017

Ex. 2

a) DH:



i	a_i	d_i	α_i	θ_i
1	0	L_1	0	θ_1^*
2	0	L_2^*	-90	-90
3	0	L_3^*	0	0
4	L_4	0	90	θ_4^*

$(\Delta\theta_4 + 90)$

$L_2^* = L_2 + \Delta L_2$

$L_3^* = L_3 + \Delta L_3$

$\theta_4^* = \Delta\theta_4 + 90$

b) Forward Kinematics:

$$A_1 = \begin{bmatrix} C_1 & -S_1 C(0) & S_1 S(0) & 0\cdot C_1 \\ S_1 & C_1 C(0) & -C_1 S(0) & 0\cdot S_1 \\ 0 & S(0) & C(0) & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} C_{-90} & -S_{-90}C_{-90} & S_{-90}S_{-90} & 0\cdot C_{-90} \\ S_{-90} & C_{-90}C_{-90} & -C_{-90}S_{-90} & 0\cdot S_{-90} \\ 0 & S_{-90} & C_{-90} & L_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & L_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} C(0) & -S(0)C(0) & S(0)S(0) & 0 \\ S(0) & C(0)C(0) & -C(0)S(0) & 0 \\ 0 & S(0) & C(0) & L_3^* \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_3^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} C_4 & -S_4C_{90} & S_4S_{90} & L_4C_4 \\ S_4 & C_4C_{90} & -C_4S_{90} & L_4S_4 \\ 0 & S_{90} & C_{90} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C_4 & 0 & S_4 & L_4C_4 \\ S_4 & 0 & C_4 & L_4S_4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_1^0 = A_1 = \begin{array}{c} \begin{array}{cccc} & & z_1 & \end{array} \\ \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \cdot \begin{array}{c} A_2 \\ \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & L_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$T_2^0 = A_1 \cdot A_2 = \begin{array}{c} \begin{array}{cccc} & & z_2 & \end{array} \\ \begin{bmatrix} S_1 & 0 & C_1 & 0 \\ -C_1 & 0 & S_1 & 0 \\ 0 & -1 & 0 & L_2^*+L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \cdot \begin{array}{c} A_3 \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_3^* \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$T_3^0 = A_1 \cdot A_2 \cdot A_3 = \begin{array}{c} \begin{array}{cccc} & & z_3 & \end{array} \\ \begin{bmatrix} S_1 & 0 & C_1 & C_1L_3^* \\ -C_1 & 0 & S_1 & S_1L_3^* \\ 0 & -1 & 0 & L_2^*+L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ \begin{array}{cccc} & & & = o_3 \end{array} \end{array} \cdot \begin{array}{c} A_4 \\ \begin{bmatrix} C_4 & 0 & S_4 & L_4C_4 \\ S_4 & 0 & C_4 & L_4S_4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$T_4^0 = A_1 \cdot A_2 \cdot A_3 \cdot A_4 = \begin{array}{c} \begin{bmatrix} S_1C_4 & C_1 & S_1S_4 & S_1C_4L_4 + C_1L_3^* \\ -C_1C_4 & S_1 & -C_1S_4 & -C_1C_4L_4 + S_1L_3^* \\ -S_4 & 0 & -C_4 & L_1 + L_2^* - S_4L_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ \begin{array}{cccc} & & & = o_4 \end{array} \end{array}$$

$\Rightarrow$ The forward kinematics of the system

c) <u>The Jacobian:</u>

$$J = \begin{bmatrix} J_v \\ \hline J_w \end{bmatrix} = \begin{array}{cccc} \text{link 1} & \text{link 2} & \text{link 3} & \text{link 4} \end{array} \left[\begin{array}{c|c|c|c} Z_0 \times \underbrace{(O_4 - O_0)}_{O_4} & Z_1 & Z_2 & Z_3 \times (O_4 - O_3) \\ \hline Z_0 & 0 & 0 & Z_3 \end{array}\right]$$

$$Z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = Z_1 \quad ; \quad Z_2 = \begin{bmatrix} C_1 \\ S_1 \\ 0 \end{bmatrix} = Z_3$$

