1
An Outline of Inertial Navigation

Navigation's Beginnings

Pioneers returning from their journeys provided travel instructions for those who wished to repeat their journeys. They wrote descriptions of their routes and made charts or maps pointing out landmarks and hazards like rivers and mountains on land, or shoals and rocks at sea. Map makers devised a global coordinate system using a grid of latitude and longitude circles, by which the position of any place on earth could be defined.

Buoys and lighthouses provided route markers for sailors close to land, but once they ventured into the featureless seas, their pathfinding became much more difficult. Magnetic compasses were known as early as the 11th century, but were not very accurate. The 14th- century navigator used the stars to find latitude, and by the 18th century, technology had provided instruments like the sextant for more accurately measuring the positions of the celestial bodies.

Finding longitude was more difficult, for the positions of the celestial bodies depend on the rotational position of the earth relative to the stars, that is, the time of day. Longitude is measured from an arbitrarily chosen zero at Greenwich, near London, England; to determine longitude, navigators need to know the time where they are and the time at Greenwich (Greenwich Mean Time, GMT). They can find local time from observations of the sun, but finding GMT means observing the satellites of Jupiter, or lunar distances [1], which are uniquely positioned at any time. But these observations require complicated calculations to yield time, so it is much better to have a clock set to GMT on board. It took the invention of the spring escapement clock in the mid-18th century to make accurate timekeeping available at sea. How the 18th-century mariner would envy today's $10 quartz watch, accurate beyond all his wildest dreams! Nowadays, GMT is broadcast by radio, worldwide.

This process of following a path on a map through predetermined latitudes and longitudes is called *navigation*, and the process of pointing the vehicle to follow a chosen path is called *guidance*. Navigation at sea uses the process of *deduced reckoning* (called "dead" reckoning), illustrated in Figure 1.1. If a ship starts from a known latitude and longitude and travels in a known direction for a known time, its position on the previously charted sea is known and can be

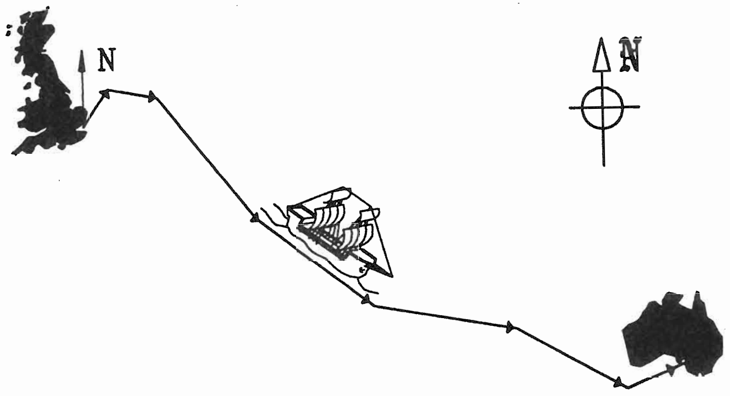

Figure 1.1. Inertial navigation is dead reckoning.

verified by sun (or other celestial body) sightings. Provided the ship's direction and speed are known accurately, the sailors will land in the correct harbor.

When aircraft began to fly over the sea, they got direction and speed from the magnetic compass and airspeed indicator. As radar evolved, the Doppler set could give more accurate ground speed. Dead reckoned courses tried to follow planned routes, corrected by celestial navigation, but airplanes fly so fast that the navigator was hard pressed to make celestial measurements quickly enough. More accurate, safer, navigation was possible when land-based radio beacons and systems like Loran and Omega became available.

By 1908, the magnetic compass had been replaced by the gyrocompass. Gyrocompasses are important marine navigation instruments, but they are not generally useful for aircraft, spacecraft and guided missiles. Because our focus is on the instruments for inertial navigators, we are not going to consider the gyrocompass in this book; they are well described in general navigation books such as Dutton's [2], by Wrigley in the *Encyclopedia Britannica* under "Gyrocompass," and by others [3,4].

Military aircraft prefer not to depend on radio beacons in time of war, for an enemy can jam them, and, conversely, aircraft can be detected by the enemy if they transmit Doppler radar speed measurement signals. They need a self-contained navigation system. If one wants to guide unmanned vehicles (such as a space probe or an intercontinental missile), one must also have a self-contained navigation system. Civil aircraft need to have reliable navigation at the lowest possible cost, and human navigators are expensive. The solution to all these needs is to automate navigation, which means that one must find a way to keep track of position and attitude continuously, in all weather conditions, in space, in the sky, and under the sea. Inertial navigation serves this purpose.

Inertial Navigation

Gyroscopes and accelerometers can provide the necessary signals for automatic navigation. Gyroscopes measure rotation, and accelerometers measure acceleration. Integrating the output from an accelerometer gives speed, and integrating speed gives distance traveled. The gyroscopes provide information on where the accelerations are directed, and therefore heading and distance are known, the essential ingredients for dead reckoning. As these instruments use the inertial properties of matter (or of light) for their operation, dead reckoning with gyros and accelerometers is called *inertial navigation*.

The first inertial navigators were used in the German V1 and V2 weapons in World War II. After the war, a group of German scientists, under Werner Von Braun, developed this technology at Redstone Arsenal, Huntsville, Alabama, for ICBMs and spacecraft, building inertial navigation systems for U.S. Army missiles.

