Control of AUVs and USVs

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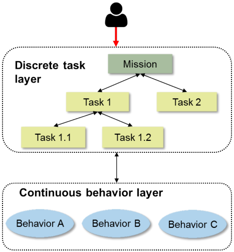
Outline

Context

Possible view of an autonomy system:

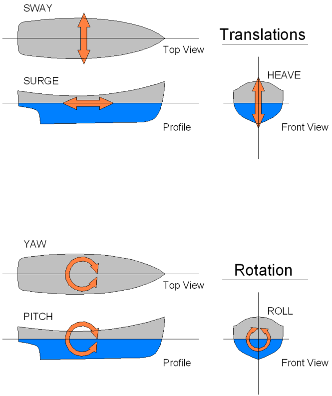
- High level planning - minutes
- Motion planning - several seconds
- Path following / guidance - < 1 sec
- Heading and speed control - < 1 sec



In addition: Decision making, navigation, sensor processing, image analysis etc.
Focus today: Guidance and control

Degrees of freedom

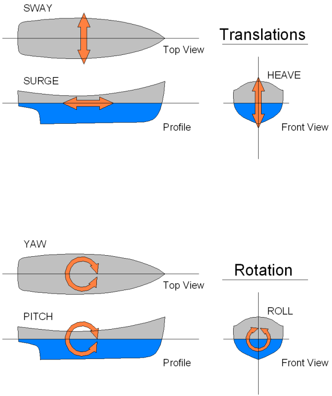
- Any marine craft has 6 DOF
- Horizontal:
  - Surge - u
  - Sway - v
  - Yaw - ψ
- Other:
  - Heave - w
  - Pitch - θ
  - Roll - ϕ



(Images from https://en.wikipedia.org/w/index.php?curid=5456221)

Degrees of freedom

- Maneuvering USVs: Yaw and surge
- Maneuvering AUVs: Also pitch
- Roll: Fast acceleration - seasick!
- Most vehicles are usually passively stabilized in roll



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Reference frames

There are many reference frames in use in maritime systems. The three most important are:

- Earth-Centered Earth-Fixed (ECEF)
  - Latitude, Longitude, Height
- North East Down (NED)
  - Why down? Right hand rule on compass.
- Body frame
  - Forward, starboard (=right), down

Velocity in BODY and NED

Velocities are usually measured in BODY, and Newtonian laws also gives them in BODY. Hence they must be converted to NED. Euler Angles:

- Roll (ϕ), pitch (θ), yaw(ψ)
- Any orientation can be decomposed into three *principal rotations* around the x (roll), y (pitch) and z (yaw) axis.

$$\boldsymbol{R}_x(\phi) \triangleq \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi) & \cos(\phi) \end{bmatrix} \tag{1}$$

$$\boldsymbol{R}_y(\theta) \triangleq \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix} \tag{2}$$

$$\boldsymbol{R}_z(\psi) \triangleq \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

Linear velocity in NED

Combine the rotations in the *zyx* convention

$$\boldsymbol{R}_b^n(\boldsymbol{\Theta}_{nb}) \triangleq R_z(\psi)R_y(\theta)R_x(\phi) \tag{4}$$

$$\boldsymbol{R}_n^b(\boldsymbol{\Theta}_{nb}) = R_x^T(\phi)R_y^T(\theta)R_z^T(\psi) \tag{5}$$

The velocity in NED is then:

$$\dot{\boldsymbol{p}}_{b/n}^n = \boldsymbol{R}_b^n(\boldsymbol{\Theta}_{nb})\boldsymbol{v}_{b/n}^b \tag{6}$$

Linear velocity in NED

Lets write it out for clarity

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} c\psi c\theta & -s\psi c\phi + c\psi s\theta s\phi & s\psi s\phi + c\psi s\theta c\phi \\ s\psi c\theta & c\psi c\phi + s\psi c\theta s\psi & -c\psi s\phi + s\psi s\theta c\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} \tag{7}$$