$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} ; \quad O_4 - O_3 = \begin{bmatrix} S_1 C_4 L_4 \\ -C_1 C_4 L_4 \\ -S_4 L_4 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$O_3 = \begin{bmatrix} C_1 L_3^* \\ S_1 L_3^* \\ L_1 + L_2^* \end{bmatrix} ; \quad O_4 = \begin{bmatrix} S_1 C_4 L_4 + C_1 L_3^* \\ -C_1 C_4 L_4 + S_1 L_3^* \\ L_1 + L_2^* - S_4 L_4 \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$$

$Z_0 \times O_4$

$$\begin{array}{ccc|cc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ 0 & 0 & 1 & 0 & 0 \\ A & B & C & A & B \end{array}$$

$$\Rightarrow \underline{-B\,\vec{i} + A\,\vec{j} + 0 \cdot \vec{k}}$$

$Z_3 \times (O_4 - O_3)$

$$\begin{array}{ccc|cc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ C_1 & S_1 & 0 & C_1 & S_1 \\ a & b & c & a & b \end{array}$$

$$\Rightarrow \underline{\underline{(S_1 \cdot c)\,\vec{i} - (C_1 \cdot c)\,\vec{j} + (C_1 b - S_1 a)\,\vec{k}}}$$

$$J = \left[\begin{array}{c|c|c|c} +C_1 C_4 L_4 - S_1 L_3^* & 0 & C_1 & -S_1 S_4 L_4 \\ S_1 C_4 L_4 + C_1 L_3^* & 0 & S_1 & +C_1 S_4 L_4 \\ 0 & 1 & 0 & (-C_1^2 C_4 L_4 - S_1^2 C_4 L_4) \\ \hline 0 & 0 & 0 & C_1 \\ 0 & 0 & 0 & S_1 \\ 1 & 0 & 0 & 0 \end{array}\right]$$

$$\Rightarrow S_1^2 + C_1^2 = 1 \Downarrow \underline{\underline{-C_4 L_4}}$$

EXERCISE 2

d)

FROM FORWARD KINEMATICS:

$$P_x = S_1 C_4 L_4 + C_1 L_3^*$$

$$P_y = -C_1 C_4 L_4 + S_1 L_3^*$$

$$P_z = L_1 + L_2^* - S_4 L_4$$

$$\Downarrow$$

(4)

$$\boxed{L_2^* = P_z - L_1 + S_4 L_4}$$

$$\boxed{L_3^* = \frac{P_y - C_1 C_4 L_4}{S_1}}$$

SUMMING & SQUARING GIVES:

$$P_x^2 = S_1^2 C_4^2 L_4^2 + 2 S_1 C_4 L_4 C_1 L_3^* + C_1^2 L_3^{*2}$$

$$P_y^2 = C_1^2 C_4^2 L_4^2 - 2 C_1 C_4 L_4 S_1 L_3^* + S_1^2 L_3^{*2}$$

$$P_x^2 + P_y^2 = \cancel{S_1^2} C_4^2 L_4^2 + \cancel{2 S_1 C_4 L_4 C_1 L_3^*} + \cancel{C_1^2} L_3^{*2} + \cancel{C_1^2} C_4^2 L_4^2 - \cancel{2 C_1 C_4 L_4 S_1 L_3^*} + \cancel{S_1^2} L_3^{*2}$$

$$P_x^2 + P_y^2 = C_4^2 L_4^2 + L_3^{*2}$$

$$C_4^2 = \frac{P_x^2 + P_y^2 - L_3^{*2}}{L_4^2}$$

$$C_4^2 = \frac{Px^2 + Py^2 - L_3^*}{L_4^2}$$

$$C_4 = \sqrt{\frac{Px^2 + Py^2 - L_3^{*2}}{L_4^2}}$$

$$C^2 + S^2 = 1$$

$$S^2 = 1 - C^2$$

$$S = \sqrt{1 - C^2}$$

3

$$\boxed{\theta_4 = \mathrm{ATAN2}\left(\frac{\sqrt{1 - \sqrt{\frac{Px^2 + Py^2 - L_3^{*2}}{L_4^2}}}}{\sqrt{\frac{Px^2 + Py^2 - L_3^{*2}}{L_4^2}}} \right)}$$

