UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Exam in: UNIK4490
Day of exam: 12th of December
Exam hours: 9.15 - 13.15 (4 hours)
This examination paper consists of 6 page(s).
Appendices: Formulas
Permitted materials: None

Make sure that your copy of this examination paper is complete before answering.

General information

This exam has seven problems from different topics of the course. Before you start answering you should read through the exam, to get an overview of the problems. In some of the problems it is asked to *briefly describe* or *briefly explain*, then please write brief. In most cases two to three sentences is sufficient.

Problem 1 - Actuators and sensors (9%)

a) (3 %) Electric drives can behave in two fundamental different ways regarding the output they generate. What are these two generators called, and what are they typically used for?

b) (6 %) Robots have both proprioceptive and exteroceptive sensors. Explain the principal difference between these two types of sensors. Then give one example of each sensors type.

Problem 2 - Independent joint control (18%)

We will now look at the control of an independent joint of robot. The transfer function of the control input $U(s)$ and the joint position $X(s)$ is

$$\frac{X(s)}{U(s)} = \frac{\frac{1}{F_m}}{s(1 + \frac{I_m}{F_m}s)} \tag{1}$$

The joint is controlled using both position and velocity feedback with the controllers

$$C_P(s) = K_P \qquad\qquad C_V = K_V \frac{1 + sT_V}{s} \tag{2}$$

The control input $U(s)$ is given as

$$U(s) = C_V(s)(C_P(s)E(s) - sX(s)) \tag{3}$$

where $E(s) = R(s) - X(s)$, and $R(s)$ is the reference input to the controller. I_m and F_m are two positive constants.

a) (6 %) Draw the block diagram of the system (model and controller) using only constants blocks, integral blocks and the control blocks $C_P(s)$ and $C_V(s)$.

b) (9 %) Find the transfer function of the system assuming that $\frac{I_m}{F_m} = T_V$. Which order is the system? Find the poles of the system as an expression. Is the system stable? Explain why/why not.

c) (3 %) When doing independent joint control, the model is divided into a linear decoupled part and a nonlinear coupled part. Explain briefly how this division is done. When designing control systems, which part is considered the model and which part is considered as a disturbance.

Problem 3 - Centralized control (24%)

a) (3 %) Briefly describe the difference between independent joint control and centralized control.

b) (9 %) A PD controller with gravity compensation is given below

$$\boldsymbol{u} = \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{K}_P\tilde{\boldsymbol{q}} - \boldsymbol{K}_D\dot{\boldsymbol{q}} \tag{4}$$

where $\tilde{\boldsymbol{q}} = \boldsymbol{q}_d - \boldsymbol{q}$, and $\boldsymbol{q}_d$ is the desired joint positions which is constant. This controller will be used to control the system

$$\boldsymbol{B}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{F}\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{u} \tag{5}$$

The Lyapunov function candidate for the controller is

$$\boldsymbol{V}(\dot{\boldsymbol{q}}, \tilde{\boldsymbol{q}}) = \frac{1}{2}\dot{\boldsymbol{q}}^T\boldsymbol{B}(\boldsymbol{q})\dot{\boldsymbol{q}} + \frac{1}{2}\tilde{\boldsymbol{q}}^T\boldsymbol{K}_P\tilde{\boldsymbol{q}} \tag{6}$$

Show that the system has a negative semi-definite $\dot{\boldsymbol{V}}$, but that the other conditions in Lyapunov's direct method is satisfied.

c) (3 %) Draw a block diagram of the PD controller with gravity compensation in (4). Draw the manipulator model as one block with $\boldsymbol{u}$ as input and $\boldsymbol{q}$ and $\dot{\boldsymbol{q}}$.

d) (3 %) What is the physical interpretation of the two terms of $\boldsymbol{V}(\dot{\boldsymbol{q}}, \tilde{\boldsymbol{q}})$ in (6)?

e) (6 %) Given the system

$$\boldsymbol{B}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{F}\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{u} \tag{7}$$

Show how it is possible to do an exact linearization of the system to obtain

$$\ddot{\boldsymbol{q}} = \boldsymbol{y} \tag{8}$$

Problem 4 - Force control (9%)

a) (3 %) Briefly explain the principal difference between indirect and direct force control.

b) (3 %) When a robot is under active compliance control and interacts with the environment, which two factors determine the actual position of the robot at steady-state?

c) (3 %) Briefly explain the differences between compliance and impedance control.

