UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Exam in:	TEK4030
Day of exam:	18th of December
Exam hours:	15.00 - 19.30 (4,5 hours)
This examination paper consists of 15 page(s).	
Appendices:	Formulas
Permitted materials:	All
	Cooperation not allowed.

Make sure that your copy of this examination paper is complete before answering.

Problem 1 - Random questions: Course overview (28%)

Seven of these questions are randomly given to each candidate using Inspera.

Information text: You only need to write one or two sentences answering the question to get full score on each of the questions in this assignment.

1) (4 %) **(Control theory)** Why is the Laplace transform useful in control theory?

> One of: Solve linear differential equations, check stability of a system

2) (4 %) **(Control theory)** For a transfer function with poles in the left half plane of the complex plane, how do the response change if the imaginary part of the poles increase?

> The response will have more oscillations

3) (4 %) **(Actuators and sensors)** Mention at least two reasons for using a transmission system in a manipulator.

> - Qualitatively changes the motion between translation and rotation
> - Quantitatively changes the speed and torque ration
> - Reduces the perceived load from the motor perspective
> - The actuators can be placed in the robot base, rather than the joints, reducing the weight of the robot.

4) (4 %) **(Actuators and sensors)** With which ratio is the perceived load friction reduced from the motor perspective when using gears?

> It is reduced bu $\frac{1}{k_r}$ where k_r is the gear reduction ratio.

5) (4 %) **(Independent joint control)** An independent joint is controlled by a PI controller (i.e. a set point controller). What do you need to add to transform the controller into a trajectory tracking controller?

> A feed forward component of the velocity and acceleration.

6) (4 %) **(Independent joint control)** Briefly explain why the Computed Torque Feedforward Controller in the textbook is not considered a centralized controller.

It does not need the measurements of the other joints, by rely on the nominal trajectory of the other joints.

7) (4 %) **(Independent joint control)** Consider the independent joint controllers in the textbook. Which performance benefit do you gain by adding more feedback variables (i.e. velocity and acceleration)?

You are able to specify more performance metrics for the controller. With only position feedback one can only design the controller with a desired disturbance rejection factor. With position, velocity and acceleration feedback one can design the controller with a desired disturbance rejection factor, damping factor and natural frequency.

8) (4 %) **(Centralized control)** Using inverse dynamics control, what happens if the model parameters contain errors?

The assumption for the controller is not met, and the nonlinear compensation and decoupling part of the controller will not have perfect compensation. Therefore the will be some coupling between the joints and the controller will have some tracking error.

9) (4 %) **(Centralized control)** When using a centralized control scheme in operational space, what happens when the manipulator is close to singularity?

For a Jacobian Transpose scheme the manipulator will get stuck, while for a Jacobian inverse scheme the control input would go towards infinity.

10) (4 %) **(Centralized control)** What is the purpose of the D term in the PD control with gravity compensation?

It provides damping to the system, and increase the stability margin. If not present and if the system has low internal friction, the system might get unstable.

11) (4 %) **(Force control)** What is the difference between passive and active compliance?

Passive compliance is achieved through a mechanical device, while active compliance is achieved using a controller.

12) (4 %) **(Force control)** What is the benefit of using impedance control compared to compliance control?

> One has more control of the manipulators interaction with the environment. In the compliance control case the manipulator behaves as a spring, while in the impedance control case the manipulator behaves like a mass-spring-damper system. This means that impedance control also can specify how the robot should interact before coming to a rest position.

13) (4 %) **(Visual servoing)** Imagine using visual servoing with four points as the image features. The four points are distributed as a square, and the difference between the desired position and actual position of the points equals a rotation of $\frac{1}{4}\pi$ around the optical axis. Using a classical image based visual servoing approach, how would the camera move to minimize the errors in the feature vector?

> In general the method will try to move the points linear in the image, resulting in a rotation around the optical axis and moving away from the points along the optical axis, and then moving inwards again.

14) (4 %) **(Visual servoing)** When designing a image based visual servoing system using point features, you experience that the interaction matrix is singular and can't be inverted. How can you overcome this problem?

> You can add more point features, thus creating an over-defined system, and then use the pseudo-inverse to invert the interaction matrix.

15) (4 %) **(Mobile robots)** Given a rear wheel drive bicycle, where the front wheel of the bicycle is perpendicular to the back wheel. What will happen when the robot starts driving?

> This is a singular case for the rear wheel drive bicycle, as the non-holonomic constraint does not permit side slip of the front wheel. Hence, the robot is unable to move.

