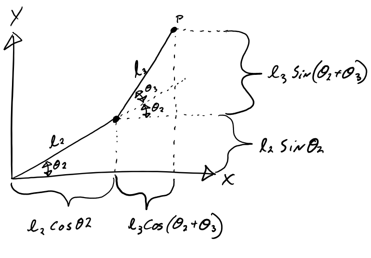

(1) $P_x = l_1 c\theta_2 + l_3 c\theta_{23}$

(2) $P_y = l_2 s\theta_2 + l_3 s\theta_{23}$

SQUARING THE EQUATIONS GIVES:

$$P_x^2 = l_2^2 c^2\theta_2 + l_3^2 c^2\theta_{23} + 2 l_2 l_3 c\theta_2 c\theta_{23}$$

$$P_y^2 = l_2^2 s\theta_2 + l_3^2 s^2\theta_{23} + 2 l_2 l_3 s\theta_2 s\theta_{23}$$

ADDING THEM TOGETHER GIVES:

$$P_x^2 + P_y^2 = l_2^2(c^2\theta_2 + s^2\theta_2) + l_3^2(c^2\theta_{23} + s^2\theta_{23}) + 2 l_2 l_3 (c\theta_2 c\theta_{23} + s\theta_2 s\theta_{23})$$

$\cos(\theta_2+\theta_3) = \cos\theta_2 \cos\theta_3 - \sin\theta_2 \sin\theta_3$

$\sin(\theta_2+\theta_3) = \sin\theta_2 \cos\theta_3 + \cos\theta_2 \sin\theta$

$$P_x^2 + P_y^2 = l_2^2 + l_3^2 + 2 l_2 l_3 (c\theta_2(c\theta_2 c\theta_3 - s\theta_2 s\theta_3) + s\theta_2(s\theta_2 c\theta_3 + c\theta_2 s\theta_3))$$

$$P_x^2 + P_y^2 = l_2^2 + l_3^2 + 2 l_2 l_3 (c^2\theta_2 c\theta_3 - \cancel{c\theta_2 s\theta_2 s\theta_3} + s^2\theta_2 c\theta_3 + \cancel{c\theta_2 s\theta_2 s\theta_3})$$

$$P_x^2 + P_y^2 = l_2^2 + l_3^2 + 2 l_2 l_3 (c\theta_3 (\cancel{c^2\theta_2 + s^2\theta_2}))$$

$$P_x^2 + P_y^2 = l_2^2 + l_3^2 + 2 l_2 l_3 c\theta_3$$

$$c\theta_3 = \frac{P_x^2 + P_y^2 - l_2^2 - l_3^2}{2 l_2 l_3} = D$$

$$\theta_3 = \mathrm{Atan2}\left(\pm\sqrt{1 - D^2}, D\right)$$

FOR θ_2 :

MULTIPLYING (1) WITH $c\theta_2$ AND (2) WITH $s\theta_2$:

$$P_x = l_1 c\theta_2 + l_3 c\theta_{23}$$

$$P_x = l_2 c\theta_2 + l_3(c\theta_2 c\theta_3 - s\theta_2 s\theta_3)$$

$$P_x = l_2 c\theta_2 + l_3 c\theta_2 c\theta_3 - l_3 s\theta_2 s\theta_3 \quad | \cdot c\theta_2$$

$$c\theta_2 P_x = l_2 c^2\theta_2 + l_3 c^2\theta_2 c\theta_3 - l_3 c\theta_2 s\theta_2 s\theta_3$$

$$s\theta_2 P_y = l_2 s^2\theta_2 + l_3 s^2\theta_2 c\theta_3 + l_3 c\theta_2 s\theta_2 s\theta_3$$

$$P_y = l_2 s\theta_2 + l_3 (s\theta_2 c\theta_3 + c\theta_2 s\theta_3)$$

$$\Big| \cdot s\theta_2$$

ADDING THEM TOGETHER GIVES:

$$c\theta_1 P_x + s\theta_1 P_y = l_2\cancel{(c^2\theta_2 + s^2\theta_2)} + l_3 c\theta_2 \cancel{(c^2\theta_2 + s^2\theta_2)}$$

(3) $$c\theta_2 P_x + s\theta_2 P_y = l_2 + l_3 c\theta_3$$

MULTIPLYING (1) WITH $-s\theta_2$ AND (2) WITH $c\theta_2$

$$P_x = l_2 c\theta_2 + l_3 c\theta_{23}$$
$$P_x = l_2 c\theta_2 + l_3 (c\theta_2 c\theta_3 - s\theta_2 s\theta_3)$$
$$P_x = l_2 c\theta_2 + l_3 c\theta_2 c\theta_3 - l_3 s\theta_2 s\theta_3 \qquad \Big| \cdot - s\theta_2$$
$$-s\theta_2 P_x = -l_2 c\theta_2 s\theta_2 - l_3 s\theta_2 c\theta_2 c\theta_3 + l_3 s^2\theta_2 s\theta_3$$

$$P_y = l_2 s\theta_2 + l_3 s\theta_{23}$$
$$P_y = l_2 s\theta_2 + l_3 (s\theta_2 c\theta_3 + c\theta_2 s\theta_3) \qquad \Big| \cdot c\theta_2$$
$$c\theta_2 P_y = l_2 s\theta_2 c\theta_2 + l_3 s\theta_2 c\theta_2 c\theta_3 + l_3 c^2\theta_2 s\theta_3$$

ADDING THEM TOGETHER GIVES:

$$-s\theta_2 P_x + c\theta_2 P_y = l_3 s\theta_3 \cancel{(c^2\theta_2 + s^2\theta_2)}$$

(4) $$-s\theta_2 P_x + c\theta_2 P_y = l_3 s\theta_3$$

MULTIPLYING (3) BY P_x AND (4) BY P_y:

$$c\theta_2 P_x^2 + s\theta_2 P_y P_x = l_2 P_x + l_3 c\theta_3 P_x$$
$$-s\theta_2 P_x P_y + c\theta_2 P_y^2 = l_3 s\theta_3 P_y$$

ADDING THEM TOGETHER GIVES:

$$c\theta_1 (P_x^2 + P_y^2) = P_x (l_2 + l_3 c\theta_3) + P_y l_3 s\theta_3$$

$$c\theta_2 = \frac{P_x (l_2 + l_3 c\theta_3) + P_y l_3 s\theta_3}{P_x^2 + P_y^2} = D2$$

$$s\theta_2 = \pm\sqrt{1 - D2^2},$$

$$\theta_2 = \mathrm{ATAN2}\left(\pm\sqrt{1-D2^2},\ D2\right)$$