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9. Force Control

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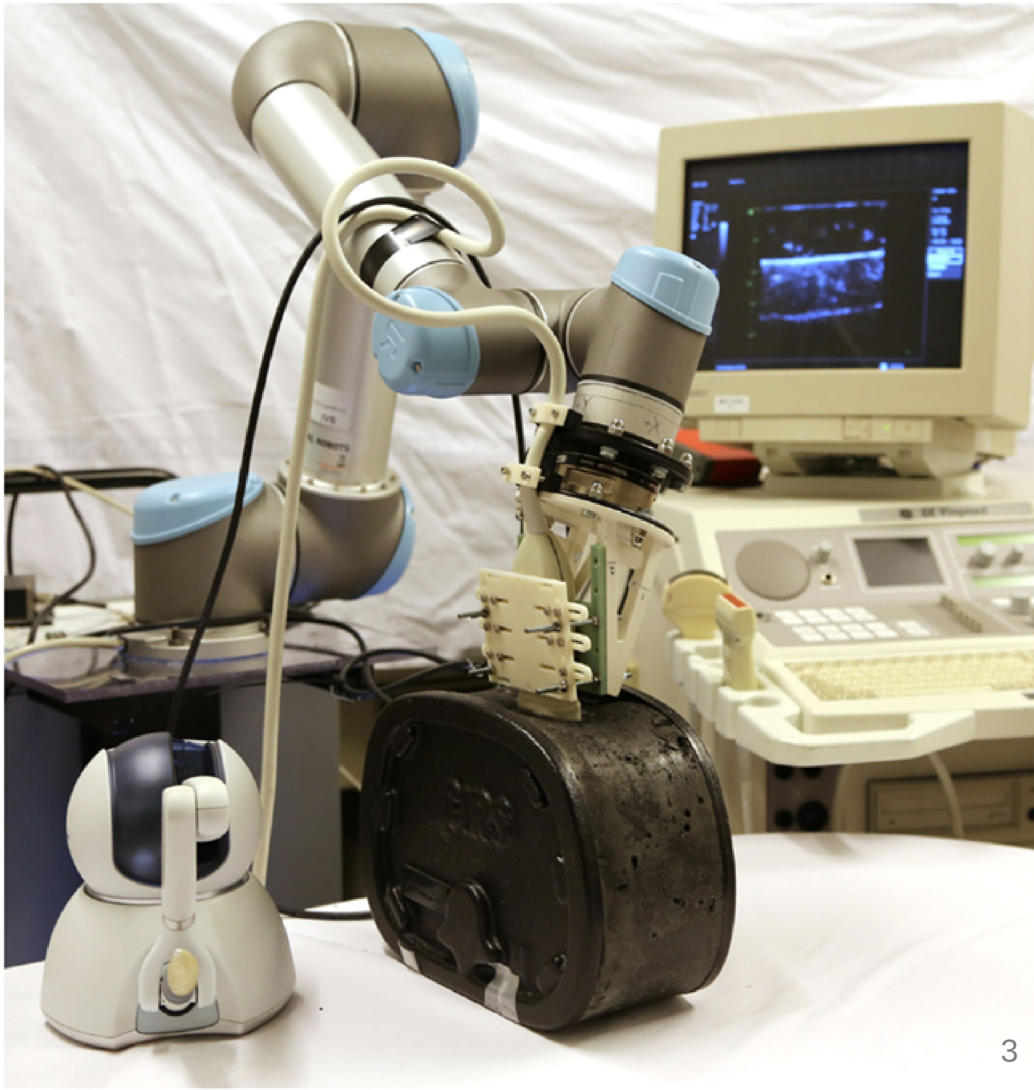
Lecture overview

- Indirect force control
  - Compliance control (PD control with gravity compensation)
  - Impedance control (Inverse dynamics control)
- Direct force control
  - Inner position loop
  - Inner velocity loop
  - Parallel force/position control

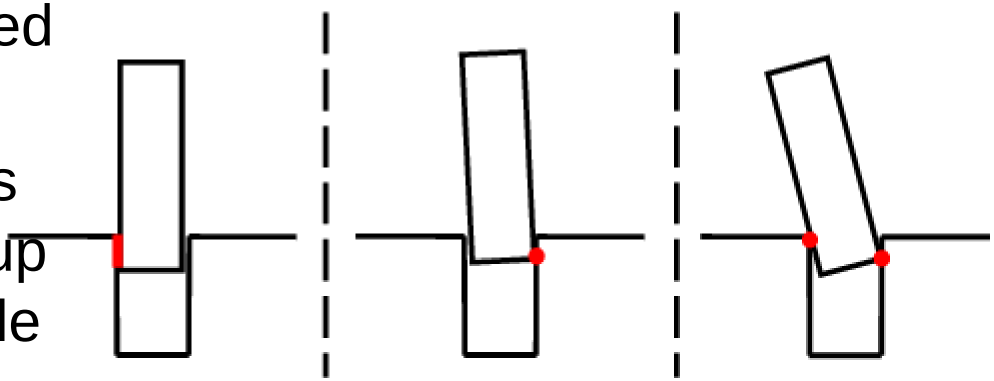
Motivation

- In motion control the manipulator was moving in free air
- Force control handle interaction between the manipulator and the environment
- Typical tasks may be polishing, machining or assembly



Example: Mating task

- Without force control everything must be modeled very accurately
- Because of planning errors and high stiffness a build-up of contact force is inevitable
- This could lead to saturation of the joint actuators or breakage of parts



(a) Line Contact (b) One-Point Contact (c) Two-Point Contact

9.2 Compliance control

- Manipulator dynamic model with contact force

$$\boldsymbol{B}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{F}\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{u} - \boldsymbol{J}^T(\boldsymbol{q})\boldsymbol{h}_e \tag{9.1}$$

