UiO : Department of Technology Systems
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8. Motion control – Independant joint control

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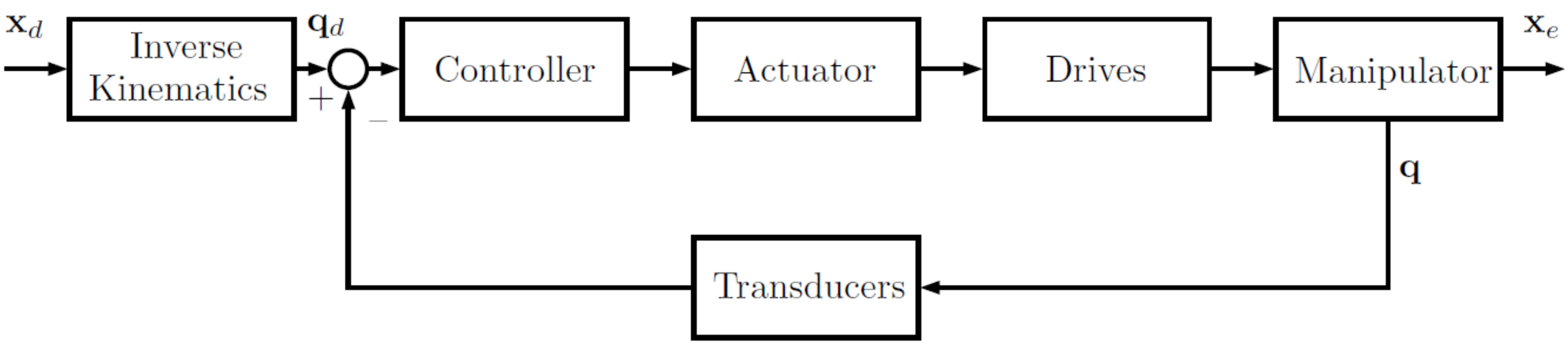
Lecture overview

- The control problem
- Joint space control
- Decentralized control
- Computed torque feedforward control

8.1 The control problem

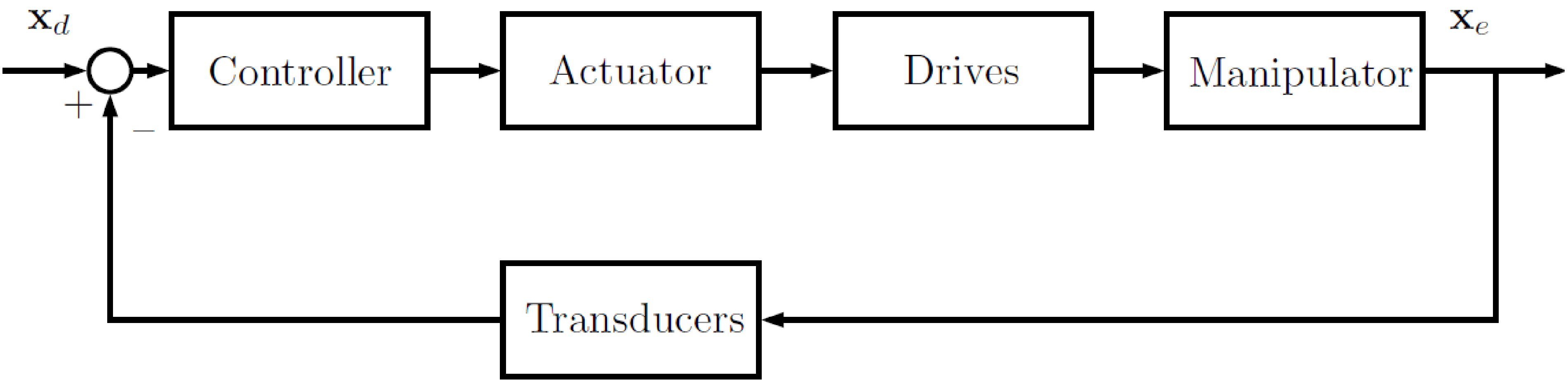
- Trajectory tracing / point-to-point control
- Mechanical design influence control scheme
- Driving system
- Two general control schemes
  - Joint space control
  - Operational space control

Joint space control



- Two subproblems
  - Inverse kinematics
  - Joint space control scheme

Operational space control



- Global approach
  - Greater algorithmic complexity
  - Inverse kinematics embedded into the feedback loop

8.2 Joint space control

- Derive dynamic model of manipulator and drives
  - Velocity (voltage) controlled system
  - Torque (current) controlled system

Derive dynamic model of manipulator and drives

$$\boldsymbol{K}_r^{-1}\boldsymbol{\tau} = \boldsymbol{K}_t\boldsymbol{i}_a \tag{8.4}$$

$$\boldsymbol{v}_a = \boldsymbol{R}_a\boldsymbol{i}_a + \boldsymbol{K}_v\dot{\boldsymbol{q}}_m \tag{8.5}$$

$$\boldsymbol{v}_a = \boldsymbol{G}_v\boldsymbol{v}_c. \tag{8.6}$$

Derive dynamic model of manipulator and drives

Inertial forces

Coriolis and centrifugal forces

$$B(q)\ddot{q} + C(q,\dot{q})\dot{q} + F_v\dot{q} + g(q) = \tau \qquad (8.1)$$

Gravity

Viscous friction

Derive dynamic model of manipulator and drives

$$\boldsymbol{B}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{F}\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{u} \tag{8.7}$$

$$\boldsymbol{F} = \boldsymbol{F}_v + \boldsymbol{K}_r\boldsymbol{K}_t\boldsymbol{R}_a^{-1}\boldsymbol{K}_v\boldsymbol{K}_r \tag{8.8}$$

$$\boldsymbol{u} = \boldsymbol{K}_r\boldsymbol{K}_t\boldsymbol{R}_a^{-1}\boldsymbol{G}_v\boldsymbol{v}_c. \tag{8.9}$$

$$\boldsymbol{\tau} = \boldsymbol{K}_r\boldsymbol{K}_t\boldsymbol{R}_a^{-1}(\boldsymbol{G}_v\boldsymbol{v}_c - \boldsymbol{K}_v\boldsymbol{K}_r\dot{\boldsymbol{q}}). \tag{8.11}$$

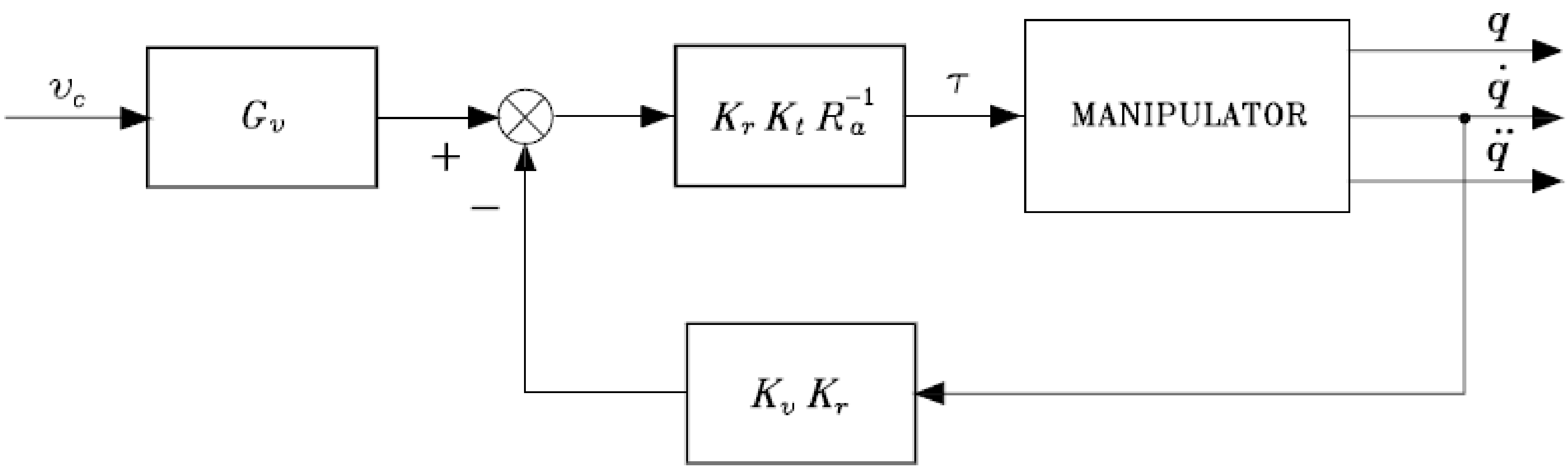
Velocity controlled system
- Deriving proportional relationship

