UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Exam in: TEK4030
Day of exam: 17th of December
Exam hours: 9.15 - 13.15 (4 hours)
This examination paper consists of 5 page(s).
Appendices: Formulas
Permitted materials: None

Make sure that your copy of this examination paper is complete before answering.

Problem 1 - Independent joint control (20%)

a) (4 %) Briefly describe the principal difference between decentralized and centralized control schemes.

b) (4 %) In independent motion control schemes, what are the main motivation for adding feed forward compensation of joint velocities and acceleration?

c) (12 %) Given the model of the manipulator with drives below

$$\boldsymbol{B}_m(\boldsymbol{q})\ddot{\boldsymbol{q}}_m + \boldsymbol{C}_m(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}}_m + \boldsymbol{F}_m\dot{\boldsymbol{q}}_m + \boldsymbol{g}_m(\boldsymbol{q}) = \boldsymbol{\tau}_m \tag{1}$$

where $\boldsymbol{q}_m$ is the vector of joint actuator displacement, $\boldsymbol{q}$ is the vector of the joint positions, $\boldsymbol{\tau}_m$ is the vector of actuator driving torque and

$$\boldsymbol{B}_m(\boldsymbol{q}) = \boldsymbol{K}_r^{-1}\boldsymbol{B}(\boldsymbol{q})\boldsymbol{K}_r^{-1} \qquad \boldsymbol{C}_m(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \boldsymbol{K}_r^{-1}\boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\boldsymbol{K}_r^{-1}$$
$$\boldsymbol{F}_m = \boldsymbol{K}_r^{-1}\boldsymbol{F}_v\boldsymbol{K}_r^{-1} \qquad \boldsymbol{g}_m(\boldsymbol{q}) = \boldsymbol{K}_r^{-1}\boldsymbol{g}(\boldsymbol{q})$$

where $\boldsymbol{K}_r$ is a diagonal matrix of gear reduction ratios. Show how the model (1) can be divided into one linear decoupled part and one nonlinear coupled part. Draw a block diagram of the system showing the division.

Problem 2 - Centralized control (20%)

a) (4 %) Briefly explain the difference between joint space control and operation space control.

b) (16 %) The following controller is a PD controller with gravity compensation in operational space

$$\boldsymbol{u} = \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{J}_A^T(\boldsymbol{q})\boldsymbol{K}_P\tilde{\boldsymbol{x}} - \boldsymbol{J}_A^T(\boldsymbol{q})\boldsymbol{K}_D\boldsymbol{J}_A(\boldsymbol{q})\dot{\boldsymbol{q}} \tag{2}$$

where $\tilde{\boldsymbol{x}} = \boldsymbol{x}_d - \boldsymbol{x}_e$. Here $\boldsymbol{x}_d$ is the desired end-effector pose and $\boldsymbol{x}_e$ is the current end-effector pose. The desired end-effector pose $\boldsymbol{x}_d$ is assumed to be constant, and this yields the time derivative of the error to be given as

$$\dot{\tilde{\boldsymbol{x}}} = -\boldsymbol{J}_A(\boldsymbol{q})\dot{\boldsymbol{q}} \tag{3}$$

The controller will be used to control the robotic system

$$\boldsymbol{B}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{F}\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{u} \tag{4}$$

Use the following Lyapunov function candidate

$$V(\dot{\boldsymbol{q}}, \tilde{\boldsymbol{x}}) = \frac{1}{2}\dot{\boldsymbol{q}}^T\boldsymbol{B}(\boldsymbol{q})\dot{\boldsymbol{q}} + \frac{1}{2}\tilde{\boldsymbol{x}}^T\boldsymbol{K}_P\tilde{\boldsymbol{x}} \tag{5}$$

and show that the system is asymptotically stable under the assumption that $\boldsymbol{K}_P$ and $\boldsymbol{K}_D$ are~~is~~ a symmetric positive definite constant matrix and that the Jacobian has full rank. Use Lyapunov's direct method. If $\dot{V}$ becomes negative-semi definite, check if the equilibrium posture has zero error. If the error is zero and the other conditions are met, the system is asymptotically stable.

Problem 3 - Force control (16%)

a) (4 %) The dynamic model of a manipulator interacting with the environment is given by

$$\boldsymbol{B}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{F}\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{u} - \boldsymbol{J}^T(\boldsymbol{q})\boldsymbol{h}_e \tag{6}$$

Do an exact linearization of the system, as done in the inverse dynamics control for motion control. Why is the resulting system not an double integrator, as in the inverse dynamics control for motion control case?

b) (4 %) Now assume that we measure the contact forces. Use the following equation to do en exact linearization of the system.

$$\boldsymbol{u} = \boldsymbol{B}(\boldsymbol{q})\boldsymbol{y} + \boldsymbol{n}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{J}^T(\boldsymbol{q})\boldsymbol{h}_e \tag{7}$$

Show that the system becomes an double integrator.

