Laplace transform

$$m\ddot{x} + f\dot{x} + kx = u$$

$$mxs^2 + fxs + kx = u$$

$$x(ms^2 + fs + k) = u$$

$$\frac{x}{u} = \frac{1}{ms^2 + fs + k}$$

5

10

Block manipulation

15

$$z_2 = z_1 \pm z_4$$
$$= z_1 \pm h_2 z_3$$

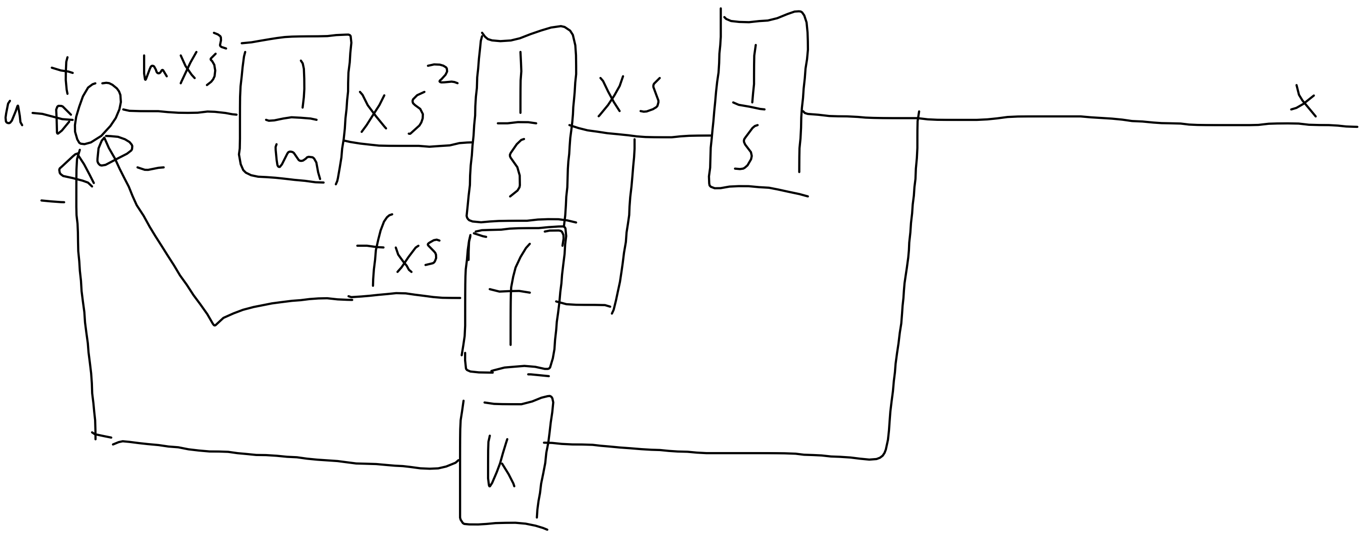
$$z_3 = h_1 z_2$$
$$= h_1 (z_1 \pm h_2 z_3)$$
$$z_3 (1 \mp h_2 h_1) = h_1 z_1$$
$$z_3 = \underbrace{\frac{h_1}{1 \mp h_2 h_1}}_{= \text{Transfer function}} z_1$$