Other U.S. groups developed IN systems, notably one under Draper at the Massachusetts Institute of Technology [5,6]. Their first aircraft inertial navigators were flown in 1949, followed in 1954 by the Navy Ship's Inertial Navigation System. The Draper Laboratory made inertial systems for Polaris submarines; one of them, the *Nautilus*, successfully navigated under the polar ice past the North Pole. In the 1960s the Apollo program took inertial guidance into space, and now inertial systems are being used in "smart" munitions. For more on the history of inertial navigation, see Wrigley's summary [7].

Maps and Reference Frames

Before we go anywhere, we need to orient ourselves in the space in which we will navigate, which we do with *maps* and *reference frames*. A map or chart is drawn to some scale so that the user can calculate distances between places. Over the distance covered by a town map we assume that the flat sheet represents a flat area of the earth, and although there may be hills and valleys in the town, the town is represented as if it were a model on a flat board. But for larger areas, the earth, being roughly a sphere, cannot be precisely represented by flat maps. Many different projections have been developed for making flat navigation maps [2], but we will leave those readers who are interested to pursue this field elsewhere.

Maps are oriented in a known direction, usually with North up the map. Navigation requires us to define frames of reference (or coordinate sets) so that we can orient ourselves in the mapped space, and in this instance "north" specifies the map's reference frame. There is a frame fixed in the "fixed stars," the *inertial frame*, that Newton postulated for his laws of inertia in 1687, and inertial navigation is navigation in this frame. This is a reference frame that is independent of the motion of the vehicle.

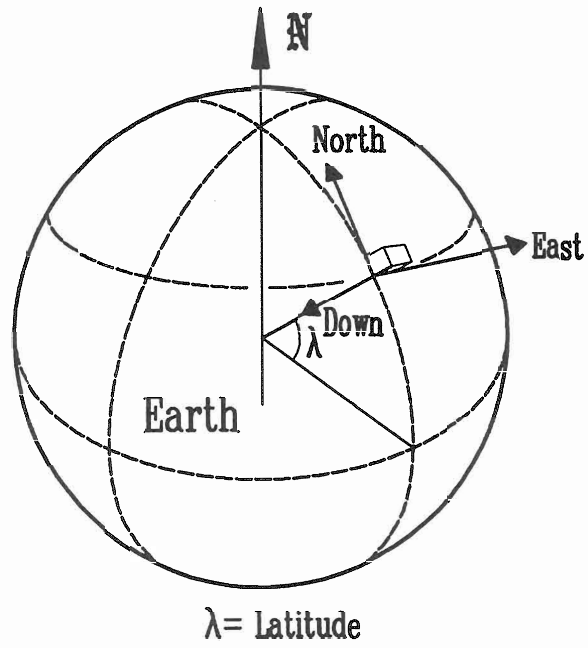

Figure 1.2. Earth-fixed axes.

On (or close to) the earth's surface it is more useful to work in a reference frame oriented to surrounding objects, a *local level* frame with North, East, and Vertically-Down as axes. In the simple (spherical homogeneous) earth local level frame, Figure 1.2, Down is a vector pointing at the center of the earth at the angle of latitude λ, North is the horizontal vector in the plane of Down and the earth's spin axis (True North), and East is the horizontal normal to that plane at the observer's site. The earth rotates once in 24 hours, so its rotation rate is 15 deg/h; the horizontal and vertical earth's rate components Ω_{eh} and Ω_{ev} in local level axes are

$$\Omega_{eh} = 15 \cos \lambda$$
$$\Omega_{ev} = 15 \sin \lambda \text{ deg/h}$$

where λ = latitude.

East is the direction in which there is no earth's rate, a fact that is used to align inertial navigation systems because it is much easier to find a null than the peak of a maximum. Gyroscopic north-finding systems find East!

Vehicles have their own axis set, Roll, Pitch, and Yaw, shown in Figure 1.3, corresponding to the conventional use of the terms.

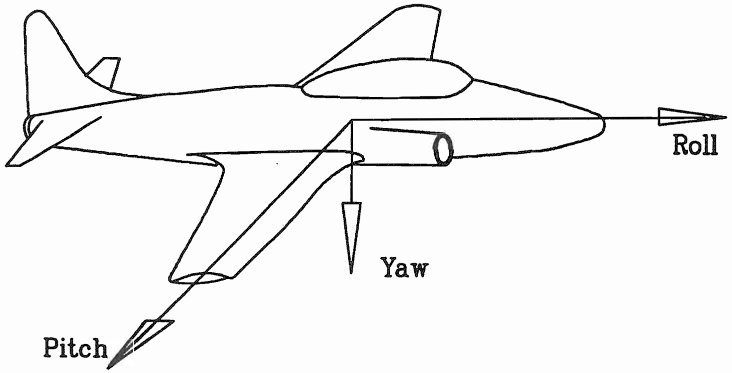

Figure 1.3. Vehicle axes set.

The Inertial Navigation Process

The inertial navigation system, INS (or unit, INU), is made from a navigation computer and a set of gyroscopes and accelerometers that measure in Newton's inertial axes, generally called *inertial sensors*. The group of inertial sensors is commonly called an *inertial measurement unit* (IMU) or an *inertial reference unit* (IRU). Once aligned to a set of reference axes (such as the "North, East, Down" set), the sensors provide distance measurements and the navigation computer carries out the continuous dead reckoning calculations.

The inertial sensors might be mounted in a set of gimbals so that they stay level and head in a fixed direction whatever the vehicle's motion. This construction is called a *navigation platform*. Alternatively, the instruments might be attached to the vehicle, in which case they measure its motion components in the vehicle axes set, and the system computes direction traveled in the reference axes by transforming the measurements from the vehicle axes to the reference axes. This is called a *strapdown* system, jargon for instruments "strapped down" to the vehicle.