Angular velocity in NED

From measured angular rates to euler angle rates:

$$\dot{\boldsymbol{\Theta}}_{nb} = \boldsymbol{T}_{\boldsymbol{\Theta}}(\boldsymbol{\Theta}_{nb})\boldsymbol{\omega}^{b}_{b/n} \tag{8}$$

Where

$$\boldsymbol{T}_{\Theta}(\boldsymbol{\Theta}_{nb}) = \begin{bmatrix} 1 & s\phi t\theta & c\phi t\theta \\ 0 & c\phi & -s\phi \\ 0 & \frac{s\phi}{c\theta} & \frac{c\phi}{c\theta} \end{bmatrix} \tag{9}$$

Kinematics

Linear + angular velocities = kinematics

$$\begin{bmatrix} \dot{\boldsymbol{p}}^n_{b/n} \\ \dot{\boldsymbol{\Theta}}_{nb} \end{bmatrix} = \begin{bmatrix} \boldsymbol{R}^n_b(\boldsymbol{\Theta}_{nb}) & \boldsymbol{0}_{3\times 3} \\ \boldsymbol{0}_{3\times 3} & \boldsymbol{T}_\Theta(\boldsymbol{\Theta}_{nb}) \end{bmatrix} \begin{bmatrix} \boldsymbol{v}^b_{b/n} \\ \boldsymbol{\omega}^b_{b/n} \end{bmatrix} \tag{10}$$

$$\dot{\boldsymbol{\eta}} = \boldsymbol{J}(\boldsymbol{\eta})\boldsymbol{\nu} \tag{11}$$

Kinematics 3 DOF

The 3 DOF kinematics in component form becomes:

$$
\begin{aligned}
\dot{x} &= u\cos(\psi) - v\sin(\psi) && \text{(12a)}\\
\dot{y} &= u\sin(\psi) + v\cos(\psi) && \text{(12b)}\\
\dot{\psi} &= r && \text{(12c)}
\end{aligned}
$$

We see that, when v (sway) is not zero, then the velocity vector of the vehicle points in a different direction from the vehicle it self. The difference is called sideslip.

Kinematics 3 DOF

- Heading angle ψ: The direction of the body.
- Course angle χ: The direction of the velocity vector.
- Sideslip angle $\beta \triangleq \chi - \psi$.

Sideslip can be found from BODY velocities as

$$\beta = \arctan(\frac{v}{u}) = \arcsin(\frac{v}{U}) \tag{13}$$

where $U \triangleq \sqrt{u^2 + v^2}$ is the total speed of the vehicle.

Kinematics 3 DOF

The 3 DOF kinematics can be written:

$$\dot{x} = U\cos(\chi) \tag{14a}$$

$$\dot{y} = U\sin(\chi) \tag{14b}$$

$$\dot{\chi} = r + \frac{\dot{u}v - \dot{v}u}{U^2} \tag{14c}$$

Simpler in $\dot{x}$ and $\dot{y}$, more difficult in χ, which now includes dynamics. It is theoretically possible for the vehicle to turn without the course changing!

Dynamics: Maneuvering VS Dynamic positioning

Maneuvering

- The ship is moving at some positive speed, which is sometimes considered constant
- No frequency dependencies in the hydrodynamic parameters (i.e. no waves)
- Most vehicles are underactuated in this domain

Dynamic positioning

- The vehicle stands stillish, but are affected by waves
- Frequency-dependent models
- Wave spectra important
- Thrusters are of more use at low speeds, hence it is more common with fully actuated vehicles

Dynamics

The complete, robot-like, dynamics equation from [Fossen, 2011]:

$$\boldsymbol{M}\dot{\boldsymbol{\nu}} + \boldsymbol{C}(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{D}(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{g}(\boldsymbol{\eta}) = \tau + \tau_{wind} + \tau_{wave} \tag{15}$$