- If the following assumptions hold
  - The elements of K_r are much greater than unity (transmission)
  - The elements of R_a are very small
  - The values of the torque τ are not too large
- Then it can be assumed that

$$\boldsymbol{\tau} = \boldsymbol{K}_r \boldsymbol{K}_t \boldsymbol{R}_a^{-1} (\boldsymbol{G}_v \boldsymbol{v}_c - \boldsymbol{K}_v \boldsymbol{K}_r \dot{\boldsymbol{q}}). \tag{8.11}$$

$$\boldsymbol{G}_v \boldsymbol{v}_c \approx \boldsymbol{K}_v \boldsymbol{K}_r \dot{\boldsymbol{q}}. \tag{8.12}$$

Velocity controlled system



Velocity controlled system
- Observations

- The joint velocities are proportional to the control voltages
- It is independent of manipulator parameters
  - Inherent robustness with respect to manipulator model
- All matrices in 8.12 are diagonal, this implies that the joint velocity og the i-th joint only depends on the i-th control velocity
  - Suitable for decentralized control

$$\boldsymbol{G}_v \boldsymbol{v}_c \approx \boldsymbol{K}_v \boldsymbol{K}_r \dot{\boldsymbol{q}}. \tag{8.12}$$

Velocity controlled system
- Drawbacks

- If the desired manpulator motion requires large joint velocities and/or accelarations
  - Assumptions no longer holds
- Another scheme is to use inverse dynamics techniques
  - This requires accurate knowledge of the manipulator dynamic model
  - Neccessary to know the evolution of the motion of all the joints
  - Centralized control structure

Torque controlled system

$$\boldsymbol{\tau} = \boldsymbol{K}_r \boldsymbol{K}_t \boldsymbol{R}_a^{-1} \boldsymbol{G}_v \boldsymbol{v}_c - \boldsymbol{K}_r \boldsymbol{K}_t \boldsymbol{R}_a^{-1} \boldsymbol{K}_v \boldsymbol{K}_r \dot{\boldsymbol{q}}. \tag{8.15}$$

- If the above equation is used the control voltages depend on torque values and joint velocities
- The relationship depends on the parameters K_t, K_v and R_a^{-1}, which are influenced by the operating conditions of the motors

Torque controlled system

- Instead considering a current control driving system

$$\boldsymbol{i}_a = \boldsymbol{G}_i \boldsymbol{v}_c, \tag{8.16}$$

- This replaces (8.5) in the previous calculations

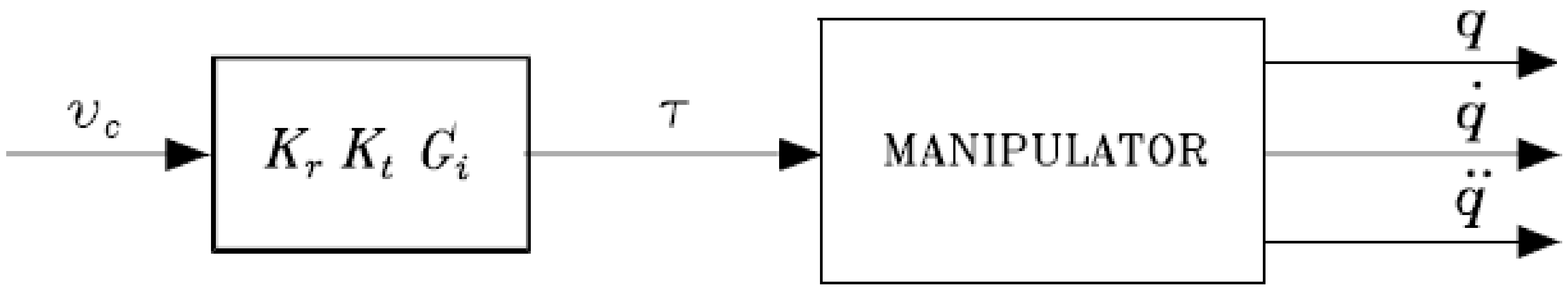
$$\boldsymbol{v}_a = \boldsymbol{R}_a \boldsymbol{i}_a + \boldsymbol{K}_v \dot{\boldsymbol{q}}_m \tag{8.5}$$

- This yields

$$\boldsymbol{\tau} = \boldsymbol{u} = \boldsymbol{K}_r \boldsymbol{K}_t \boldsymbol{G}_i \boldsymbol{v}_c \tag{8.17}$$

- The system is now torque controlled
- Redused dependency of motor parameters

Torque controlled system



Summary

- Velocity controlled generator
  - Decentralized control systems
  - Feedback control systems
  - Velocity controlled generator
- Torque controlled generator
  - Centralized control
  - Feedforward of inverse dynamics
  - Still requires feedback of the trajectory

8.3 Decentralized control

- Controlling each joint independently
- Easier control schemes
- Single input/single output system

Deriving a linear decoupled model

Based on the dynamic model

$$\boldsymbol{B}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{F}_v\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau} \tag{8.1}$$

Inserting

$$\boldsymbol{K}_r\boldsymbol{q} = \boldsymbol{q}_m, \tag{8.2}$$

Yields

$$\boldsymbol{K}_r^{-1}\boldsymbol{B}(\boldsymbol{q})\boldsymbol{K}_r^{-1}\ddot{\boldsymbol{q}}_m + \boldsymbol{K}_r^{-1}\boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\boldsymbol{K}_r^{-1}\dot{\boldsymbol{q}}_m + \boldsymbol{K}_r^{-1}\boldsymbol{F}_v\boldsymbol{K}_r^{-1} + \boldsymbol{K}_r^{-1}\boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau}_m. \tag{8.18}$$

Error in above equation: q_dot missing for viscous friction

Deriving a linear decoupled model

- Diagonal elements of B consists of constant and configuration dependent terms

$$\boldsymbol{B}(\boldsymbol{q}) = \bar{\boldsymbol{B}} + \Delta\boldsymbol{B}(\boldsymbol{q}) \tag{8.19}$$

- $\bar{B}$ - Diagonal matrix with constant elements representing average inertia at each joint
- $\Delta B(q)$ – Configuration dependant terms

Deriving a linear decoupled model

- Using this yields a simplified model

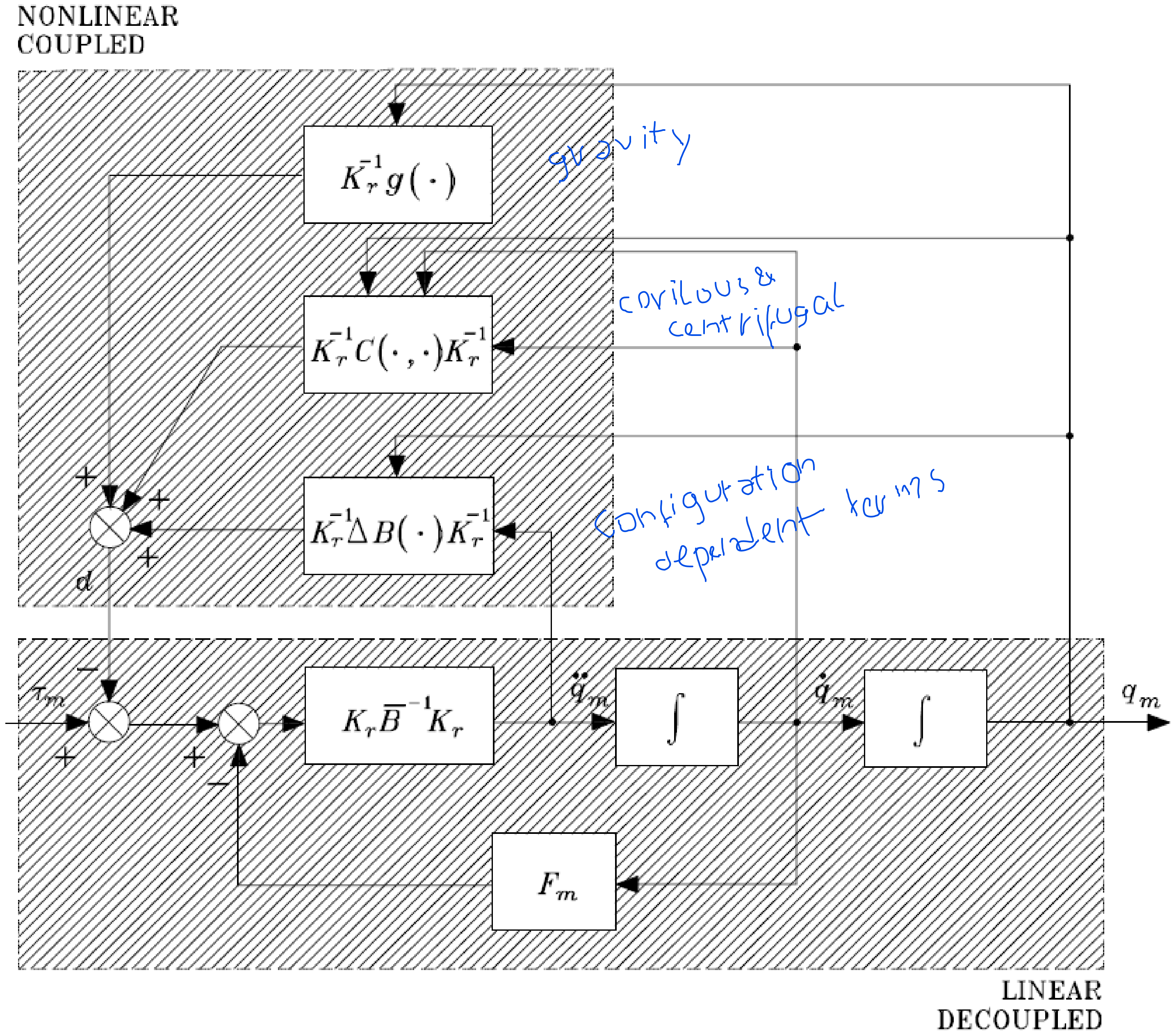
$$\boldsymbol{K}_r^{-1}\bar{\boldsymbol{B}}\boldsymbol{K}_r^{-1}\ddot{\boldsymbol{q}}_m + \boldsymbol{F}_m\dot{\boldsymbol{q}}_m + \boldsymbol{d} = \boldsymbol{\tau}_m \tag{8.20}$$