- where $\boldsymbol{h}_e$ is a 6x1 vector of forces and torques
- Assuming Operational Space control, and PD control with gravity compensation
- If $\boldsymbol{h}_e \neq \boldsymbol{0}$, the controller no longer ensures that the end effector reaches its desired pose
- An equilibrium is found in

$$\boldsymbol{J}_A^T(\boldsymbol{q})\boldsymbol{K}_P\widetilde{\boldsymbol{x}} = \boldsymbol{J}^T(\boldsymbol{q})\boldsymbol{h}_e. \tag{9.2}$$

$$J = T_A(\phi) J_A. \qquad (3.66)$$

Compliance control

- Assuming full rank Jacobian (i.e. the Jacobian is invertible) the error is given as

$$\widetilde{x} = K_P^{-1} T_A^T(x_e) h_e = K_P^{-1} h_A \qquad (9.3)$$

- Where $T_A^T(x_e) h_e = h_A$, and T_A is the transformation matrix between the geometric and analytic Jacobian
- This shows that the manipulator behaves like a generalized spring in the operational space
- The compliance of the spring is K_P^{-1}

Remarks

- The transform between the geometric and analytic Jacobian is given by

$$\boldsymbol{v}_e = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{T}(\boldsymbol{\phi}_e) \end{bmatrix} \dot{\boldsymbol{x}}_e = \boldsymbol{T}_A(\boldsymbol{\phi}_e)\dot{\boldsymbol{x}}_e \qquad (3.65)$$

- Assuming $\boldsymbol{K}_P^{-1}$ is diagonal, it can be recognized that
  - Linear compliance is configure independent
  - Torsional compliance depends on the end-effector configuration
- Rewriting (9.3)

$$\boldsymbol{h}_A = \boldsymbol{K}_P \widetilde{\boldsymbol{x}} \qquad (9.4)$$

- Now $\boldsymbol{K}_P$ represents stiffness

9.2.1 Passive compliance

- Consider two elastically coupled rigid bodies R and S
- The elementary displacement is

$$d\boldsymbol{x}_{r,s} = \begin{bmatrix} d\boldsymbol{o}_{r,s} \\ \boldsymbol{\omega}_{r,s}dt \end{bmatrix} = \boldsymbol{v}_{r,s}dt \tag{9.5}$$

- Where

$$\begin{aligned} \boldsymbol{v}_{r,s} &= \boldsymbol{v}_s - \boldsymbol{v}_r \\ d\boldsymbol{o}_{r,s} &= \boldsymbol{o}_s - \boldsymbol{o}_r \\ \boldsymbol{\omega}_{r,s} &= \boldsymbol{\omega}_s - \boldsymbol{\omega}_r \end{aligned}$$

- The displacement yields an elastic force (spring model)

$$\boldsymbol{h}_s = \begin{bmatrix} \boldsymbol{f}_s \\ \boldsymbol{\mu}_s \end{bmatrix} = \begin{bmatrix} \boldsymbol{K}_f & \boldsymbol{K}_c \\ \boldsymbol{K}_c^T & \boldsymbol{K}_\mu \end{bmatrix} \begin{bmatrix} d\boldsymbol{o}_{r,s} \\ \boldsymbol{\omega}_{r,s}dt \end{bmatrix} = \boldsymbol{K}d\boldsymbol{x}_{r,s}, \tag{9.6}$$

Passive compliance

$$\boldsymbol{h}_s = \begin{bmatrix} \boldsymbol{f}_s \\ \boldsymbol{\mu}_s \end{bmatrix} = \begin{bmatrix} \boldsymbol{K}_f & \boldsymbol{K}_c \\ \boldsymbol{K}_c^T & \boldsymbol{K}_\mu \end{bmatrix} \begin{bmatrix} d\boldsymbol{o}_{r,s} \\ \boldsymbol{\omega}_{r,s} dt \end{bmatrix} = \boldsymbol{K} d\boldsymbol{x}_{r,s}, \tag{9.6}$$

- The 6x6 matrix $\boldsymbol{K}$ represents a stiffness matrix
- $\boldsymbol{K}_f$ and $\boldsymbol{K}_\mu$ is the translational and rotational stiffness
- $\boldsymbol{K}_c$ is the coupling stiffness
- Same mapping can be made for the compliance C

$$d\boldsymbol{x}_{r,s} = \boldsymbol{C} \boldsymbol{h}_s. \tag{9.7}$$

Remarks

- In real elastic systems K_c is generally non-symmetric
- There are devices that can make K_c symmetric or null, such as Remote Center of Compliance (RCC) devices
- The aim at introducing a passive compliance device is to facilitate the execution of a task (for instance assembly or peg in a hole)
- For the peg in a hole task the device ensures high stiffness in the insertion direction and high compliance along the other directions
- The downside with such devices is low versatility

9.2.2 Active compliance

- Active compliance is achieved through the control of the manipulator
- Want to have a controller that has the same relationship as in passive compliance

$$\boldsymbol{h}_s = \begin{bmatrix} \boldsymbol{f}_s \\ \boldsymbol{\mu}_s \end{bmatrix} = \begin{bmatrix} \boldsymbol{K}_f & \boldsymbol{K}_c \\ \boldsymbol{K}_c^T & \boldsymbol{K}_\mu \end{bmatrix} \begin{bmatrix} d\boldsymbol{o}_{r,s} \\ \boldsymbol{\omega}_{r,s} dt \end{bmatrix} = \boldsymbol{K} d\boldsymbol{x}_{r,s}, \tag{9.6}$$

- We would like to specify the compliance of the manipulator

Active compliance

- Redefining the error to be in the desired frame
- Transformation matric between current end-effector pose (e) and desired pose (d)

$$\boldsymbol{T}_e^d = (\boldsymbol{T}_d)^{-1}\boldsymbol{T}_e = \begin{bmatrix} \boldsymbol{R}_e^d & \boldsymbol{o}_{d,e}^d \\ \boldsymbol{0}^T & 1 \end{bmatrix}, \qquad (9.8)$$

- Where $\boldsymbol{R}_e^d = \boldsymbol{R}_d^T \boldsymbol{R}_e$ and $\boldsymbol{o}_{d,e}^d = \boldsymbol{R}_d^T(\boldsymbol{o}_e - \boldsymbol{o}_d)$.
- New operational space error is

$$\widetilde{\boldsymbol{x}} = -\begin{bmatrix} \boldsymbol{o}_{d,e}^d, \\ \boldsymbol{\phi}_{d,e} \end{bmatrix} \qquad (9.9)$$

- Where $\phi_{d,e}$ are the Euler angles extracted from the rotation matrix $\boldsymbol{R}_e^d$