Block diagram, mass-spring-damper

16

$$mXs^2 + fXs + kX = u$$

$$Xs^2 = \frac{1}{m}\left(u - fXs - kX\right)$$



Roots and zeros example

18

$$H(s) = \frac{2s^2+4s+4}{s^2+5s+6} = 2\,\frac{s^2+2s+2}{s^2+5s+6}$$

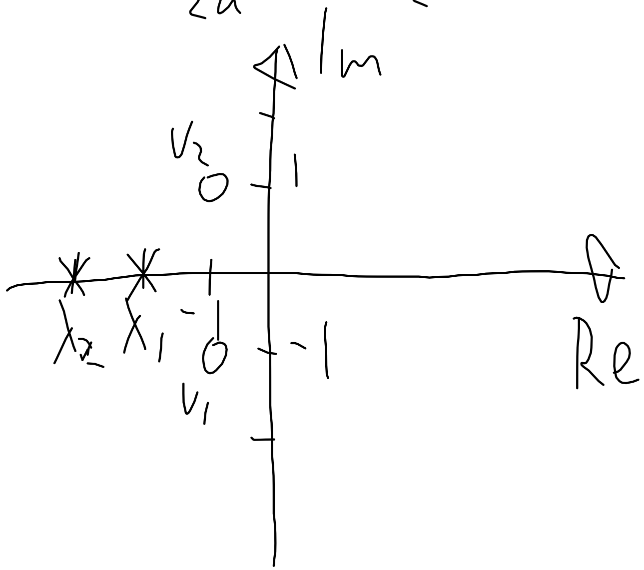
$$= 2\,\frac{(s+1+j)(s+1-j)}{(s+2)(s+3)}$$

$$ax^2+bx+c=0$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$$

$\nu_1 = -1-j \qquad \nu_2 = -1+j$

$\lambda_1 = -2 \qquad \lambda_2 = -3$



Root locus plot

(24)

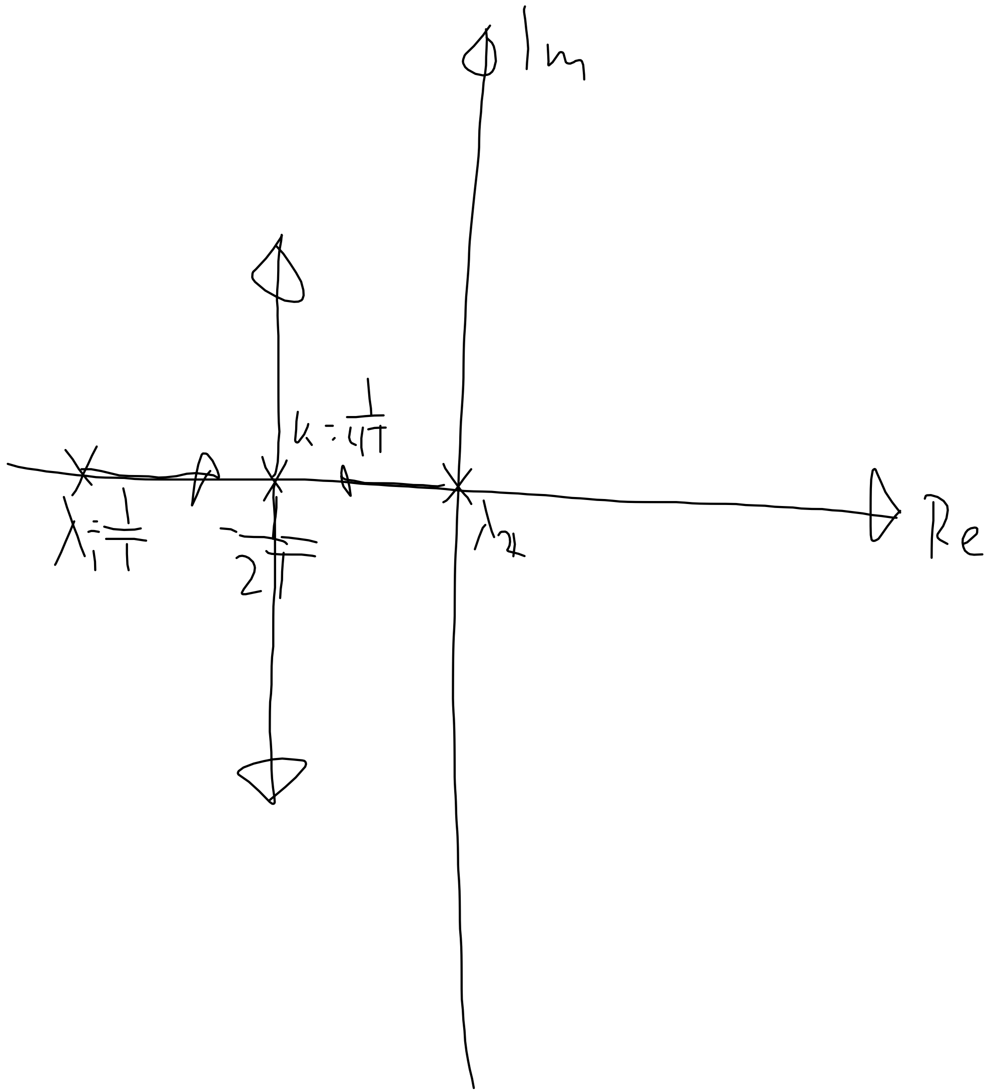
$$H(s) = \frac{1}{\frac{T}{K}s^2 + \frac{1}{K}s + 1} = \frac{1}{\frac{T}{K}(s-\lambda_1)(s-\lambda_2)}$$

