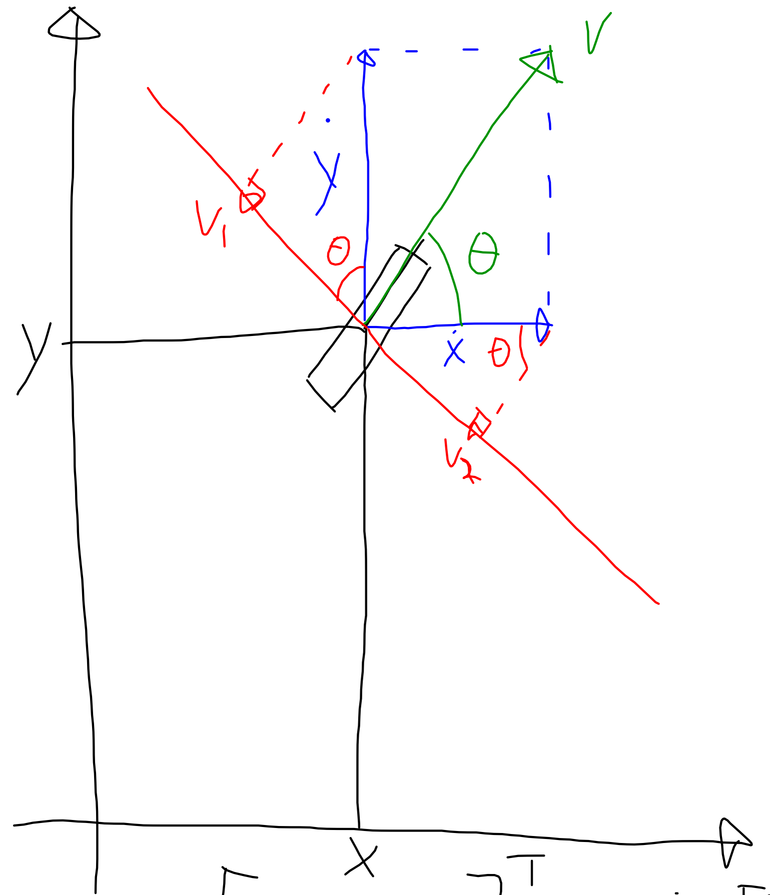
11.2.1 Unicycle (slides 16-18)



$$q = [x, y, \theta]^T \Rightarrow \dot{q} = [\dot{x}, \dot{y}, \dot{\theta}]^T$$

$$v_1 = v_2$$

$$\cos\theta = \frac{v_1}{\dot{y}} \Rightarrow v_1 = \dot{y}\cos\theta$$

$$\sin\theta = \frac{v_2}{\dot{x}} \Rightarrow v_2 = \dot{x}\sin\theta$$

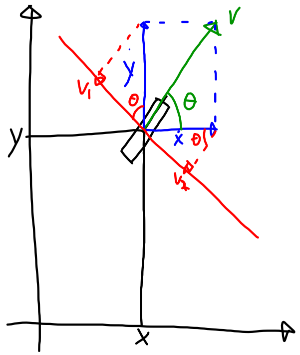
$$\dot{y}\cos\theta = \dot{x}\sin\theta$$

$$\dot{x}\sin\theta - \dot{y}\cos\theta = 0$$

$$[\sin\theta, -\cos\theta, 0]\,\dot{q} = 0 \quad (11.12)$$

$$A^T(q)\dot{q} = 0$$

constraint



Kinematic model

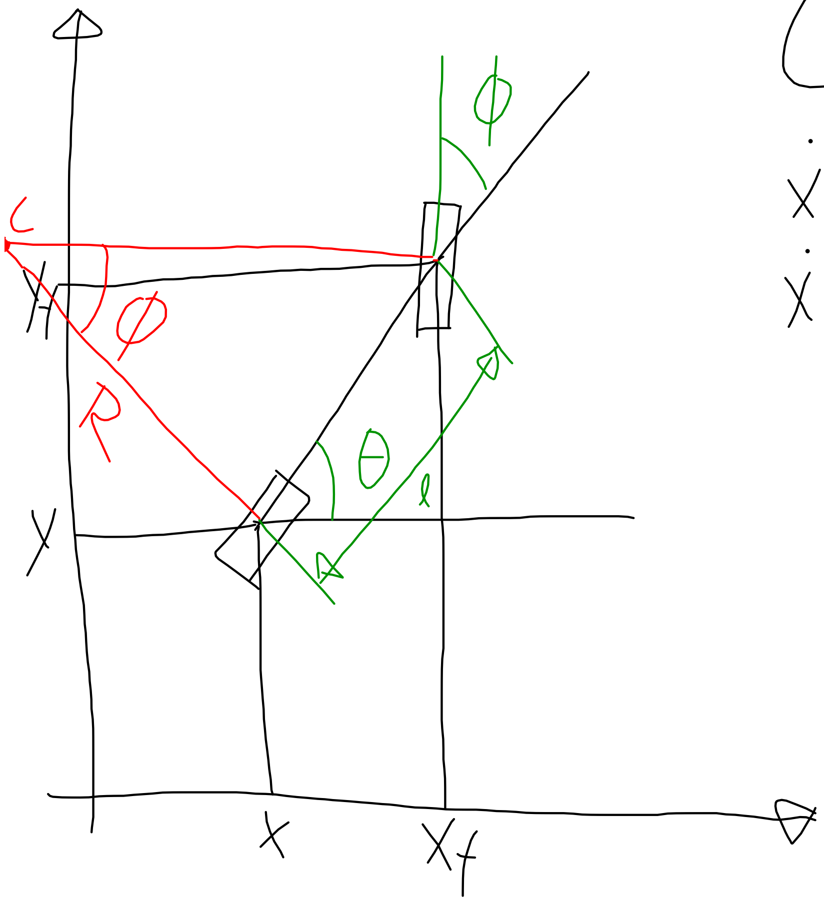
$$\cos\theta = \frac{\dot{x}}{v} \qquad \sin\theta = \frac{\dot{y}}{v}$$

