Problem 2 - b)

Transferfunction $\frac{X}{R} = ?$

$$\frac{X}{U} = \frac{1/F_m}{s\left(1 + \frac{I_m}{F_m} s\right)} \quad ; \quad U = C_A\left(C_p \varepsilon - s^2 X\right) \quad ; \quad \varepsilon = (R - X)$$

$$X = \frac{1/F_m}{s\left(1 + \frac{I_m}{F_m} s\right)} \cdot C_A\left(C_p \varepsilon - s^2 X\right) \quad ; \quad C_A = k_A \frac{1 + s T_A}{s} \quad ; \quad C_p = k_p$$

$$= \frac{1/F_m}{s\underbrace{\cancel{\left(1 + \frac{I_m}{F_m} s\right)}}_{T_A}} \cdot k_A \frac{\cancel{(1 + s T_A)}}{s}\left(k_p (R - X) - s^2 X\right) \quad ; \quad T_A = \frac{I_m}{F_m}$$

$$= \frac{1}{s^2} \cdot \frac{1}{F_m} \cdot k_A\left(k_p (R - X) - s^2 X\right)$$

$$X s^2 F_m = k_A k_p R - k_A k_p X - k_A s^2 X$$

$$X s^2 F_m + k_A k_p X + k_A s^2 X = k_A k_p R$$

$$X\left(s^2 F_m + s^2 k_A + k_A k_p\right) = k_A k_p R$$

$$X\left(s^2 (F_m + k_A) + k_A k_p\right) = k_A k_p R$$

↳ Transferfunction: $$\frac{X}{R} = \frac{k_A k_p}{s^2 (F_m + k_A) + k_A k_p}$$

↳ This is the system where it has the second order.

↳ poles are given as ; $$s = \frac{-0 \pm \sqrt{0^2 - 4 \cdot (F_m + k_A) \cdot (k_A k_p)}}{2 (F_m + k_A)}$$

Problem 2 - Independent joint control

For simplicity, $X(s) = X$ and $E = R - X$

$U(s) = U$

$C_P(s) = C_P$

$C_A(s) = C_A$

$U(s) = U$

$$U = C_A(C_P E - s^2 X)$$

$$\frac{X}{U} = \frac{1/F_m}{s(1 + \frac{I_m}{F_m} s)}$$

$$X s(1 + \frac{I_m}{F_m} s) = \frac{1}{F_m} U$$

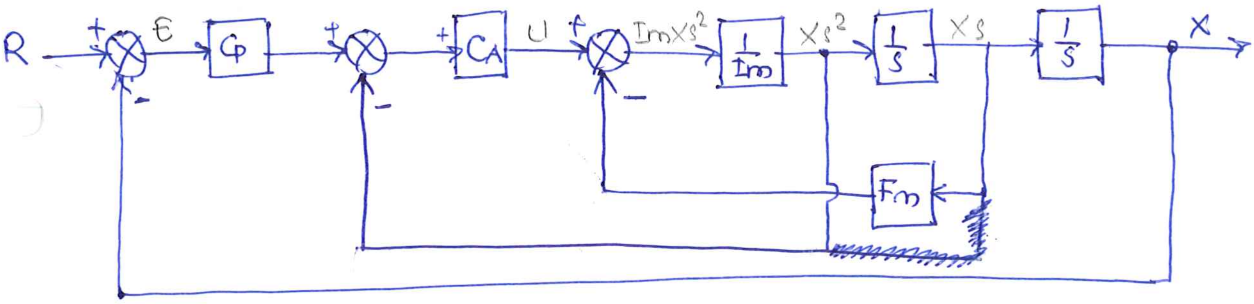
$$X s + X \frac{I_m}{F_m} s^2 = \frac{1}{F_m} U$$

$$X \frac{I_m}{F_m} s^2 = \frac{1}{F_m} U - X s$$

$$X s^2 = \frac{F_m}{I_m} \left(\frac{1}{F_m} U - X s \right)$$

$$= \frac{1}{I_m} U - \frac{F_m}{I_m} X s$$

$$= \frac{1}{I_m} (U - F_m X s)$$



Problem 3 - centralized control

$u = g(q) + K_P \tilde{q} - K_{D^2} \ddot{q}$; $\tilde{q} = q_d - q$

The dynamic equation is ,

$B(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = u \Rightarrow B\ddot{q} = u - C(q,\dot{q})\dot{q} - g(q)$ —— (1)