$$\lambda_{1,2} = -\frac{1}{2T}\left(1 \pm \sqrt{1-4TK}\right)$$

$K = 0$

$$\lambda_{1,2} = -\frac{1}{2T}(1 \pm 1) \Rightarrow \lambda_1 = -\frac{1}{T} \quad \lambda_2 = 0$$

$$1 - 4TK = 0 \Rightarrow 1 = 4TK \Rightarrow K = \frac{1}{4T}$$


Im
Re
$k = \frac{1}{4T}$
$\lambda_1 = -\frac{1}{T}$
$-\frac{1}{2T}$
λ_2

Transforming to state space system. (29)

$$m\ddot{x}_1 + f\dot{x}_1 + kx_1 = u$$

$$\underline{x} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \Rightarrow \underline{\dot{x}} = Ax + Bu$$

I $\dot{x}_1 = x_2 \Rightarrow \ddot{x}_1 = \dot{x}_2$

II $m\dot{x}_2 + f y_2 + kx_1 = u$

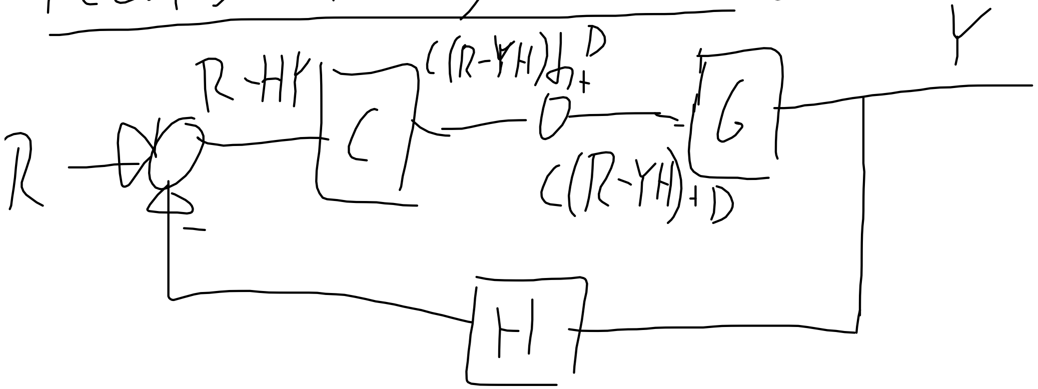
$$\dot{x}_2 = \frac{1}{m}\left(-f x_2 - kx_1 + u\right)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{f}{m} \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}}_{B} u$$

Feedback system

$GC(R-YH)+GD$

30



$$Y = GCR - GCYH + GD$$

$$Y(1+GCH) = GCR + GD$$

$$Y = \frac{GC}{\underbrace{1+GCH}_{W}} R + \frac{G}{\underbrace{1+GCH}_{W_D}} D$$