- Viscous friction

$$\boldsymbol{F}_m = \boldsymbol{K}_r^{-1}\boldsymbol{F}_v\boldsymbol{K}_r^{-1} \tag{8.21}$$

- Configuration dependent contribution

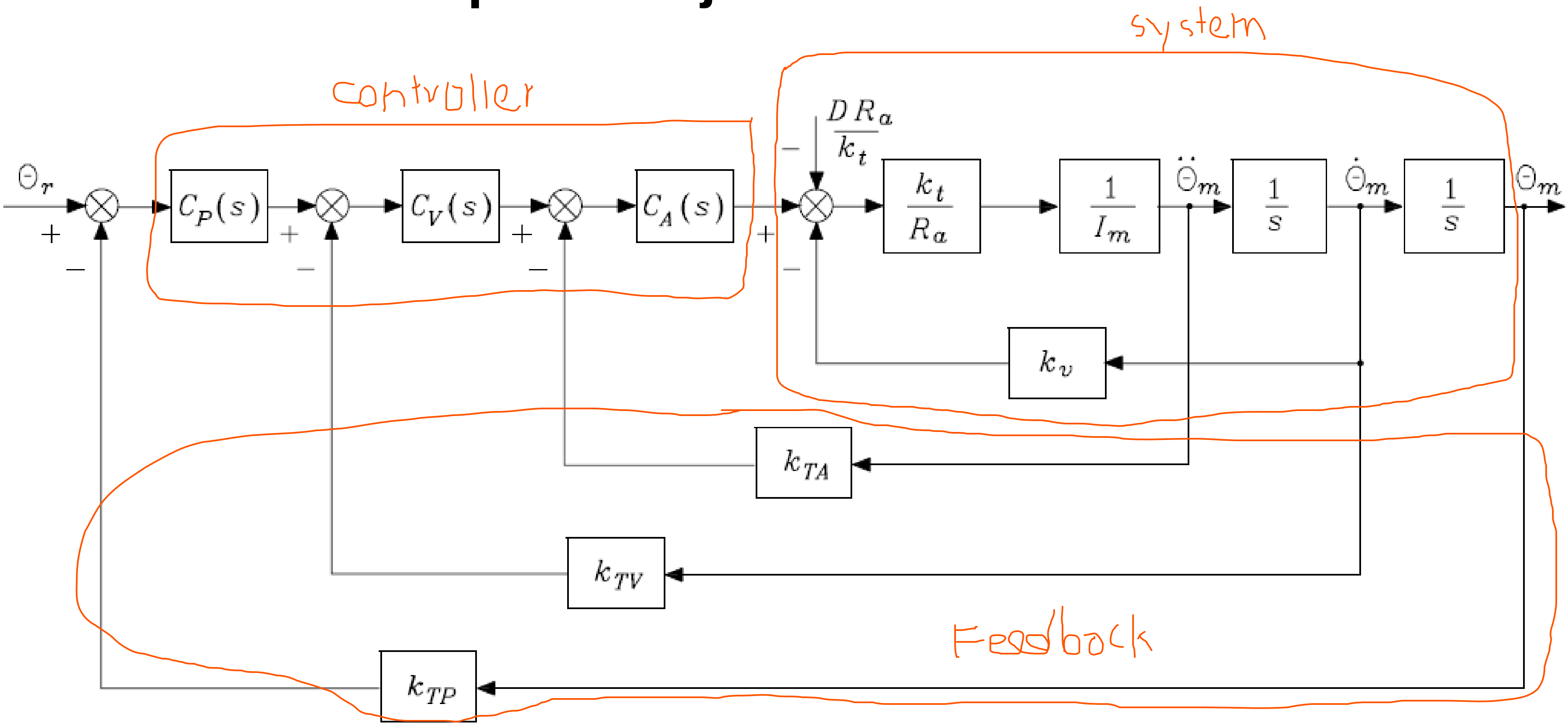
$$\boldsymbol{d} = \boldsymbol{K}_r^{-1}\Delta\boldsymbol{B}(\boldsymbol{q})\boldsymbol{K}_r^{-1}\ddot{\boldsymbol{q}}_m + \boldsymbol{K}_r^{-1}\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\boldsymbol{K}_r^{-1}\dot{\boldsymbol{q}}_m + \boldsymbol{K}_r^{-1}\boldsymbol{g}(\boldsymbol{q}) \tag{8.22}$$


NONLINEAR
COUPLED
$K_r^{-1}g(\cdot)$
gravity
$K_r^{-1}C(\cdot,\cdot)K_r^{-1}$
corilous &
centrifugal
$K_r^{-1}\Delta B(\cdot)K_r^{-1}$
configuration
dependent terms
d
τ_m
$K_r\overline{B}^{-1}K_r$
$\ddot{q}_m$
$\int$
$\dot{q}_m$
$\int$
q_m
F_m
LINEAR
DECOUPLED

8.3.1 Independant Joint Control

- An effective rejection of the disturbance d is ensured by
  - A large amplifier gain before the point of intervention of the disturbance innblanding
  - Integral action in the controller to cancel gravitational components
- This suggest a proportional-integral (PI) controller

General independent joint control



PI-controller

$$C(s) = K_c \frac{1 + sT_c}{s}; \qquad (8.23)$$

- Offers zero error at steady state for a constant disturbance
- Stabilizing with a real root at $s = -1/T_c$

