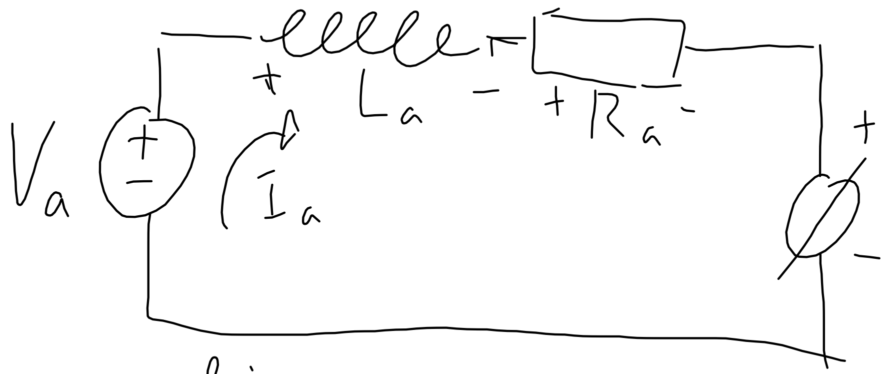
5.2.1 Electric drives

16

Modelling of electric drives

- Electrical balance
- Mechanical balance
- Power amplifier

Electrical balance



$$V_a = \frac{di_a}{dt} L_a + R_a i_a + V_g$$

$$V_a = (sL_a + R_a) I_a + V_g \quad (5.1)$$

$$V_g = k_v \omega_m \quad (5.2)$$

V_a - armature voltag

I_a - armature current

V_g - back electromotiv force

L_a - armature inducta

R_a - armature resistan

k_v - voltage constant

ω_m - Angular velocity of the motor

Mechanical balance

$\tau_{tot} = I\dot{\omega}$

$\tau_m - F_m \omega_m - \tau_l = I_m \dot{\omega}_m$

$\tau_m = (sI_m + F_m)\omega_m + \tau_l \quad (5.3)$

$\tau_m = K_t I_a \quad (5.4)$

τ_m - driving torque

τ_l - load reaction torque

I_m - moment of inertia

F_m - Viscous friction

K_t - torque constant

Power amplifier

$$\frac{V_a}{V_c} = \frac{G_v}{1+sT_v} \quad (5.5)$$

V_c - control voltage

G_v - voltage gain

T_v - time constant

C_i - current regulator

K_i - current feedback

$$V_a = \frac{G_v}{1+sT_v} C_i \left(V_c' - k_i I_a \right)$$

Steady-state · general approximations

$$V_a = (R_a + \cancel{sL_a})I_a + k_v\omega_m = R_a I_a + k_v \omega_m$$

$$\tau_m = (F_m + \cancel{sJ_m})\omega_m + \tau_l = F_m\omega_m + \tau_l$$

$$V_a = \frac{G_v}{1+\cancel{sT_v}} C_i (V_c' - k_i I_a) = G_v C_i (V_c' - k_i I_a)$$

$$\tau_m = k_t I_a \Rightarrow I_a = \frac{\tau_m}{k_t}$$

$s = 0$

Velocity controlled generator

Assumptions

$K_i = 0$ $\qquad C_i = 1$ $\qquad \tau_1 = 0$

$$F_m \ll \frac{K_v K_t}{R_a}$$

Inserting the assumptions

$$V_a = G_v V_c' = R_a I_a + K_v \omega_m$$

$$\tau_m = K_t I_a = F_m \omega_m \Rightarrow I_a = \frac{F_m}{K_t} \omega_m$$

$$G_v V_c' = R_a \frac{F_m}{K_t} \omega_m + K_v \omega_m$$

$$= K_v \left(\underline{\frac{R_a}{K_t K_v} F_m} + 1 \right) \omega_m$$

$$\omega_m \approx \frac{G_v}{K_v} V_c'$$

$$\frac{F_m}{\frac{K_t K_v}{R_a}} \approx 0$$

Torque controlled generator

Assumptions

$k_i \neq 0 \qquad C_i = 1 \qquad k_i >> R_a$

Inserting the assumptions

$$V_a = G_v (V_c' - k_i I_a) = R_a I_a + k_v \omega_m$$

$$G_v \left(V_c' - k_i \frac{\tau_m}{k_t}\right) = R_a \frac{\tau_m}{k_t} + k_v \omega_m$$

$$G_v V_c' = \frac{\cancel{R_a} + G_v k_i}{k_t} \tau_m + k_v \omega_m$$

$$= \frac{k_i}{k_t} G_v \tau_m + k_v \omega_m$$

$$\tau_m = \frac{k_t}{k_i}\left(V_c' - \frac{k_v}{G_v} \omega_m\right)$$

Reduced order models

Velocity controlled generator assumptions

$\frac{L_a}{R_a} \to 0$ Electrical time constant << Mechanical time constant

$$F_m << \frac{K_v K_t}{R_a}$$

$T_v = 0$ $K_i = 0$ $C_i = 1$

Reduced order models

Torque controlled generator assumptions

$k k_i >> R_a$ Large current loop gain

$k = G_v C_i$

$$\frac{K_v \omega_m}{k k_i} \approx 0$$

$$\frac{L_a}{R_a} \to 0 \qquad T_v = 0$$

General equation for generators

$$M(s) = \frac{K_m}{s(1 + T_m s)}$$

Velocity $K_m = \frac{1}{K_v}$ $\quad T_m = \frac{R_a I_m}{K_v K_t}$

Torque $K_m = \frac{K_t}{K_i F_m}$ $\quad T_m = \frac{I_m}{F_m}$

Transmission effects (5.2.3)

24

$$r_m \theta_m = r\theta \Rightarrow \frac{r}{r_m} = \frac{\theta_m}{\theta} = K_r = \frac{\omega_m}{\omega}$$

r, r_m - radius of spur gears

$$r_m \omega_m = r\omega$$

θ - load position

ω - load velocity

$$\tau = f \cdot r$$

$$\tau_m = I_m \dot{\omega}_m + F_m \omega_m + f r_m$$

$$f r = I \dot{\omega} + F\omega + \tau_l$$

$$\tau_m = I_m \dot{\omega}_m + F_m \omega_m + \frac{r_m}{r}\left(J \dot{\omega} + F \omega + \tau_l\right)$$

$$= I_m \dot{\omega}_m + F_m \omega_m + \frac{1}{k_m}\left(J \frac{\dot{\omega}_m}{k_m} + F \frac{\omega_m}{k_m}\right) + \frac{\tau_l}{k_m}$$

$$= \underbrace{\left(I_m + \frac{J}{k_m^2}\right)}_{I_{eq}} \dot{\omega}_m + \underbrace{\left(F_m + \frac{F}{k_m^2}\right)}_{F_{eq}} \omega_m + \frac{\tau_l}{k_m}$$

Weekly exercises

5.1 (5.2)

Exam 2017