To navigate inertially, we first measure the accelerations in the directions of the navigation axes, and if our instruments are not perfect, we might compensate their readings by removing biases or scale factor errors (defined in Chapter 2), perhaps known as a function of the system's measured temperature.

Second, to find the vehicle's vertical acceleration, we subtract gravity from the "Down" accelerometer output, perhaps using a gravity model to allow for the variation of gravity with latitude and longitude. The earth's rotation causes a centripetal acceleration that is greatest (about 3 m*g*) at the equator and zero at the poles. In addition, the earth is not a sphere; rotation has flattened it, causing

a mass concentration at the equator, so that the value of g varies with latitude. Precise navigators, like those in nuclear submarines, carry gravity gradiometers to correct for gravity vector errors; because of this unevenness in mass distribution, the vector direction is not necessarily along an earth's radius [8,9]. In space we can use Newton's Law of Gravitation to compute the local gravity, but once on a foreign planet, we lose accuracy until we can map its gravity fields.

Third, we integrate the accelerations over a known time, once to get the velocity, twice to get the distance traveled. As integration is the process of summing the outputs at frequent, known intervals, we must know the time interval accurately for it enters as a squared term in the distance computation.

Fourth, we measure the rotation rates either from gimbal motions in a stabilized platform, or directly with gyroscopes in a strapdown system. We then compensate for gyro bias and possibly scale factor errors, and determine a new heading.

Fifth, we compensate for earth's rotation if we are in local level axes, for otherwise the platform would be space stabilized and would seem to tilt in the vehicle axes set.

Finally, the combined distance and heading data give us an updated dead reckoned position to display. Then we go back to the beginning and do it all over again, until the end of the journey.

Inertial Platforms

An inertial platform uses gyros to maintain the accelerometers in a fixed attitude, i.e., the gimbaled platform serves to define the directions for the measurements of acceleration. A single-axis platform, Figure 1.4, consists of a gyro mounted so that its sensing axis or *input axis* (IA) is along the axis of the platform, which is set into the vehicle in bearings. Electrical power for the gyro comes in, and the gyro's output goes out, through slip rings. The platform can be driven around its axis by an electric torque motor. The gyro output, which indicates an unwanted platform rotation, drives the torque motor through a servo amplifier; it basically provides the torque to overcome the friction in the slip rings and bearings.

If the platform had no gyro and the vehicle rotated around the platform axis, the platform's inertia would tend to keep the platform aligned in space. But the platform would slowly accelerate due to the torque transmitted through the bearings and slip rings, and the inertial reference would be lost. By adding the gyro, we provide a means of sensing any platform rotation with respect to inertial space, down to the limit of the gyro's resolution. The gyro does not measure anything in the inertial platform; it is only used to maintain a fixed position (null operation), and so does not need to be able to measure large rotation rates.

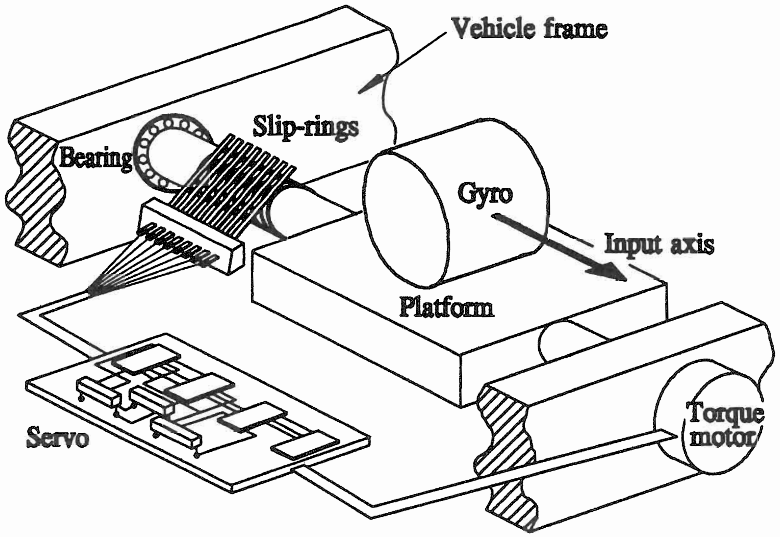

Figure 1.4. A single-axis platform.

For navigation in three dimensions we must expand our platform to three axes by adding gimbals, Figure 1.5. Here we have shown gyros with two sensing axes each (*two degree of freedom* gyros) and also schematically show a group of three accelerometers. We have also provided a *pickoff* on each gimbal axis, a device that measures the angle between two gimbals; the torquers, slip rings, and three servo loops are there but are not shown. If the gimbals are aligned with the vehicle axes before setting out, then the gimbals, being gyro stabilized, define the attitude and heading of the vehicle during the journey.

The gyros alone will try to maintain the platform aligned in inertial space. If the platform is operating in local level coordinates, the navigation computer must keep the platform horizontal. It does this by sending command signals to the gyros which otherwise would fight the gimbal motion. The roll and pitch accelerometers can be used to level the platform if we know that it is not accelerating. The relationship between the sensors, the gimbals and the navigation computer is shown schematically in Figure 1.6.