Analysing position feedback controller

$$C_P(s) = K_P \frac{1 + sT_P}{s} \qquad C_V(s) = 1 \qquad C_A(s) = 1$$

$$k_{TV} = k_{TA} = 0.$$

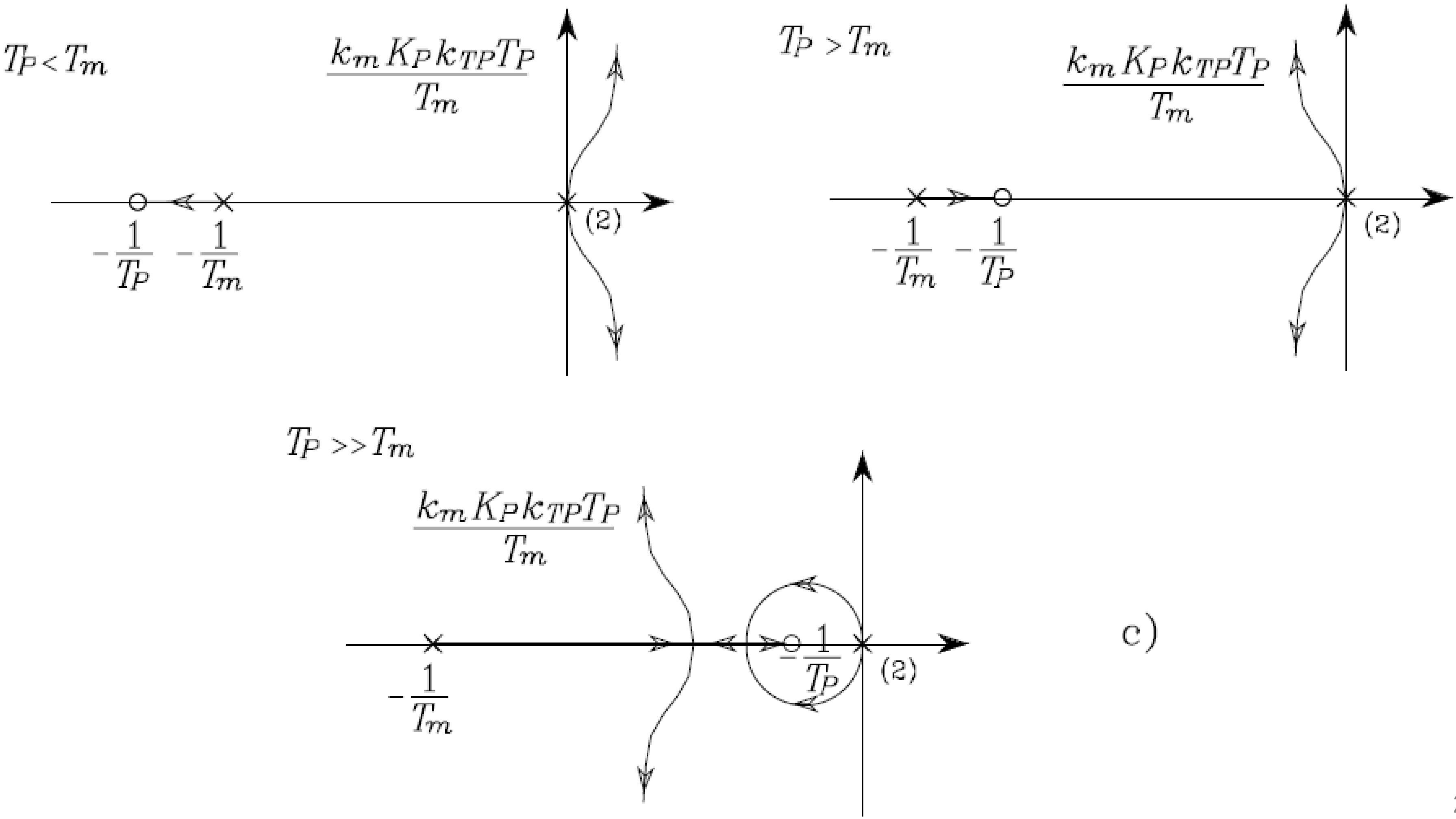
Position feedback transfer functions

$$\frac{\Theta_m(s)}{\Theta_r(s)} = \frac{\dfrac{1}{k_{TP}}}{1 + \dfrac{s^2(1+sT_m)}{k_m K_P k_{TP}(1+sT_P)}}, \tag{8.24}$$

$$W(s) = \frac{\dfrac{1}{k_{TP}}(1+sT_P)}{\left(1 + \dfrac{2\zeta s}{\omega_n} + \dfrac{s^2}{\omega_n^2}\right)(1+s\tau)},$$

$$k_m = \frac{1}{k_v} \qquad T_m = \frac{R_a I_m}{k_v k_t} \tag{5.12}$$

Root locus plots



Position feedback disturbance

$$\frac{\Theta_m(s)}{D(s)} = -\frac{\dfrac{sR_a}{k_t K_P k_{TP}(1+sT_P)}}{1+\dfrac{s^2(1+sT_m)}{k_m K_P k_{TP}(1+sT_P)}}, \tag{8.25}$$

Disturbance rejection factor

$$X_R = K_P k_{TP}$$

The rejection is determined by the gain Kp. It is not advisable to increase Kp too much, as this could lead to oscillations.

Analysing position and velocity feedback controller

$$C_P(s) = K_P \qquad C_V(s) = K_V \frac{1 + sT_V}{s} \qquad C_A(s) = 1$$

$$k_{TA} = 0;$$

Forward path

$$P(s) = \frac{k_m K_P K_V (1 + sT_V)}{s^2(1 + sT_m)},$$

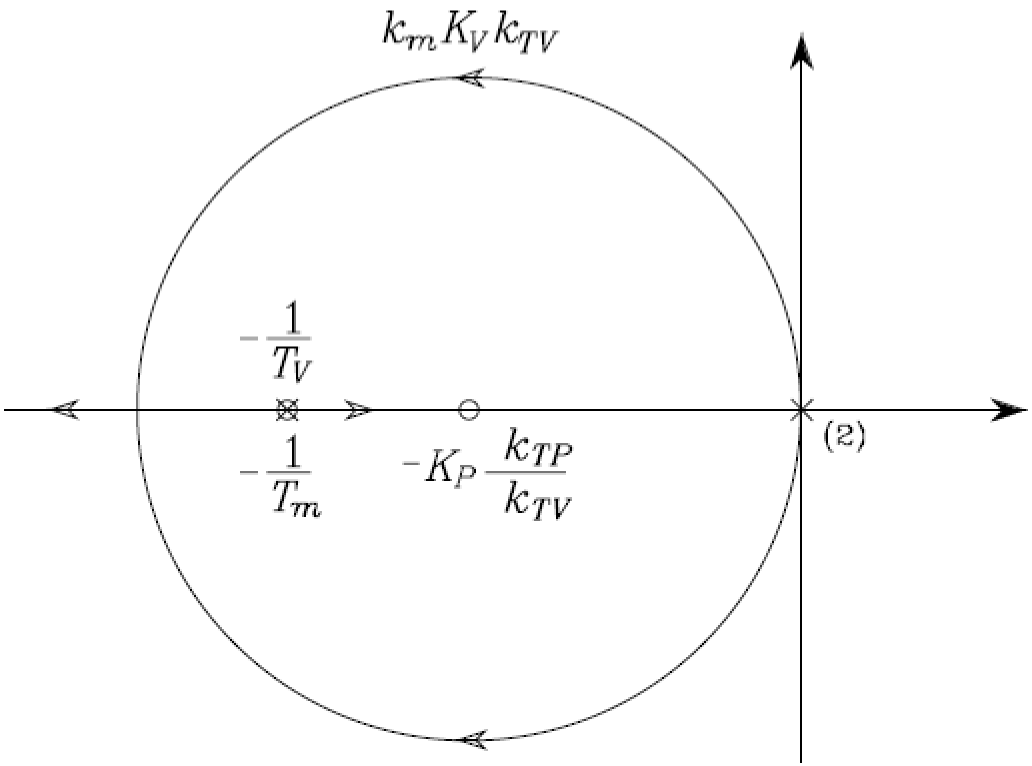
Choose $T_V = T_m,$ to cancel pole

Position and velocity feedback transfer functions

$$\frac{\Theta_m(s)}{\Theta_r(s)} = \frac{\dfrac{1}{k_{TP}}}{1 + \dfrac{s k_{TV}}{K_P k_{TP}} + \dfrac{s^2}{k_m K_P k_{TP} K_V}}, \tag{8.28}$$

$$W(s) = \frac{\dfrac{1}{k_{TP}}}{1 + \dfrac{2\zeta s}{\omega_n} + \dfrac{s^2}{\omega_n^2}}. \tag{8.29}$$

Root locus plot



Position and velocity feedback disturbance and control design

$$\frac{\Theta_m(s)}{D(s)} = -\frac{\dfrac{sR_a}{k_t K_P k_{TP} K_V (1 + sT_m)}}{1 + \dfrac{sk_{TV}}{K_P k_{TP}} + \dfrac{s^2}{k_m K_P k_{TP} K_V}}, \tag{8.32}$$

Disturbance rejection factor

$$X_R = K_P k_{TP} K_V \tag{8.33}$$

Designing control parameters

$$K_V k_{TV} = \frac{2\zeta\omega_n}{k_m} \tag{8.30}$$

$$K_P k_{TP} K_V = \frac{\omega_n^2}{k_m}. \tag{8.31}$$

Position, velocity and acceleration feedback controller

$$C_P(s) = K_P \qquad C_V(s) = K_V \qquad C_A(s) = K_A \frac{1 + sT_A}{s}.$$

Position, velocity and acceleration feedback controller transfer functions

$$\frac{\Theta_m(s)}{\Theta_r(s)} = \frac{\dfrac{1}{k_{TP}}}{1 + \dfrac{s k_{TV}}{K_P k_{TP}} + \dfrac{s^2(1 + k_m K_A k_{TA})}{k_m K_P k_{TP} K_V K_A}}, \tag{8.35}$$

