Inverse kinematics - Algebraic Solution

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FROM FORWARD KINEMATICS, WE HAVE

$$(I) \quad P_x = C_1C_2L_3^* + C_1S_2L_2 - S_1L_{OFF}$$

$$(II) \quad P_y = S_1C_2L_3^* + S_1S_2L_2 + C_1L_{OFF}$$

$$(III) \quad P_z = -S_2L_3^* + C_2L_2 + L_1$$

MULTIPLYING (I) WITH S_1 AND (II) WITH C_1 GIVES

$$(1) \quad S_1P_x = S_1C_1C_2L_3^* + S_1C_1S_2L_2 - S_1^2L_{OFF}$$

$$(2) \quad C_1P_y = S_1C_1C_2L_3^* + S_1C_1S_2L_2 + C_1^2L_{OFF}$$

AND FURTHER TAKING (2)-(1) LEADS TO

$$(3) \quad C_1P_y - S_1P_x = L_{OFF}$$

WHICH IS AN EQUATION WITH ONLY ONE VARIABLE, AND GIVES θ_1.

WE HAVE

$$a\cos(\theta) + b\sin(\theta) = c \iff \theta = \mathrm{ATAN2}(b, a) \pm \mathrm{ATAN2}\left(\sqrt{a^2+b^2-c^2}, c\right) \quad (\#)$$

THUS

$$\underline{\underline{\theta_1 = \text{ATAN2}(-P_x, P_y) \pm \text{ATAN2}\left(\sqrt{P_Y^2 + P_X^2 + L_{OFF}^2},\ L_{OFF}\right)}}$$

USING THE SAME TRICK AS ABOVE, WE CAN SOLVE FOR θ_2

MULTIPLYING (I) WITH C_1 AND (II) WITH S_1 LEADS TO

$$(4)\quad C_1 P_X = C_1^2 C_2 L_3^* + C_1^2 S_2 L_2 - C_1 S_1 L_{OFF}$$

$$(5)\quad S_1 P_Y = S_1^2 C_2 L_3^* + S_1^2 S_2 L_2 + C_1 S_1 L_{OFF}$$

AND GIVES

$$(6)\qquad C_1 P_X + S_1 P_Y = C_2 L_3^* + S_2 L_2$$

SQUARING (3) AND (6) GIVES FURTHER

$$(7)\quad S_1^2 P_X^2 + C_1^2 P_Y^2 - 2 S_1 C_1 P_X P_Y = L_{OFF}^2$$

$$(8)\quad C_1^2 P_X^2 + S_1^2 P_Y^2 + 2 S_1 C_1 P_X P_Y = (C_2 L_3^* + S_2 L_2)^2$$

AND THEN ADDING (7) AND (8)

$$P_X^2 + P_Y^2 = L_{OFF}^2 + (C_2 L_3^* + S_2 L_2)^2$$

$$(9) \quad \underbrace{\sqrt{P_X^2 + P_Y^2 + L_{OFF}^2}}_{P_1} = \pm\, C_2 L_3^* + S_2 L_2$$

FROM (III) WE HAVE

$$(10) \quad \underbrace{P_Z - L_1}_{P_2} = -S_2 L_3^* + C_2 L_2$$

MULTIPLYING THE POSITIVE VERSION OF (9) BY S_2, AND (10) WITH C_2

$$P_1 S_2 = C_2 S_2 L_3^* + S_2^2 L_2$$

$$P_2 C_2 = -S_2 C_2 L_3^* + C_2^2 L_2$$

AND THEN ADDING TOGETHER GIVES

$$L_2 = P_1 S_2 + P_2 C_2$$

SO ACCORDING TO (#)

$$\underline{\underline{\theta_2^+ = \mathrm{ATAN2}\left(\sqrt{P_X^2 + P_Y^2 + L_{OFF}^2},\ P_Z - L_1\right) \pm \mathrm{ATAN2}\left(\sqrt{(P_Z - L_1)^2 + P_X^2 + P_Y^2 + L_{OFF}^2 + L_2^2},\ L_2\right)}}$$

SIMILARLY, MULTIPLYING THE NEGATIVE VERSION OF (9) BY $-S_2$, (10) WITH C_2 AND ADDING TOGETHER GIVES

$$L_2 = -P_1 S_2 + P_2 C_2$$

LEADING TO

$$\underline{\underline{\theta_2^- = \mathrm{ATAN2}\left(-\sqrt{P_x^2 + P_y^2 + L_{OFF}^2},\ P_z - L_1\right) \pm \mathrm{ATAN2}\left(\sqrt{(P_z - L_1)^2 + P_x^2 + P_y^2 + L_{OFF}^2 + L_2^2},\ L_2\right)}}$$

ACCORDING TO (#).

FINALLY L_3^* CAN BE SOLVED BY SQUARING AND SUMMING (6) AND (III)

$$(P_z - C_2 L_2 - L_1)^2 = (-S_2 L_3^*)^2$$

$$(C_1 P_x + S_1 P_y - S_2 L_2)^2 = (C_2 L_3^*)^2$$

SO

$$L_3^{*2} = (P_z - C_2 L_2 - L_1)^2 + (C_1 P_x + S_1 P_y - S_2 L_2)^2$$

WHERE

$$L_3^* = d_3 + L_3$$

AND LASTLY WE HAVE

$$\underline{\underline{d_3 = \pm\sqrt{(P_z - C_2 L_2 - L_1)^2 + (C_1 P_x + S_1 P_y - S_2 L_2)^2} - L_3}}$$