Heading and Attitude Reference Systems

Rather than use an expensive inertial navigator, many aircraft use a simplified version that indicates the direction of flight (the heading) and the attitude in roll and pitch. This Heading and Attitude Reference System (HARS) is a platform with three stabilized axes and two horizontal accelerometers. The accelerometers

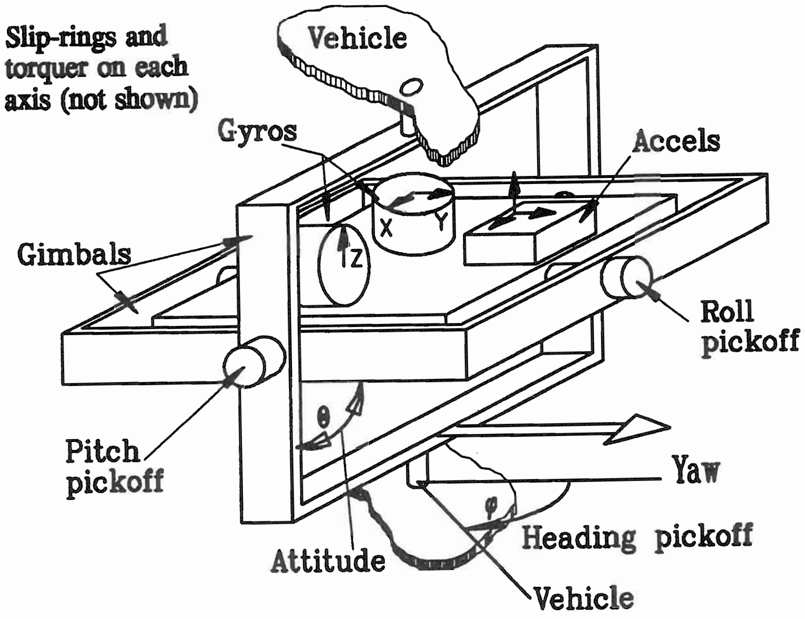

Figure 1.5. A three-axis platform.

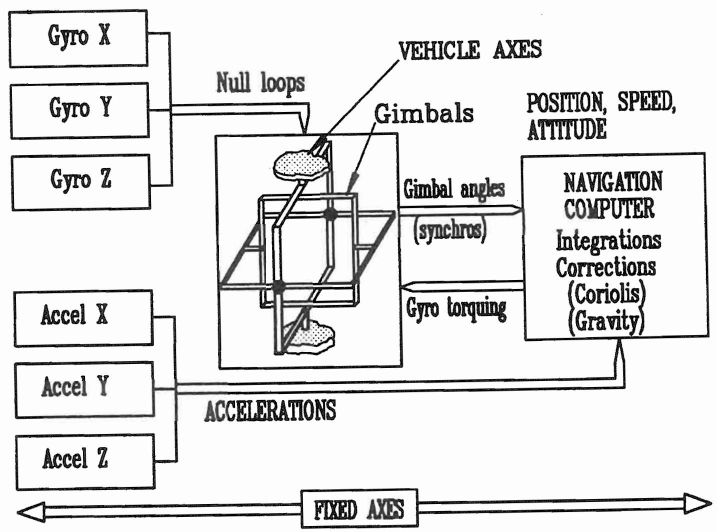

Figure 1.6. The basic platform system.

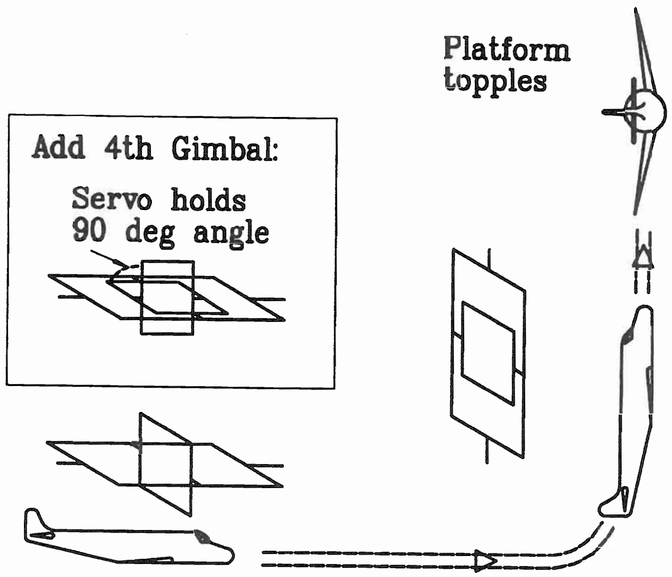

Figure 1.7. Gimbal lock.

provide a vertical reference to set the initial level before takeoff, whereas a flux valve (a kind of magnetic compass) can be used to set the magnetic heading. This heading can be corrected to True North using the local value of the magnetic variation. Alternatively, the heading can be set by gyrocompassing (page 15). Once under way, the HARS provides flight information under all visibility conditions.

Schuler Tuning

Imagine that we have a pendulum hanging in the vehicle that we are navigating, intended to provide a vertical reference. But as we accelerate, the pendulum tilts, giving a false vertical indication. Schuler showed that this would not occur with a pendulum of 84 minutes period, and we can make a compact compound pendulum with this period using an ordinary accelerometer and a servo loop with the correct characteristics. Correcting an inertial system so that it does not tilt when accelerated is known as *Schuler tuning* it.