- $\boldsymbol{M}$: Mass and *added mass* matrix
- $\boldsymbol{C}$: Coriolis and centripetal matrix (stuff that happens because we turn)
- $\boldsymbol{D}$: Damping matrix
- $\boldsymbol{g}$: Gravitational effects
- τ: Forces and moments from actuators
- τ_{wind} and τ_{wave}: Forces and moments from environment

Dynamics

3 DOF case with linear damping:

- Roll, pitch and heave is zero
- We are floating or neutrally buoyant: ignore gravity
- We assume linear damping: $\boldsymbol{D}(\boldsymbol{\nu})\boldsymbol{\nu} = \boldsymbol{D}\boldsymbol{\nu}$
- Ship design: port-starboard symmetry, BODY frame located in center of gravity

Dynamics

3 DOF case: 10 parameters

$$\boldsymbol{M} = \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & m_{23} \\ 0 & m_{23} & m_{33} \end{bmatrix} \tag{16}$$

$$\boldsymbol{D} = \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & d_{23} \\ 0 & d_{32} & d_{33} \end{bmatrix} \tag{17}$$

$$\boldsymbol{C}(\boldsymbol{\nu}) = \begin{bmatrix} 0 & 0 & -m_{22}\,v - m_{23}\,r \\ 0 & 0 & m_{11}\,u \\ m_{22}\,v + m_{23}\,r & -m_{11}\,u & 0 \end{bmatrix} \tag{18}$$

Dynamics

How to find the parameters?

- Ask your local hydrodynamics expert
- Experiments and curve fitting
  - Turning circle
  - Kempf's zigzag maneuver
  - Pull-out maneuver
  - Stopping trials
- The dark side: can machine learning be used?

Complete vehicle model in 3DOF

Vehicle model of a ship with rudder and propeller in component form.

$$\dot{x} = u\cos(\psi) - v\sin(\psi), \tag{19a}$$

$$\dot{y} = u\sin(\psi) + v\cos(\psi), \tag{19b}$$

$$\dot{\psi} = r, \tag{19c}$$

$$\dot{u} = F_u(u, v, r) + \tau_u, \tag{19d}$$

$$\dot{v} = X(u)r + Y(u)v, \tag{19e}$$

$$\dot{r} = F_r(u, v, r) + \tau_r. \tag{19f}$$

Note: Underactuation in sway

Complete vehicle model in 3DOF

A quick look on $F_u(v,r)$, $F_r(u,v,r)$, $X(u)$ and $Y(u)$..

$$F_u(u,v,r) \triangleq \frac{1}{m_{11}}(m_{22}v + m_{23}r)r - \frac{d_{11}}{m_{11}}u \tag{20}$$

$$X(u) \triangleq \frac{m_{23}^2 - m_{11}m_{33}}{m_{22}m_{33} - m_{23}^2}u + \frac{d_{33}m_{23} - d_{23}m_{33}}{m_{22}m_{33} - m_{23}^2} \tag{21}$$

$$Y(u) \triangleq \frac{(m_{22} - m_{11})m_{23}}{m_{22}m_{33} - m_{23}^2}u - \frac{d_{22}m_{33} - d_{32}m_{23}}{m_{22}m_{33} - m_{23}^2} \tag{22}$$

$$\begin{aligned} F_r(u,v,r) \triangleq & \frac{m_{23}d_{22} - m_{22}(d_{32} + (m_{22} - m_{11})u)}{m_{22}m_{33} - m_{23}^2}v \\ & + \frac{m_{23}(d_{23} - m_{11}u) - m_{22}(d_{33} + m_{23}u)}{m_{22}m_{33} - m_{23}^2}r \end{aligned} \tag{23}$$

Surge and Yaw controllers

Surge and yaw

$$\dot{u} = F_u(u, v, r) + \tau_u, \tag{24}$$
$$\dot{r} = F_r(u, v, r) + \tau_r. \tag{25}$$

can be controlled using feedback linearizing controllers:

$$\tau_u = -F_u(u, v, r) + \frac{d_{11}}{m_{11}} u_d + \dot{u}_d - k_u(u - u_d), \tag{26}$$
$$\tau_r = -F_r(u, v, r) + \ddot{\psi} - k_\psi(\psi - \psi_d) - k_r(\dot{\psi} - \dot{\psi}_d), \tag{27}$$

k_u, k_ψ and k_r are constant, positive gains. Also possible: PID control.