16) (4 %) **(Mobile robots)** Why is it not possible to use the inverse dynamics control scheme used for manipulators for non-holonomic systems?

> The non-holonomic constraints pose as constraints also in the dynamic equation of the system, leaving too few degrees of freedom to control to cancel the dynamics of the system.

17) (4 %) **(Mobile robots)** You need your unicycle robot to follow a arbitrary path, that could contain discontinuities in velocity and acceleration. Which controller from the textbook would you choose, and why?

Input/output linearization, as this method is stable for arbitrary paths, and even paths with discontinuities in velocity and acceleration. However, the robot would not perfectly follow the path.

18) (4 %) **(Motion planning)** What is the main disadvantage with the artificial potential fields motion planning method?

It risks getting stuck at a local minima.

Problem 2 - Independent joint control (15%)

We will now look at the control of an independent joint of robot. The transfer function of the control input $U(s)$ and the joint position $X(s)$ is

$$\frac{X(s)}{U(s)} = \frac{\frac{1}{F_m}}{s(1+\frac{I_m}{F_m}s)} \tag{1}$$

The joint is controlled using both position and acceleration feedback with the controllers

$$C_P(s) = K_P \qquad\qquad C_A(s) = K_A\frac{1+sT_A}{s} \tag{2}$$

The control input $U(s)$ is given as

$$U(s) = C_A(s)(C_P(s)E(s) - s^2X(s)) \tag{3}$$

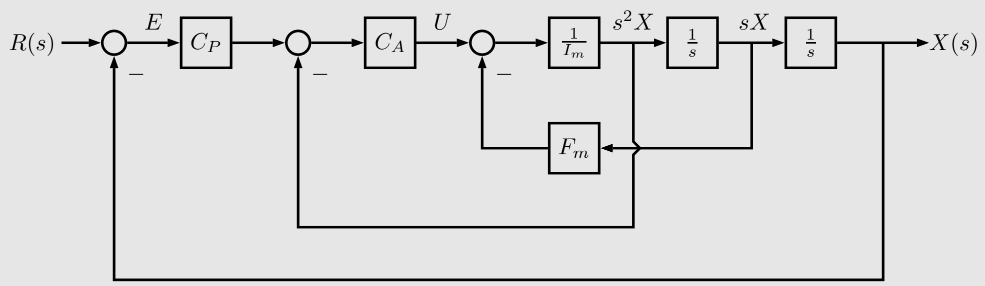
where $E(s) = R(s) - X(s)$, and $R(s)$ is the reference input to the controller. I_m and F_m are two positive constants.

a) (6 %) Draw the block diagram of the system (model and controller) using only constants blocks, integral blocks and the control blocks $C_P(s)$ and $C_A(s)$.

$$\frac{X(s)}{U(s)} = \frac{1}{s(F_m + I_m s)}$$

$$I_m s^2 X + F_m sX = U$$

$$I_m s^2 X = U - F_m sX$$



b) (9 %) Find the transfer function of from $R(s)$ to $X(s)$ assuming that $\frac{I_m}{F_m} = T_A$. Which order is the system? Find the poles of the system as an expression. Is the system stable? Explain why/why not.

Writing the full expression of the transfer function

$$\begin{aligned} X(s) &= \frac{\frac{1}{F_m}}{s(1+T_A s)} U \\ &= \frac{\frac{1}{F_m}}{s(1+T_A s)} C_A(s)(C_P(s)E(s) - s^2 X(s)) \\ &= \frac{\frac{1}{F_m}}{s\cancel{(1+T_A s)}} K_A \frac{\cancel{1+T_A s}}{s}(K_P(R-X) - s^2 X(s)) \\ F_m s^2 X &= K_A(K_P(R-X) - s^2 X(s)) \\ &= K_A K_P R - K_A K_P X - K_A s^2 X(s) \\ (F_m + K_A)s^2 X + K_A K_P X &= K_A K_P R \\ \frac{X}{R} &= \frac{K_A K_P}{(F_m + K_A)s^2 + K_A K_P} \end{aligned}$$

The system is of second order. Finding the poles by setting the characterstic polynomial to zero

$$\begin{aligned} (F_m + K_A)s^2 + K_A K_P &= 0 \\ s^2 &= -\frac{K_A K_P}{F_m + K_A} \\ s &= \pm\sqrt{-\frac{K_A K_P}{F_m + K_A}} \end{aligned}$$

The system is marginally stable, as it has two complec conjugate poles with zero real part.