Problem 5 - Visual servoing (12%)

a) (3 %) Explain the principal difference between image-based and position-based visual servoing.

b) (3 %) In the equation

$$\dot{\boldsymbol{s}} = \boldsymbol{J}_s \boldsymbol{v}_o^c + \boldsymbol{L}_s \boldsymbol{v}_c^c \tag{9}$$

what are the names of $\boldsymbol{J}_s$ and $\boldsymbol{L}_s$ and what does $\boldsymbol{v}_o^c$ and $\boldsymbol{v}_c^c$ represent?

c) (6 %) For image-based visual servoing we have the following relations

$$\dot{\boldsymbol{s}} = \boldsymbol{L}_s \boldsymbol{v}_c^c \qquad\qquad \boldsymbol{v}_c^c = \boldsymbol{J}_c \dot{\boldsymbol{q}} = \begin{bmatrix} \boldsymbol{R}_c & 0 \\ 0 & \boldsymbol{R}_c \end{bmatrix} \boldsymbol{J} \dot{\boldsymbol{q}} \tag{10}$$

We assume that our robot functions as an ideal positioning device, i.e. $\boldsymbol{q}(t) \approx \boldsymbol{q}_r(t)$, where $\boldsymbol{q}_r(t)$ is the reference input. Find a control law that gives the following error dynamics

$$\dot{\boldsymbol{e}}_s + \boldsymbol{K}_s \boldsymbol{e}_s = 0 \tag{11}$$

where the error definition is $\boldsymbol{e}_s = \boldsymbol{s}_d - \boldsymbol{s}$. The desired image features $\boldsymbol{s}_d$ are constant. The control law should be on the form $\dot{\boldsymbol{q}}_r = \ldots$

Problem 6 - Mobile robots (18%)

a) (3 %) What is a non-holonomic constraint and why are they relevant for mobile robotics?

b) (3 %) Briefly describe one of the methods for path planning for mobile robots

c) (3 %) Briefly explain the principal difference between trajectory tracking and posture regulation

d) (9 %) Draw a figure showing the configuration variables of a unicycle. Find an expression of the non-holonomic constraint of the unicycle and derive the kinematic model.

Problem 7 - Control of AUV and USV (10%)

A three degree of freedom maneuvering model of an USV is given by

$$\dot{x} = u\cos(\psi) - v\sin(\psi), \tag{12a}$$
$$\dot{y} = u\sin(\psi) + v\cos(\psi), \tag{12b}$$
$$\dot{\psi} = r, \tag{12c}$$
$$\dot{u} = F_u(v,r) - \frac{d_{11}}{m_{11}}u + \tau_u, \tag{12d}$$
$$\dot{v} = X(u)r + Y(u)v, \tag{12e}$$
$$\dot{r} = F_r(u,v,r) + \tau_r. \tag{12f}$$

Here, x and y are the north and east position in the NED frame, and ψ is the vehicle heading. The velocities u and v are forward (surge) and sideways (sway) velocities in

BODY, while r is the heading rate. The functions $F_u(v, r)$, $F_r(u, v, r)$, $X(u)$ and $Y(u)$ are general nonlinar term. The details of these are not required here. The damping term d_{11} and m_{11} are positive constants. Finally, the vehicle is controlled in surge and yaw rate through the control inputs τ_u and τ_r.

a) (2.5 %) Assume that u and v are known. Find an expression for the sideslip angle β of the vehicle, using u and v.

b) (2.5 %) Find an expression for the total speed U of the vehicle.

c) (2.5 %) Assume that the sideslip is $\beta = 30\,°$, and that the vehicle heading is $\psi = 90\,°$. What is the course χ of the vehicle?

d) (2.5 %) Assume that the total speed U and the course χ are known. Rewrite equations (12a) and (12b) using U and χ.

A Formulas

Solution for quadratic equations

$$ax^2 + bx + c = 0 \qquad \Rightarrow \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Properties of dynamic models of robots

Given the following dynamic system

$$\boldsymbol{B}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{F}\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{u}$$

The following properties hold

$$\begin{aligned} \dot{\boldsymbol{B}} - 2\boldsymbol{C} &= \boldsymbol{0} \\ \boldsymbol{B} &= \boldsymbol{B}^T \\ \boldsymbol{q}^T \boldsymbol{B} \boldsymbol{q} &> 0 \\ \boldsymbol{q}^T \boldsymbol{F} \boldsymbol{q} &> 0 \end{aligned}$$

Linear algebra

A matrix is symmetric if it satisfies the condition

$$\boldsymbol{A} = \boldsymbol{A}^T$$

For symmetric matrices the following holds

$$\boldsymbol{x}^T \boldsymbol{A} \boldsymbol{y} = \boldsymbol{y}^T \boldsymbol{A} \boldsymbol{x}$$

Trigonometry

Definitions

$$\sin\theta = \frac{opposite}{hypotenuse} \qquad \cos\theta = \frac{adjacent}{hypotenuse}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{opposite}{adjacent}$$

Addition formulas

$$\begin{aligned} \sin(x \pm y) &= \sin x \cos y \pm \cos x \sin y \\ \cos(x \pm y) &= \cos x \cos y \mp \sin x \sin y \end{aligned}$$