UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Exam in: UNIK4490
Day of exam: 12th of December
Exam hours: 9.15 - 13.15 (4 hours)
This examination paper consists of 11 page(s).
Appendices: Formulas
Permitted materials: None

Make sure that your copy of this examination paper is complete before answering.

General information

This exam has seven problems from different topics of the course. Before you start answering you should read through the exam, to get an overview of the problems. In some of the problems it is asked to *briefly describe* or *briefly explain*, then please write brief. In most cases two to three sentences is sufficient.

Problem 1 - Actuators and sensors (9%)

a) (3 %) Electric drives can behave in two fundamental different ways regarding the output they generate. What are these two generators called, and what are they typically used for?

- Velocity-controlled generator, primarily used for independent joint control
- Torque-controlled generator, primarily used for centralized joint control

b) (6 %) Robots have both proprioceptive and exteroceptive sensors. Explain the principal difference between these two types of sensors. Then give one example of each sensors type.

Proprioceptive sensors measure the internal state of the robot, while exteroceptive provide the robot with knowledge of the surrounding environment.
Proprioceptive sensors(one of the below gives full score)

- Position encoder (position transducer)
- Position resolver (position transducer)
- DC tachometer (velocity transducer)
- AC tachometer (velocity transducer)

Exteroceptive sensors(one of the below gives full score)

- Force sensor
- Sonar
- Laser/Lidar
- Vision sensor/camera

Problem 2 - Independent joint control (18%)

We will now look at the control of an independent joint of robot. The transfer function of the control input $U(s)$ and the joint position $X(s)$ is

$$\frac{X(s)}{U(s)} = \frac{\frac{1}{F_m}}{s(1 + \frac{I_m}{F_m}s)} \tag{1}$$

The joint is controlled using both position and velocity feedback with the controllers

$$C_P(s) = K_P \qquad\qquad C_V = K_V \frac{1 + sT_V}{s} \tag{2}$$

The control input $U(s)$ is given as

$$U(s) = C_V(s)(C_P(s)E(s) - sX(s)) \tag{3}$$

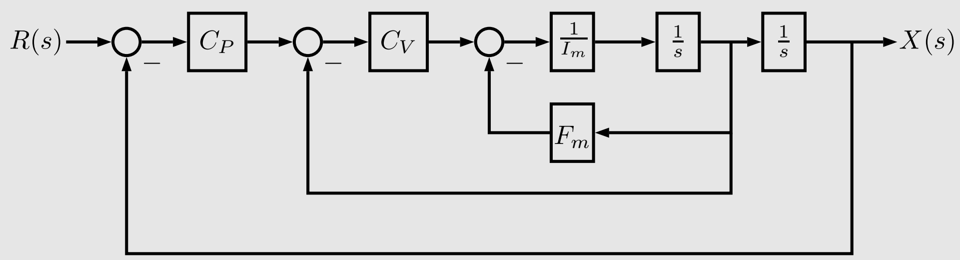
where $E(s) = R(s) - X(s)$, and $R(s)$ is the reference input to the controller. I_m and F_m are two positive constants.

a) (6 %) Draw the block diagram of the system (model and controller) using only constants blocks, integral blocks and the control blocks $C_P(s)$ and $C_V(s)$.

First transforming (1) to easier make the block diagram.

$$\begin{aligned}
X &= \frac{\frac{1}{F_m}}{s(1 + \frac{I_m}{F_m}s)} U \qquad\qquad |\cdot F_m \\
X &= \frac{1}{s(F_m + I_m s)} U \\
I_m s^2 X + F_m s X &= U \\
I_m s^2 X &= U - F_m s X
\end{aligned}$$

Then draw diagram based on (1) and (3).



b) (9 %) Find the transfer function of the system assuming that $\frac{I_m}{F_m} = T_V$. Which order is the system? Find the poles of the system as an expression. Is the system stable? Explain why/why not.

