UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Exam in:	TEK4030
Day of exam:	18th of December
Exam hours:	15.00 - 19.30 (4,5 hours)

This examination paper consists of 6 page(s).

Appendices:	Formulas
Permitted materials:	All Cooperation not allowed.

Make sure that your copy of this examination paper is complete before answering.

Problem 1 - Random questions: Course overview (28%)

Seven of these questions are randomly given to each candidate using Inspera.

Problem 2 - Independent joint control (15%)

We will now look at the control of an independent joint of robot. The transfer function of the control input $U(s)$ and the joint position $X(s)$ is

$$\frac{X(s)}{U(s)} = \frac{\frac{1}{F_m}}{s(1 + \frac{I_m}{F_m}s)} \tag{1}$$

The joint is controlled using both position and acceleration feedback with the controllers

$$C_P(s) = K_P \qquad\qquad C_A(s) = K_A \frac{1 + sT_A}{s} \tag{2}$$

The control input $U(s)$ is given as

$$U(s) = C_A(s)(C_P(s)E(s) - s^2X(s)) \tag{3}$$

where $E(s) = R(s) - X(s)$, and $R(s)$ is the reference input to the controller. I_m and F_m are two positive constants.

a) (6 %) Draw the block diagram of the system (model and controller) using only constants blocks, integral blocks and the control blocks $C_P(s)$ and $C_A(s)$.

b) (9 %) Find the transfer function of from $R(s)$ to $X(s)$ assuming that $\frac{I_m}{F_m} = T_A$. Which order is the system? Find the poles of the system as an expression. Is the system stable? Explain why/why not.

Problem 3 - Centralized control (18%)

A PD^2 controller with gravity compensation is given below

$$\boldsymbol{u} = \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{K}_P\tilde{\boldsymbol{q}} - \boldsymbol{K}_{D^2}\ddot{\boldsymbol{q}} \tag{4}$$

where $\tilde{\boldsymbol{q}} = \boldsymbol{q}_d - \boldsymbol{q}$, and $\boldsymbol{q}_d$ is the desired joint positions which are constant. This controller will be used to control the system

$$\boldsymbol{B}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{u} \tag{5}$$

The Lyapunov function candidate for the controller is

$$\boldsymbol{V}(\dot{\boldsymbol{q}}, \tilde{\boldsymbol{q}}) = \frac{1}{2}\dot{\boldsymbol{q}}^T\boldsymbol{B}(\boldsymbol{q})\dot{\boldsymbol{q}} + \frac{1}{2}\tilde{\boldsymbol{q}}^T\boldsymbol{K}_P\tilde{\boldsymbol{q}} + \frac{1}{2}\dot{\boldsymbol{q}}^T\boldsymbol{K}_{D^2}\dot{\boldsymbol{q}} \tag{6}$$

a) (10 %) Show that it is not possible to use Lyapunov's direct method to prove the stability of the system.

b) (8 %) Modify the control law in (4), by adding one or more terms, so that the system becomes stable. Show that the system is stable using Lyapunov's direct method.

Problem 4 - Force control (5%)

See Inspera for this assignment.

Problem 5 - Visual servoing (10%)

In this problem we are going to use polar coordinates for point features in the image. In polar coordinates a image point is written as $\boldsymbol{p}(r, \phi)$, where r is the distance from the point to the principal point, and ϕ is the angle from the X-axis to a line joining the principal point to the image point. These are defined as

$$r = \sqrt{X^2 + Y^2} = \sqrt{\frac{x_c^2}{z_c^2} + \frac{y_c^2}{z_c^2}} \qquad \phi = \tan^{-1}\frac{Y}{X} = \tan^{-1}\frac{y_c}{x_c} \tag{7}$$

The transform between polar and image coordinates are given as

$$X = r\cos\phi \qquad Y = r\sin\phi \tag{8}$$

a) (10 %) Find the interaction matrix $\boldsymbol{L}_s$ for the feature vector $\boldsymbol{s} = [r\ \phi]^T$.

The following formulas might prove useful

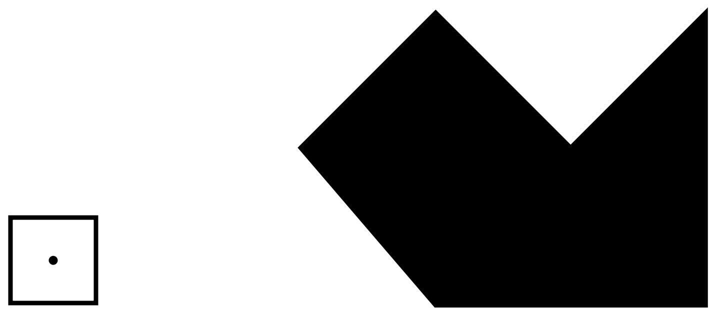
$$\frac{d}{dx}\sqrt{x} = \frac{1}{2}\frac{1}{\sqrt{x}} \qquad \frac{d}{dx}\frac{1}{x} = -\frac{1}{x^2} \tag{9}$$

$$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2} \qquad \sin^2 x + \cos^2 x = 1 \tag{10}$$

$$\frac{\frac{y}{x^2}}{1+\frac{y^2}{x^2}} = \frac{y}{x^2+y^2} \qquad \frac{\frac{1}{x}}{1+\frac{y^2}{x^2}} = \frac{x}{x^2+y^2} \tag{11}$$