Active compliance

- Assuming that the desired pose is constant and time differentiating the error yields

$$\dot{\tilde{\boldsymbol{x}}} = -\boldsymbol{T}_A^{-1}(\phi_{d,e}) \begin{bmatrix} \boldsymbol{R}_d^T & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{R}_d^T \end{bmatrix} \boldsymbol{v}_e \qquad (9.12)$$

- Where $\boldsymbol{v}_e = [\,\dot{\boldsymbol{o}}_e^T \quad \boldsymbol{\omega}_e^T\,]^T = \boldsymbol{J}(\boldsymbol{q})\dot{\boldsymbol{q}}$
- We now find the analytic Jacobian corresponding to the new error definition

$$\dot{\tilde{\boldsymbol{x}}} = -\boldsymbol{J}_{A_d}(\boldsymbol{q}, \tilde{\boldsymbol{x}})\dot{\boldsymbol{q}}, \qquad (9.13)$$

- Where

$$\boldsymbol{J}_{A_d}(\boldsymbol{q}, \tilde{\boldsymbol{x}}) = \overbrace{\boldsymbol{T}_A^{-1}(\phi_{d,e})}^{\text{the mat: translate btw geo and an: J}} \begin{bmatrix} \boldsymbol{R}_d^T & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{R}_d^T \end{bmatrix} \boldsymbol{J}(\boldsymbol{q}) \qquad (9.14)$$

Active compliance

- Setting up a new controller similar to the PD controller with gravity compensation

$$\boldsymbol{u} = \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{J}_{A_d}^T(\boldsymbol{q}, \widetilde{\boldsymbol{x}})(\boldsymbol{K}_P\widetilde{\boldsymbol{x}} - \boldsymbol{K}_D\boldsymbol{J}_{A_d}(\boldsymbol{q}, \widetilde{\boldsymbol{x}})\dot{\boldsymbol{q}}). \tag{9.15}$$

- Asymptotic stability can be proven assuming that $\boldsymbol{K}_P$ and $\boldsymbol{K}_D$ are symmetric positive definite matrices and h_e = 0
- In presence of interaction forces the equilibrium is

$$\boldsymbol{J}_{A_d}^T(\boldsymbol{q})\boldsymbol{K}_P\widetilde{\boldsymbol{x}} = \boldsymbol{J}^T(\boldsymbol{q})\boldsymbol{h}_e; \tag{9.16}$$

- Assuming full rank Jacobian yields

$$\boldsymbol{h}_e^d = \boldsymbol{T}_A^{-T}(\phi_{d,e})\boldsymbol{K}_P\widetilde{\boldsymbol{x}}. \tag{9.17}$$

Comparing with position control

$$u = g(q) + J_A^T(q)K_P\widetilde{x} - J_A^T(q)K_D J_A(q)\dot{q} \qquad (8.110)$$

$$u = g(q) + J_{A_d}^T(q,\widetilde{x})(K_P\widetilde{x} - K_D J_{A_d}(q,\widetilde{x})\dot{q}). \qquad (9.15)$$

- The difference in the control laws is only how the error is defined
- In (8.110) the error is referred to in the base frame
- In (9.5) the error is referred to in the desired frame

Active compliance

- To compare this result with the passive compliance case the equation must be rewritten in terms of elementary displacements
- Assuming XYZ Euler angles yields

$$\boldsymbol{h}_e = \boldsymbol{K}_P d\boldsymbol{x}_{e,d}, \tag{9.19}$$

- $\boldsymbol{K}_P$ is the active stiffness corresponding to a generalized spring

- Rewriting the equation

$$d\boldsymbol{x}_{e,d} = \boldsymbol{K}_P^{-1} \boldsymbol{h}_e, \tag{9.20}$$

- $\boldsymbol{K}_P^{-1}$ is the active compliance

Interacting with the environment

- Selection of $\boldsymbol{K}_P$ must take into account the geometrical and mechanical features of the environment
- The controller performance is highly affected by the properties of the environment
- The elastic force applied by the end-effector on the environment is given by

$$\boldsymbol{h}_e = \boldsymbol{K} d\boldsymbol{x}_{r,e} \qquad (9.21)$$

- Where $d\boldsymbol{x}_{r,e}$ is the displacement of the environment rest position and the environment and $\boldsymbol{K}$ is the stiffness matrix of the environment

Interacting with the environment

- The contact force at equilibrium can be found to be

$$\boldsymbol{h}_e = \left(\boldsymbol{I}_6 + \boldsymbol{K}\boldsymbol{K}_P^{-1}\right)^{-1}\boldsymbol{K}\,d\boldsymbol{x}_{r,d}. \tag{9.22}$$

- The end-effector pose error can be found to be

$$d\boldsymbol{x}_{e,d} = \boldsymbol{K}_P^{-1}\left(\boldsymbol{I}_6 + \boldsymbol{K}\boldsymbol{K}_P^{-1}\right)^{-1}\boldsymbol{K}\,d\boldsymbol{x}_{r,d}, \tag{9.23}$$

- The error depends on
  - The environment rest position
  - The desired pose
- The interaction between environment and manipulator is influenced by the mutual weight of the respective compliance features

Interacting with the environment

- Generally
  - Choose high values of K_P for directions where the environment must comply (manipulator error will be low)
  - Choose low values of K_P for directions where the manipulator must comply (manipulator error will be high)