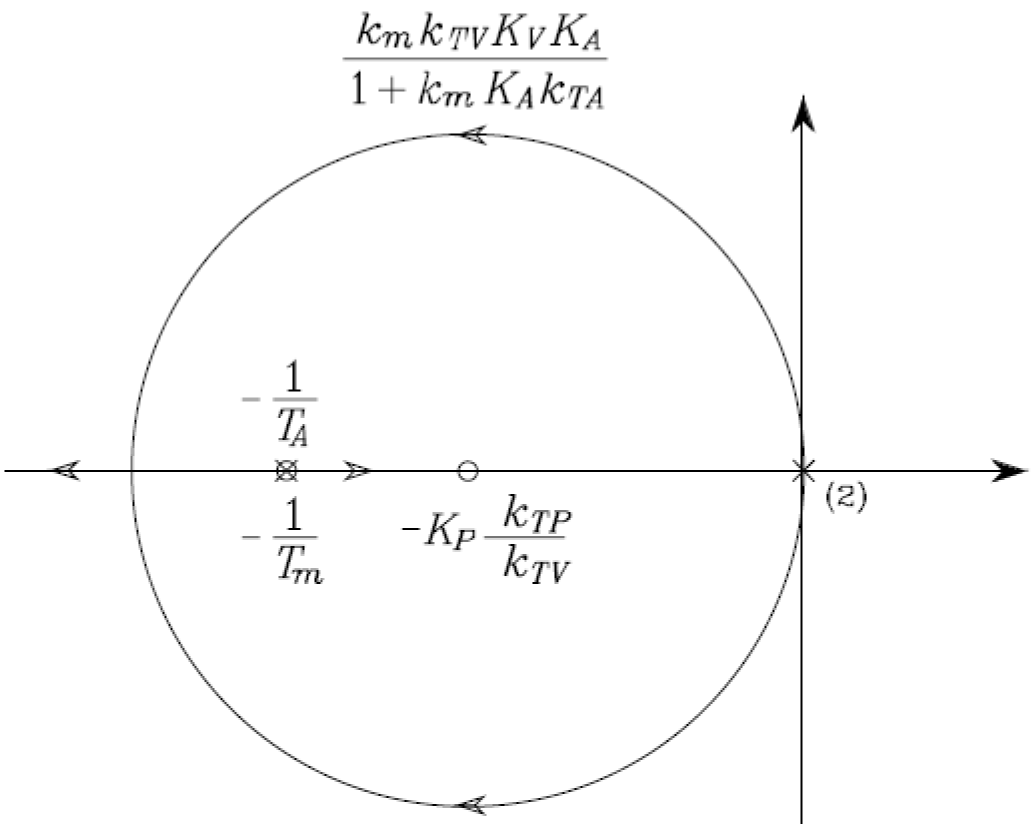
Assuming

$$T_A = T_m,$$

or

$$k_m K_A k_{TA} T_A \gg T_m \qquad k_m K_A k_{TA} \gg 1.$$

Root locus plot



Position, velocity and acceleration disturbance and control design

$$\frac{\Theta_m(s)}{D(s)} = -\frac{\dfrac{sR_a}{k_t K_P k_{TP} K_V K_A (1 + sT_A)}}{1 + \dfrac{sk_{TV}}{K_P k_{TP}} + \dfrac{s^2(1 + k_m K_A k_{TA})}{k_m K_P k_{TP} K_V K_A}}. \tag{8.36}$$

Disturbance rejection factor

$$X_R = K_P k_{TP} K_V K_A,$$

Designing control parameters

$$\frac{2K_P k_{TP}}{k_{TV}} = \frac{\omega_n}{\zeta} \tag{8.39}$$

$$k_m K_A k_{TA} = \frac{k_m X_R}{\omega_n^2} - 1 \tag{8.40}$$

$$K_P k_{TP} K_V K_A = X_R. \tag{8.41}$$

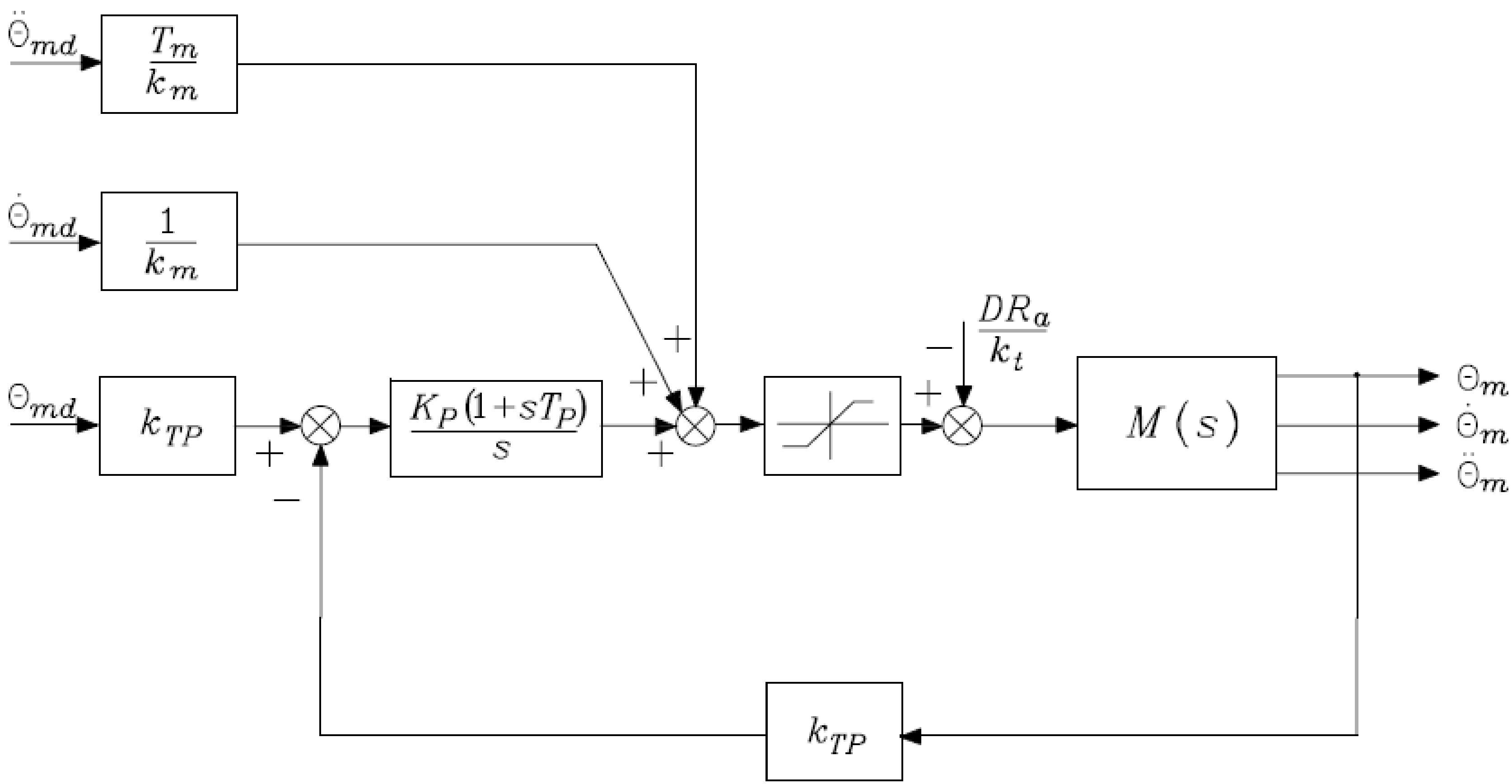
Implementation issues

- Position and velocity are easily available
- Acceleration is more expensive to measure
- May use filters to estimate acceleration
- This may degenerate the performance

