Interaction matrix 10.3.2 (slides 9-14)

We want to find the relationship between image features and the operational space.

Definitions

Image feature $s = \begin{bmatrix} X \\ Y \end{bmatrix}$

$V^c_{c,o}$ - velocity of the camera with respect to the object in the camera fram

$$V^c_{c,o} = \begin{bmatrix} \dot{o}^c_{c,o} \\ R^T_c(\omega_o - \omega_c) \end{bmatrix} \quad (10.16)$$

$$o^c_{c,c} = R^T_c(o_o - o_c)$$

$$o^c_{c,o} = R^c_b(o^b_o - o^b_c)$$

Absolute velocity of camera and object

$$V_c^c = \begin{bmatrix} R_c^T \dot{O}_c \\ R_c^T \omega_c \end{bmatrix} \qquad V_o^c = \begin{bmatrix} R_c^T \dot{O}_o \\ R_c^T \omega_o \end{bmatrix}$$

We want to find the image Jacobian J_s

$$\dot{s} = J_s(s, T_o^c)\, v_{c,o}^c \quad (10.17)$$

Partition (10.17) into two parts

$$o^c_{c,0} = R_c^T (o_0 - o_c)$$

$$\dot{o}^c_{c,0} = R_c^T(\dot{o}_0 - \dot{o}_c) + \dot{R}_c^T (o_0 - o_c)$$

$$= \cdots + \left(S(\omega)R_c\right)^T (o_0 - o_c)$$

$$= \cdots + R_c^T \, S^T(\omega)\,(o_0 - o_c)$$

$$= \cdots - R_c^T \, S(\omega)(o_0 - o_c)$$

$$= \cdots - R_c^T \left(\omega \times (o_0 - o_c)\right)$$

$$= \cdots + R_c^T \left((o_0 - o_c) \times \omega\right)$$

$$= \cdots + \left(R_c^T (o_0 - o_c)\right) \times R_c^T \omega$$

$$= \cdots + o^c_{c,0} \times (R^T_c \omega_c) = \cdots + S(o^c_{c,0}) R_c^T \omega_c$$

$$\dot{R} = \omega^\times R$$

$$\dot{R} = S(\omega)R$$

$$(AB)^T = B^T A^T$$

$$S^T(\omega) = -S(\omega)$$

$$S(a)b = a \times b$$

$$a \times b = -b \times a$$

$$R(a \times b) = (Ra) \times (Rb)$$

$$v^c_{c,o} = \begin{bmatrix} \dot{o}^c_{c,o} \\ R_c^T(\omega_o - \omega_c) \end{bmatrix} = \begin{bmatrix} R_c^T \dot{o}_o - R_c^T \dot{o}_c + S(o^c_{c,o}) R_c^T \omega_c \\ R_c \omega_o - R_c^T \omega_c \end{bmatrix}$$

$$v_o^c$$

$$= v_o^c + \underbrace{\begin{bmatrix} -I & S(o^c_{c,o}) \\ 0 & -I \end{bmatrix}}_{\Gamma(o^c_{c,o})} \underbrace{\begin{bmatrix} R_c^T \dot{o}_c \\ R_c^T \omega_c \end{bmatrix}}_{v_c^c} = v_o^c + \Gamma(o^c_{c,o}) v_c^c$$

Insert into (10.17)

$$\dot{s} = J_s(s, T_o^c)\, v_{c,o}^c$$

$$\dot{s} = J_s v_o^c + \underbrace{J_s \Gamma(O_{c,o}^c)}_{L_s \text{ - interaction matrix}} v_c^c$$

For fixed objects

$$\dot{s} = L_s v_c^c$$

Interaction matrix of a point (slides 15-17)

Consider a point p which can be represented in the camera frame as

$$r_c^c = (R_c^b)^T (p^b - o_c^b)$$

where p is the position in the base frame. Now we will find L_s for a point.

We choose the feature vector

$$s(r_c^c) = \frac{1}{z_c}\begin{bmatrix} x_c \\ y_c \end{bmatrix} = \begin{bmatrix} X \\ Y \end{bmatrix}$$

$$\boxed{\dot{s} = L_s v_c^c}$$

$$r_c^c = [x_c, y_c, z_c]^T$$

$$\dot{s} = \frac{\partial s(r_c^c)}{\partial r_c^c}\dot{r}_c^c \qquad \text{(chain rule)}$$

$$\frac{\partial s(r_c^i)}{\partial r_c^i} = \frac{1}{z_c}\begin{array}{c}\begin{bmatrix} 1 & 0 & -\frac{x_c}{z_c} \\ 0 & 1 & -\frac{y_c}{z_c} \end{bmatrix} \\ \begin{array}{ccc} \frac{\partial}{\partial x_c} & \frac{\partial}{\partial y_c} & \frac{\partial}{\partial z_z} \end{array}\end{array}$$

$$s(r_c^i) = \frac{1}{z_c}\begin{bmatrix} x_c \\ y_c \end{bmatrix}^T$$

$$\frac{\partial}{\partial z_c}\frac{1}{z_c} = -\frac{1}{z_c^2}$$

$$\dot{r}_c^i = -R_c^T \dot{o}_c + S(r_c^i) R_c^T \omega_c \quad (p \text{ is constant})$$

$$= \begin{bmatrix} -I & S(r_c^i) \end{bmatrix} \underbrace{\begin{bmatrix} R_c^T \dot{o}_c \\ R_c^T \omega_c \end{bmatrix}}_{V_c^c} = \begin{bmatrix} -I & S(r_c^i) \end{bmatrix}^T V_c^c$$

$$\dot{s} = \underbrace{\frac{1}{Z_c}\begin{bmatrix} 1 & 0 & -X \\ 0 & 1 & -Y \end{bmatrix}\begin{bmatrix} -1 & 0 & 0 & 0 & -z_c & y_c \\ 0 & -1 & 0 & z_c & 0 & -x_c \\ 0 & 0 & -1 & -y_c & x_c & 0 \end{bmatrix}}_{L_s} V_c^c$$

$$\dot{s} = L_s v_c^c$$