UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Exam in: TEK4030
Day of exam: 4th of December
Exam hours: 9.15 - 13.15 (4 hours)
This examination paper consists of 5 page(s).
Appendices: Formulas
Permitted materials: None

Make sure that your copy of this examination paper is complete before answering.

Problem 1 - Actuators and sensors (8%)

a) (4 %) What is the effect of having a high gear reduction ratio?

b) (4 %) When modeling an electric drive what are the three main components that the model is derived from?

Problem 2 - Independent joint control (16%)

Assume that we have the following dynamic model for a singe joint of a manipulator

$$I\ddot{\theta} + F\dot{\theta} + d = u \tag{1}$$

where I is the moment of intertia, F is the viscous friction coefficient, d is the disturbance, and θ is the joint position.

a) (4 %) Transform the above model into the Lapace domain and draw the block diagram of the system using only constant and intregral blocks.

b) (4 %) Use a propotional controller to control the joint to a constant desired position θ_d. Find the transfer function of the system and add the controller to the block diagram of the system.

c) (4 %) Find the roots of the system. Is the system stable? Why/why not?

d) (4 %) Will the error of the system go to zero? Why/why not?

Problem 3 - Centralized control (24%)

a) (4 %) In operational space control there are two general schemes for controlling the robot. What are the two schemes called?

b) (4 %) Briefly explain the difference between computed torque feedforward control and inverse dynamics control.

c) (16 %) A robotic manipulator has the following dynamic model

$$\boldsymbol{B}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{u} \tag{2}$$

where $\boldsymbol{q}$ is the joint position, $\dot{\boldsymbol{q}}$ is the joint velocities, $\ddot{\boldsymbol{q}}$ is the joint accelerations and $\boldsymbol{u}$ is the control input. The controller is given by

$$\boldsymbol{u} = \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{K}_P\tilde{\boldsymbol{q}} \tag{3}$$

where $\tilde{\boldsymbol{q}} = \boldsymbol{q}_d - \boldsymbol{q}$, and $\boldsymbol{q}_d$ is the desired joint positions which is constant. The Lyapunov function candidate for the controller is

$$\boldsymbol{V}(\dot{\boldsymbol{q}}, \tilde{\boldsymbol{q}}) = \frac{1}{2}\dot{\boldsymbol{q}}^T\boldsymbol{B}(\boldsymbol{q})\dot{\boldsymbol{q}} + \frac{1}{2}\tilde{\boldsymbol{q}}^T\boldsymbol{K}_P\tilde{\boldsymbol{q}} \tag{4}$$

Check the stability of the system using Lyapunovs direct method. What causes the system to be stable, or what lacks in the controller or dynamic model to prove the stability?

Problem 4 - Force control (16%)

a) (4 %) Briefly explain the difference between impedance and admittance control.

b) (4 %) The control law for active compliance is very similar to the operational space PD controller with gravity compensation. What is the principal difference?

c) (8 %) The dynamic model of a manipulator interacting with the environment is given by

$$\boldsymbol{B}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{F}\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{u} - \boldsymbol{J}^T(\boldsymbol{q})\boldsymbol{h}_e \tag{5}$$

Assume that the following controller is used to do an exact linearization

$$\boldsymbol{u} = \boldsymbol{B}(\boldsymbol{q})\boldsymbol{y} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{F}\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{J}^T(\boldsymbol{q})\boldsymbol{h}_e \tag{6}$$

Assume that only position variables are controlled, implying that the analytic and geometric Jacobian are the same. Find the ~~error~~ system dynamics using the following equation

$$\boldsymbol{y} = \boldsymbol{J}^{-1}(\boldsymbol{q})\boldsymbol{M}_d^{-1}\left(-\boldsymbol{K}_D\dot{\boldsymbol{x}}_e + \boldsymbol{K}_P(\boldsymbol{x}_F - \boldsymbol{x}_e) - \boldsymbol{M}_d\dot{\boldsymbol{J}}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}}\right) \tag{7}$$

where $\boldsymbol{x}_e$ is the end-effector position, and $\boldsymbol{x}_F$ a reference to be related to the force error.