- 3 points for finding the transfer function
- 1 points for stating the correct order of the system
- 3 points for finding the poles of the system
- 2 points for explaining the stability of the system

Problem 3 - Centralized control (18%)

A PD^2 controller with gravity compensation is given below

$$\boldsymbol{u} = \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{K}_P \tilde{\boldsymbol{q}} - \boldsymbol{K}_{D^2} \ddot{\boldsymbol{q}} \tag{4}$$

where $\tilde{\boldsymbol{q}} = \boldsymbol{q}_d - \boldsymbol{q}$, and $\boldsymbol{q}_d$ is the desired joint positions which are constant. This controller will be used to control the system

$$\boldsymbol{B}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{u} \tag{5}$$

The Lyapunov function candidate for the controller is

$$\boldsymbol{V}(\dot{\boldsymbol{q}}, \tilde{\boldsymbol{q}}) = \frac{1}{2}\dot{\boldsymbol{q}}^T\boldsymbol{B}(\boldsymbol{q})\dot{\boldsymbol{q}} + \frac{1}{2}\tilde{\boldsymbol{q}}^T\boldsymbol{K}_P\tilde{\boldsymbol{q}} + \frac{1}{2}\dot{\boldsymbol{q}}^T\boldsymbol{K}_{D^2}\dot{\boldsymbol{q}} \tag{6}$$

a) (10 %) Show that it is not possible to use Lyapunov's direct method to prove the stability of the system.

Finding $\dot{\boldsymbol{V}}(\dot{\boldsymbol{q}}, \tilde{\boldsymbol{q}})$

$$\begin{aligned}
\dot{\boldsymbol{V}}(\dot{\boldsymbol{q}}, \tilde{\boldsymbol{q}}) =& \frac{1}{2}\ddot{\boldsymbol{q}}^T\boldsymbol{B}(\boldsymbol{q})\dot{\boldsymbol{q}} + \frac{1}{2}\dot{\boldsymbol{q}}^T\dot{\boldsymbol{B}}(\boldsymbol{q})\dot{\boldsymbol{q}} + \frac{1}{2}\dot{\boldsymbol{q}}^T\boldsymbol{B}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \frac{1}{2}\dot{\tilde{\boldsymbol{q}}}^T\boldsymbol{K}_P\tilde{\boldsymbol{q}} + \frac{1}{2}\tilde{\boldsymbol{q}}^T\boldsymbol{K}_P\dot{\tilde{\boldsymbol{q}}} \\
&+ \frac{1}{2}\ddot{\boldsymbol{q}}^T\boldsymbol{K}_{D^2}\dot{\boldsymbol{q}} + \frac{1}{2}\dot{\boldsymbol{q}}^T\boldsymbol{K}_{D^2}\ddot{\boldsymbol{q}} \\
\dot{\boldsymbol{V}}(\dot{\boldsymbol{q}}, \tilde{\boldsymbol{q}}) =& \frac{1}{2}\dot{\boldsymbol{q}}^T\dot{\boldsymbol{B}}(\boldsymbol{q})\dot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^T\boldsymbol{B}(\boldsymbol{q})\ddot{\boldsymbol{q}} - \dot{\boldsymbol{q}}^T\boldsymbol{K}_P\tilde{\boldsymbol{q}} + \dot{\boldsymbol{q}}^T\boldsymbol{K}_{D^2}\ddot{\boldsymbol{q}}
\end{aligned}$$

Inserting for $\boldsymbol{B}(\boldsymbol{q})\ddot{\boldsymbol{q}}$

$$\dot{\boldsymbol{V}}(\dot{\boldsymbol{q}}, \tilde{\boldsymbol{q}}) = \frac{1}{2}\dot{\boldsymbol{q}}^T\dot{\boldsymbol{B}}(\boldsymbol{q})\dot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^T\left(\boldsymbol{u} - \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} - \boldsymbol{g}(\boldsymbol{q})\right) - \dot{\boldsymbol{q}}^T\boldsymbol{K}_P\tilde{\boldsymbol{q}} + \dot{\boldsymbol{q}}^T\boldsymbol{K}_{D^2}\ddot{\boldsymbol{q}}$$

