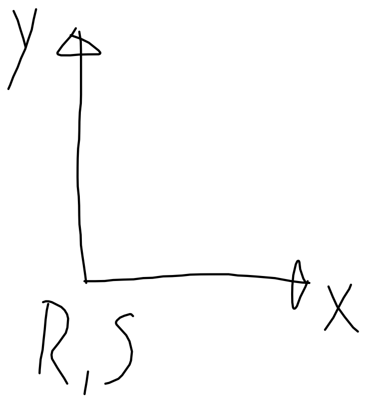
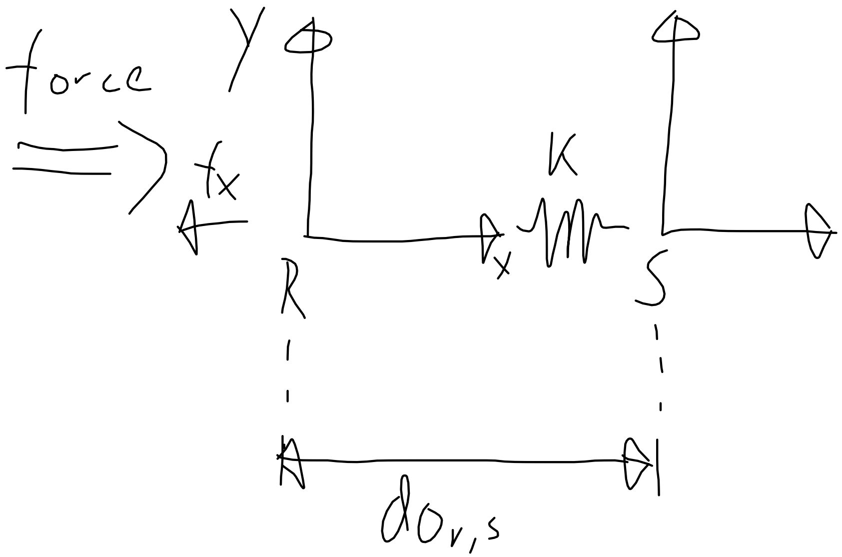
9.2.1 Passive compliance (8)





Spring model

$$f_{x,s} = K(x_s - x_r)$$

Example 9.1: Two-Link planar arm 20

$O_r = [x_r, 0]^T$ - environment rest position

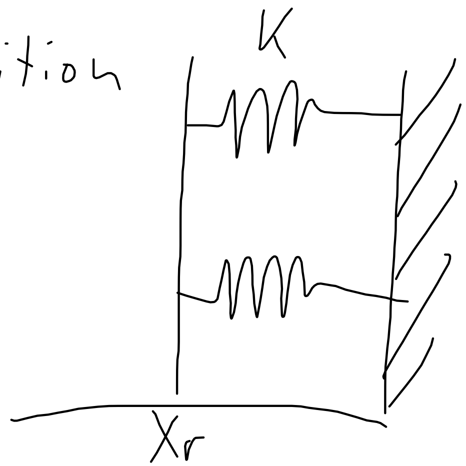
$K = \mathrm{diag}(K_x, 0)$ - —— " —— stiffness

$f_e = [f_x, 0]^T$ - End effector force

$O_e = [x_e, y_e]^T$ - —— " —— position

$O_d = [x_d, y_d]^T$ - Desired position

$K_p = \mathrm{diag}(k_{px}, k_{py})$ - P-control gain



Insert into (9.22)

$$h_e = (I_2 + K K_p^{-1})^{-1} K \, dx_{r,d}$$

$$= \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} K_x & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{k_{px}} & 0 \\ 0 & \frac{1}{k_{py}} \end{bmatrix} \right)^{-1} \begin{bmatrix} K_x & 0 \\ 0 & 0 \end{bmatrix} dx_{r,d}$$

$$= \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} \frac{K_x}{k_{px}} & 0 \\ 0 & 0 \end{bmatrix} \right)^{-1} \begin{bmatrix} K_x & 0 \\ 0 & 0 \end{bmatrix} dx_{r,d}$$

$$= \begin{bmatrix} \frac{1}{1 + \frac{K_x}{k_{px}}} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} K_x & 0 \\ 0 & 0 \end{bmatrix} dx_{r,d} = \begin{bmatrix} \frac{k_{px} K_x}{k_{px} + K_x} & 0 \\ 0 & 0 \end{bmatrix} dx_{r,d} = \begin{bmatrix} \frac{k_{px} K_x}{k_{px} + K_x}(x_d - x_r) \\ 0 \end{bmatrix}$$

Insert into (9.23)

$$dx_{e,d} = K_p^{-1}\left(I + K K_p^{-1}\right)^{-1} K\, dx_{r,d}$$

$$= K_p^{-1} h_e$$

$$= \begin{bmatrix} \frac{1}{k_{px}} & 0 \\ 0 & \frac{1}{k_{py}} \end{bmatrix} \begin{bmatrix} \frac{k_{px} k_x}{k_{px} + k_x}(x_d - x_r) \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\cancel{k_{px}}} \cdot \frac{\cancel{k_{px}} k_x}{k_{px} + k_x}(x_d - x_r) \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{k_x}{k_{px} + k_x}(x_d - x_r) \\ 0 \end{bmatrix}$$

$$dx_{e,d} = O_d - O_e$$

$$O_e = O_d - dx_{e,d}$$

$$= \begin{bmatrix} x_d \\ y_d \end{bmatrix} - \begin{bmatrix} \frac{K_x}{K_{px} + K_x}(x_d - x_r) \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{K_{px} + \cancel{K_x}}{K_{px} + K_x} x_d - \cancel{\frac{K_x}{K_{px} + K_x} x_d} + \frac{K_x}{K_{px} + K_x} x_r \\ y_d \end{bmatrix}$$

$$= \begin{bmatrix} \frac{K_{px} x_d + K_x x_r}{K_{px} + K_x} \\ y_d \end{bmatrix}$$

If $\frac{K_{px}}{K_x} >> 1$

Arm has much greater stiffness than the environment

$$x_e \approx x_d \qquad f_x \approx K_x (x_d - x_r)$$

// x_e = arm pushes the environment
// x_d = desired position

If $\frac{K_x}{K_{px}} >> 1$

The environment has much greater stiffness than the arm

$$x_e \approx x_r \qquad f_x \approx K_{px} (x_d - x_r)$$

// x_r = environment rest position

9.3 Impedane control (25)

$$B(q)\ddot{q} + n(q,\dot{q}) = u - J^T(q)h_e$$

$$u = By + n + J^T h_e$$

$$\cancel{B}\ddot{q} + \cancel{n} = \cancel{B}y + \cancel{n} + \cancel{J^T h_e} - \cancel{J^T h_e}$$

$$\ddot{q} = y$$