Example

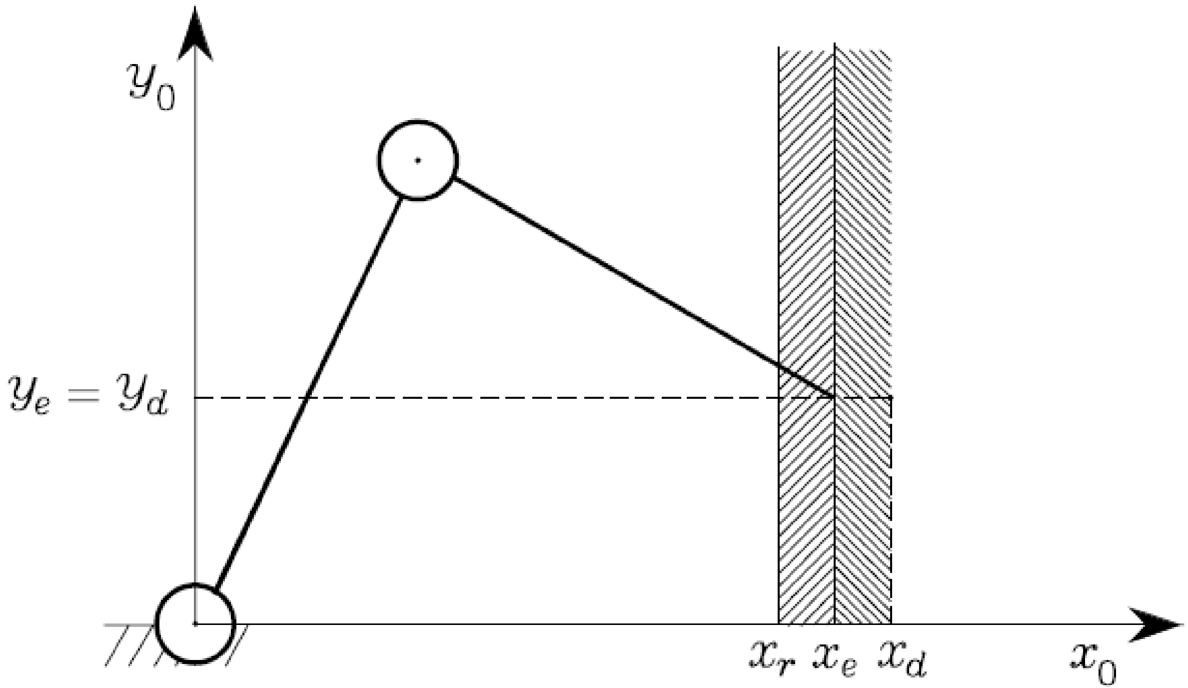


Fig. 9.1. Two-link planar arm in contact with an elastically compliant plane

9.3 Impedance control

- Now we will analyze interaction between the manipulator and the environment using inverse dynamics control
- The inverse dynamics control use the control law

$$\boldsymbol{u} = \boldsymbol{B}(\boldsymbol{q})\boldsymbol{y} + \boldsymbol{n}(\boldsymbol{q}, \dot{\boldsymbol{q}}),$$

- With end-effector forces the dynamics of the system is described by

$$\ddot{\boldsymbol{q}} = \boldsymbol{y} - \boldsymbol{B}^{-1}(\boldsymbol{q})\boldsymbol{J}^{T}(\boldsymbol{q})\boldsymbol{h}_e \tag{9.27}$$

- As opposed to $\ddot{q} = y$ when no external forces was present

Impedance control

- Choosing y conceptually analogous to operational space inverse dynamics control

$$y = J_A^{-1}(q)M_d^{-1}\big(M_d\ddot{x}_d + K_D\dot{\widetilde{x}} + K_P\widetilde{x} - M_d\dot{J}_A(q,\dot{q})\dot{q}\big) \qquad (9.28)$$

- Inserting this into the dynamic model yields

$$M_d\ddot{\widetilde{x}} + K_D\dot{\widetilde{x}} + K_P\widetilde{x} = M_d B_A^{-1}(q)h_A, \qquad (9.29)$$

- Where

$$B_A(q) = J_A^{-T}(q)B(q)J_A^{-1}(q)$$

- This is the inertia matrix of the manipulator in the operational space
- It is configuration dependent and is positive definite if J_A has full rank

Comparing with motion control

$$y = J_A^{-1}(q)\big(\ddot{x}_d + K_D\dot{\widetilde{x}} + K_P\widetilde{x} - \dot{J}_A(q,\dot{q})\dot{q}\big) \qquad (8.114)$$

$$y = J_A^{-1}(q)M_d^{-1}\big(M_d\ddot{x}_d + K_D\dot{\widetilde{x}} + K_P\widetilde{x} - M_d\dot{J}_A(q,\dot{q})\dot{q}\big) \qquad (9.28)$$

- Added matrix M_d

Impedance control

$$\boldsymbol{M}_d \ddot{\widetilde{\boldsymbol{x}}} + \boldsymbol{K}_D \dot{\widetilde{\boldsymbol{x}}} + \boldsymbol{K}_P \widetilde{\boldsymbol{x}} = \boldsymbol{M}_d \boldsymbol{B}_A^{-1}(\boldsymbol{q}) \boldsymbol{h}_A, \tag{9.29}$$

- The above model establishes a relationship though a generalized mechanical impedance between
  - The vector of forces $\boldsymbol{M}_d \boldsymbol{B}_A^{-1} \boldsymbol{h}_A$
  - The vector of displacement in operational space $\widetilde{\boldsymbol{x}}$
- The impedance is characterized by
  - Mass matrix $\boldsymbol{M}_d$
  - Damping matrix $\boldsymbol{K}_D$
  - Stiffness matrix $\boldsymbol{K}_P$

Impedance control

- The presence of B_A^{-1} makes the system coupled
- If it is wished to have a decoupled system the contact forces must be measures
- We now use the control law

$$u = B(q)y + n(q,\dot{q}) + J^T(q)h_e \qquad (9.30)$$

$$y = J_A^{-1}(q)M_d^{-1}\big(M_d\ddot{x}_d + K_D\dot{\tilde{x}} + K_P\tilde{x} - M_d\dot{J}_A(q,\dot{q})\dot{q} - h_A\big),$$

- With error free force measurements this yields

$$M_d\ddot{\tilde{x}} + K_D\dot{\tilde{x}} + K_P\tilde{x} = h_A. \qquad (9.32)$$

- For compliance control we had

$$h_A = K_P\tilde{x} \qquad (9.4)$$

$$\boldsymbol{h}_A = \boldsymbol{T}_A{}^T(\boldsymbol{x}_e)\boldsymbol{h}_e$$

Remarks

- The addition of the term $\boldsymbol{J}^T\boldsymbol{h}_e$ exactly compensates for the contact forces
- This renders the manipulator indefinitely stiff
- The term $-\boldsymbol{J}_A^{-1}\boldsymbol{M}_d^{-1}\boldsymbol{h}_A$ was introduced to get linear impedance with regard to $\boldsymbol{h}_A$
- The controller allows complete characterization of the system dynamics though an active impedance specified by $\boldsymbol{M}_d, \boldsymbol{K}_D, \boldsymbol{K}_P$
- With regards to $\boldsymbol{h}_e$ the impedance depends on the end-effector orientation
  - Selection of impedance parameters may become difficult
  - Problems in the neighborhood of representation singularities