Gimbal Lock

Three axis platforms have a limitation that can cause them to "lock up" during aerobatic maneuvers. This *gimbal lock* can be caused by a set of aircraft movements that cause two of the gimbal axes to become aligned, followed by a

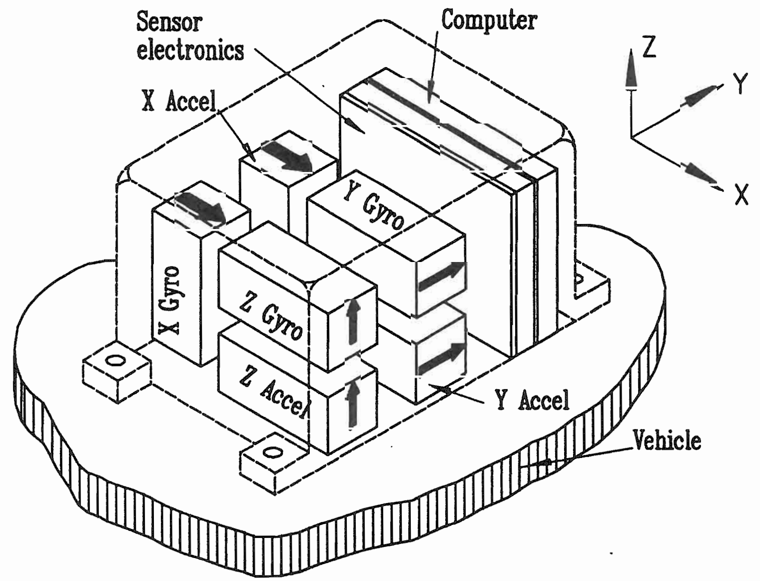

Figure 1.8. The strapdown system

rotation in the plane of the aligned axes. How this happens is much easier to see than to read, so Figure 1.7 shows a sequence that tumbles the platform. A plane flying straight and level climbs vertically. Now the roll gimbal is aligned with the pitch gimbal, and a turn over the wing, around the aircraft yaw axis, forces the pitch gimbal over and makes the system lose alignment. A fourth gimbal [10], a duplicate roll gimbal, can be added to overcome this limitation, although it adds considerably to the system's cost and size.

For further reading, books by Markey and Hovorka [11] and Savant [12] describe inertial systems in more detail.

Strapdown Systems

The platform system's complex mechanical construction goes against the trend in machine design. Since the 1960s, technical advances in electronics and optics have reduced their size and cost, while there has been little progress in reducing the cost of mechanisms, apart from robotic fabrication. Generally, lower costs and higher reliability have come from replacing mechanisms with electronics.

Navigation systems have benefitted from this technological culture; the strapdown system is the outcome. The strapdown system replaces gimbals with a computer that simulates their presence electronically. In the strapdown system the gyroscopes and accelerometers are rigidly mounted to the vehicle structure so that they move with the vehicle, as shown in Figure 1.8. Now, unlike the platform gyros, the strapdown gyros must **measure** the angles turned, up to the maximum rotation rate expected. Airplanes can experience short-term rates up to 400 deg/s, so a gyro with 0.01 deg/h performance has a dynamic range of 10^8 - two orders above the platform gyro.

As the vehicle travels, the gyros measure the yaw, pitch, and roll angles turned in a short time (say, 0.01 s), and pass them to a computer that uses them to resolve the accelerometer outputs into the navigation axes set. Simplifying to two dimensions, we can easily express the coordinate transformation between the navigation axis set (x_{nav}, y_{nav}) and the body axis set (x_{body}, y_{body}), at a time when the z-axis gyro has measured that there is an angle θ between them. The accelerometers fixed in the body axes record accelerations a_x, a_y. Converting to navigation axes we get

$$(a_x)_{nav} = a_x \cos\theta - a_y \sin\theta \tag{1.1}$$

$$(a_y)_{nav} = a_x \sin\theta + a_y \cos\theta$$

When adding the third (z) axis we must use more complicated transformations as the rotations are non-commutative. Typical systems use direction cosines, and the transformation between the inertial set [X_i, Y_i, Z_i] and the body axes set [X_b, Y_b, Z_b] is expressed as follows:

$$r^i = \begin{vmatrix} r_x \\ r_y \\ r_z \end{vmatrix} = C_b^i r^b$$

$$C_b^i = \begin{vmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{vmatrix}$$

where c_{ij} are the direction cosines between the j^{th} axis in the inertial frame and the k^{th} axis in the body frame; Britting [13] describes the necessary computations. The sensors and computer in a strapdown system are arranged as shown schematically in Figure 1.9.

System Alignment

The first step in inertial navigation is aligning the platform to the navigation axes or, equivalently, initializing the axes in the strapdown system's computer. We can level the platform or measure the attitude of the vehicle for a strapdown system, using accelerometers. We can find North using a magnetic compass (which is not very accurate), by astronomical sightings (which means we must be able to see stars), or by *gyrocompassing*. In some cases, such as that of missiles suspended under the wing of a fighter plane, we can align the missiles' strapdown systems

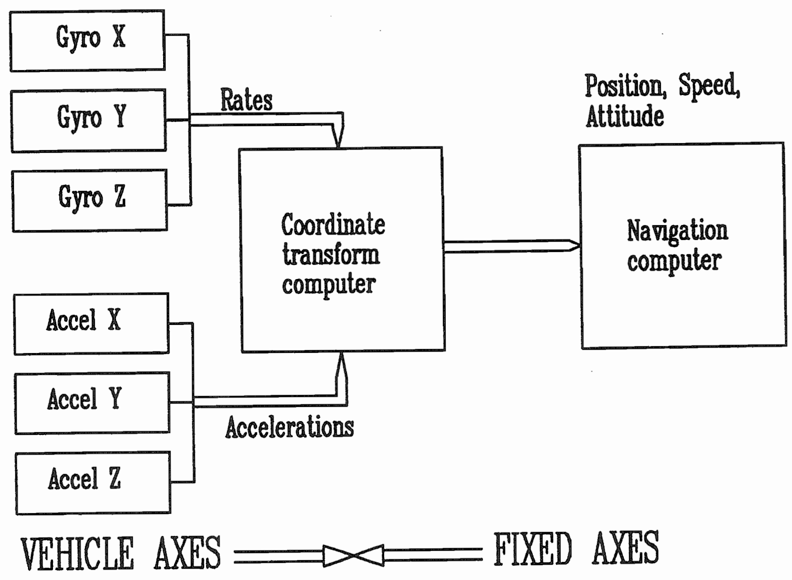

Figure 1.9. The basic strapdown system.

by referring their outputs to those of a master navigator in the fighter, called *transfer alignment*.