Inserting (3) and the controllers into (1)

$$X(s) = \frac{\frac{1}{F_m}}{s(1+\frac{I_m}{F_m}s)} K_V \frac{1+sT_V}{s} \left(K_P(R-X) - sX\right)$$

Inserting assumption $\frac{I_m}{F_m} = T_V$

$$\begin{aligned} X(s) &= \frac{\frac{1}{F_m}}{s\cancel{(1+T_V s)}} K_V \frac{\cancel{1+sT_V}}{s} \left(K_P(R-X) - sX\right) \\ &= \frac{\frac{1}{F_m}}{s} K_V \frac{1}{s} \left(K_P(R-X) - sX\right) & |\cdot s \\ s^2 X &= \frac{1}{F_m} K_V \left(K_P(R-X) - sX\right) \end{aligned}$$

Rearranging to get $X(s)$ on one side and $R(s)$ on the other

$$\begin{aligned} &F_m s^2 X + K_V sX + K_V K_P X = K_V K_P R \\ &\frac{X}{R} = \frac{K_V K_P}{F_m s^2 + K_V s + K_V K_P} \end{aligned}$$

The transfer function is found above. The system is of second order. The poles are given as

$$s = \frac{-K_V \pm \sqrt{K_V^2 - 4F_M K_V K_P}}{2F_m}$$

The system is stable as long as K_V is positive and larger than $\sqrt{K_V^2 - 4F_m K_V K_P}$.

c) (3 %) When doing independent joint control, the model is divided into a linear decoupled part and a nonlinear coupled part. Explain briefly how this division is done. When designing control systems, which part is considered the model and which part is considered as a disturbance.

All configuration dependent parts of the dynamic equation is put into a nonlinear coupled part, which is considered as disturbance to the system. The linear decoupled part is used as the manipulator model.

Problem 3 - Centralized control (24%)

a) (3 %) Briefly describe the difference between independent joint control and centralized control.

In independent joint control each joint is controlled only using measurements from that joint. All effect from other joints are considered as disturbance. In centralized control the control law is using measurements from all the joints and uses a model based on all the joints.

b) (9 %) A PD controller with gravity compensation is given below

$$\boldsymbol{u} = \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{K}_P\tilde{\boldsymbol{q}} - \boldsymbol{K}_D\dot{\boldsymbol{q}} \tag{4}$$

where $\tilde{\boldsymbol{q}} = \boldsymbol{q}_d - \boldsymbol{q}$, and $\boldsymbol{q}_d$ is the desired joint positions which is constant. This controller will be used to control the system

$$\boldsymbol{B}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{F}\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{u} \tag{5}$$

The Lyapunov function candidate for the controller is

$$\boldsymbol{V}(\dot{\boldsymbol{q}},\tilde{\boldsymbol{q}}) = \frac{1}{2}\dot{\boldsymbol{q}}^T\boldsymbol{B}(\boldsymbol{q})\dot{\boldsymbol{q}} + \frac{1}{2}\tilde{\boldsymbol{q}}^T\boldsymbol{K}_P\tilde{\boldsymbol{q}} \tag{6}$$

Show that the system has a negative semi-definite $\dot{\boldsymbol{V}}$, but that the other conditions in Lyapunov's direct method is satisfied.

The three conditions that are satisfied are

$$\begin{aligned}
&\boldsymbol{V}(\dot{\boldsymbol{q}},\tilde{\boldsymbol{q}}) > 0 && \forall\, \dot{\boldsymbol{q}},\tilde{\boldsymbol{q}} \neq 0\\
&\boldsymbol{V}(\dot{\boldsymbol{q}},\tilde{\boldsymbol{q}}) = 0 && \dot{\boldsymbol{q}},\tilde{\boldsymbol{q}} = 0\\
&\boldsymbol{V}(\dot{\boldsymbol{q}},\tilde{\boldsymbol{q}}) \to \infty && ||\dot{\boldsymbol{q}},\tilde{\boldsymbol{q}}|| \to \infty
\end{aligned}$$

The last condition $\dot{\boldsymbol{V}}(\dot{\boldsymbol{q}},\tilde{\boldsymbol{q}}) < 0\ \forall\ \dot{\boldsymbol{q}},\tilde{\boldsymbol{q}} \neq 0$ is not satisfied (as stated in the problem). Taking the time derivative of (6) to show that.