Impedance control

- To avoid these problems we redesign $\boldsymbol{y}$ as a function of the same operational space error as in compliance control (9.9), (i.e. in the desired frame)
- Assuming a time varying desired frame and taking the time derivative yields

$$\dot{\widetilde{\boldsymbol{x}}} = -\boldsymbol{J}_{A_d}(\boldsymbol{q}, \widetilde{\boldsymbol{x}})\dot{\boldsymbol{q}} + \boldsymbol{b}(\widetilde{\boldsymbol{x}}, \boldsymbol{R}_d, \dot{\boldsymbol{o}}_d, \boldsymbol{\omega}_d), \tag{9.33}$$

- Where $\boldsymbol{b}(\widetilde{\boldsymbol{x}}, \boldsymbol{R}_d, \dot{\boldsymbol{o}}_d, \boldsymbol{\omega}_d) = \begin{bmatrix} \boldsymbol{R}_d^T \dot{\boldsymbol{o}}_d + \boldsymbol{S}(\omega_d^d)\boldsymbol{o}_{d,e}^d \\ \boldsymbol{T}^{-1}(\phi_{d,e})\boldsymbol{\omega}_d^d \end{bmatrix}.$

Impedance control

- Then computing the time derivative again yields

$$\ddot{\widetilde{\boldsymbol{x}}} = -\boldsymbol{J}_{A_d}\ddot{\boldsymbol{q}} - \dot{\boldsymbol{J}}_{A_d}\dot{\boldsymbol{q}} + \dot{\boldsymbol{b}}, \tag{9.34}$$

- Using these results together with the controller

$$\boldsymbol{y} = \boldsymbol{J}_{A_d}^{-1}\boldsymbol{M}_d^{-1}\left(\boldsymbol{K}_D\dot{\widetilde{\boldsymbol{x}}} + \boldsymbol{K}_P\widetilde{\boldsymbol{x}} - \boldsymbol{M}_d\dot{\boldsymbol{J}}_{A_d}\dot{\boldsymbol{q}} + \boldsymbol{M}_d\dot{\boldsymbol{b}} - \boldsymbol{h}_e^d\right), \tag{9.35}$$

- Yields

$$\boldsymbol{M}_d\ddot{\widetilde{\boldsymbol{x}}} + \boldsymbol{K}_D\dot{\widetilde{\boldsymbol{x}}} + \boldsymbol{K}_P\widetilde{\boldsymbol{x}} = \boldsymbol{h}_e^d, \tag{9.36}$$

- This controller have a linear impedance with regards to the force vector $\boldsymbol{h}_e^d$, and is independent of robot configuration