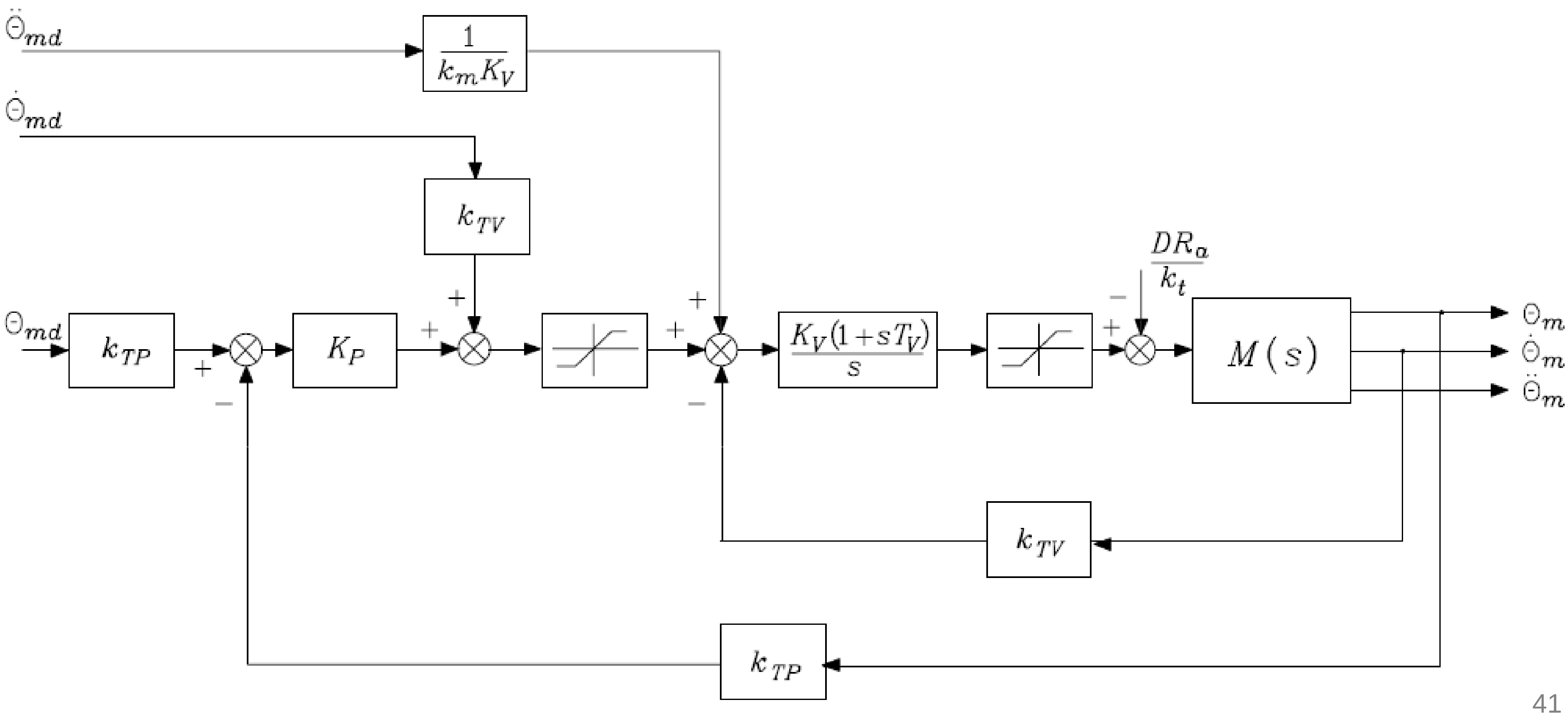
8.3.2 Decentralized Feedforward Compensation

- Tracking performance of previous shemes are degraded if the trajectory has high values of speed and acceleration
- To handle this problem we introduce a decentralized feedforward compensation

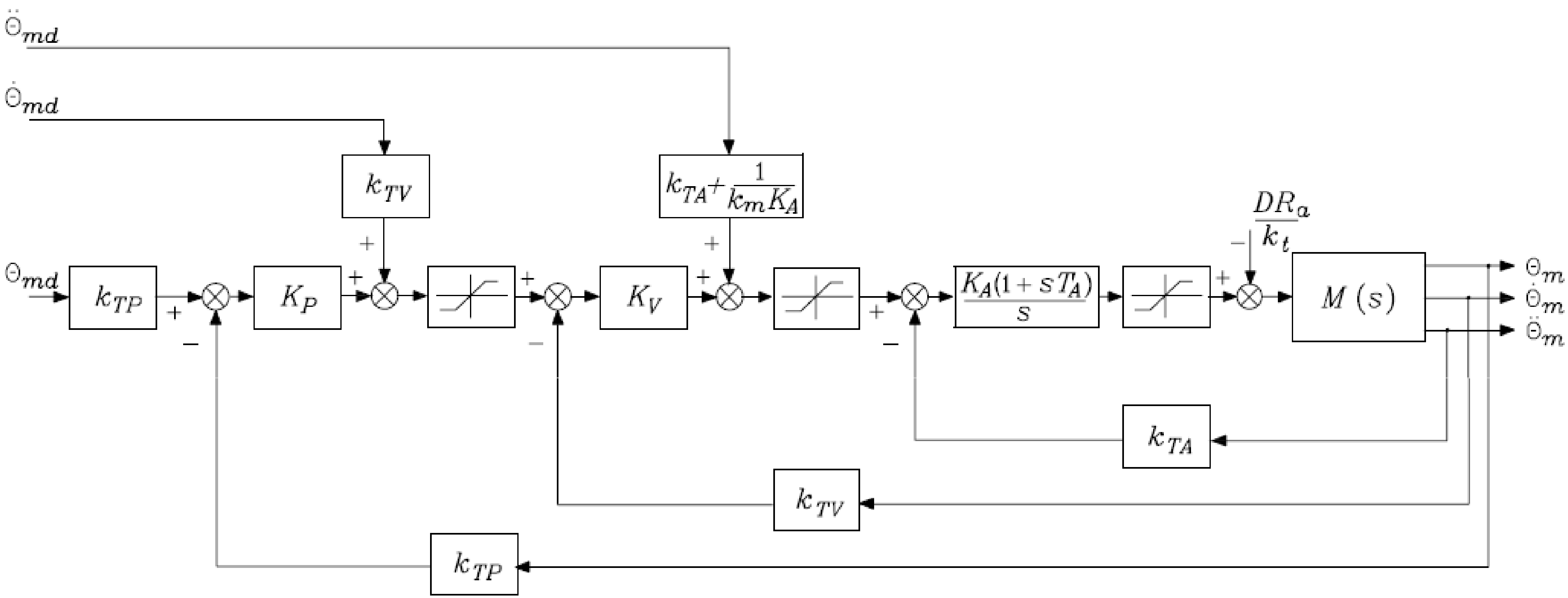
Position feedback with decentralized feedforward