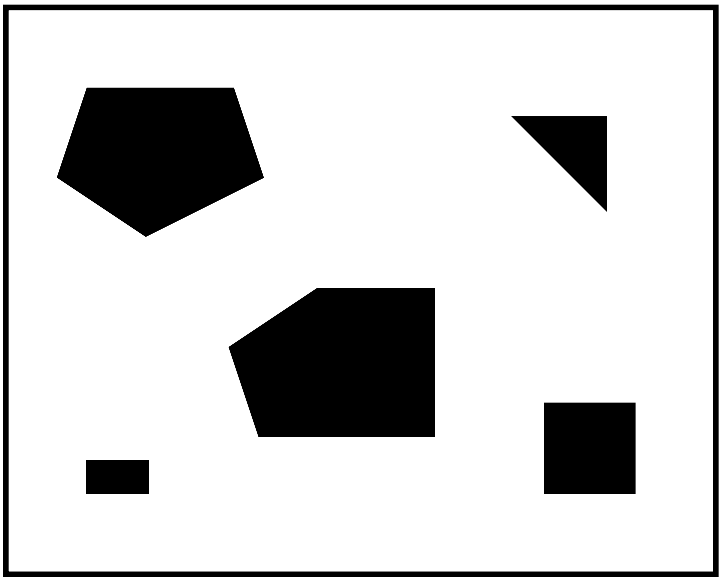
Problem 6 - Motion planning (12%)

a) (6 %) Given a square robot that can translate in any direction and can rotate around its center, and the an obstacle, as seen below. Draw the C-obstacle.



Robot $\mathcal{B}$ Obstacle $\mathcal{O}$

b) (6 %) Given the obstables in the figure below, draw an approximation of a generalized Voronoi diagram and explain how you determined where the graph edges should be.



Problem 7 - Control of AUV and USV (12%)

A three degree of freedom maneuvering model of an USV is given by

$$\dot{x} = u\cos(\psi) - v\sin(\psi), \tag{12a}$$
$$\dot{y} = u\sin(\psi) + v\cos(\psi), \tag{12b}$$
$$\dot{\psi} = r, \tag{12c}$$
$$\dot{u} = F_u(v,r) - \frac{d_{11}}{m_{11}}u + \tau_u, \tag{12d}$$
$$\dot{v} = X(u)r + Y(u)v, \tag{12e}$$
$$\dot{r} = F_r(u,v,r) + \tau_r. \tag{12f}$$

Here, x and y are the north and east position in the NED frame, and ψ is the vehicle heading. The velocities u and v are forward (surge) and sideways (sway) velocities in BODY, while r is the heading rate. The functions $F_u(v,r)$, $F_r(u,v,r)$, $X(u)$ and $Y(u)$ are general nonlinar term. The details of these are not required here. The damping term d_{11} and m_{11} are positive constants. Finally, the vehicle is controlled in surge and yaw rate through the control inputs τ_u and τ_r.

Sway dynamics

a) (4 %) Consider the underactuated sway dynamics given in (12e). It is common to assume that $Y(u)$ is negative. By looking only at (12e), would you say that this is a reasonable assumption? Why or why not?

Lookahead-based line of sight guidance

Consider an USV controlled by the lookahead-based line of sight guidance law to follow a straight line path. The desired heading is then given by $\psi_d = \tan^{-1}(-\frac{e}{\Delta})$ where e is the cross-track error, i.e. the distance from the path.

b) (4 %) Draw a sketch showing the most important parameters of the guidance law, including the vehicle and the path. Assume that the sway v is zero.

c) (4 %) If the sway v is not zero, how would you adjust the guidance law to compensate for the induced sideslip?

A Formulas

Solution for quadratic equations

$$ax^2 + bx + c = 0 \qquad \Rightarrow \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Properties of dynamic models of robots
Given the following dynamic system

$$\boldsymbol{B}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{F}\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{u}$$

The following properties hold

$$\begin{aligned} \dot{\boldsymbol{q}}^T \left(\dot{\boldsymbol{B}} - 2\boldsymbol{C}\right) \dot{\boldsymbol{q}} &= \boldsymbol{0} \\ \boldsymbol{B} &= \boldsymbol{B}^T \\ \boldsymbol{q}^T \boldsymbol{B} \boldsymbol{q} &> 0 \\ \boldsymbol{q}^T \boldsymbol{F} \boldsymbol{q} &> 0 \end{aligned}$$

Linear algebra
A matrix is symmetric if it satisfies the condition

$$\boldsymbol{A} = \boldsymbol{A}^T$$

For symmetric matrices the following holds

$$\boldsymbol{x}^T \boldsymbol{A} \boldsymbol{y} = \boldsymbol{y}^T \boldsymbol{A} \boldsymbol{x}$$

A skew-symmetric matrix is from the vector $\boldsymbol{x} = [x \; y \; z]^T$ is defined as

$$\boldsymbol{S}(\boldsymbol{x}) = \begin{bmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix} \tag{13}$$

Trigonometry
Definitions

$$\sin\theta = \frac{opposite}{hypotenuse} \qquad \cos\theta = \frac{adjacent}{hypotenuse}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{opposite}{adjacent}$$

Addition formulas

$$\begin{aligned} \sin(x \pm y) &= \sin x \cos y \pm \cos x \sin y \\ \cos(x \pm y) &= \cos x \cos y \mp \sin x \sin y \end{aligned}$$

Derivatives

$$\frac{d}{dx} \sin x = \cos x \tag{14}$$

$$\frac{d}{dx} \cos x = -\sin x \tag{15}$$