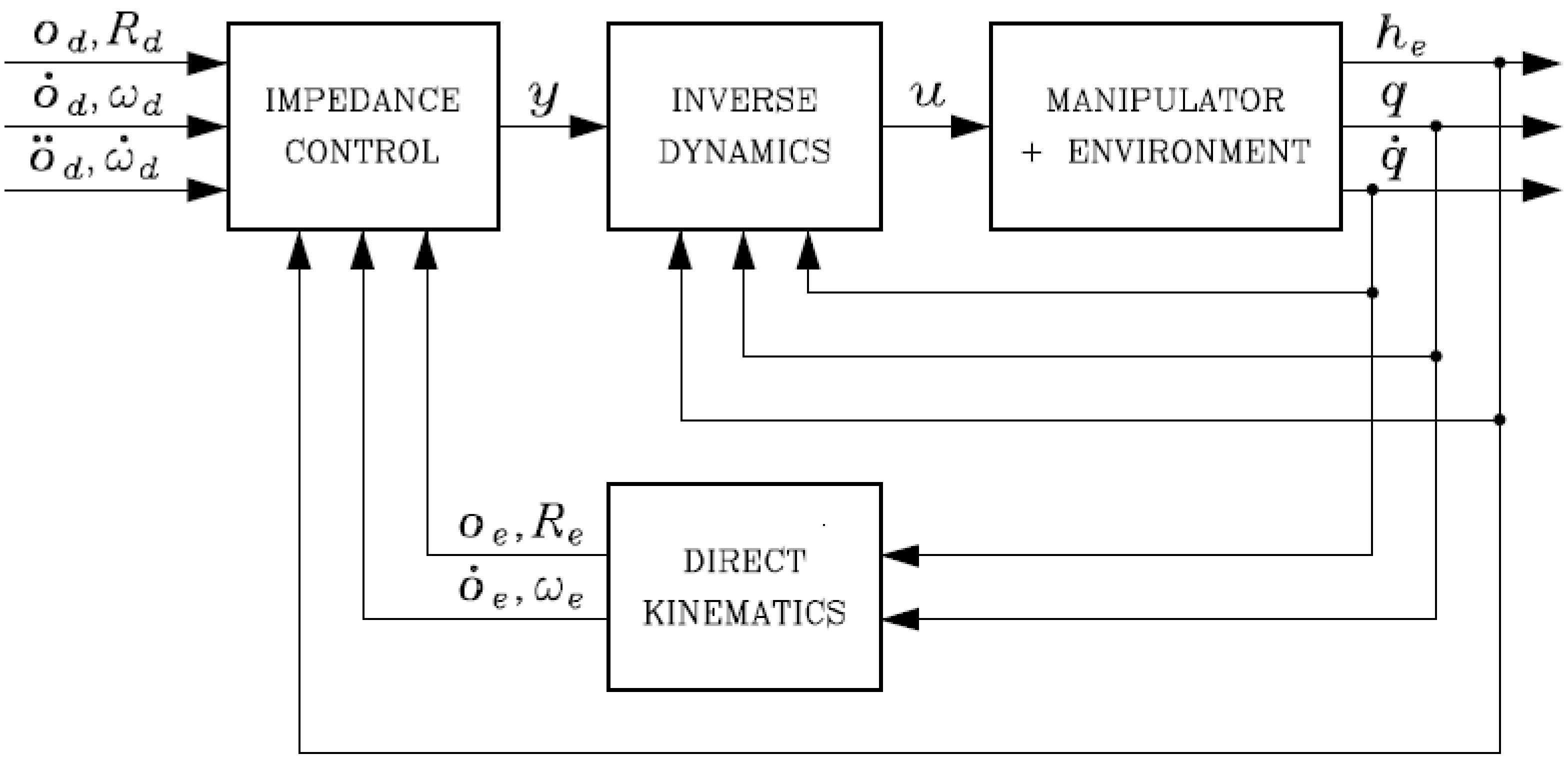


Fig. 9.2. Block scheme of impedance control

Passive impedance

- Similar to passive compliance, devices can be created to have impedance properties
- In the case where a system has both passive and active impedance, the combined mechanical system has the two impedances in parallel and the dynamic behavior is dependent on the relative weight of the impedances

Remarks on impedance control

- In absence of interaction with the environment the control is equal inverse dynamics position control
- The parameters of the control must take into account both the position control performance and the desired impedance properties
- For instance lowering $\boldsymbol{K}_P$ may be desired for compliant behavior, but will degrade position control

Admittance control

- Separation of impedance and motion control
- This also separates the parameters for the two controls
- Impedance control finds the ideal behavior of the end-effector, and the motion control use this as input
- Admittance is the inverse of impedance

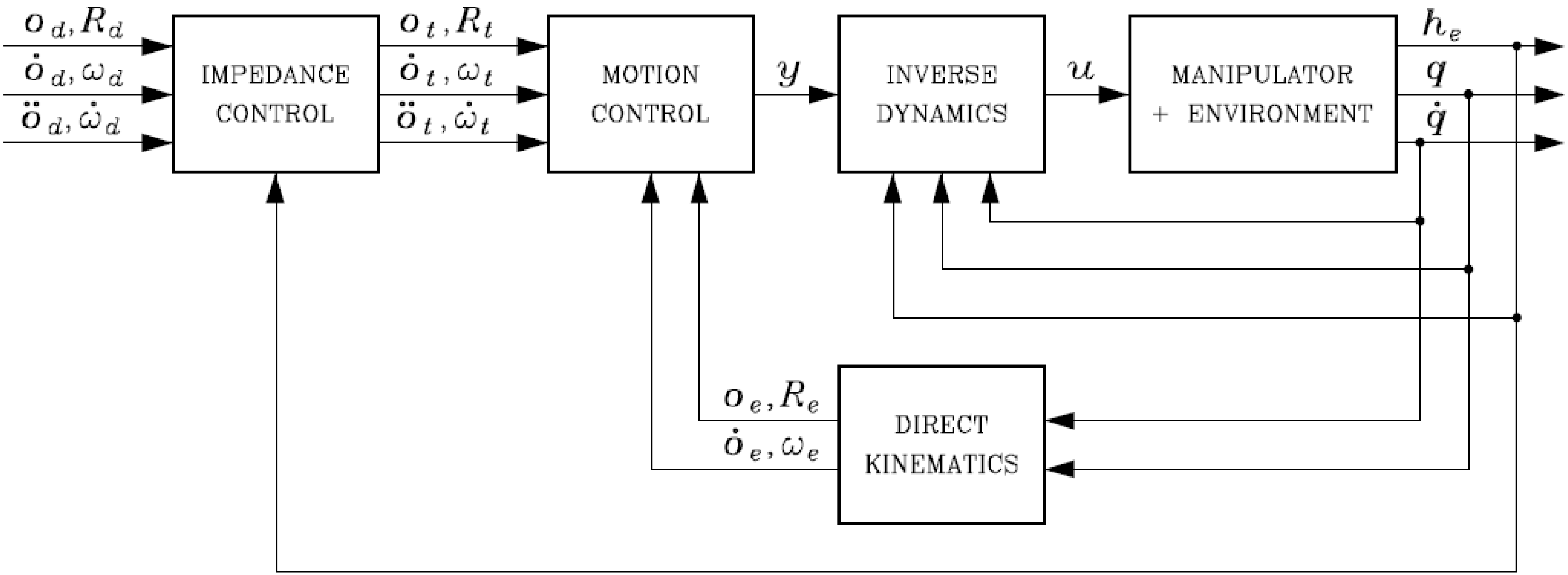


Fig. 9.5. Block scheme of admittance control

9.4 Force control

- Both compliance and impedance control are indirect force controllers
  - They act on the desired pose of the end-effector
- If it is desired to accurately control the contact force direct force control must be used
  - These methods want to stabilize the force error
- Force measurements have noise, making them unsuitable for derivative action
- Velocity is often used as a damping term

Force control

- Force control is realized by closing an outer force regulation feedback loop
- Inverse dynamics position control will be used to realize force control
- Force control is only meaningful in those direction that will have interaction with the environment
- Assumptions
  - Only using position variables
  - Using the elastic model $\boldsymbol{f}_e = \boldsymbol{K}(\boldsymbol{x}_e - \boldsymbol{x}_r)$ for the environment
  - The axes attached to the environment in rest position are parallel with the axes of the base frame

9.4.1 Force control with inner position loop

- Using inverse dynamics control

$$\boldsymbol{u} = \boldsymbol{B}(\boldsymbol{q})\boldsymbol{y} + \boldsymbol{n}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{J}^T(\boldsymbol{q})\boldsymbol{h}_e \tag{9.30}$$

- And then the controller

$$\boldsymbol{y} = \boldsymbol{J}^{-1}(\boldsymbol{q})\boldsymbol{M}_d^{-1}\big(-\boldsymbol{K}_D\dot{\boldsymbol{x}}_e + \boldsymbol{K}_P(\boldsymbol{x}_F - \boldsymbol{x}_e) - \boldsymbol{M}_d\dot{\boldsymbol{J}}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}}\big) \tag{9.39}$$

- Where $\boldsymbol{x}_F$ is a position reference to be related to a force error
- Note: no compensating action on time derivatives of $\boldsymbol{x}_F$
- Since we are only using position: $\boldsymbol{J}_A(\boldsymbol{q}) = \boldsymbol{J}(\boldsymbol{q})$.