c) (8 %) Now we will use the following controller on the system

$$\boldsymbol{y} = \boldsymbol{J}_{A_d}^{-1}\boldsymbol{M}_d^{-1}\left(\boldsymbol{K}_D\dot{\tilde{\boldsymbol{x}}} + \boldsymbol{K}_P\tilde{\boldsymbol{x}} - \boldsymbol{M}_d\dot{\boldsymbol{J}}_{A_d}\dot{\boldsymbol{q}} + \boldsymbol{M}_d\dot{\boldsymbol{b}} - \boldsymbol{h}_e^d\right) \tag{8}$$

The acceleration error of system is given as

$$\ddot{\tilde{\boldsymbol{x}}} = -\boldsymbol{J}_{A_d}\ddot{\boldsymbol{q}} - \dot{\boldsymbol{J}}_{A_d}\dot{\boldsymbol{q}} + \dot{\boldsymbol{b}} \tag{9}$$

Find the error dynamics of the system. What kind of controller is this?

Problem 4 - Tele-operations (8%)

a) (4 %) What possible advantages are there in using tele-operations in surgery? Two advantages are sufficient.

b) (4 %) What does it mean that a tele-operated system is transparent?

Problem 5 - Visual servoing (8%)

a) (4 %) Briefly explain the principal difference between eye-in-hand and eye-to-hand configuration for a visual system.

b) (4 %) Image based visual servoing is based on the following relation between change in image features and the velocities of an object and the camera

$$\dot{\boldsymbol{s}} = \boldsymbol{J}_s\boldsymbol{v}_o^c + \boldsymbol{L}_s\boldsymbol{v}_c^c \tag{10}$$

What assumption is made in the above equation in order to use this relation to image based visual servoing. Use the assumption to simplify the above equation.

Problem 6 - Mobile robots (12%)

a) (4 %) Draw a sketch showing all the parameters of the bicycle model. This includes the position of the back wheel (x, y), position of the front wheel (x_f, y_f), the orientation of the vehicle (θ), the orientation of the front wheel (ϕ), the spacing between the wheels (l) and the instantaneous center of rotation (C).

b) (8 %) Find the non-holonomic contraints for the bicycle model expressed in the Pfaffian form. The generic non-holonomic constraint for a wheel is given as

$$v_x \sin\gamma - v_y \cos\gamma = 0 \tag{11}$$

where v_x is the wheel's velocity in along the x-axis, v_y is the wheel's velocity in along the y-axis, and γ is the wheel's orientation. You may also use the following relation

$$\sin\theta \sin(\theta + \phi) + \cos\theta \cos(\theta + \phi) = \cos\phi \tag{12}$$

Problem 7 - Motion planning (16%)

a) (4 %) Given an obstacle in a mobile robot's workspace, briefly explain how we can find the obstacle's configuration space image (i.e. the $\mathcal{C}$-obstacle). What is the benefit of representing obstacles in configuration space?

b) (12 %) In the course these four methods for motion planning was covered:

- Retraction
- Probabilistic roadmap method
- Bidirectional rapidly-exploring random tree
- Artificial potential fields

Choose one and explain the concept of the method. Also specify if the method can be considered a single- or multi-quarry method.

A Formulas

Solution for quadratic equations

$$ax^2 + bx + c = 0 \qquad \Rightarrow \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Properties of dynamic models of robots
Given the following dynamic system

$$\boldsymbol{B}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{F}\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{u}$$

The following properties hold

$$\begin{aligned} \dot{\boldsymbol{q}}^T \left(\dot{\boldsymbol{B}} - 2\boldsymbol{C}\right) \dot{\boldsymbol{q}} &= \boldsymbol{0} \\ \boldsymbol{B} &= \boldsymbol{B}^T \\ \boldsymbol{q}^T \boldsymbol{B} \boldsymbol{q} &> 0 \\ \boldsymbol{q}^T \boldsymbol{F} \boldsymbol{q} &> 0 \end{aligned}$$

Linear algebra
A matrix is symmetric if it satisfies the condition

$$\boldsymbol{A} = \boldsymbol{A}^T$$

For symmetric matrices the following holds

$$\boldsymbol{x}^T \boldsymbol{A} \boldsymbol{y} = \boldsymbol{y}^T \boldsymbol{A} \boldsymbol{x}$$

Trigonometry
Definitions

$$\sin\theta = \frac{opposite}{hypotenuse} \qquad \cos\theta = \frac{adjacent}{hypotenuse}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{opposite}{adjacent}$$

Addition formulas

$$\begin{aligned} \sin(x \pm y) &= \sin x \cos y \pm \cos x \sin y \\ \cos(x \pm y) &= \cos x \cos y \mp \sin x \sin y \end{aligned}$$

Derivatives

$$\frac{d}{dx}\sin x = \cos x \tag{13}$$

$$\frac{d}{dx}\cos x = -\sin x \tag{14}$$