Problem 5 - Tele-operations (8%)

a) (4 %) Explain what haptic guidance is with regards to tele-operation.

b) (4 %) The relationship between the forces and velocities of a master and slave system is often represented using a hybrid matrix, as given below

$$\begin{bmatrix} f_m \\ -v_s \end{bmatrix} = \boldsymbol{H} \begin{bmatrix} v_m \\ f_s \end{bmatrix} \tag{8}$$

How does the hybrid matrix for a transparent system look like? What does this imply for the relationship between the master and slave system?

Problem 6 - Visual servoing (12%)

a) (8 %) Assume we have a non-moving point P that with respect to the camera fram c is defined as

$$\boldsymbol{r}_c^c = \boldsymbol{R}_c^T(\boldsymbol{p} - \boldsymbol{o}_c) \tag{9}$$

where $\boldsymbol{p}$ is the point in with respect to the base frame and $\boldsymbol{o}_c$ is the origin of the camera frame with respect to the base frame. The normalized image coordinates X and Y are used as a feature vector, which yields

$$\boldsymbol{s}(\boldsymbol{r}_c^c) = \frac{1}{z_c}\begin{bmatrix} x_c \\ y_c \end{bmatrix} = \begin{bmatrix} X \\ Y \end{bmatrix} \tag{10}$$

where $\boldsymbol{r}_c^c = [x_c \; y_c \; z_c]$.

Find the interaction matrix $\boldsymbol{L}_s(\boldsymbol{s}, z_c)$ for the point P. Since $\boldsymbol{p}$ is constant you may use the relation

$$\dot{\boldsymbol{r}}_c^c = [-\boldsymbol{I} \; \boldsymbol{S}(\boldsymbol{r}_c^c)]\boldsymbol{v}_c^c \tag{11}$$

b) (4 %) If you have a set of points, as given in a), how would you do visual servoing?

Problem 7 - Mobile robots (16%)

a) (4 %) Name two mobile robots that are kinematically equivalent to a unicycle.

b) (4 %) What is the differential flatness property? Does the unicycle and bicycle have this property?

c) (4 %) Briefly explain the input/output linearization trajectory tracking method, and state one benefit with this method.

d) (4 %) What is odometric localization? Mention one method to do odometric localization, and briefly explain how it works.

A Formulas

Solution for quadratic equations

$$ax^2 + bx + c = 0 \qquad \Rightarrow \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Properties of dynamic models of robots

Given the following dynamic system

$$\boldsymbol{B}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{F}\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{u}$$

The following properties hold

$$\begin{aligned} \dot{\boldsymbol{q}}^T \left(\dot{\boldsymbol{B}} - 2\boldsymbol{C}\right) \dot{\boldsymbol{q}} &= \boldsymbol{0} \\ \boldsymbol{B} &= \boldsymbol{B}^T \\ \boldsymbol{q}^T \boldsymbol{B} \boldsymbol{q} &> 0 \\ \boldsymbol{q}^T \boldsymbol{F} \boldsymbol{q} &> 0 \end{aligned}$$

Linear algebra

A matrix is symmetric if it satisfies the condition

$$\boldsymbol{A} = \boldsymbol{A}^T$$

For symmetric matrices the following holds

$$\boldsymbol{x}^T \boldsymbol{A} \boldsymbol{y} = \boldsymbol{y}^T \boldsymbol{A} \boldsymbol{x}$$

A skew-symmetric matrix is from the vector $\boldsymbol{x} = [x\ y\ z]^T$ is defined as

$$\boldsymbol{S}(\boldsymbol{x}) = \begin{bmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix} \tag{12}$$

Trigonometry

Definitions

$$\sin\theta = \frac{opposite}{hypotenuse} \qquad \cos\theta = \frac{adjacent}{hypotenuse}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{opposite}{adjacent}$$

Addition formulas

$$\begin{aligned} \sin(x \pm y) &= \sin x \cos y \pm \cos x \sin y \\ \cos(x \pm y) &= \cos x \cos y \mp \sin x \sin y \end{aligned}$$

Derivatives

$$\frac{d}{dx} \sin x = \cos x \tag{13}$$

$$\frac{d}{dx} \cos x = -\sin x \tag{14}$$