Using $\dot{\boldsymbol{B}} - 2\boldsymbol{C} = \boldsymbol{0}$ and inserting for $\boldsymbol{u}$

$$\begin{aligned}
\dot{\boldsymbol{V}}(\dot{\boldsymbol{q}}, \tilde{\boldsymbol{q}}) &= \dot{\boldsymbol{q}}^T\left(\cancel{\boldsymbol{g}(\boldsymbol{q})} + \cancel{\boldsymbol{K}_P\tilde{\boldsymbol{q}}} - \cancel{\boldsymbol{K}_{D^2}\ddot{\boldsymbol{q}}} - \cancel{\boldsymbol{g}(\boldsymbol{q})}\right) - \cancel{\dot{\boldsymbol{q}}^T\boldsymbol{K}_P\tilde{\boldsymbol{q}}} + \cancel{\dot{\boldsymbol{q}}^T\boldsymbol{K}_{D^2}\ddot{\boldsymbol{q}}} \\
\dot{\boldsymbol{V}}(\dot{\boldsymbol{q}}, \tilde{\boldsymbol{q}}) &= 0
\end{aligned}$$

The last condition $\dot{\boldsymbol{V}}(\dot{\boldsymbol{q}}, \tilde{\boldsymbol{q}}) < 0 \ \forall \ \dot{\boldsymbol{q}}, \tilde{\boldsymbol{q}} \neq 0$ is not satisfied (as stated in the problem), and therefore we cannot prove stability of the system.

- 2 points for correctly taking the time derivative of $\boldsymbol{V}$
- 3 points for successfully inserting for $\boldsymbol{B}(\boldsymbol{q})\ddot{\boldsymbol{q}}$
- 2 points for successfully using $\dot{\boldsymbol{B}} - 2\boldsymbol{C} = \boldsymbol{0}$
- 2 points for successfully inserting for $\boldsymbol{u}$
- 1 point for stating why it is not possible to prove stability

b) (8 %) Modify the control law in (4), by adding one or more terms, so that the system becomes stable. Show that the system is stable using Lyapunov's direct method.

Adding i D-term to the controller, thus the controller now is

$$\boldsymbol{u} = \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{K}_P\tilde{\boldsymbol{q}} - \boldsymbol{K}_D\dot{\boldsymbol{q}} - \boldsymbol{K}_{D^2}\ddot{\boldsymbol{q}}$$

Now inserting the controller into the time derivative of the Lyapunov candidate function, reusing some of the calculation from the exercise above.

$$\begin{aligned}\dot{\boldsymbol{V}}(\dot{\boldsymbol{q}}, \tilde{\boldsymbol{q}}) &= \dot{\boldsymbol{q}}^T\left(\boldsymbol{u} - \boldsymbol{g}(\boldsymbol{q})\right) - \dot{\boldsymbol{q}}^T\boldsymbol{K}_P\tilde{\boldsymbol{q}} + \dot{\boldsymbol{q}}^T\boldsymbol{K}_{D^2}\ddot{\boldsymbol{q}} \\ &= \dot{\boldsymbol{q}}^T\left(\cancel{\boldsymbol{g}(\boldsymbol{q})} + \cancel{\boldsymbol{K}_P}\tilde{\boldsymbol{q}} - \boldsymbol{K}_D\dot{\boldsymbol{q}} - \cancel{\boldsymbol{K}_{D^2}}\ddot{\boldsymbol{q}} - \cancel{\boldsymbol{g}(\boldsymbol{q})}\right) - \cancel{\dot{\boldsymbol{q}}^T\boldsymbol{K}_P}\tilde{\boldsymbol{q}} + \cancel{\dot{\boldsymbol{q}}^T\boldsymbol{K}_{D^2}}\ddot{\boldsymbol{q}} \\ &= -\dot{\boldsymbol{q}}^T\boldsymbol{K}_D\dot{\boldsymbol{q}}\end{aligned}$$

We now have a negative semi-definite $\dot{\boldsymbol{V}}(\dot{\boldsymbol{q}}, \tilde{\boldsymbol{q}})$, as $\dot{\boldsymbol{V}}$ is independent of $\tilde{\boldsymbol{q}}$. We have to check the states at equilibrium, i.e. when $\dot{\boldsymbol{q}} = 0$ and $\ddot{\boldsymbol{q}} = 0$. Insering this into the dynamics of the system yields

$$\begin{gathered}\cancel{\boldsymbol{B}(\boldsymbol{q})}\ddot{\boldsymbol{q}} + \cancel{\boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})}\dot{\boldsymbol{q}} + \cancel{\boldsymbol{g}(\boldsymbol{q})} = \cancel{\boldsymbol{g}(\boldsymbol{q})} + \boldsymbol{K}_P\tilde{\boldsymbol{q}} - \cancel{\boldsymbol{K}_D}\dot{\boldsymbol{q}} - \cancel{\boldsymbol{K}_{D^2}}\ddot{\boldsymbol{q}} \\ \boldsymbol{K}_P\tilde{\boldsymbol{q}} = 0\end{gathered}$$

As long as $\boldsymbol{K}_P$ is invertable the error goes to zero and the system is stable. It is also required to check the remaining three conditions of the Lyapunov direct method.