The dark side: Machine learning

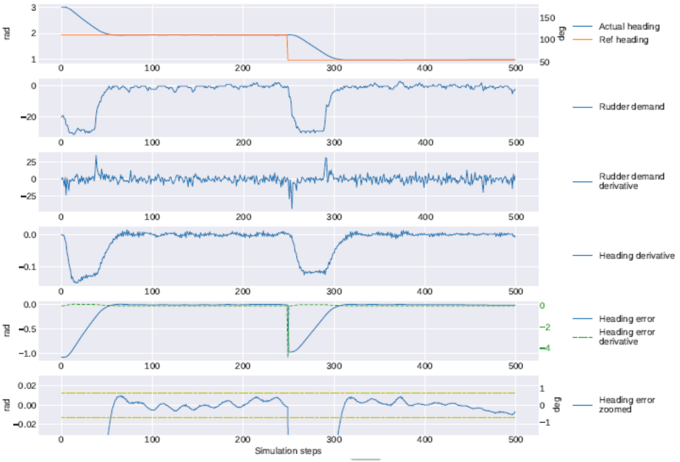
- Finding a controller = finding a function
- Can use machine learning to find parameters or function
- Problem with guarantees
- Difficult to find cost function
- Rely on a good model
- Example: Summer intern Marius Sleire Rundhovde

The dark side: Machine learning

$$
\begin{aligned}
20 \cdot R(\cdot) = {} & e^{\frac{-64}{\pi}\delta_\psi^2} + e^{-10000\delta_\psi^2} && \text{Base reward} \\
& - \frac{1}{12500} \cdot \dot{u}_{steering}^2 && \text{Steering derivative penalty} \\
& + \begin{cases} \frac{1}{3} & \text{if } \delta_{\psi,t} \leq \text{boundary and } \dot{\delta}_{\psi,t} \leq 0.005 \\ 0 & \text{otherwise} \end{cases} && \text{Low error derivative reward} \\
& - \begin{cases} \frac{1}{5} & \text{if } \delta_{\psi,t} > \text{boundary and } \dot{\delta}_{\psi,t} \leq \dot{\delta}_{\psi,t-1} \\ 0 & \text{otherwise} \end{cases} && \text{Error not decreasing penalty} \\
& - \min\left(\frac{1}{25}\left(1 - \sum_{i=0}^{9} |\Delta u_{steering,t-i}|\right),\ 0\right) && \text{Sum change steering penalty}
\end{aligned}
$$

Example reward function - From Marius Sleire Rundhovde

The dark side: Machine learning



Heading controller using deep learning - From Marius Sleire Rundhovde

Guidance

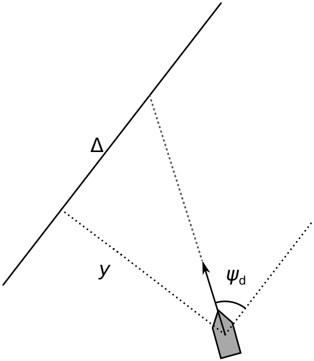
- Heading and surge are assumed to be perfectly controlled
- Heading and surge references are usually generated by a guidance system
  - Path following
  - Trajectory tracking
  - Target following
- A guidance system is usually reactive. A deliberate approach would be motion planning, MPC etc.
- Can either control course χ or heading ψ, depending on the measurements and controller available