Gyrocompassing

To gyrocompass with a platform, a gyro with its input axis horizontal is connected by a servo to a platform torquer that will rotate the gyro about the vertical (yaw) axis. The servo is designed to drive the sensed rotation to zero, so the gimbal will swing around until the gyro's axis points East. But if the gyro has a bias - an output signal when there is no rotation input - it will point a little off East, enough so that the earth's rotation rate component cancels the bias. Then the platform is slewed 180°, and again servoed to null. This time the bias has not changed sign but the earth's rotation component has, so, by combining the East and West readings of platform heading, both North and the gyro bias can be deduced, a big advantage for the platform system. (In passing, note that some systems rotate continuously, the "rotating azimuth" mechanization, so that instrument biases are modulated at the rotation rate and filtered out. This improves performance for given instrument errors and alignment accuracy.)

A strapdown system cannot rotate the gyros except by swinging the entire vehicle, usually impossible to do over the half-circle necessary for East-West gyrocompassing. All one can do is measure the three components of earth's rotation with the gyros and compute where North must be, assuming that the gyro bias has not changed since it was last measured. To find North to an accudirections (usually orthogonal) in the vehicle, they can be placed so that they best use the space available.

2. Ruggedness. The simpler structure better withstands shock and vibration, and, being lighter, is easier to shock mount than a platform.
3. Reliability. There are no gimbal magnetics, no slip rings, and no bearings. The electronics that replace them are inherently more reliable.

Disadvantages:

1. Alignment. Strapdown systems are difficult to align because they cannot be easily moved. Transfer alignment is suitable for tactical systems.
2. Sensor calibration. Again, the immobility means that the sensors cannot be calibrated in the system. Therefore, they must be stable, a burden on the sensor design. Strapdown systems rely on sensor models, using real-time compensation of inertial errors and thermal effects.
3. Motion induced errors. The body motions induce unique sensor errors (torquer errors, anisoinertia, output axis angular acceleration), which can be compensated to some degree.
4. Accelerometer errors. Bias errors accumulate, and strapdown accelerometers may be subjected to components of gravity as the vehicle rolls and pitches, reducing the accuracy of the vehicle acceleration measurement and exciting cross-axis errors.
5. The strapdown computer. Not needed in the platform system, the computer must be fast enough to do all the strapdown calculations in a few milliseconds. In a typical tactical system, a bandwidth of 100 Hz demands that sensor compensation and coordinate transformation must be done in less than 0.01 s. This requires well-crafted program code.

The particular benefits of strapdown systems are well illustrated in the following story. One of the first strapdown systems fielded was used in the Apollo Lunar Module (LM) moon lander, as a backup to the primary platform system. The Apollo 13 spaceship had just left the earth when a fuel cell in the main ship (the service module) exploded, drastically reducing the amount of electrical power available. The ship was committed to a trajectory to the moon; it could not change direction much once in space. Mission Control decided to let the ship pass around the moon and come back to earth without landing on the moon. Because of the shortage of electrical power, the astronauts switched off the command module and used the LM. Everything possible was switched off, so that the three astronauts could stay warm enough; even so, the cabin got unpleasantly cold, down to 4°C (40°F). The platform navigator was turned off, and the crew relied entirely on the lower-power strapdown system, operating below its design temperature, for navigation around the moon and back to earth. They then left the LM and used the command module's navigator through re-entry to a safe landing [14,15].

Star Trackers

In applications where a missile may not have time for an accurate ground alignment before firing, or in cases where flight time is very long, the guidance system may be provided with a star tracker to update its alignment. This is a type of telescope with an optical detector at its focus, which can be precisely pointed at a spot in the heavens where a star is known to be. Once flying high enough that the atmospheric turbulence, smoke, and haze do not obscure the view, the tracker locks onto the pre-selected star and uses the star's known position to correct for misalignment in the inertial navigator.

As star trackers are expensive and require a window in the vehicle, they tend to be used only in long range missiles, reconnaissance planes, and long range bombers. They also need a clear view of the sky. Radio systems are all-weather navigation aids. Omega gives 2-4-nautical mile position accuracy but no velocity information, whereas satellite systems provide 15-m position accuracy and velocity data.

Satellite systems have the advantage over ground-based transmitters that they are radiating almost vertically down, so that satellite signals are much less affected by hills and the curvature of the earth. The newest United States satellite system is the Navstar *Global Positioning System* (GPS) and is accompanied by the similar Russian GLONASS system.

The Global Positioning System

The Navstar Global Positioning System (GPS) is a space-based position and navigation system that can provide three-dimensional position to an accuracy a few meters, anywhere on or near the earth, to those with the proper receivers. Although it is primarily a military system and provides the highest accuracy only to those who know its secret codes, it provides a lower accuracy (15-30 m) signal to all users. The precise encrypted "P" code and the coarser clear/acquisition "C/A" signal are transmitted on separate frequencies.