Force control with inner position loop

- This leads to the dynamic system

$$\boldsymbol{M}_d \ddot{\boldsymbol{x}}_e + \boldsymbol{K}_D \dot{\boldsymbol{x}}_e + \boldsymbol{K}_P \boldsymbol{x}_e = \boldsymbol{K}_P \boldsymbol{x}_F,$$

- This system describes how the robot is taken to $\boldsymbol{x}_F$
- Let $\boldsymbol{f}_d$ denote a constant force reference

$$\boldsymbol{x}_F = \boldsymbol{C}_F(\boldsymbol{f}_d - \boldsymbol{f}_e), \qquad (9.41)$$

- where $\boldsymbol{C}_F$ is a diagonal compliance matrix
- The elements give control action along the operational space directions

Force control with inner position control loop

- Under the assumption of elastically compliant environment

$$\boldsymbol{M}_d\ddot{\boldsymbol{x}}_e + \boldsymbol{K}_D\dot{\boldsymbol{x}}_e + \boldsymbol{K}_P(\boldsymbol{I}_3 + \boldsymbol{C}_F\boldsymbol{K})\boldsymbol{x}_e = \boldsymbol{K}_P\boldsymbol{C}_F(\boldsymbol{K}\boldsymbol{x}_r + \boldsymbol{f}_d). \qquad (9.42)$$

- Under the assumption that a stable equilibrium is reached (i.e. $\boldsymbol{f}_e = \boldsymbol{f}_d$)

$$\boldsymbol{K}\boldsymbol{x}_e = \boldsymbol{K}\boldsymbol{x}_r + \boldsymbol{f}_d. \qquad (9.44)$$

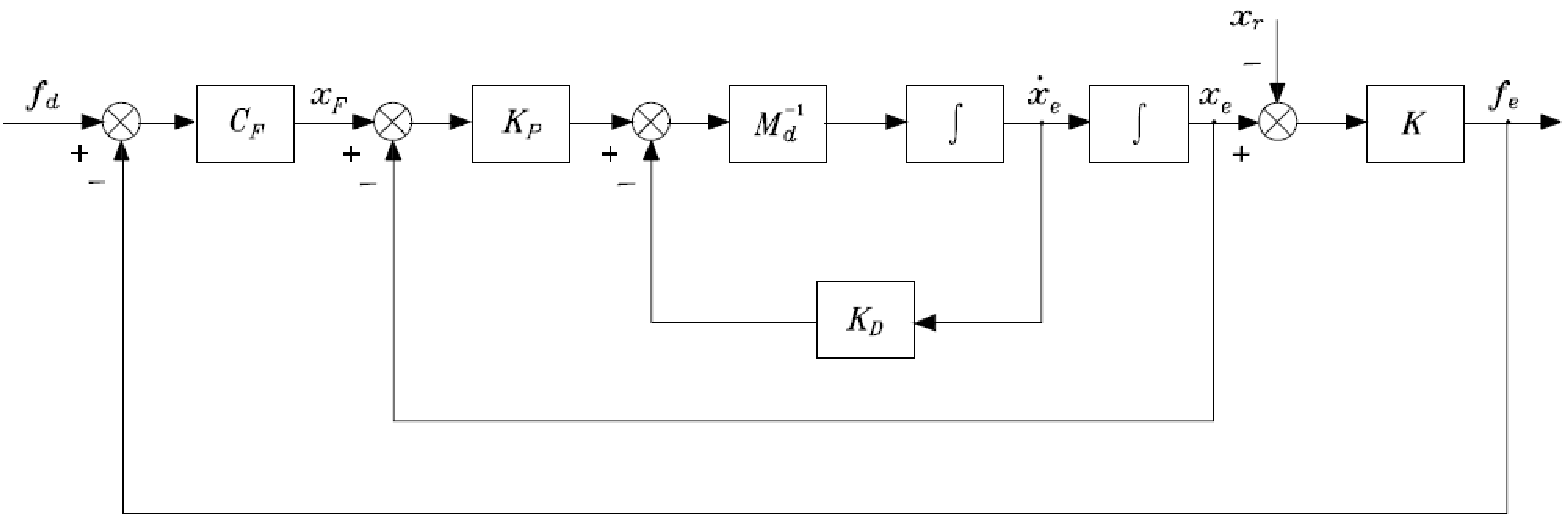


Fig. 9.6. Block scheme of force control with inner position loop

Remarks

- If C_F has only proportional action, then f_e can not reach f_d
- If we add integral action it is possible to reach the desired force at steady state and to reject the effects of x_r on f_e
- We use

$$C_F = K_F + K_I \int^t (\cdot)\, d\varsigma. \tag{9.43}$$

- The control parameters are K_D, K_P, K_F, K_I
- The system is of third order

9.4.2 Force control with inner velocity loop

- If the position feedback loop is opened, $\boldsymbol{x}_F$ will represent a velocity reference
- Then an integration relationship exist between $\boldsymbol{x}_F$ and $\boldsymbol{x}_e$.
- Only need proportional controller to reach desired force at steady state
- Choosing

$$\boldsymbol{y} = \boldsymbol{J}^{-1}(\boldsymbol{q})\boldsymbol{M}_d^{-1}\left(-\boldsymbol{K}_D\dot{\boldsymbol{x}}_e + \boldsymbol{K}_P\boldsymbol{x}_F - \boldsymbol{M}_d\dot{\boldsymbol{J}}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}\right), \qquad (9.45)$$

- yields

$$\boldsymbol{x}_F = \boldsymbol{K}_F(\boldsymbol{f}_d - \boldsymbol{f}_e) \qquad (9.46)$$

- And the dynamics

$$\boldsymbol{M}_d\ddot{\boldsymbol{x}}_e + \boldsymbol{K}_D\dot{\boldsymbol{x}}_e + \boldsymbol{K}_P\boldsymbol{K}_F\boldsymbol{K}\boldsymbol{x}_e = \boldsymbol{K}_P\boldsymbol{K}_F(\boldsymbol{K}\boldsymbol{x}_r + \boldsymbol{f}_d). \qquad (9.47)$$