$$\begin{aligned}
\dot{\boldsymbol{V}}(\dot{\boldsymbol{q}},\tilde{\boldsymbol{q}}) &= \frac{1}{2}\ddot{\boldsymbol{q}}^T\boldsymbol{B}(\boldsymbol{q})\dot{\boldsymbol{q}} + \frac{1}{2}\dot{\boldsymbol{q}}^T\dot{\boldsymbol{B}}(\boldsymbol{q})\dot{\boldsymbol{q}} + \frac{1}{2}\dot{\boldsymbol{q}}^T\boldsymbol{B}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \frac{1}{2}\dot{\tilde{\boldsymbol{q}}}^T\boldsymbol{K}_P\tilde{\boldsymbol{q}} + \frac{1}{2}\tilde{\boldsymbol{q}}^T\boldsymbol{K}_P\dot{\tilde{\boldsymbol{q}}}\\
\dot{\boldsymbol{V}}(\dot{\boldsymbol{q}},\tilde{\boldsymbol{q}}) &= \frac{1}{2}\dot{\boldsymbol{q}}^T\dot{\boldsymbol{B}}(\boldsymbol{q})\dot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^T\boldsymbol{B}(\boldsymbol{q})\ddot{\boldsymbol{q}} - \dot{\boldsymbol{q}}^T\boldsymbol{K}_P\tilde{\boldsymbol{q}}
\end{aligned}$$

Inserting for $\boldsymbol{B}(\boldsymbol{q})\ddot{\boldsymbol{q}}$

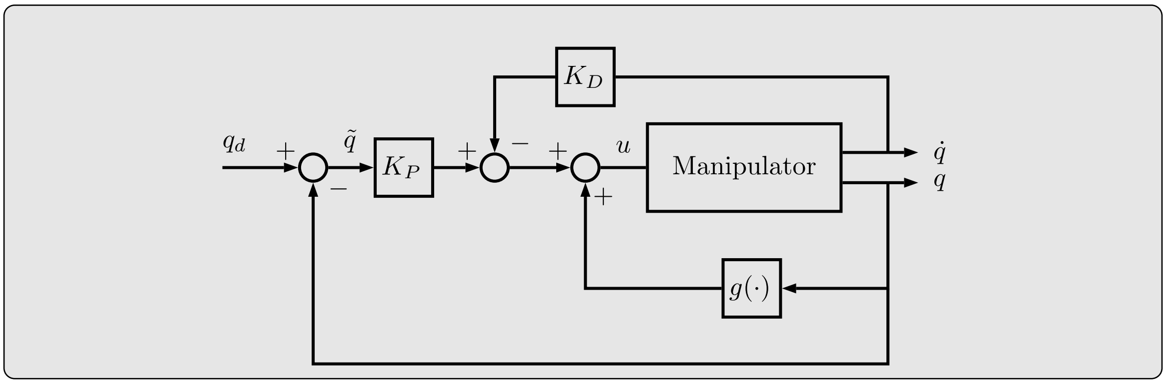
$$\dot{\boldsymbol{V}}(\dot{\boldsymbol{q}},\tilde{\boldsymbol{q}}) = \frac{1}{2}\dot{\boldsymbol{q}}^T\dot{\boldsymbol{B}}(\boldsymbol{q})\dot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^T\left(\boldsymbol{u} - \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} - \boldsymbol{F}\dot{\boldsymbol{q}} - \boldsymbol{g}(\boldsymbol{q})\right) - \dot{\boldsymbol{q}}^T\boldsymbol{K}_P\tilde{\boldsymbol{q}}$$

Using $\dot{\boldsymbol{B}} - 2\boldsymbol{C} = \boldsymbol{0}$ and inserting for $\boldsymbol{u}$

$$\begin{aligned}
\dot{\boldsymbol{V}}(\dot{\boldsymbol{q}},\tilde{\boldsymbol{q}}) &= \dot{\boldsymbol{q}}^T\left(\cancel{\boldsymbol{g}(\boldsymbol{q})} + \cancel{\boldsymbol{K}_P\tilde{\boldsymbol{q}}} - \boldsymbol{K}_D\dot{\boldsymbol{q}} - \boldsymbol{F}\dot{\boldsymbol{q}} - \cancel{\boldsymbol{g}(\boldsymbol{q})}\right) - \cancel{\dot{\boldsymbol{q}}^T\boldsymbol{K}_P\tilde{\boldsymbol{q}}}\\
\dot{\boldsymbol{V}}(\dot{\boldsymbol{q}},\tilde{\boldsymbol{q}}) &= -\dot{\boldsymbol{q}}^T\left(\boldsymbol{K}_D + \boldsymbol{F}\right)\dot{\boldsymbol{q}}
\end{aligned}$$