↑ +7%

↓ +3%

$$T_4^0 = \begin{bmatrix} R_{11} & R_{12} & R_{13} & Px \\ R_{21} & R_{22} & R_{23} & Py \\ R_{31} & R_{32} & R_{33} & Pz \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^0 = T_1^0 \, T_2^1 \, T_3^2 \, T_4^3$$

IDENTITY MATRIX

$$(T_1^0)^{-1} T_4^0 = \cancel{(T_1^0)^{-1} T_1^0} \, T_2^1 \, T_3^2 \, T_4^3$$

$$(T_1)^{-1} T^0_4 = T^1_2 \, T^2_3 \, T^3_4$$

$$(A_1)^{-1} T^0_4 = A_2 A_3 A_4$$

```
[ r11*cos(T1) + r21*sin(T1), r12*cos(T1) + r22*sin(T1), r13*cos(T1) + r23*sin(T1), Px*cos(T1) + Py*sin(T1)]
[ r21*cos(T1) - r11*sin(T1), r22*cos(T1) - r12*sin(T1), r23*cos(T1) - r13*sin(T1), Py*cos(T1) - Px*sin(T1)]
[                       r31,                       r32,                       r33,                  Pz - L1]
[                         0,                         0,                         0,                        1]

=

[        0, 1,        0,                L3d]
[ -cos(T4), 0, -sin(T4),        -L4*cos(T4)]
[ -sin(T4), 0,  cos(T4), L2d - L4*sin(T4)]
[        0, 0,        0,                  1]
```

$$\Downarrow$$

$$P_x C_1 + P_y S_1 = L_3^*$$

$$\boxed{P_y C_1 - P_x S_1 = -L_4 C_4}$$

$$P_z - L_1 = L_2^* - L_4 S_4$$

$$C_4 = \frac{P_x S_1 - P_y C_1}{L_4}$$

FROM OVER

$$\boxed{L_3^* = \frac{P_y - C_1 C_4 L_4}{S_1}}$$

$$L_3^* = \frac{P_y - C_1\left(\frac{P_x S_1 - P_y C_1}{\cancel{L_4}}\right)\cancel{L_4}}{S_1}$$

$$L_3^* = \frac{P_Y - P_X S_1 C_1 - P_Y C_1^2}{S_1}$$

2

$$\boxed{L_3^* = \frac{P_Y(1 - C_1^2) - P_X S_1 C_1}{S_1}}$$

EXERCISE 2

e)

YOU CAN FIND THE SINGULARITIES BY TAKING THE DETERMINANT OF J_V.

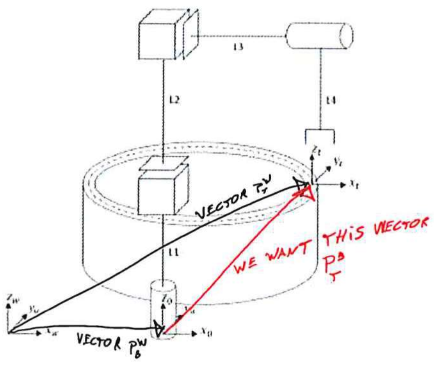
NOT REQUIRED TO ANSWER: HERE WE HAVE A 3x4 MATRIX OF J_V. TO GET IT SQUARE (FOR TAKING THE DERMINANT), YOU CAN TAKE $J_V^T J_V$, AND THEN TAKE THE DETERMINANT.

