Groupsession5, velocity kinematics

Saturday, February 24, 2018 1:55 PM

Schedule:
- Velocity kinematics
  - Basics
  - Velocities of a rigid body
  - Deriving Jacobian
- 2014 exam - jacobian Co-Op
- (Menti)
- Weekly tasks

- Explain why we do velocity kinematics.
  - Why? We want to find the velocities of the end-effector, relative to the base. At what linear velocities does it translate, and at what angular velocities does it rotate?
  - The problem is that we can only measure the change in joint values.
  - How do we set up an equation for these velocities?
  -
    - **Therefore, we want to come up with the following mappings:**

      $$v_n^0 = J_v \dot{q}$$
      $$\omega_n^0 = J_\omega \dot{q}$$

      - **Thus J_v and J_ω are 3xn matrices**
    - **we can combine these into the following:** $\xi = J\dot{q}$
      - **where:**

      $$\xi = \begin{bmatrix} v_n^0 \\ \omega_n^0 \end{bmatrix}$$
      $$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix}$$

    - ***J* is called the Jacobian**
      - **6x*n* where *n* is the number of joints**

- We want to understand how to set up such jacobian matrix, but then we first need to understand velocities for rigid bodies.
  - What is velocities, in general? Distance over time…
  - Practical example with a stick… Where the stick is the rigid body, and my hand is first a translational joint and then a rotational joint. At rotational; draw a rotating disc and illustrate the velocity of an arbitrary point on the rim.

- **When a rigid body rotates about a fixed axis, every point moves in a circle**
  - **Let *k* represent the fixed axis of rotation, then the angular velocity is:**

$$\omega = \dot{\theta}\hat{k}$$

  - **The velocity of any point on a rigid body due to this angular velocity is:** $v = \omega \times r$

○

  - **Where *r* is the vector from the axis of rotation to the point**

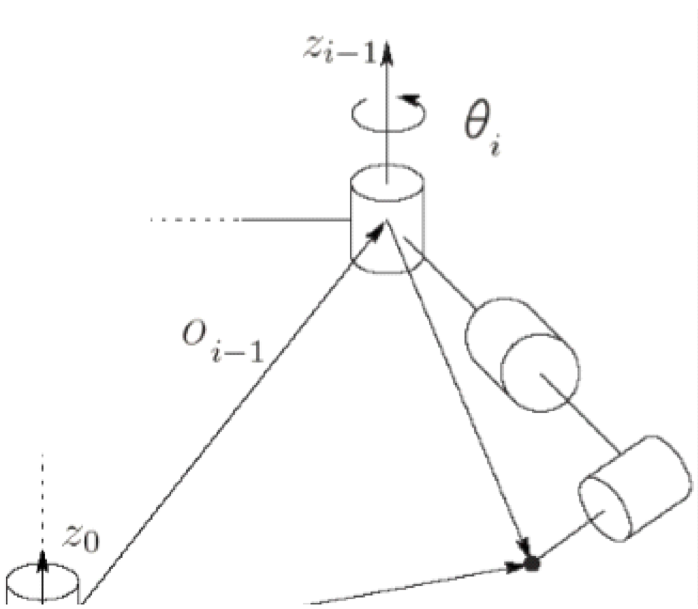
- **When a rigid body translates, all points attached to the body have the same velocity**

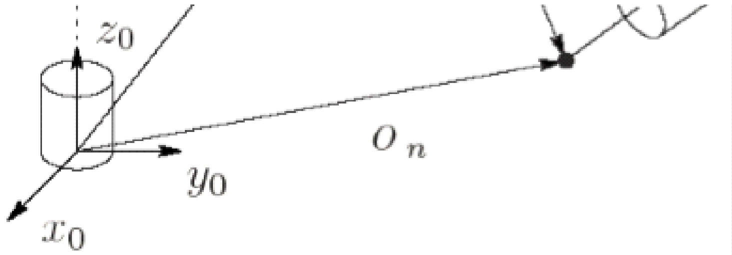
○ The problem in velocity kinematics is that all we know is the values of our joint variables. We thus want to define a transformation from changes in joint variables to changes in velocities at the tip, relative to base.
  - The transformation need to represent how the change in each joint contribute to the change at our tip.
  - Open page 137
    - We have to look at the change in a joint angle q_i, while all other joints angles are considered constant. Show a practical exercise of looking at the coordinate axis in my hand while moving one joint in my arm and keeping the others constant. Repeat the exercise while moving to joints, to prove the point that the movement of the tip is now different.
  - How to calculate the changes in one joint while keeping all other joints still? There are two ways:
    - Either by taking the cross product of different coordinate axis.
      - **The i^{th} column of J_v is given by:**

$$J_{v_i} = \begin{cases} z_{i-1} \times (o_n - o_{i-1}) & \text{for } i \text{ revolute} \\ z_{i-1} & \text{for } i \text{ prismatic} \end{cases}$$

The i^{th} column of J_ω is given by:

$$J_{\omega_i} = \begin{cases} z_{i-1} & \text{for } i \text{ revolute} \\ 0 & \text{for } i \text{ prismatic} \end{cases}$$

      - 



- Or using partial derivatives. You might already know the general Jacobian matrix from precursory math courses? Look at cheatsheet page four!

$$\mathbf{J} = \begin{bmatrix} \dfrac{\partial \mathbf{f}}{\partial x_1} & \cdots & \dfrac{\partial \mathbf{f}}{\partial x_n} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f_1}{\partial x_1} & \cdots & \dfrac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial f_m}{\partial x_1} & \cdots & \dfrac{\partial f_m}{\partial x_n} \end{bmatrix}$$

- Both of these methods result in us having a matrix, our jacobian. With this tool we can represent the contribution of all the change in each joint variables to the change in end-effector position, relative to base.

- How to use this tool - this jacobian-matrix. In this explanation we will have show the equation for velocity, and then explain why we use Jacobian.

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Jacobian Matrix

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \left[\frac{dh(q)}{dq} \right]_{6\times n} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_n \end{bmatrix}_{n\times 1}$$

Jacobian is a function of q, it is not a constant!

$$\begin{bmatrix} \dfrac{\partial h_1}{\partial q_1} & \dfrac{\partial h_1}{\partial q_2} & \cdots & \dfrac{\partial h_1}{\partial q_n} \end{bmatrix}$$

Jacobian Matrix

- $$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \left[\frac{dh(q)}{dq}\right]_{6\times n} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_n \end{bmatrix}_{n\times 1}$$

  Jacobian is a function of q, it is not a constant!

  $$J = \left(\frac{dh(q)}{dq}\right)_{6\times n} = \begin{bmatrix} \frac{\partial h_1}{\partial q_1} & \frac{\partial h_1}{\partial q_2} & \cdots & \frac{\partial h_1}{\partial q_n} \\ \frac{\partial h_2}{\partial q_1} & \frac{\partial h_2}{\partial q_2} & \cdots & \frac{\partial h_2}{\partial q_n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial h_6}{\partial q_1} & \frac{\partial h_6}{\partial q_2} & \cdots & \frac{\partial h_6}{\partial q_n} \end{bmatrix}_{6\times n}$$

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