When it is complete, GPS will consist of 18 satellites, 3 each in 6 orbital planes. They will orbit at a height of 20,200 km (10,900 nautical miles) with a 12-h period; the orbits are inclined at 55°, as illustrated in Figure 1.11. The satellites already in orbit are demonstrating that the system will easily meet its design objectives.

In operation, each satellite broadcasts both the P and C/A signals at precisely known times, timed by each satellite's on-board atomic clock. Each receiver has a clock, too, so it can measure the time at which it receives the signal from the satellite. As the speed at which the radio wave travels is known, the distance between the satellite and the receiver is found by multiplying that speed by the time difference between satellite transmission and user reception. Each satellite's

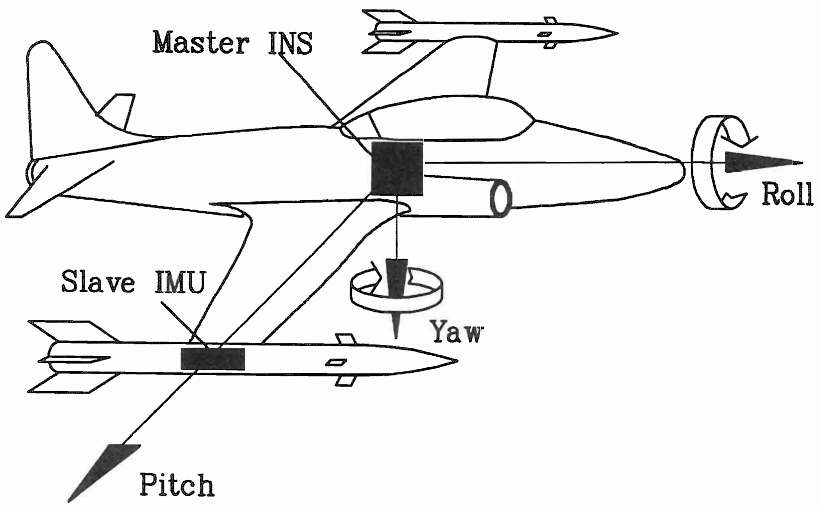

Figure 1.10. Transfer alignment.

racy of 1 mr, one must measure one thousandth of the horizontal earth's rate. At Boston, Massachusetts, for example, HER = 10 deg/h, so for 1 mr alignment accuracy we must know the gyro bias to 0.01 deg/h, a demanding requirement met only in expensive gyros.

Transfer Alignment

Another way to align a strapdown system is to transfer alignment from a master system. A fighter aircraft can initialize wing-carried missiles, for example, by carrying out maneuvers designed to allow the fighter's master navigator to send alignment information to the missile, as shown in Figure 1.10. If the fighter flies straight and level, the master and slave velocity outputs can be matched by adjusting the accelerometer bias, and controlled rolls or turns can calibrate the slave system's gyros.

Advantages and Disadvantages of Platform Systems

Advantages:

1. Simpler gyros. Because the sensor platform rotates only at the small rates needed to keep it level, the gyros need only a small dynamic range. A maximum rate of 3 deg/s would suffice for a gyro of 0.01 deg/h performance (e.g., for an aircraft navigator), a range of 10^6. Further, gyro torquer errors do not lead to attitude error. The lack of gyro rotations means that there are no anisoinertia and output axis angular acceleration errors to minimize in the design.
2. Higher accuracy. Because the accelerometer axes are always well defined, the platform navigator can be very accurate; the North and East accelerometers see no component of gravity and measure only the vehicle accelerations. The vertical accelerometer, though, measures the vehicle's vertical motion in the presence of $1g$, therefore less accurately. In an aircraft this causes altitude errors, which can be compensated with a barometer signal.
3. Self-alignment by gyrocompassing.
4. Sensor calibration by platform rotations. The other sensor biases are obtained by orienting the platform with each major axis vertical in turn, provided there is enough time.

Disadvantages:

1. Complexity and cost. The gimbal structure and its bearings must be stiff so that the accelerometer axes remain defined even under vehicle vibrations, but the bearings and slip rings must have as little friction as possible. As a result, the gimbal structure is an elaborate, precisely made mechanism which cannot avoid being expensive.
2. Gimbal magnetics. Each gimbal must have a pickoff and torquer. The pickoffs (synchros) measure the intergimbal angles with arc-sec resolution over a full revolution, a range of 10^6. When the system is first switched on, the torquers need to provide enough torque to accelerate the platform inertia so that it can gyrocompass and align itself to Level and North in a reasonable time. The torquers also must not leak magnetic flux, for that could upset the sensors as the gimbals move around them.
3. Reliability. The bearings and slip rings tend to wear, degrading alignment and performance.