- 1 point for adding the correct additional term
- 3 points for finding the new time derivative of the Lyapunov candidate function
- 2 points for checking the equilibrium
- 2 point for checking the additional Lyapunov direct method conditions.

Problem 4 - Force control (5%)

a) (5 %) *(This exercise has many versions in the original exam, where the parameter values has changed)*
Given a robot with compliance control moving in the x-axis in the operational space and interact with the environment. The stiffness of the environment in this direction is 500 N/m, and the manipulator control gain (stiffness) is 1500 N/m. The environment rest position in the x-direction of the operational space is 1 m, and the desired end effector position in the x-direction of the operational space is 2 m. What is the final end effector position?

This is very similar to Example 9.1 in the textbook, and using the following formula from that example

$$o_x = \frac{k_{P_x} x_d + k_x x_r}{k_{P_x} + k_x} = \frac{1500 \cdot 2 + 500 \cdot 1}{1500 + 500} = \frac{3500}{2000} = 1,75m \tag{7}$$

Parameter	1	2	3	4
k_x	500	1500	1000	1000
k_{P_x}	1500	500	500	4000
x_r	1	1	1	2
x_d	2	2	3	5
o_x	1,75	1,25	1,67	4,4

Problem 5 - Visual servoing (10%)

In this problem we are going to use polar coordinates for point features in the image. In polar coordinates a image point is written as $\boldsymbol{p}(r, \phi)$, where r is the distance from the point to the principal point, and ϕ is the angle from the X-axis to a line joining the principal point to the image point. These are defined as

$$r = \sqrt{X^2 + Y^2} = \sqrt{\frac{x_c^2}{z_c^2} + \frac{y_c^2}{z_c^2}} \qquad \phi = \tan^{-1}\frac{Y}{X} = \tan^{-1}\frac{y_c}{x_c} \tag{8}$$

The transform between polar and image coordinates are given as

$$X = r\cos\phi \qquad Y = r\sin\phi \tag{9}$$

a) (10 %) Find the interaction matrix $\boldsymbol{L}_s$ for the feature vector $\boldsymbol{s} = [r\ \phi]^T$.

The following formulas might prove useful

$$\frac{d}{dx}\sqrt{x} = \frac{1}{2}\frac{1}{\sqrt{x}} \qquad \frac{d}{dx}\frac{1}{x} = -\frac{1}{x^2} \tag{10}$$

$$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2} \qquad \sin^2 x + \cos^2 x = 1 \tag{11}$$

$$\frac{\frac{y}{x^2}}{1+\frac{y^2}{x^2}} = \frac{y}{x^2+y^2} \qquad \frac{\frac{1}{x}}{1+\frac{y^2}{x^2}} = \frac{x}{x^2+y^2} \tag{12}$$

Using equation (10.25) in the textbook:

$$\dot{\boldsymbol{s}} = \frac{\partial \boldsymbol{s}(\boldsymbol{r}_c^c)}{\partial \boldsymbol{r}_c^c}\dot{\boldsymbol{r}}_c^c \tag{13}$$

The last term is given in the textbook, so we only need to find the first term. First expressing $\boldsymbol{s}$ as a function of $\boldsymbol{r}_c^c$

$$\boldsymbol{s}(\boldsymbol{r}_c^c) = \begin{bmatrix} \sqrt{\frac{x_c^2}{z_c^2} + \frac{y_c^2}{z_c^2}} \\ \tan^{-1} \frac{y_c}{x_c} \end{bmatrix} = \begin{bmatrix} \frac{1}{z_c}\sqrt{x_c^2 + y_c^2} \\ \tan^{-1} \frac{y_c}{x_c} \end{bmatrix} \tag{14}$$