$\dot{\boldsymbol{V}}(\dot{\boldsymbol{q}},\tilde{\boldsymbol{q}})$ negative for all $\dot{\boldsymbol{q}} \neq 0$, but is independent of $\tilde{\boldsymbol{q}}$ and therefor negative semi-definite.

c) (3 %) Draw a block diagram of the PD controller with gravity compensation in (4). Draw the manipulator model as one block with $\boldsymbol{u}$ as input and $\boldsymbol{q}$ and $\dot{\boldsymbol{q}}$.



d) (3 %) What is the physical interpretation of the two terms of $\boldsymbol{V}(\dot{\boldsymbol{q}}, \tilde{\boldsymbol{q}})$ in (6)?

The first term is kinetic energy and the second term is potential energy

e) (6 %) Given the system

$$\boldsymbol{B}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{F}\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{u} \tag{7}$$

Show how it is possible to do an exact linearization of the system to obtain

$$\ddot{\boldsymbol{q}} = \boldsymbol{y} \tag{8}$$

Setting

$$\boldsymbol{n}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{F}\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q})$$

Then using the control law

$$\boldsymbol{u} = \boldsymbol{B}(\boldsymbol{q})\boldsymbol{y} + \boldsymbol{n}(\boldsymbol{q}, \dot{\boldsymbol{q}})$$

yields

$$\begin{aligned} \boldsymbol{B}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{n}(\boldsymbol{q}, \dot{\boldsymbol{q}}) &= \boldsymbol{B}(\boldsymbol{q})\boldsymbol{y} + \boldsymbol{n}(\boldsymbol{q}, \dot{\boldsymbol{q}}) \\ \boldsymbol{B}(\boldsymbol{q})\ddot{\boldsymbol{q}} &= \boldsymbol{B}(\boldsymbol{q})\boldsymbol{y} \\ \ddot{\boldsymbol{q}} &= \boldsymbol{y} \end{aligned}$$

Problem 4 - Force control (9%)

a) (3 %) Briefly explain the principal difference between indirect and direct force control.

Indirect force control methods alter the position of the manipulator based on external force, but do not regulate on the force error. Direct force control methods tries to reach a desired force, thus controlling the force directly.

b) (3 %) When a robot is under active compliance control and interacts with the environment, which two factors determine the actual position of the robot at steady-state?

> The compliance of the robot and the compliance of the environment.

c) (3 %) Briefly explain the differences between compliance and impedance control.

> In compliance control the manipulator works as a virtual spring, while in impedance control the manipulator works as a virtual mass-spring-damper system.

Problem 5 - Visual servoing (12%)

a) (3 %) Explain the principal difference between image-based and position-based visual servoing.

> Position-based visual servoing estimates the pose of object in the scene and control the manipulator using that pose (i.e. pose as feedback variable). Image-based visual servoing control the manipulator by using image features (i.e. image features as feedback variable).

b) (3 %) In the equation

$$\dot{\boldsymbol{s}} = \boldsymbol{J}_s \boldsymbol{v}_o^c + \boldsymbol{L}_s \boldsymbol{v}_c^c \tag{9}$$

what are the names of $\boldsymbol{J}_s$ and $\boldsymbol{L}_s$ and what does $\boldsymbol{v}_o^c$ and $\boldsymbol{v}_c^c$ represent?

> $\boldsymbol{J}_s$ is the image Jacobian and $\boldsymbol{L}_s$ is the interaction matrix. $\boldsymbol{v}_o^c$ is the object velocity on the camera frame and $\boldsymbol{v}_c^c$ is the camera velocity in the camera frame.

c) (6 %) For image-based visual servoing we have the following relations

$$\dot{\boldsymbol{s}} = \boldsymbol{L}_{\boldsymbol{s}} \boldsymbol{v}_c^c \qquad\qquad \boldsymbol{v}_c^c = \boldsymbol{J}_c \dot{\boldsymbol{q}} = \begin{bmatrix} \boldsymbol{R}_c & 0 \\ 0 & \boldsymbol{R}_c \end{bmatrix} \boldsymbol{J} \dot{\boldsymbol{q}} \tag{10}$$