Feed forward

$$(1) \quad -YHC + RC + RF - D_c + D$$

$$Y = GRC - GYHC + GRF + G(D - D_c)$$

$$Y(1 + GHC) = GRC + GRF + G(D - D_c)$$

$$Y = \frac{GC + GF}{1 + GHC} R + \frac{G(D - D_c)}{1 + GHC}$$

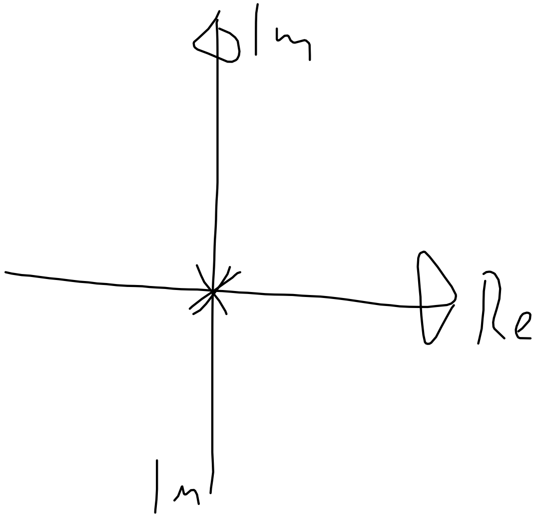
$$H = \frac{1}{K_d} = H_0 \qquad F = \frac{K_d}{G}$$

$$Y = \frac{GC + \cancel{G}\frac{K_d}{\cancel{G}}}{1 + CG\frac{1}{K_d}} R + \underbrace{\frac{G(D - D_c)}{1 + CG H_0}}_{D_{tot}}$$

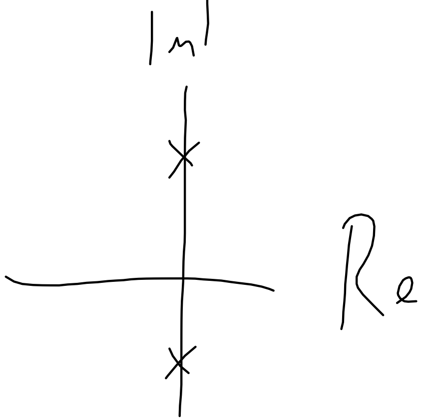
$$= K_d \frac{\cancel{\frac{1}{K_d} GC + 1}}{\cancel{1 + CG\frac{1}{K_d}}} R + D_{tot}$$

$$Y = K_d R + D_{tot} \qquad W = \frac{1 + GCH}{CG}$$

Stability in the frequency domain 30

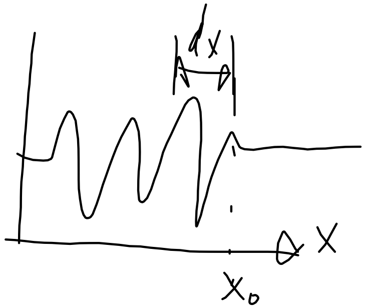


Unstable



marginally stable

Lyapunov example



$E = \frac{1}{2}k(dx)^2$

$$m\ddot{x}_1 + b\dot{x}_1^3 + kx_1 = u$$

$$\boxed{\dot{x}_1 = x_2} \Rightarrow \ddot{x}_1 = \dot{x}_2$$

$$m\dot{x}_2 + b x_2^3 + kx_1 = u$$

$$\boxed{\dot{x}_2 = -\frac{b}{m}x_2^3 - \frac{k}{m}x_1 + u} \qquad u = 0$$

$$V = \frac{1}{2}mv^2 + \frac{1}{2}k(dx)^2 = \frac{1}{2}\underline{x}^T \begin{bmatrix} k & 0 \\ 0 & m \end{bmatrix} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\underline{x}} = \frac{1}{2}\underline{x}^T Q \underline{x}$$

$$= \frac{1}{2}mx_2^2 + \frac{1}{2}kx_1^2 =$$

$$V = \frac{1}{2} \underline{x}^T Q \underline{x}$$

$$\boxed{\begin{aligned} Q &= Q^T \\ y^T Q x &= x^T Q y \end{aligned}}$$

$$\dot{V} = \frac{1}{2} \dot{\underline{x}}^T Q \underline{x} + \frac{1}{2} \underline{x}^T Q \dot{\underline{x}}$$

$$= \underline{x}^T Q \dot{\underline{x}}$$

$$= \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} k & 0 \\ 0 & m \end{bmatrix} \begin{bmatrix} x_2 \\ -\frac{b}{m} x_2^3 - \frac{k}{m} x_1 \end{bmatrix}$$

$$= \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} k x_2 \\ -b x_2^3 - k x_1 \end{bmatrix} = k x_1 x_2 - b x_2^4 - k x_1 x_2 = -b x_2^4$$