$$\begin{aligned} \dot{x} &= \cos\theta \cdot v \\ \dot{y} &= \sin\theta \cdot v \\ \dot{\theta} &= \omega \end{aligned} \Rightarrow \begin{array}{ccccccc} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} & = & \begin{bmatrix} \cos\theta \\ \sin\theta \\ 0 \end{bmatrix} & v & + & \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & \omega \\ \dot{q} & & g_1 & & & g_2 & \end{array} \tag{11.13}$$

$$\dot{q} = G(q)\,u$$

$$u = [v, \omega]^T$$

11.2.2 Bicycle model (slides 20-24)



Constraints

$$\dot{x}\sin\theta - \dot{y}\cos\theta = 0 \quad (11.16)$$

$$\dot{x}_f\sin(\theta+\phi) - \dot{y}_f\cos(\theta+\phi) = 0 \quad (11.15)$$

Transform the constraints to the same frame

Using the rigid body constraints

$$x_f = x + l\cos\theta$$

$$y_f = y + l\sin\theta$$

Time differentiating

$$\dot{x}_f = \dot{x} - l\sin\theta\cdot\dot{\theta}$$

$$\dot{y}_f = y_f + l\cos\theta\cdot\dot{\theta}$$

Insert into (11.15)

$$(\dot{x} - l\sin\theta\,\dot{\theta})\sin(\theta+\phi) - (\dot{y} + l\cos\theta\,\dot{\theta})\cos(\theta+\phi) = 0$$

$$\dot{x}\sin(\theta+\phi) - \dot{y}\cos(\theta+\phi) - l\dot{\theta}\left(\sin\theta\sin(\theta+\phi) + \cos\theta\cos(\theta+\phi)\right)$$

$$= \cos(\theta-\theta-\phi) = \cos\phi$$

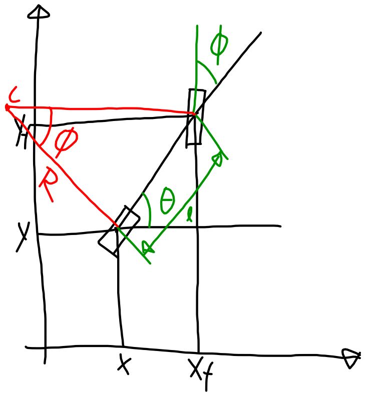
$$\dot{x}\sin(\theta+\phi) - \dot{y}\cos(\theta+\phi) - l\dot{\theta}\cos\phi = 0$$

$$\cos(-x) = \cos x$$

$$\underbrace{\begin{bmatrix} \sin\theta & -\cos\theta & 0 & 0 \\ \sin(\theta+\phi) & -\cos(\theta+\phi) & -l\cos\phi & 0 \end{bmatrix}}_{A^T(q)} \underbrace{\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix}}_{\dot{q}} = 0$$

$$q = [x \; y \; \theta \; \phi]^T$$

Rear wheel drive kinematic model



Same cartesian velocity as the unicycle

Rotation velocity

$$\tan\phi = \frac{l}{R}$$

$$\dot{\theta} = \frac{v}{R} = \frac{v}{l}\tan\phi$$

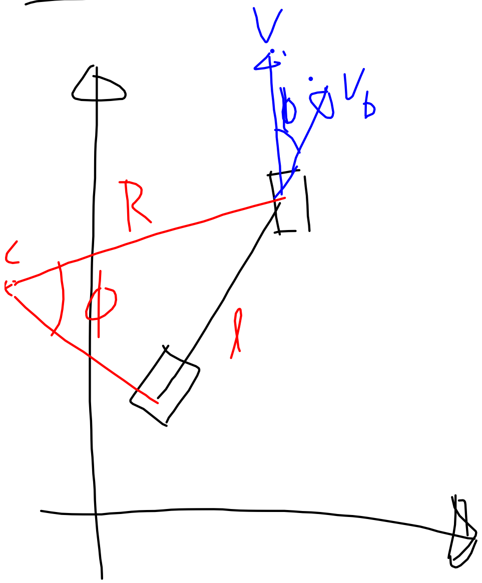
$$\frac{1}{R} = \frac{\tan\phi}{l}$$

$$\dot{\phi} = \omega$$

$$\underbrace{\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix}}_{\dot{q}} = \underbrace{\begin{bmatrix} \cos\theta \\ \sin\theta \\ \frac{\tan\phi}{l} \\ 0 \end{bmatrix}}_{g_1} v + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}}_{g_2} \omega \qquad (11.19)$$

$$\dot{q} = G(q)u$$

Front wheel drive kinematic model



$$\sin\phi = \frac{l}{R} \Rightarrow \frac{1}{R} = \frac{\sin\phi}{l}$$

$$\dot{\theta} = \frac{v}{R} = \frac{v}{l}\sin\phi$$

$$\cos\phi = \frac{v_b}{v}$$

$$v_b = v\cos\phi$$

$$\dot{x} = \cos\theta\, v_b = \cos\theta \cos\phi\, v$$
$$\dot{y} = \sin\theta\, v_b = \sin\theta \cos\phi\, v$$

$$\underbrace{\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix}}_{\dot{q}} = \underbrace{\begin{bmatrix} \cos\theta\cos\phi \\ \sin\theta\cos\phi \\ \sin\phi/l \\ 0 \end{bmatrix}}_{g_1} v + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}}_{g_2} \omega$$