Now we find the partial derivative

$$\begin{aligned}
\frac{\partial \boldsymbol{s}(\boldsymbol{r}_c^c)}{\partial \boldsymbol{r}_c^c} &= \begin{bmatrix} \frac{1}{2z_c}\frac{1}{\sqrt{x_c^2+y_c^2}}2x_c & \frac{1}{2z_c}\frac{1}{\sqrt{x_c^2+y_c^2}}2y_c & -\frac{1}{z_c^2}\sqrt{x_c^2+y_c^2} \\ -\frac{1}{1+\frac{Y^2}{X^2}}\frac{Y}{X^2}\frac{1}{z_c} & \frac{1}{1+\frac{Y^2}{X^2}}\frac{1}{X}\frac{1}{z_c} & 0 \end{bmatrix} && (15) \\
&= \begin{bmatrix} \frac{x_c}{z_c^2}\frac{1}{r} & \frac{y_c}{z_c^2}\frac{1}{r} & -\frac{r}{z_c} \\ -\frac{1}{z_c}\frac{Y}{X^2+Y^2} & \frac{1}{z_c}\frac{X}{X^2+Y^2} & 0 \end{bmatrix} && (16) \\
&= \begin{bmatrix} \frac{1}{z_c}\cos\phi & \frac{1}{z_c}\sin\phi & -\frac{r}{z_c} \\ -\frac{1}{z_c}\frac{Y}{r^2} & \frac{1}{z_c}\frac{X}{r^2} & 0 \end{bmatrix} && (17) \\
&= \begin{bmatrix} \frac{1}{z_c}\cos\phi & \frac{1}{z_c}\sin\phi & -\frac{r}{z_c} \\ -\frac{1}{z_c r}\sin\phi & \frac{1}{z_c r}\cos\phi & 0 \end{bmatrix} && (18)
\end{aligned}$$

Now we need to multiply this with $\dot{\boldsymbol{r}}_c^c$, which is given in (10.26) in the textbook.

$$\begin{aligned}
\dot{\boldsymbol{s}} &= \begin{bmatrix} \frac{1}{z_c}\cos\phi & \frac{1}{z_c}\sin\phi & -\frac{r}{z_c} \\ -\frac{1}{z_c r}\sin\phi & \frac{1}{z_c r}\cos\phi & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 0 & -z_c & y_c \\ 0 & -1 & 0 & z_c & 0 & -x_c \\ 0 & 0 & -1 & -y_c & x_c & 0 \end{bmatrix} \\
&= \begin{bmatrix} \frac{1}{z_c}\cos\phi & \frac{1}{z_c}\sin\phi & -\frac{r}{z_c} & \sin\phi + \frac{r}{z_c}y_c & -\cos\phi - \frac{r}{z_c}x_c & \frac{y_c}{z_c}\cos\phi - \frac{x_c}{z_c}\sin\phi \\ -\frac{1}{z_c r}\sin\phi & \frac{1}{z_c r}\cos\phi & 0 & \frac{z_c}{z_c r}\cos\phi & \frac{z_c}{z_c r}\sin\phi & -\frac{y_c}{z_c r}\sin\phi - \frac{x_c}{z_c r}\cos\phi \end{bmatrix} \\
&= \begin{bmatrix} \frac{1}{z_c}\cos\phi & \frac{1}{z_c}\sin\phi & -\frac{r}{z_c} & (1+r^2)\sin\phi & -(1+r^2)\cos\phi & 0 \\ -\frac{1}{z_c r}\sin\phi & \frac{1}{z_c r}\cos\phi & 0 & \frac{1}{r}\cos\phi & \frac{1}{r}\sin\phi & -1 \end{bmatrix}
\end{aligned}$$

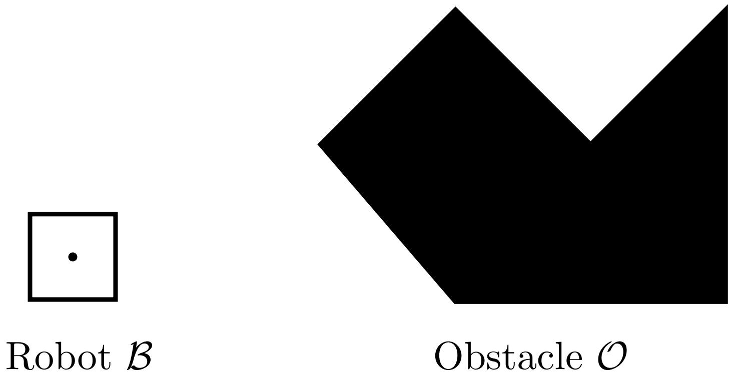
The assignment is inpired by Peter Corke *Robotics, Vision and control* (DOI: 10.1007/978-3-642-20144-8), and the result is consisten with (16.9) in that book.