The Lyapunov function

$V(\dot{q},\tilde{q}) = \frac{1}{2}\dot{q}^T B(q)\dot{q} + \frac{1}{2}\tilde{q}^T K_P \tilde{q} + \frac{1}{2}\dot{q}^T K_{D^2} \dot{q}$

$\dot{V}(\dot{q},\tilde{q}) = \frac{1}{2}\ddot{q}^T B(q)\dot{q} + \frac{1}{2}\dot{q}^T \dot{B}(q)\dot{q} + \frac{1}{2}\dot{q}^T B(q)\ddot{q}$

$+ \frac{1}{2}\dot{\tilde{q}}^T K_P \tilde{q} + \frac{1}{2}\tilde{q}^T K_P \dot{\tilde{q}}$

$+ \frac{1}{2}\ddot{q}^T K_{D^2} \dot{q} + \frac{1}{2}\dot{q}^T K_{D^2} \ddot{q}$

$\ddot{q}^T K_{D^2} \dot{q}$

$= \dot{q}^T B(q)\ddot{q} + \frac{1}{2}\dot{q}^T \dot{B}(q)\dot{q}$

$- \dot{q}^T K_P \tilde{q}$; $\tilde{q} = q_d - q$

$+ \ddot{q}^T K_{D^2} \dot{q}$; $\dot{\tilde{q}} = -\dot{q}$

q

(1) $\Rightarrow B\ddot{q} = u - C(q,\dot{q})\dot{q} - g(q)$

$= \dot{q}^T (u - C(q,\dot{q})\dot{q} - g(q)) + \frac{1}{2}\dot{q}^T \dot{B}(q)\dot{q}$

$- \dot{q}^T K_P \tilde{q} + \ddot{q}^T K_{D^2} \dot{q}$

$= \frac{1}{2}\dot{q}^T \underbrace{(\dot{B}(q) - 2C(q,\dot{q}))}_{0}\dot{q} + \dot{q}^T(u - g(q)) - \dot{q}^T K_P \tilde{q}$

$+ \ddot{q}^T K_{D^2} \dot{q}$

$= \dot{q}^T (\cancel{g(q)} + \cancel{K_P \tilde{q}} - K_{D^2}\ddot{q} - \cancel{g(q)}) - \cancel{\dot{q}^T K_P \tilde{q}} + \ddot{q}^T K_{D^2} \dot{q}$

$= -\dot{q}^T K_{D^2} \ddot{q} + \ddot{q}^T K_{D^2} \dot{q}$

$= -\dot{q}^T K_{D^2} \ddot{q} + \dot{q}^T K_{D^2} \ddot{q}$

$= 0$

$\dot{V}(\dot{q},\tilde{q})$ negative for all $\dot{q} \neq 0$, unstable

Problem 3 - centralized control

By adding

$$u = g(q) + K_P \tilde{q} - K_D^2 \dot{q}' - K_D \dot{q}$$

(1) would be =>

$$B \dot{q} = \dot{q}^T (\cancel{g(q)} \cancel{+ K_P \tilde{q}} \cancel{- K_D^2 \ddot{q}} - K_D \dot{q} \cancel{- g(q)}) \cancel{- \dot{q}^T K_P \tilde{q}} \cancel{+ \ddot{q}^T K_D^2 \dot{q}}$$

$$= - \dot{q}^T K_D \dot{q}$$

$\dot{V}(\dot{q}, \tilde{q})$ negativ for all $\dot{q} \neq 0$, but is independent of $\tilde{q}$ and therefore negativ semidefinit.