Path following of straight lines

- A typical Hugin or Odin mission consists of a set of waypoints
- The waypoints can be made either by the user or by a high-level autonomy system
- The vehicle is affected by current, which is often constant or very slowly varying
- Current compensation: Either use course control, or compensate by adding integral effect
- Good idea to use both

Line of Sight guidance law

- Aim toward a point Δ m along the path
- Assume no disturbances
- Coordinate system rotated so that the x-axis is aligned with the desired path
- $\mathcal{P} \triangleq \{(x, y) \in \mathbb{R}^2 : y = 0\}$



The LOS guidance law

Line of Sight guidance law

Desired heading:

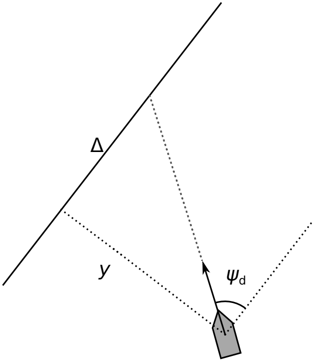
$$\psi_d = -\tan^{-1}(\frac{y}{\Delta}) \qquad (28)$$

If we add sideslip compensation:

$$\psi_d = -\tan^{-1}(\frac{y}{\Delta}) - \beta \qquad (29)$$

Which gives the desired course:

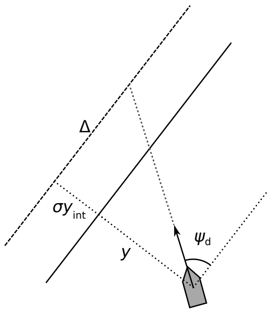
$$\chi_d = -\tan^{-1}(\frac{y}{\Delta}) \qquad (30)$$



The LOS guidance law

Integral Line of Sight guidance law

- Nominal: Aim toward a point Δ m along the path
- Integrate the cross track error to counter disturbances
- The integral effect makes the vessel aim towards a parallel path
- Makes it possible to sideslip along the path
- First presented in [Børhaug et al., 2008]



The ILOS guidance law

Necessary assumptions

- **Assumption 1:** The ocean current constant, irrotational and bounded. Hence, there exists a constant $V_{\max} \geq 0$ such that $V_{\max} \geq \sqrt{V_x^2 + V_y^2}$.
- **Assumption 2:** The vehicle can move faster than the current
- **Assumption 3:** $Y(u) \leq -Y_{\text{min}} < 0$

$$\dot{x} = u\cos(\psi) - v\sin(\psi) \tag{31a}$$
$$\dot{y} = u\sin(\psi) + v\cos(\psi) \tag{31b}$$
$$\dot{v} = X(u)r + Y(u)v \tag{31c}$$

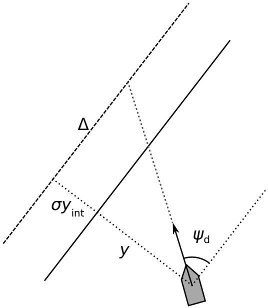
Integral Line of Sight guidance law

Guidance law:

$$\psi_d \triangleq -\tan^{-1}\left(\frac{y + \sigma y_{\mathrm{int}}}{\Delta}\right) \tag{32a}$$

$$\dot{y}_{\mathrm{int}} \triangleq \frac{\Delta y}{(y + \sigma y_{\mathrm{int}})^2 + \Delta^2} \tag{32b}$$

The integral term growth rate will decrease for large cross-track errors y.



The ILOS guidance law

Summary

- Reference frames and DOFs
- Kinematics: From BODY to NED
- Dynamics
- 3DOF example
- Guidance and control
  - LOS guidance

Børhaug, E., Pavlov, A., and Pettersen, K. Y. (2008).
Integral LOS control for path following of underactuated marine surface vessels in the presence of constant ocean currents.
In *Proc. 47th IEEE Conference on Decision and Control*, pages 4984–4991, Cancun, Mexico.

Fossen, T. I. (2011).
Handbook of marine craft hydrodynamics and motion control.
John Wiley & Sons.