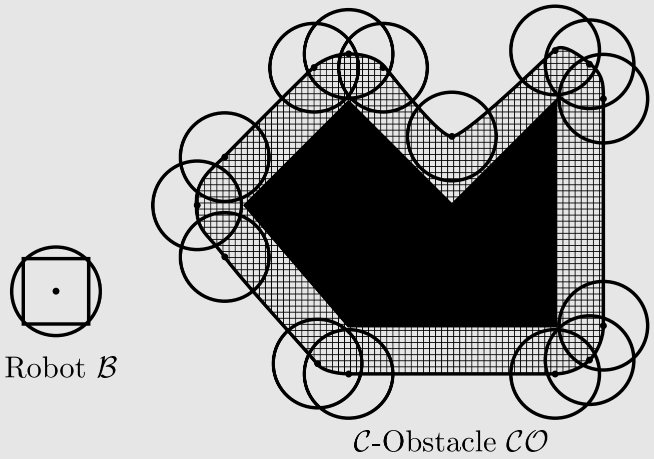
- 1 points for identifying that one need to use (10.25) in the textbook
- 2 points for finding $\boldsymbol{s}(\boldsymbol{r}_c^c)$
- 3 points for for finding the correct partial derivative
- 1 points for identifying that one can use (10.26) in the textbook
- 3 points for finding the correct $\dot{\boldsymbol{s}}$

There are other ways to find the correct result.

Problem 6 - Motion planning (12%)

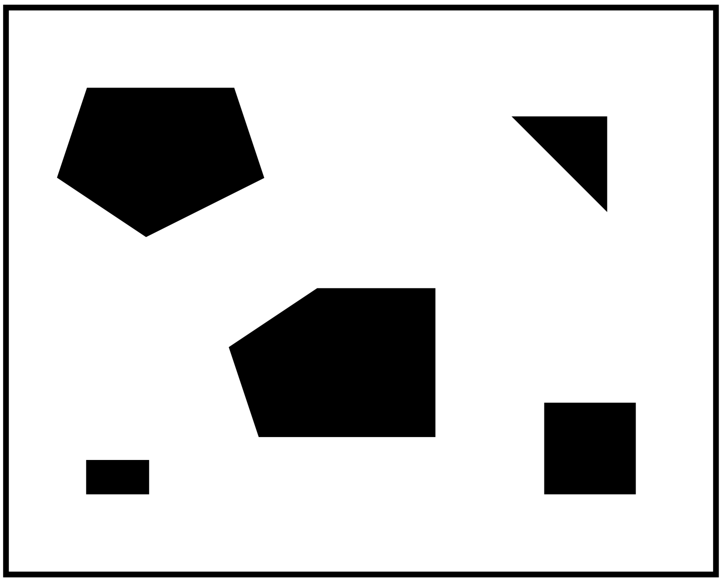
a) (6 %) Given a square robot that can translate in any direction and can rotate around its center, and the an obstacle, as seen below. Draw the C-obstacle.

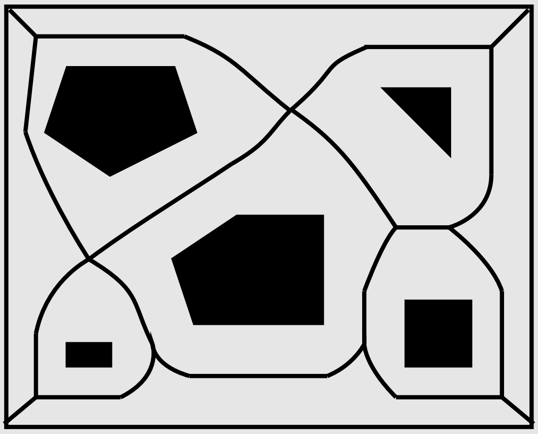




Above is an estimated C-obstacle. As the robot can rotate a circle with a diameter of the diagonal of the square that represents the robot will cover all possible headings of the robot.

b) (6 %) Given the obstables in the figure below, draw an approximation of a generalized Voronoi diagram and explain how you determined where the graph edges should be.





An estimate is sufficient. The edges should be drawn at the position where that have maxmum clearance to the obtacles.

Problem 7 - Control of AUV and USV (12%)

A three degree of freedom maneuvering model of an USV is given by

$$\dot{x} = u\cos(\psi) - v\sin(\psi), \tag{19a}$$
$$\dot{y} = u\sin(\psi) + v\cos(\psi), \tag{19b}$$
$$\dot{\psi} = r, \tag{19c}$$
$$\dot{u} = F_u(v, r) - \frac{d_{11}}{m_{11}}u + \tau_u, \tag{19d}$$
$$\dot{v} = X(u)r + Y(u)v, \tag{19e}$$
$$\dot{r} = F_r(u, v, r) + \tau_r. \tag{19f}$$

Here, x and y are the north and east position in the NED frame, and ψ is the vehicle heading. The velocities u and v are forward (surge) and sideways (sway) velocities in BODY, while r is the heading rate. The functions $F_u(v,r)$, $F_r(u,v,r)$, $X(u)$ and $Y(u)$ are general nonlinar term. The details of these are not required here. The damping term d_{11} and m_{11} are positive constants. Finally, the vehicle is controlled in surge and yaw rate through the control inputs τ_u and τ_r.