Problem 4 - Force control

$$O_e = \begin{bmatrix} \dfrac{K_{px}\, x_d + K_x x_r}{K_x + K_{px}} \\ \\ y_d \end{bmatrix}$$; where

$K_{px} = 500 \; [\frac{N}{m}]$

$K_x = 1000 \; [\frac{N}{m}]$

$x_d = 3 \; [m]$

$x_r = 1 \; [m]$

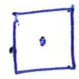
$$= \frac{(500 \frac{N}{m})(3\,m) + (1000 \frac{N}{m})(1\,m)}{(1000 \frac{N}{m}) + (500 \frac{N}{m})}$$

$$= 1.67 \; [m]$$

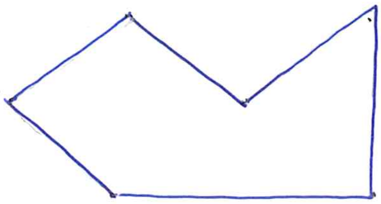
$$O_e = \begin{bmatrix} 1.67 \; m \\ 0 \; m \end{bmatrix}$$; yd suppose to be 0m

Problem 6 - Motion planning

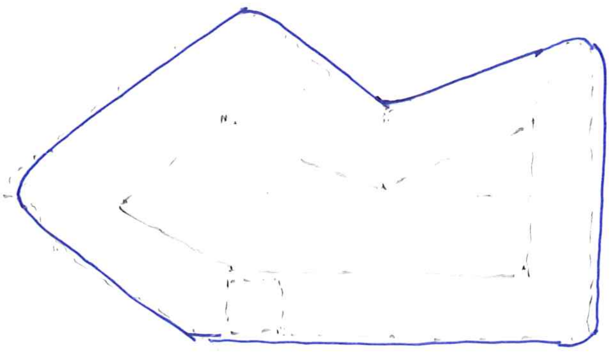
a) Draw the C-obstacle



Robot B



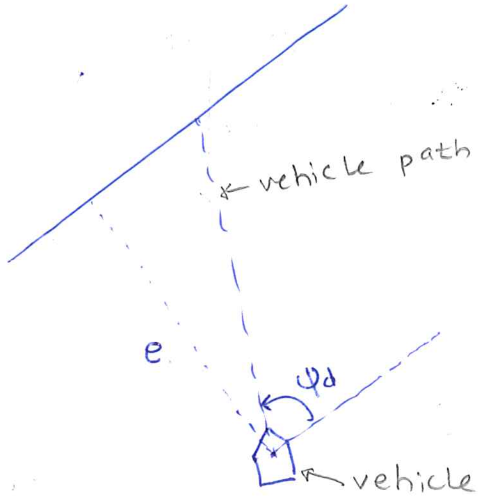
Obstacle O

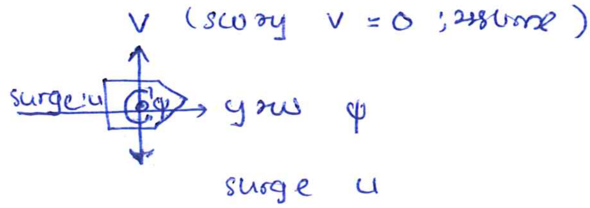


C - obstacle

Problem 7

b) Draw the sketch showing the most important parameters of the guidance Law





where e is the cross track error and

ψ_d is the yaw

desired heading:

$$\psi_d = \tan^{-1}\left(-\left(\frac{e}{\Delta}\right)\right)$$

c) If the sway v is not zero, I would adjust the guidance Law to compensate for the induced sideslip,

$$\psi_d = -\tan^{-1}\left(\frac{e}{\Delta}\right) - \beta \quad , \text{ where } \beta = \text{sideslip angle} \triangleq \chi - \psi$$

where $\beta = \arctan\left(\frac{v}{u}\right) = \arcsin\left(\frac{v}{U}\right)$

$U \triangleq \sqrt{u^2 + v^2}$

χ = the direction of the velocity vector

u = surge = u

a) By Looking at 12 e,

$\dot{v} = X(u)r + Y(u)v$,

and this is underactuation in sway (comon to assume that $Y(u)$ is negative).

The function $Y(u_b)$ basically satisfies,

$$Y(u_b) < 0$$

If the assumption does not satisfies, then there will be prescence a small disturbance in sway and it would Lead an increasing sway motion. But mostly it would not be the case for commercial vessels.