Advantages and Disadvantages of Strapdown Systems

Advantages:

1. Simple structure, low cost. Strapdown systems are lighter, simpler, cheaper, and easily configured for odd-shaped spaces. As it is only necessary to mount the sensors so that their sensing axes point in known

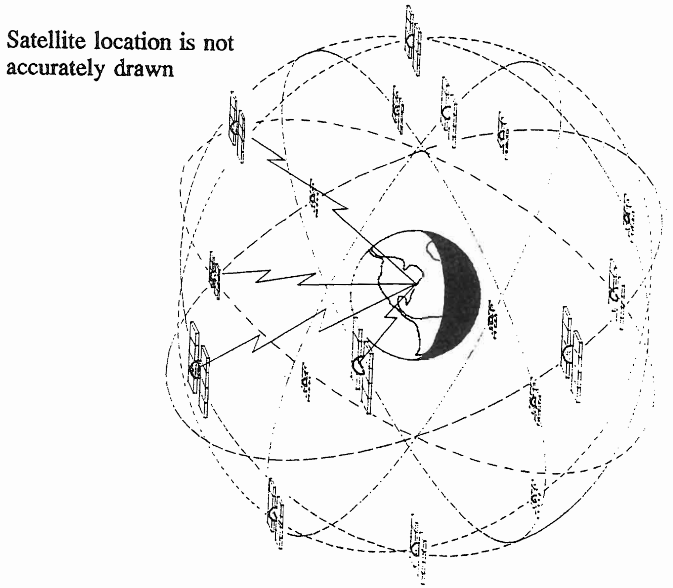

Figure 1.11. The Global Positioning System satellites.

position is known, so the user now knows position along the line of sight to the satellite. Users take data from three satellites at different angles in the sky to fix their position in three dimensions.

Now the user's clock might not be very accurate, and it is to everyone's advantage to allow it to be inexpensive, so each satellite also broadcasts the precise time in its message. As we now need a separate piece of data to decode time, we read data from a fourth satellite and compute position and accurate time. The system has been designed so that four satellites will always be visible from anywhere on the earth.

The satellites' positions can drift slightly in orbit, from irregularities in gravity, from solar pressure and so on, so fixed ground stations observe them and send orbit data to a master control center in Colorado Springs. From there a message is sent to each satellite, telling it how much it is deviating from its predicted orbit, and providing small time corrections to its clock. The satellite incorporates these correction messages (*ephemeris* data) into its transmissions. All of this is naturally done in a computer in the user's receiver, and some companies make receivers small enough to carry in the hand [16,17]. It is possible to make a receiver for both GPS and GLONASS [18,19]. The C/A code is used as a coarse acquisition signal for those privileged with access to the P code, but it is the only signal available to commercial users. However, the C/A code gives 30 m accuracy, so it is still a valuable aid.

GPS will allow more accurate navigation, but will existing charts and maps be accurate enough to realize the benefit? Roeber [20] thinks not. However,

GPS can itself be used for map making and geodetic surveys. It will be used on test ranges to check out inertial navigation systems and will facilitate air traffic control and search and rescue missions. It will change the inertial navigation field for all except strategic weapon guidance, for such a device is exactly what an inertial navigation system needs for a partner.

The INS gives very accurate attitude and distance measurements for a short time, but gyro and accelerometer errors accumulate after a while and navigation errors grow. The GPS system has exactly the opposite characteristic, for it can give accurate position fixes periodically to correct the INS, but cannot tell you quickly if you're turning or rolling; the two together complement one another [21]. Should the vehicle lose the satellite signals for a while, perhaps because the vehicle has banked and a wing hides the antenna from the satellite or because a ground vehicle has entered a tunnel, the INS can carry on until the receiver re-acquires the satellite signal. For military, use the INS can keep a missile on target even if the GPS signal is obliterated by powerful jammers.

Applications of Inertial Navigation

The most accurate systems, with heading error less than 0.001 deg/h (one revolution in 40 years!) are found in submarines and ICBMs, which need navigation with errors measured in tens of feet over long mission times. We might call these "1 mile a day" systems. Their accelerometers resolve gravity to a few μg while measuring vehicle accelerations of the order of $10g$, a dynamic range of 10^7.

Next, aircraft systems, with errors of less than 1 mile an hour, are found in all civil aircraft that cross the oceans, and in long range military aircraft. While all the 1-mile-a-day systems are platform systems using mechanical sensors, laser gyro strapdown systems are dominating the 1-mile-an-hour market now. Less expensive heading and attitude reference systems (HARS) are used in short range military aircraft, and, frequently, in long range planes, to back up the INS. HARS often use mechanical gyros, but laser gyro versions are appearing as laser gyros get cheaper.

Tactical weapons such as short range missiles, Harpoon, Tomahawk, and Phoenix being U.S. examples, exclusively use strapdown systems. They presently use only mechanical sensors, but that will change as the new optical (laser) gyros and micromachined accelerometers complete development. These are described in later chapters.

Conclusions

For a deeper study of inertial systems, the books by Broxmeyer [22], Draper et al. [23], Leondes [24], McClure [25], O'Donnell [26], and Parvin [27] could be consulted. The guidance of spacecraft uses special techniques, described by Hynoff [28].

GPS will have a tremendous impact on the navigation field, for GPS and strapdown systems go together well. The strapdown processor can support the GPS, and the INS can tell the GPS receiver where to look for satellites and maintain navigation while satellites are obscured.

There is one very important exception, though, and that is the guidance of strategic weapons (including aircraft and submarines). In a war serious enough for strategic weapons to be used, one could not rely on the constellation of GPS satellites to survive. They would either be destroyed or their signals would be jammed. So strategic systems must remain self-contained (perhaps aided with a star-tracker), and the accuracy demanded of these systems is difficult to meet with strapdown gyros.

In the 1960s, when inertial navigation systems began to be used, their instruments were electro-mechanical and very sophisticated; they were expensive, large, and fairly fragile [29]. In the 1980s the gyros were more likely to be optical, using lasers; they were less expensive, smaller, and more rugged. In the 1990s gyros and accelerometers will continue to get cheaper, smaller, and more robust, based on technologies described in the later chapters of this book.

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