AT JOINT SPACE SINGULARITY, INFINITE INVERSE KINEMATICS SOLUTIONS MAY EXIST, ALSO SMALL CARTESIAN MOTIONS MAY REQUIRE INFINITE JOINT VELOCITIES, CAUSING A PROBLEM (UNWANTED VELOCITY, TORQUE OR FORCE). WE OFTEN GET JOINT SPACE SINGULARITY WHEN WE GET ALIGNMENT OF THE ROBOT AXES IN SPACE. F.EKS. ROTATION OF ONE CAN BE CANCELLED BY COUNTER ROTATION OF THE OTHER. ALSO CERTAIN KINEMATIC ALIGNMENTS SPESIFIC TO EACH MANIPULATOR CAN CAUSE THESE.

WORKSPACE SINGULARITIES HAPPENS WHEN THE ROBOT IS FULLY EXSTENDED (AND THE END EFFECTOR IS AT IT'S OUTER-MOST PLACE IN SPACE). HERE THE ROBOT LOOSES ONE OR MORE DEGREES OF FREEDOM (RESTRICTED MOVEMENT/MOBILITY IS REDUCED), OR THE ROBOT CANNOT MOVE IN ANY DIRECTION/GETS STUCK.

EXERCISE 2

F)



(WEE NEED THE POINT OF THE TASK COORDINATE FRAME EXPRESSED IN THE BASE COORDINATE FRAME, IN ORDER TO USE THE INVERSE KINEMATIC EQUATIONS FOR CALCULATING THE JOINT CONFIGURATION)

THE CALCULATIONS OVER THE GREEN LINE IS NOT REQUIRED, BUT GOOD FOR SHOWING UNDERSTANDING OF THE TRANSFORMATIONS

$$T_t^B = T_W^B \, T_t^W$$

$$T_t^B = \left(T_B^W\right)^{-1} T_t^W$$

$$T_B^W = \begin{bmatrix} 1 & 0 & 0 & x_B \\ 0 & 1 & 0 & y_B \\ 0 & 0 & 1 & z_B \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\left(T_B^W\right)^{-1} = \begin{bmatrix} 1 & 0 & 0 & -x_B \\ 0 & 1 & 0 & -y_B \\ 0 & 0 & 1 & -z_B \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_t^W = \begin{bmatrix} 1 & 0 & 0 & x_t \\ 0 & 1 & 0 & y_t \\ 0 & 0 & 1 & z_t \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & x_B \\ 0 & 1 & 0 & y_B \\ 0 & 0 & 1 & z_B \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & x_t \\ 0 & 1 & 0 & y_t \\ 0 & 0 & 1 & z_t \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_t^B = \begin{bmatrix} 1 & 0 & 0 & x_t+x_B \\ 0 & 1 & 0 & y_t+y_B \\ 0 & 0 & 1 & z_t+z_B \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

ONLY REQIRED TO WRITE THIS:

$$\Downarrow$$

$$P_t^B = (x_t + x_B,\ y_t + y_B,\ z_t + z_B)$$

THE X, Y AND Z VALUES OF THE POINT OVER WILL BE INSERTED INTO THE INVERSE KINEMATICS EQUATIONS, AND THEN THE JOINT CONFIGURATION WILL PUT THE TCP (TOOL CENTER POINT) IN (0,0,0) IN THE TARGET $\{t\}$ COORDINATE FRAME.

EXERCISE 3

$$J = \left[\begin{array}{c:c:c:c} C_1C_4L_4 - S_1L_3^* & 0 & C_1 & -S_1S_4L_4 \\ S_1C_4L_4 + C_1L_3^* & 0 & S_1 & +C_1S_4L_4 \\ 0 & 1 & 0 & -C_4L_4 \\ \hdashline 0 & 0 & 0 & C_1 \\ 0 & 0 & 0 & S_1 \\ 1 & 0 & 0 & 0 \end{array}\right]$$

L_3^* BECOMES L_3, AND COLUMN 3 & 4 GOES AWAY.