Position and velocity feedback with decentralized feedforward



Position, velocity and acceleration feedback with decentralized feedforward



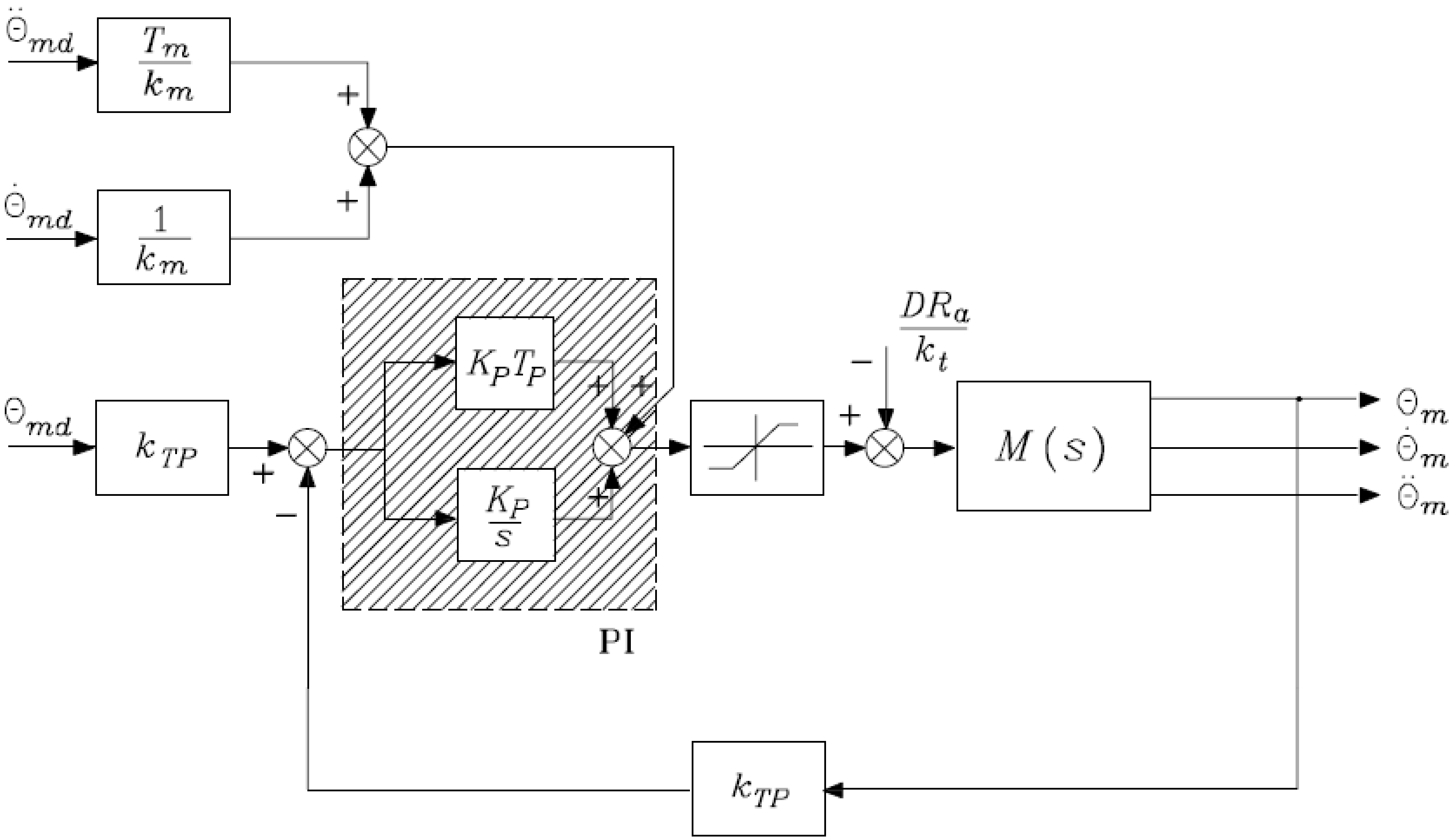
Remarks

- Computing time derivative of the trajectory is generally not a problem
- As the number of nested feedback loops increases, less knowledge og the model is required
- Saturation blocks are placed to limit physical quantities during trancients

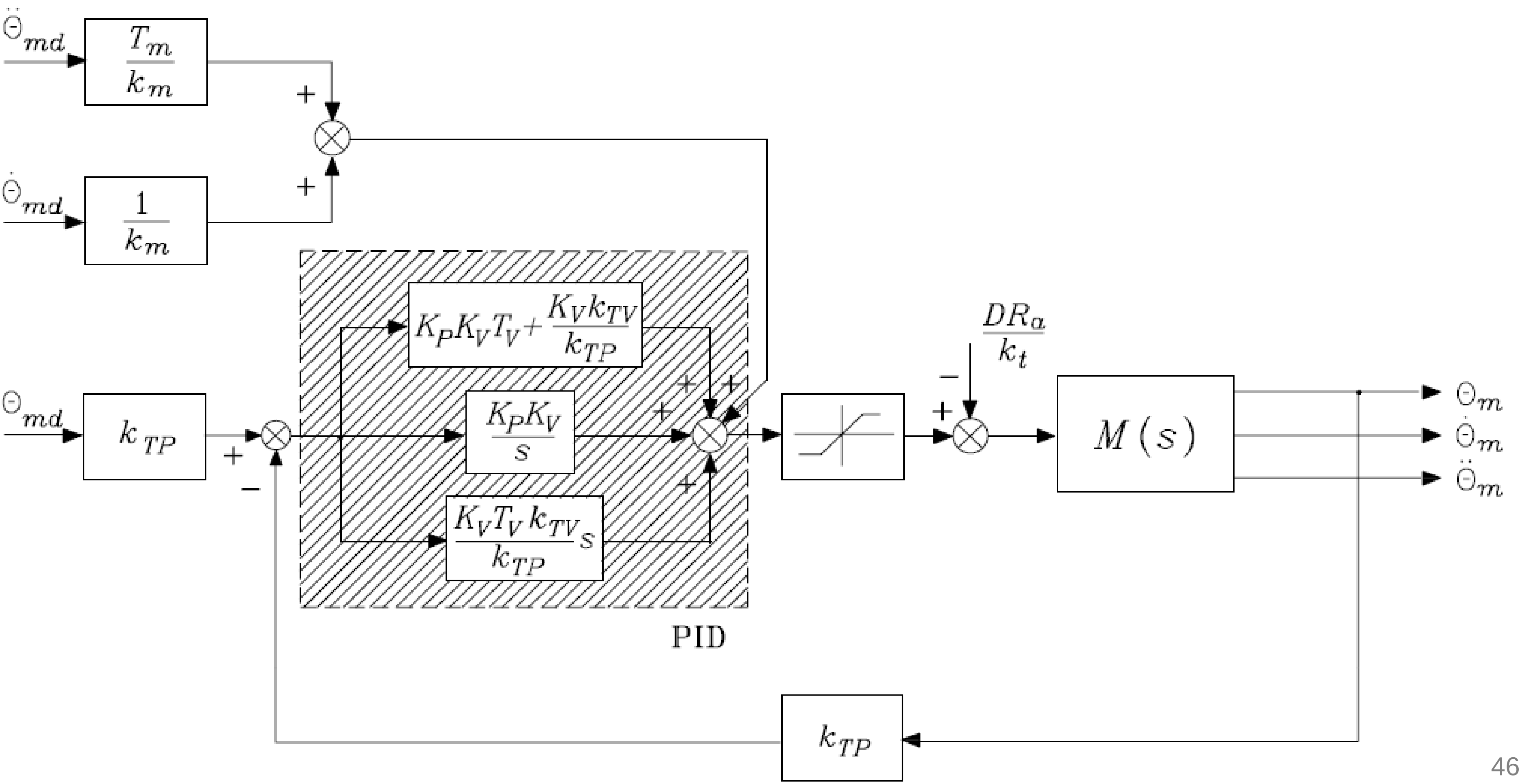
Using only position feedback for decentralized feedforward compensation

- The three control schemes may be reduced to only use position feedback and regulators with standard actions
- The two solution are equivalent in terms of disturbance rejection and trajectory tracking
- Tuning the controllers become less streight forwards
- The posibility of setting saturation blocks is lost

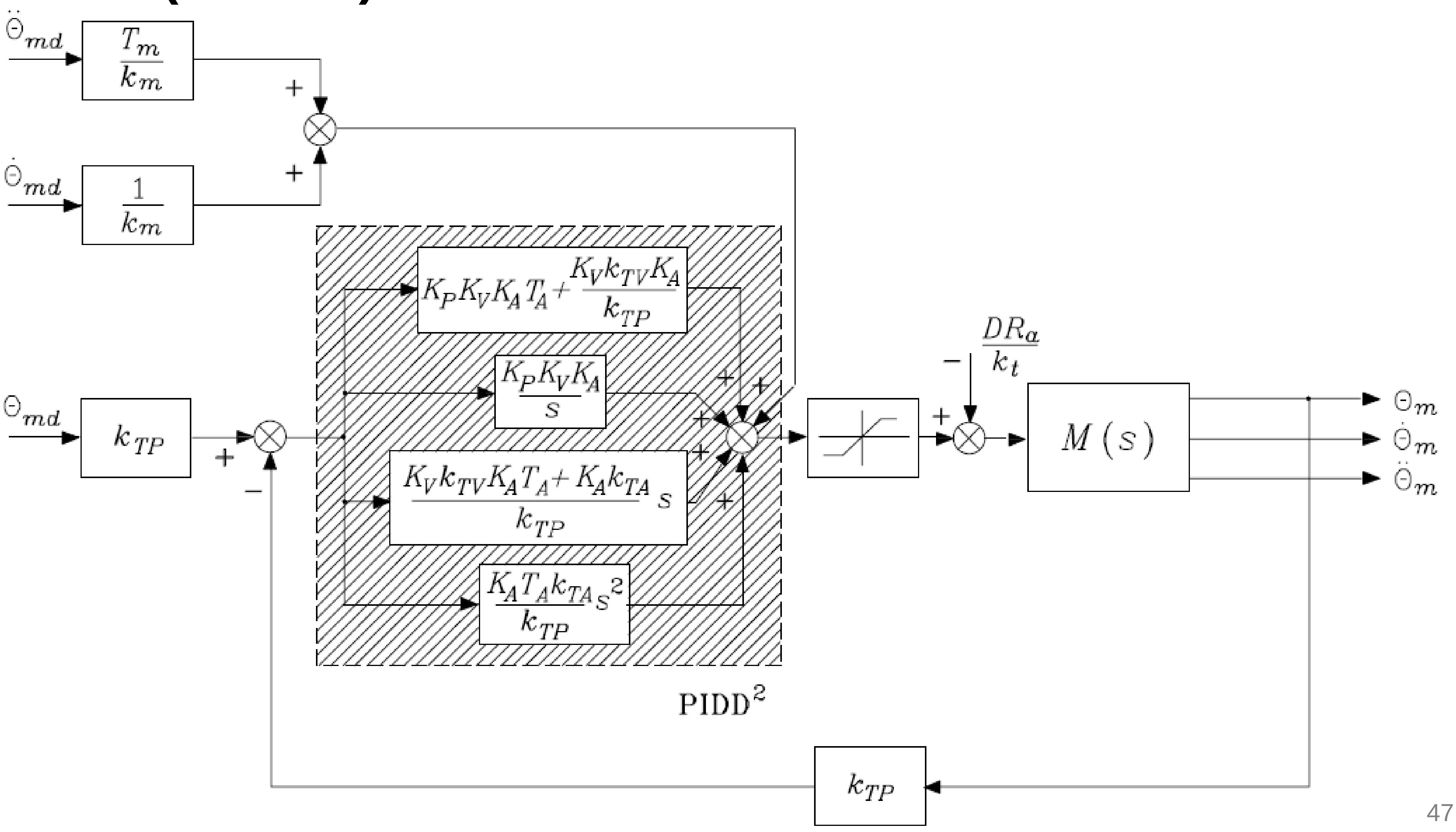
Position feedback (PI)



