TERM PROJECT FOR TEK4040 MATHEMATICAL MODELLING OF DYNAMIC SYSTEMS

Autum 2020

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This project is for those of you that have not got a personal project approved by me. You may use MatLab or another program to do the simulations. The purpose of the project is to demonstrate the rotation of a rigid body.

The paper must consist of a report describing the mathematical equations and a listing of the programs used in the simulations and the figures. In addition you must send me the three animation files.

You are suppost to work approximately one week on the project, but this will depend on how experienced programmer you are.

1. Calculate the inertial matrix, J^b, for a brick with dimensions 5, 10 and 20 cm (along the x-, y- and z-axis) when the density is 2 kg/dm^3. Place the b-frame in the center of mass with axis parallell to the edges.
2. Define and find the kinetic and angular momentum ellipsoids. Draw the ellipsoids with a common center (3D, opaque and with two different colours) for three cases which give intersection lines near the three axis (the kinetic energy ellipsoid is constant). See the attachment for how to choose the three cases.
3. Solve the Euler equations numerically for the same three cases (with initial values on the intersection lines). Draw the solutions in the same figures as above.
4. Write the kinematic equations for 3-2-1 Euler angles and solve the kinematic and angular momuntum equations numerically for the three cases above. Make three animations showing how the brick rotates as seen from inertial space.

If you have any questions you may phone (92494675) or mail me (anders.rodningsby@ffi.no).

Deadline for the project is monday November 30 (you may mail me the report and animation les, but ask me for a receipt since **only the students who get the project approved can take the exam**)

Attachment: How to choose tangular velocity near the main axis

Attachment: How to choose angular velocity near the main axis

We want the body to rotate near the main axes $\vec{b}_3$ with a periode of $T = 1$ s:

$$\begin{aligned} \tilde{\omega}_3 &= \frac{2\pi}{T} = 2\pi && (1) \\ \underline{\omega}_b^{\boldsymbol{ib}} &= [0; 0; \tilde{\omega}_3] && (2) \end{aligned}$$

This gives a kinetic energy of

$$K_0 = \frac{1}{2} J_{33} \tilde{\omega}_3^2 = 2\pi^2 J_{33} \tag{3}$$

In a similar fashion we may calculate the angular velocity which gives the same kinetic energy about the main axis $\vec{b}_1$ and $\vec{b}_2$:

$$\begin{aligned} \tilde{\omega}_1^2 &= 2\frac{K_0}{J_{11}} && (4) \\ \tilde{\omega}_2^2 &= 2\frac{K_0}{J_{22}} && (5) \end{aligned}$$

In order to calculate the angular velocity $\underline{\omega}_b^{\boldsymbol{ib}}$ near the main axes $\vec{b}_1$we look at the equation of the kinetic energy ellipsoid in the plane defined by the $\vec{b}_1$ and $\vec{b}_3$ axis:

$$\frac{\omega_1^2}{2\frac{K_0}{J_{11}}} + \frac{0^2}{2\frac{K_0}{J_{22}}} + \frac{\omega_3^2}{2\frac{K_0}{J_{33}}} = 1 \tag{6}$$

or expressed by pure axes rotation

$$\begin{aligned} \frac{\omega_1^2}{\tilde{\omega}_1^2} + \frac{\omega_3^2}{\tilde{\omega}_3^2} &= 1 && (7) \\ \omega_1 &= \tilde{\omega}_1 \sqrt{1 - \frac{\omega_3^2}{\tilde{\omega}_3^2}} && (8) \end{aligned}$$

By choosing $\omega_3 = \tilde{\omega}_3/10$ we can calculate the corresponding ω_1 which gives $\underline{\omega}_b^{\boldsymbol{ib}} = [\omega_1; 0; \omega_3]$. This angular velocity is "near" the main axes $\vec{b}_1$.

The angular velocities "near" $\vec{b}_2$ and $\vec{b}_3$ may be calculated in similar fashions.