$$J_{FORENKLET} = \left[\begin{array}{c:c} -S_1L_3 & 0 \\ +C_1L_3 & 0 \\ 0 & 1 \\ \hdashline 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{array}\right]$$

$$\mathcal{L} = K - P$$

VARIABEL

$$P = mgh \quad , \quad h = L_1 + L_2^* \quad , \quad L_2^* = L_2 + \Delta L_2$$

$$P = mg(L_1 + L_2^*)$$

$$K = \frac{1}{2} m V \cdot V^T = \frac{1}{2} m V^2$$

$$\xi = J_v(q)\dot{q}$$

3×2 2×1

$$
\begin{array}{cc} 3\times 2 & 2\times 1 \\
\begin{bmatrix} -S_1L_3 & 0 \\ +C_1L_3 & 0 \\ 0 & 1 \end{bmatrix} & \begin{bmatrix} \dot{\theta}_1 \\ \Delta\dot{L}_2 \end{bmatrix} \end{array} = \begin{bmatrix} -S_1L_3\dot{\theta}_1 \\ C_1L_3\dot{\theta}_1 \\ \Delta\dot{L}_2 \end{bmatrix} = \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}
$$

$$V^2 = V_x^2 + V_y^2 + V_z^2$$

$$V^2 = S_1^2L_3^2\dot{\theta}_1^2 + C_1^2L_3^2\dot{\theta}_1^2 + \Delta\dot{L}_2^2$$

$$V^2 = L_3^2\dot{\theta}_1^2 + \Delta\dot{L}_2^2$$

$$K = \frac{mL_3^2\dot{\theta}_1^2}{2} + \frac{m\Delta\dot{L}_2^2}{2}$$

$$\mathcal{L} = \frac{mL_3^2\dot{\theta}_1^2}{2} + \frac{m\Delta\dot{L}_2^2}{2} - mgL_1 - mgL_2 - mg\Delta L_2$$

$$\tau_1 = \frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{\theta}_1} - \frac{\partial\mathcal{L}}{\partial\theta_1}$$

$$\frac{\partial\mathcal{L}}{\partial\theta_1} = 0$$

$$\frac{\partial\mathcal{L}}{\partial\dot{\theta}_1} = mL_3\dot{\theta}_1$$

$$\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{\theta}_1} = mL_3\ddot{\theta}_1$$

$$\tau_1 = mL_3\ddot{\theta}_1$$

$$\tau_2 = \frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\Delta\dot{L}_2} - \frac{\partial\mathcal{L}}{\partial\Delta L_2}$$

$$\frac{\partial\mathcal{L}}{\partial\Delta L_2} = -mg$$

$$\frac{\partial\mathcal{L}}{\partial\Delta\dot{L}_2} = m\Delta\dot{L}_2$$

$$\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\Delta\dot{L}_2} = m\Delta\ddot{L}_2$$

$$\tau_2 = m\Delta\ddot{L}_2 + mg$$

b)

NO $C(q,\dot{q})$ MATRIX

(INGEN SINGEL-DERIVERT, DERFOR INGEN $C(q,\dot{q})$ MATRISE)

$$M\begin{bmatrix} mL_3 & 0 \\ 0 & m \end{bmatrix}\begin{bmatrix} \ddot{\theta}_1 \\ \Delta\ddot{L}_2 \end{bmatrix} + G\begin{bmatrix} 0 \\ mg \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

EXERCISE 4

a)

IT'S A PD CONTROLLER

b)

YOU CAN ADD AN INTEGRAL TERM.
THE NEW CONTROLLER WILL BE
A "PID" CONTROLLER.