Position and velocity feedback (PID)



Position, velocity and acceleration feedback (PIDD^2)



8.4 Computed Torque Feedforward Control

- Define the tracking error

$$e(t) = \vartheta_{md}(t) - \vartheta_m(t)$$

- Then looking at the error dynamics

$$a_2'\ddot{e} + a_1'\dot{e} + a_0'e + a_{-1}'\int^t e(\varsigma)d\varsigma = \frac{R_a}{k_t}d.$$

- The trajectory is asymtotically tracked only if the disturbance term is zero

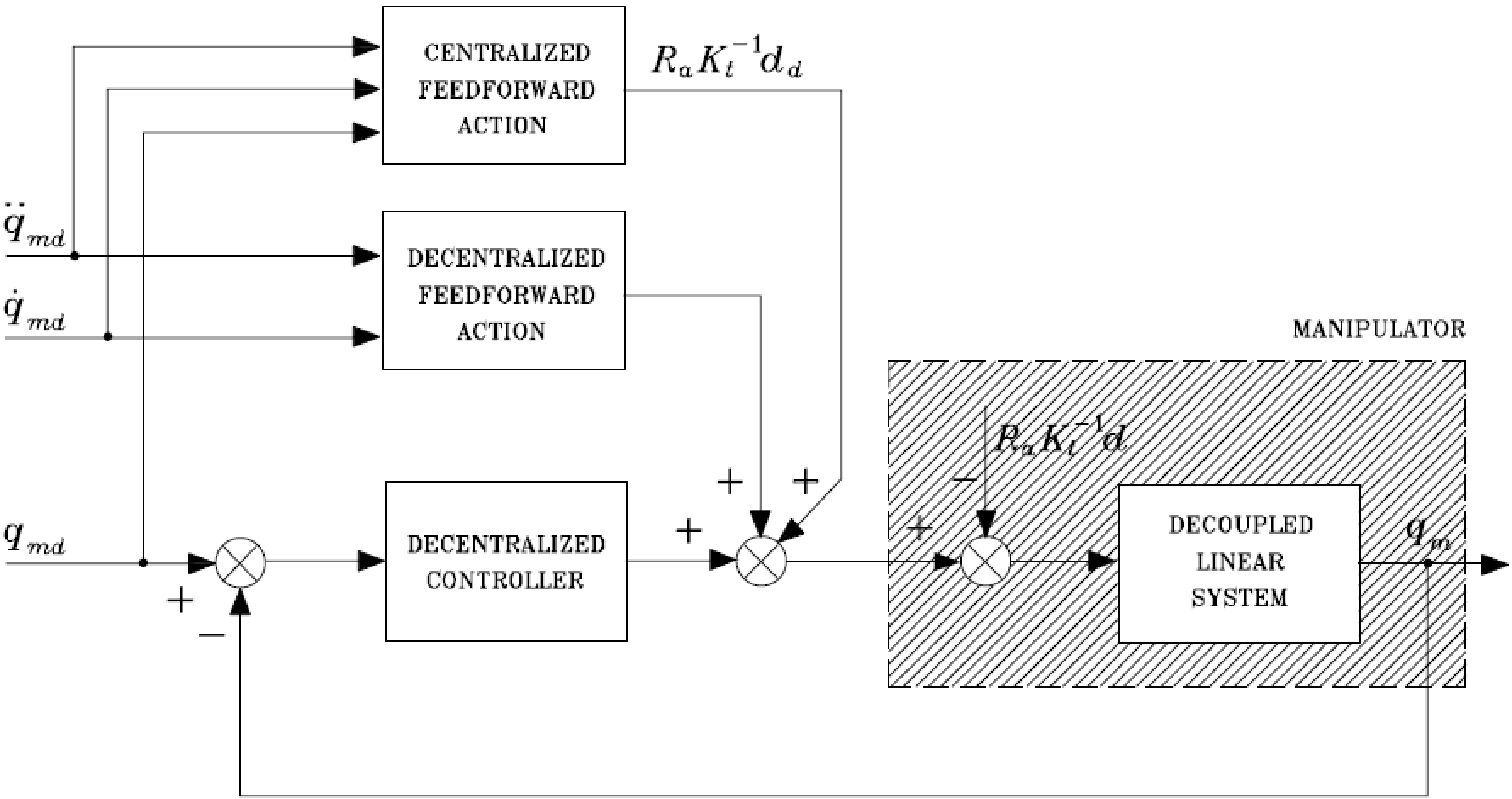
- Error dynamics is how the error evloves over time, ekspressed as a differential equation

Introducing a centralized feedforward action

- The disturbance was introduced as a simplification of the robot's dynamics
- Using an inverse model strategy a feedforward action may be introduced

$$d_d = K_r^{-1} \Delta B(q_d) K_r^{-1} \ddot{q}_{md} + K_r^{-1} C(q_d, \dot{q}_d) K_r^{-1} \dot{q}_{md} + K_r^{-1} g(q_d), \quad (8.45)$$

- D_d represents the required torque to compensate for the disturbance (hence the name computed torque)


CENTRALIZED FEEDFORWARD ACTION
$R_a K_t^{-1} d_d$
$\ddot{q}_{md}$
DECENTRALIZED FEEDFORWARD ACTION
$\dot{q}_{md}$
MANIPULATOR
$R_a K_t^{-1} d$
+
+
−
+
q_{md}
+
DECENTRALIZED CONTROLLER
+
DECOUPLED LINEAR SYSTEM
q_m
+
−

Remarks

- The centralized feedforward action compensate for nonlinear coupling terms due to inertia, Coriolis, centrifugal and gravitational forces.
- Possible to use a partial feedforward action with most relevant contributions
  - Gravitational forces
  - Inertia
- For repetetive trajectories the feedforward action may be computed offline

Overview of controllers

Tracking type	Controller	Feedback	Feedforward	Chapter
Regulation	PI (position)	Position	-	8.3.1
Regulation	PI (velocity)	Position, Velocity	-	8.3.1
Regulation	PI (acceleration)	Position, Velocity, Acceleration	-	8.3.1
Trajectory tracking	PI (position)	Position	Velocity, Acceleration	8.3.2
Trajectory tracking	PI (velocity)	Position, Velocity	Velocity, Acceleration	8.3.2
Trajectory tracking	PI (acceleration)	Position, Velocity, Acceleration	Velocity, Acceleration	8.3.2
Trajectory tracking	PI (position)	Position	Velocity, Acceleration	8.3.2
Trajectory tracking	PID (position)	Position	Velocity, Acceleration	8.3.2
Trajectory tracking	PIDD^2 (position)	Position	Velocity, Acceleration	8.3.2
Trajectory tracking	PIDD^2 (acceleration)	Position	Velocity, Acceleration, Inverse model	8.4

Summary

- Three decentralized control schemes for point-to-point control
- Three decentralized control schemes for trajectory tracking using decentralized feedforward compensation
- Three decentralized control schemes for trajectory tracking using only position feedback and regulators with standard action
- One decentralized control schemes for trajectory tracking also using a centralized feedforward action

Exercises

- 8.1-8.3
- 8.4-8.6 (not completely covered by the lecture)