We assume that our robot functions as an ideal positioning device, i.e. $\boldsymbol{q}(t) \approx \boldsymbol{q}_r(t)$, where $\boldsymbol{q}_r(t)$ is the reference input. Find a control law that gives the following error dynamics

$$\dot{\boldsymbol{e}}_s + \boldsymbol{K}_s \boldsymbol{e}_s = 0 \tag{11}$$

where the error definition is $\boldsymbol{e}_s = \boldsymbol{s}_d - \boldsymbol{s}$. The desired image features $\boldsymbol{s}_d$ are constant. The control law should be on the form $\dot{\boldsymbol{q}}_r = \ldots$

Two possible ways to finding the control law. (1) use the desired error dynamics to find the control law (2) postulate the controller from the textbook and prove that it gives the desired error dynamics. Showing the solution using (1) here.

$$\begin{aligned}\dot{\boldsymbol{e}}_s &= \dot{\boldsymbol{s}}_d - \dot{\boldsymbol{s}}\\ &= 0 - \boldsymbol{L_s}\boldsymbol{J}_c\dot{\boldsymbol{q}}\\ &= -\boldsymbol{J}_L\dot{\boldsymbol{q}}\end{aligned}$$

Inserting this into the error dynamics equation

$$\begin{aligned}\dot{\boldsymbol{e}}_s + \boldsymbol{K}_s\boldsymbol{e}_s &= 0\\ -\boldsymbol{J}_L\dot{\boldsymbol{q}} + \boldsymbol{K}_s\boldsymbol{e}_s &= 0\\ \boldsymbol{J}_L\dot{\boldsymbol{q}} &= \boldsymbol{K}_s\boldsymbol{e}_s\\ \dot{\boldsymbol{q}} &= \boldsymbol{J}_L^{-1}\boldsymbol{K}_s\boldsymbol{e}_s\end{aligned}$$

Since $\boldsymbol{q}(t) \approx \boldsymbol{q}(t)$ we set

$$\dot{\boldsymbol{q}}_r = \boldsymbol{J}_L^{-1}\boldsymbol{K}_s\boldsymbol{e}_s$$

Problem 6 - Mobile robots (18%)

a) (3 %) What is a non-holonomic constraint and why are they relevant for mobile robotics?

Non-holonomic constraint is a constraint that cannot be expressed in the configuration variables alone, but includes the time derivative of the configuration variables. Most mobile robots have these constraints.

b) (3 %) Briefly describe one of the methods for path planning for mobile robots

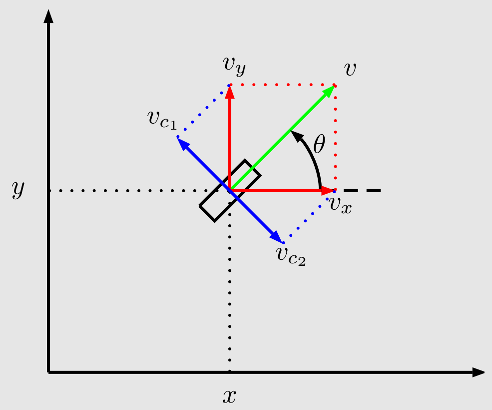
Describe on of the methods

- Planning via Cartesian polynomials (p. 492)
- Planning via the chained form (p. 493)
- Planning via parameterized inputs (p494)

c) (3 %) Briefly explain the principal difference between trajectory tracking and posture regulation

With trajectory tracking a path with associated timing law must be asymptotically tracked, while with posture regulation the robot must reach a desired final posture from a given initial posture (i.e. the posture in between are note regarded).

d) (9 %) Draw a figure showing the configuration variables of a unicycle. Find an expression of the non-holonomic constraint of the unicycle and derive the kinematic model.



The above figure shows the three configuration variables x, y and θ. The wheel of the unicycle only drive forwards and backwards, thus there can be no velocity perpendicular to the driving direction. This means that the velocities v_{c_1} and v_{c_2}, indicated by blue arrows, must be equal. These are found using trigonometry

$$\cos\theta = \frac{v_{c_1}}{v_y} \qquad \Rightarrow \qquad v_{c_1} = v_y \cos\theta$$
$$\sin\theta = \frac{v_{c_2}}{v_x} \qquad \Rightarrow \qquad v_{c_2} = v_x \sin\theta$$