TO GET A PID CONTROLLER,
JUST ADD $+\frac{C_3}{s}$ IN THE BOKS
WHERE C_1 IS.

$$\overbrace{C_1(\theta^d(s)-\theta(s))}^{P} + \overbrace{\frac{C_3}{s}(\theta^d(s)-\theta(s))}^{I} - D(s) - \overbrace{C_2 s\theta(s)}^{D} - Bs\theta(s) - K\theta(s) = Js^2\theta(s)$$

$$C_1\theta^d(s) - C_1\theta(s) + \frac{C_3}{s}\theta^d(s) - \frac{C_3}{s}\theta(s) - D(s) - C_2 s\theta(s) - Bs\theta(s) - K\theta(s) = Js^2\theta(s)$$

$$C_1\theta^d(s) + \frac{C_3}{s}\theta^d(s) = Js^2\theta(s) + C_1\theta(s) + C_2 s\theta(s) + Bs\theta(s) + K\theta(s) + D(s) + \frac{C_3}{s}\theta(s)$$

$$\theta^d(s)\left(C_1 + \frac{C_3}{s}\right) = \left(Js^2 + Bs + K + C_2 s + C_1 + \frac{C_3}{s}\right)\theta(s) + D(s)$$

$\theta^d(s)$ AND $D(s)$ IS STEP INPUT :

$$D(s) = \frac{D}{s}$$

$$\theta^d(s) = \frac{\Omega^d}{s}$$

$$\frac{\Omega^d}{s}\left(C_1 + \frac{C_3}{s}\right) = \left(Js^2 + Bs + K + C_2 s + C_1 + \frac{C_3}{s}\right)\theta(s) + \frac{D}{s}$$

$$\theta(s) = \frac{\frac{\Omega^d C_1}{s} + \frac{\Omega^d C_3}{s^2} - \frac{D}{s}}{Js^2 + Bs + K + C_2 s + C_1 + \frac{C_3}{s}}$$

FINAL VALUE THEOREM:

$$\lim_{t \to \infty} \theta(t) = \lim_{s \to 0} s\theta(s)$$

$$= \lim_{s \to 0} \frac{\cancel{s}\frac{\Omega^d C_1}{\cancel{s}} + \cancel{s}\frac{\Omega^d C_3}{s^{\cancel{2}}} - \cancel{s}\frac{D}{\cancel{s}}}{Js^2 + Bs + K + C_2 s + C_1 + \frac{C_3}{s}}$$

$$= \lim_{s \to 0} \frac{\Omega^d C_1 + \frac{\Omega^d C_3}{s} - D}{Js^2 + Bs + K + C_2 s + C_1 + \frac{C_3}{s}} \quad \begin{matrix} | \cdot s \\ | \cdot s \end{matrix}$$

$$= \lim_{s \to 0} \frac{s\Omega^d C_1 + \Omega^d C_3 - Ds}{Js^3 + Bs^2 + Ks + C_2 s^2 + C_1 s + C_3}$$

$$= \lim_{s \to 0} \frac{\cancel{s\Omega^d C_1} + \Omega^d C_3 - \cancel{Ds}}{\cancel{Ds^3} + \cancel{Bs^2} + \cancel{Ks} + \cancel{C_2 s^2} + \cancel{C_1 s} + C_3}$$

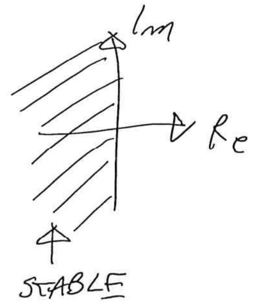
$$= \frac{\Omega^d C_3}{C_3}$$

$$= \underline{\underline{\Omega^d}}$$

BECAUSE OF THE "I" PART WE CAN SEE THAT WE END UP WITH GETTING THE DESIRED ANGLE (Ω^d IS THE STEP REFERENCE VERSION OF θd), SO WE HAVE NO STEADY STATE ERROR. THAT IS THE WHOLE MEANING OF THE INTEGRAL TERM, TO REMOVE STEADY STATE ERROR.

c)

THE STABILITY OF A CONTROL SYSTEM CAN BE INVESTIGATED BY FINDING THE POLES OF THE CHARACTERISTIC POLYNOM OF THE SYSTEM, AND SEE IF THEY ARE IN THE LEFT HALF PLANE:



d)

OUR DESIRED SYSTEM IS CALLED A "CRITICALLY DAMPED" SYSTEM. $\zeta = 1$