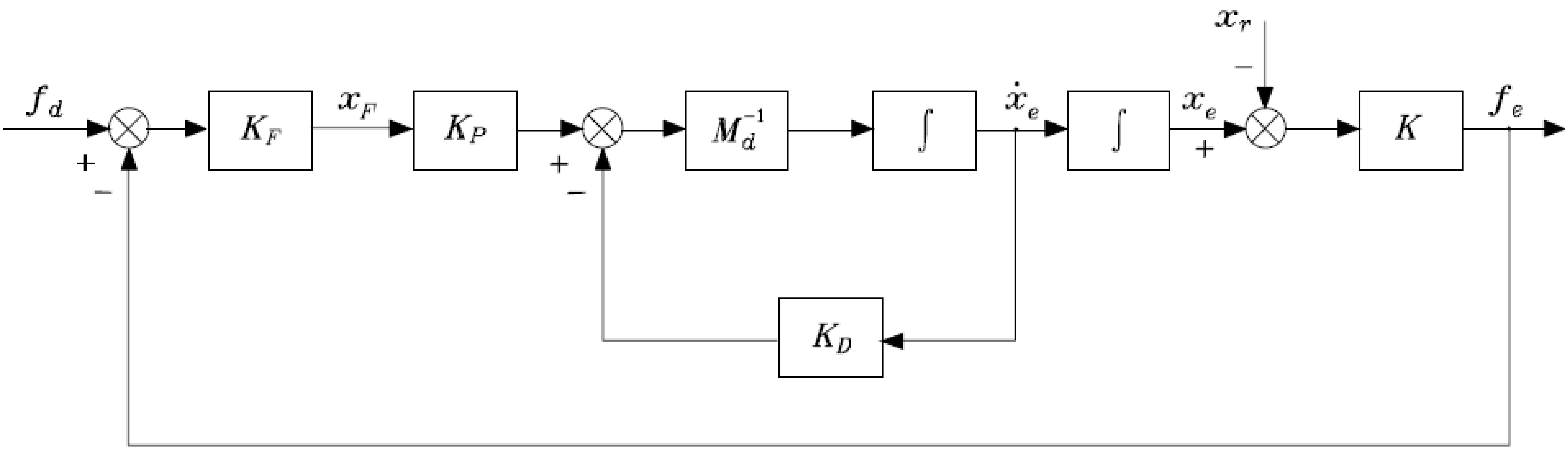


Fig. 9.7. Block scheme of force control with inner velocity loop

Remarks

- The system is of second order and the control design is simplified
- The absence of integral action in the force controller does not ensure reduction of the effects due to unmodelled dynamics

9.4.3 Parallel force/position control

- If it is desired to specify a desired pose x_d the force control scheme with inner position loop can be expanded

$$y = J^{-1}(q)M_d^{-1}\left(-K_D\dot{x}_e + K_P(\widetilde{x} + x_F) - M_d\dot{J}_A(q,\dot{q})\dot{q}\right) \qquad (9.48)$$

- The equilibrium is pose

$$x_e = x_d + C_F\left(K(x_r - x_e) + f_d\right)$$

- If C_F has integral action the force reference is reached at steady state, at the cost of the position
- If the motion is unconstrained the position reference is reached

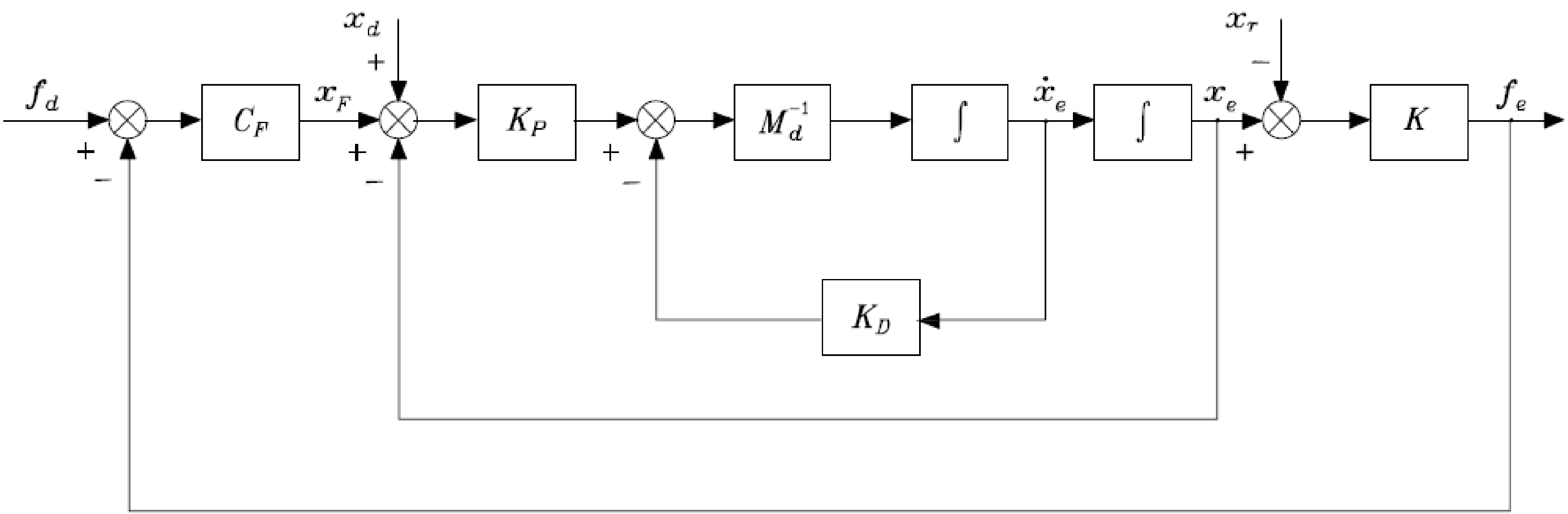


Fig. 9.8. Block scheme of parallel force/position control

Summary

- Indirect force control
  - Compliance control (PD control with gravity compensation)
  - Impedance control (Inverse dynamics control)
- Direct force control
  - Inner position loop
  - Inner velocity loop
  - Parallel force/position control

Exercises

- 9.2 (9.1)
- Exam 2017 exercise 4