Deriving a dynamic model of manipulators and drives (ch. 8.2, slides 7-10)

Control the robot so that

$$q(t) = q_d(t)$$

- Transmission
- Actuator model (electric drive)
- Manipulator model

Transmission

Assume rigid and no backlash

$$\underset{n\times n}{K_r}\,\underset{n}{\underline{q}} = \underset{n}{\underline{q}_m} \qquad (8.2)$$

$\underline{q}$ – joint displacement (position)

$\underline{q}_m$ – joint actuator displacement (position)

K_r – gear reduction ratio

$$\underline{\tau}_m = K_r^{-1}\,\underline{\tau} \qquad (8.3)$$

Actuator model (electric drive)

Use the model from chapter 5, but generalize it for n driving systems

$$V_a = (R_a + sL_a) I_a + V_g \qquad (5.1)$$

$$V_g = k_v \omega_m \qquad (5.2)$$

$$\tau_m = k_t I_a \qquad (5.4)$$

8.3 and 5.4

$$\underset{n\times n}{K_r^{-1}}\,\underset{n}{\underline{\tau}} = \underset{n\times n}{K_t}\,\underset{n}{\underline{i}_a} \qquad (8.4)$$

5.1 and 5.2 give (disregarding induction)

$$\underline{v}_a = R_a \underline{i}_a + K_v \dot{q}_m \qquad (8.5)$$

Using a simplified power amplifier model

$$\underset{n}{\underline{v}_a} = \underset{n\times n}{G_v}\,\underset{n}{\underline{v}_c} \qquad (8.6)$$

$$G_v = \begin{bmatrix} G_{v1} & 0 & \cdots & 0 \\ 0 & G_{v2} & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & G_{vn} \end{bmatrix}$$

Dynamic manipulator model

$$\underbrace{B(\underline{q})\ddot{\underline{q}}}_{\text{Inertial forces}} + \underbrace{C(\underline{q},\dot{\underline{q}})\dot{\underline{q}}}_{\text{Coriolis and centrifugal forces}} + \underbrace{F_v\dot{\underline{q}}}_{\text{Viscous friction}} + \underbrace{\underline{g}(\underline{q})}_{\text{Gravity}} = \underline{\tau}$$

Combine 8.5 and 8.6

$$G_v \underline{v_c} = R_a \underline{i_a} + K_v \underline{\dot{q}_m}$$

Insert 8.2

$$G_v \underline{v_c} = R_a \underline{i_a} + K_v K_r \underline{\dot{q}}$$

Solve for $\underline{i_a}$

$$R_a \underline{i_a} = G_v \underline{v_c} - K_v K_r \underline{\dot{q}}$$

$$\underline{i_a} = R_a^{-1} (G_v \underline{v_c} - K_v K_r \underline{\dot{q}})$$

$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{a} & 0 \\ 0 & \frac{1}{b} \end{bmatrix}$$

Insert into 8.4

$$K_r^{-1} \underline{\tau} = K_t R_a^{-1} (G_v \underline{v_c} - K_v K_r \dot{\underline{q}})$$

$$\underline{\tau} = K_r K_t R_a^{-1} (G_v v_c - K_v K_r \dot{\underline{q}})$$

manipulator torque

manipulator velocities

Insert into dynamic eqation of manipulator

$$B\ddot{\underline{q}} + C\dot{\underline{q}} + F_v\dot{\underline{q}} + g = K_r K_t R_a^{-1}(G_v \underline{v}_c - K_v K_r \dot{\underline{q}})$$

$$B\ddot{\underline{q}} + C\dot{\underline{q}} + \underbrace{F_v\dot{\underline{q}} + K_r K_t R_a^{-1} K_v K_r \dot{\underline{q}}}_{F\dot{\underline{q}}} + \underline{g} = \underbrace{K_r K_t R_c^{-1} G_v \underline{v}_c}_{u}$$

$$\boxed{B\ddot{\underline{q}} + C\dot{\underline{q}} + F\dot{\underline{q}} + \underline{g} = u}$$

Derive linear decoupled model (Ch. 8.3, slide 19-22)

Based on the dynamic model

$$B\ddot{q} + C\dot{q} + F_v\dot{q} + g = \tau$$

Inserting $\dot{q} = K_r^{-1}\dot{q}_m$ $\quad \tau_m = K_r^{-1}\tau \iff K_r\tau_m = \tau$

$$K_r^{-1}BK_r^{-1}\ddot{q}_m + K_r^{-1}CK_r^{-1}\dot{q}_m + K_r^{-1}F_vK_r^{-1}\dot{q}_m + K_r^{-1}g = \tau_m$$

Dividing B into two parts

$$B(q) = \bar{B} + \Delta B(q)$$

$\bar{B}$ - Average inertia, constant elements

ΔB - Configuration dependant terms

Rewrite dynamics as

$$K_r^{-1}\bar{B}K_r^{-1}\ddot{q}_m + F_m\dot{q}_m + d = \tau_m$$

$$F_m = K_r^{-1}F_v K_r^{-1}$$

$$d(q_m, \dot{q}_m, \ddot{q}_m)$$

$$d = K_r^{-1}\Delta B K_r^{-1}\ddot{q}_m + K_r^{-1}CK_r^{-1}\dot{q}_m + K_r^{-1}g$$