Setting the two velocities equals each other, and using the configuration vector $\boldsymbol{q} = [xy\theta]^T$ yields

$$\begin{aligned} v_x \sin\theta &= v_y \cos\theta \\ v_x \sin\theta - v_y \cos\theta &= 0 \\ [\sin\theta \ \ -\cos\theta \ 0]\dot{\boldsymbol{q}} &= \boldsymbol{A}^T(\boldsymbol{q})\dot{\boldsymbol{q}} = 0 \end{aligned}$$

which is the non-holonomic constraint for the system.
The kinematics equations can be found by finding the basis $\boldsymbol{G}(\boldsymbol{q})$ of the null space of $\boldsymbol{A}^T(\boldsymbol{q})$. The kinematics is then $\dot{\boldsymbol{q}} = \boldsymbol{G}(\boldsymbol{q})\boldsymbol{u}$. Or the kinematics can be found using trigonometry on the above figure. This yields

$$\cos\theta = \frac{v_x}{v} \qquad \Rightarrow \qquad v_x = v\cos\theta$$
$$\sin\theta = \frac{v_y}{v} \qquad \Rightarrow \qquad v_y = v\sin\theta$$

Then the rotation rate is set to be the input ω. This yields

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$

Problem 7 - Control of AUV and USV (10%)

A three degree of freedom maneuvering model of an USV is given by

$$\dot{x} = u\cos(\psi) - v\sin(\psi), \tag{12a}$$
$$\dot{y} = u\sin(\psi) + v\cos(\psi), \tag{12b}$$
$$\dot{\psi} = r, \tag{12c}$$
$$\dot{u} = F_u(v,r) - \frac{d_{11}}{m_{11}}u + \tau_u, \tag{12d}$$
$$\dot{v} = X(u)r + Y(u)v, \tag{12e}$$
$$\dot{r} = F_r(u,v,r) + \tau_r. \tag{12f}$$

Here, x and y are the north and east position in the NED frame, and ψ is the vehicle heading. The velocities u and v are forward (surge) and sideways (sway) velocities in BODY, while r is the heading rate. The functions $F_u(v,r)$, $F_r(u,v,r)$, $X(u)$ and $Y(u)$ are general nonlinar term. The details of these are not required here. The damping term d_{11} and m_{11} are positive constants. Finally, the vehicle is controlled in surge and yaw rate through the control inputs τ_u and τ_r.

a) (2.5 %) Assume that u and v are known. Find an expression for the sideslip angle β of the vehicle, using u and v.

$\beta = \text{atan2}(v,u)$

b) (2.5 %) Find an expression for the total speed U of the vehicle.

$U = \sqrt{u^2+v^2}$

c) (2.5 %) Assume that the sideslip is $\beta = 30\,°$, and that the vehicle heading is $\psi = 90\,°$. What is the course χ of the vehicle?

$\chi = \psi + \beta = 120\,°$

d) (2.5 %) Assume that the total speed U and the course χ are known. Rewrite equations (12a) and (12b) using U and χ.

$$\dot{x} = U\cos(\chi),$$
$$\dot{y} = U\sin(\chi),$$

A Formulas

Solution for quadratic equations

$$ax^2 + bx + c = 0 \qquad \Rightarrow \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Properties of dynamic models of robots
Given the following dynamic system

$$\boldsymbol{B}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{F}\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{u}$$

The following properties hold

$$\begin{aligned} \dot{\boldsymbol{B}} - 2\boldsymbol{C} &= \boldsymbol{0} \\ \boldsymbol{B} &= \boldsymbol{B}^T \\ \boldsymbol{q}^T\boldsymbol{B}\boldsymbol{q} &> 0 \\ \boldsymbol{q}^T\boldsymbol{F}\boldsymbol{q} &> 0 \end{aligned}$$

Linear algebra
A matrix is symmetric if it satisfies the condition

$$\boldsymbol{A} = \boldsymbol{A}^T$$

For symmetric matrices the following holds

$$\boldsymbol{x}^T\boldsymbol{A}\boldsymbol{y} = \boldsymbol{y}^T\boldsymbol{A}\boldsymbol{x}$$

Trigonometry
Definitions

$$\sin\theta = \frac{opposite}{hypotenuse} \qquad \cos\theta = \frac{adjacent}{hypotenuse}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{opposite}{adjacent}$$

Addition formulas

$$\begin{aligned} \sin(x \pm y) &= \sin x \cos y \pm \cos x \sin y \\ \cos(x \pm y) &= \cos x \cos y \mp \sin x \sin y \end{aligned}$$