Sway dynamics

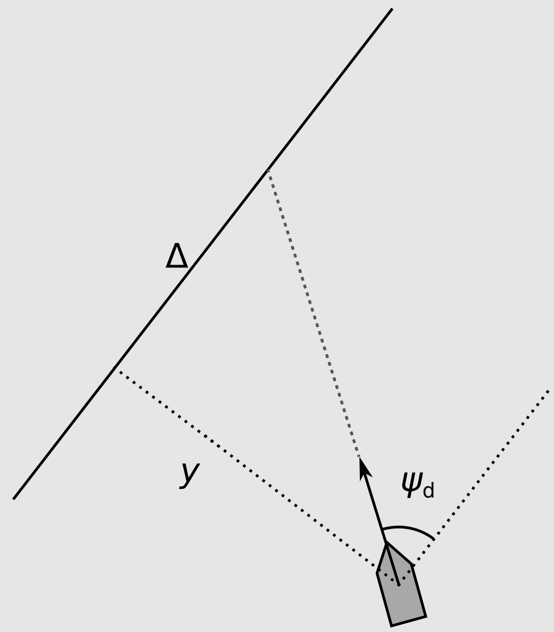
a) (4 %) Consider the underactuated sway dynamics given in (19e). It is common to assume that $Y(u)$ is negative. By looking only at (19e), would you say that this is a reasonable assumption? Why or why not?

> It is a reasonable assumption. We can notice this by considering what happens if $r = 0$ and $v \neq 0$. The system then becomes unstable, that is a small perturbation in sway induces a further increase in sway, which is unphysical.

Lookahead-based line of sight guidance

Consider an USV controlled by the lookahead-based line of sight guidance law to follow a straight line path. The desired heading is then given by $\psi_d = \tan^{-1}(-\frac{e}{\Delta})$ where e is the cross-track error, i.e. the distance from the path.

b) (4 %) Draw a sketch showing the most important parameters of the guidance law, including the vehicle and the path. Assume that the sway v is zero.



c) (4 %) If the sway v is not zero, how would you adjust the guidance law to compensate for the induced sideslip?

The old desired heading, ψ_d is then really the desired course. That is to say $\psi_d = \chi_d = \hat{\psi}_d + \beta$, where $\hat{\psi}_d$ is the new desired heading. Hence $\hat{\psi}_d = \psi_d - \beta$ Another possibility is to add an integral effect to the controller.

A Formulas

Solution for quadratic equations

$$ax^2 + bx + c = 0 \qquad \Rightarrow \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Properties of dynamic models of robots
Given the following dynamic system

$$\boldsymbol{B}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{F}\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{u}$$

The following properties hold

$$\begin{aligned} \dot{\boldsymbol{q}}^T \left(\dot{\boldsymbol{B}} - 2\boldsymbol{C}\right) \dot{\boldsymbol{q}} &= \boldsymbol{0} \\ \boldsymbol{B} &= \boldsymbol{B}^T \\ \boldsymbol{q}^T \boldsymbol{B} \boldsymbol{q} &> 0 \\ \boldsymbol{q}^T \boldsymbol{F} \boldsymbol{q} &> 0 \end{aligned}$$

Linear algebra
A matrix is symmetric if it satisfies the condition

$$\boldsymbol{A} = \boldsymbol{A}^T$$

For symmetric matrices the following holds

$$\boldsymbol{x}^T \boldsymbol{A} \boldsymbol{y} = \boldsymbol{y}^T \boldsymbol{A} \boldsymbol{x}$$

A skew-symmetric matrix is from the vector $\boldsymbol{x} = [x\ y\ z]^T$ is defined as

$$\boldsymbol{S}(\boldsymbol{x}) = \begin{bmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix} \tag{20}$$

Trigonometry
Definitions

$$\sin\theta = \frac{opposite}{hypotenuse} \qquad \cos\theta = \frac{adjacent}{hypotenuse}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{opposite}{adjacent}$$

Addition formulas

$$\begin{aligned} \sin(x \pm y) &= \sin x \cos y \pm \cos x \sin y \\ \cos(x \pm y) &= \cos x \cos y \mp \sin x \sin y \end{aligned}$$

Derivatives

$$\frac{d}{dx} \sin x = \cos x \tag{21}$$

$$\frac{d}{dx} \cos x = -\sin x \tag{22}$$