F15 N.2.

Questions Friday 11. dec
13.15 - 16.00

From Part B:

$$\text{②} \quad \underline{f}^b = m\,\dot{\underline{v}}^{ibb} + m\,S(\underline{w}_b^i)\,\underline{v}^{ib}$$

Write out eq. ② and compare with (13-73)

$$\begin{bmatrix} f_x^b \\ f_y^b \\ f_z^b \end{bmatrix} = m \begin{bmatrix} \dot{V}_x^{ibb} \\ \dot{V}_y^{ibb} \\ \dot{V}_z^{ibb} \end{bmatrix} + m \begin{bmatrix} 0 & -W_z & W_y \\ W_z & 0 & -W_x \\ -W_y & W_x & 0 \end{bmatrix} \begin{bmatrix} V_x^{ib} \\ V_y^{ib} \\ V_z^{ib} \end{bmatrix}$$

$$\begin{bmatrix} f_x^b \\ f_y^b \\ f_z^b \end{bmatrix} = m \begin{bmatrix} \dot{V}_x + W_y V_z - W_z V_y \\ \dot{V}_y + W_z V_x - W_x V_z \\ \dot{V}_z + W_x V_y - W_y V_x \end{bmatrix}$$

$$\underline{w}_b^{ib} = [P, Q, R]^T, \quad \underline{v}^{ib} = [U, V, W]^T$$

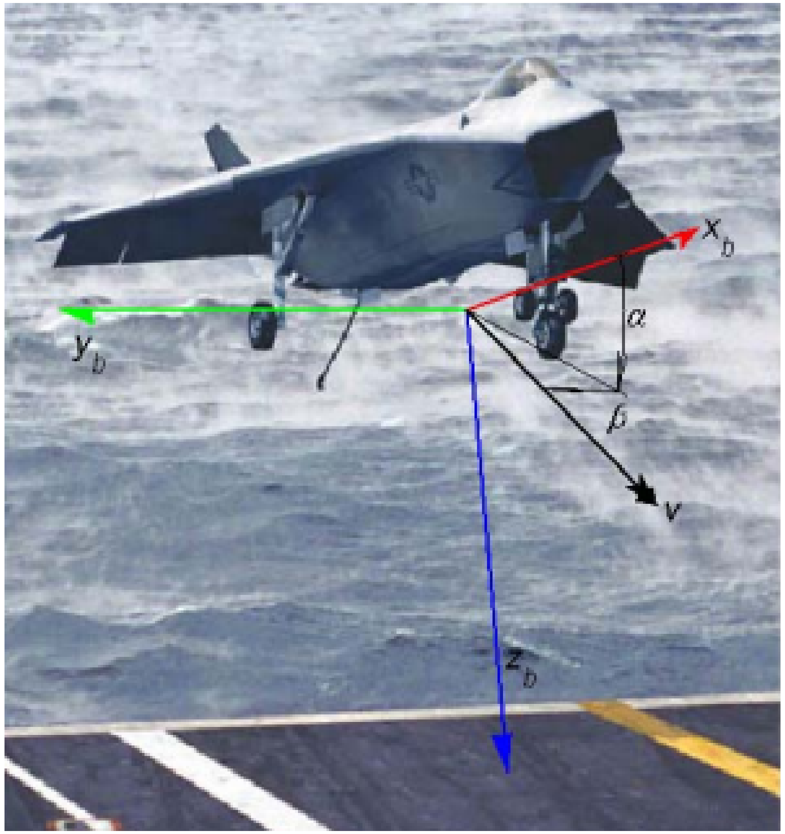
$$\underline{f}^b = [F_x + T_x + g_x,\; F_y + g_y,\; F_z + g_z]^T$$

F_x, F_y, F_z : aerodynamic forces

T_x : thrust from engine

g_x, g_y, g_z : gravity comp. in $\mathcal{F}^b$

$$\underline{g}^b = \begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix} = R_i^b \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} \underline{g}^i, \quad g = 9.81\,m/s^2$$



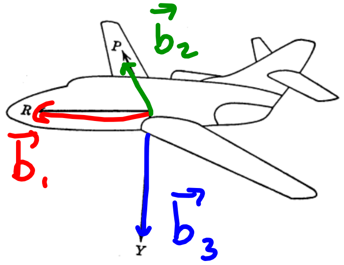


Figure 3.2 Body frame.

P : pitch

R : roll

Y : yaw

α: angle of attac

β: sideslip angle

V_m: speed

From the figure and $\underline{v}^{ib} = [U, V, W]$

$$V_m = \sqrt{U^2 + V^2 + W^2} = \|\underline{v}^{ib}\|$$

$$\alpha = \arctan\left(\frac{W}{U}\right)$$

$$\beta = \arcsin\left(\frac{V}{V_m}\right)$$

We have a uniqe relation between $\underline{v}^{ib}$ and V_m, α, β

We will write N.2 with V_m, α, β, and the law of angular momentum with orientation / attitude given by 3-2-1 Euler angles $\underline{\Theta} = [\phi, \theta, \psi]$

$$\dot{V}_m = \frac{1}{m}\{\cos\alpha\cos\beta(F_x + g_x + T_x) + \sin\beta(F_y + g_y) + \sin\alpha\cos\beta(F_z + g_z)\}$$

$$\dot{\alpha} = Q + \frac{1}{V_m\cos\beta}\{-PV_m\cos\alpha\sin\beta - RV_m\sin\alpha\sin\beta - \sin\alpha(F_x + g_x + T_x) + \cos\alpha(F_z + g_z)\} \qquad (13\text{-}74)$$

$$\dot{\beta} = P\sin\alpha - R\cos\alpha + \frac{1}{mV_m}\{-\cos\alpha\sin\beta(F_x + g_x + T_x) + \cos\beta(F_y + g_y) - \sin\alpha\sin\beta(F_z + g_z)\}$$

○: aero. dyn. forces

Newtons 2. law (N.2) ②

$$\dot{\Theta} = \cos\Phi Q - \sin\Phi R$$
$$\dot{\Phi} = P + \sin\Phi\tan\Theta Q + \cos\Phi\tan\Theta R$$
$$\dot{\Psi} = \sin\Phi\sec\Theta Q + \cos\Phi\sec\Theta R$$

Derivative of the Euler angles, $\underline{\Theta} = [\Phi, \Theta, \Psi]$

$\dot{\underline{\Theta}} = D_b^a(\underline{\Theta})\, \underline{w}_b^{ib}$ (A-105)

$$\dot{P} = -\frac{I_{zz} - I_{yy}}{I_{xx}} QR + \frac{L}{I_{xx}}$$
$$\dot{Q} = -\frac{I_{xx} - I_{zz}}{I_{yy}} PR + \frac{M}{I_{yy}}$$
$$\dot{R} = -\frac{I_{yy} - I_{xx}}{I_{zz}} PQ + \frac{N}{I_{zz}}$$

Law of angular momentum ①

$\underline{n}_c^b = [L, M, N]$

$\underline{F}^b = [F_x, F_y, F_z]$

Aero. dyn. forces rep. in $\mathcal{F}^b$

T_x: thrust

The aero dynamical forces depends on speed, angle of attac, sideslip angle and fin control deflections. $M_m = \frac{V_m}{V_0}$, where V_0 is speed of sound.

$$F_x = k_F \rho V_m^2 C_x$$
$$F_y = k_F \rho V_m^2 C_y$$
$$F_z = k_F \rho V_m^2 C_z$$
$$L = k_M \rho V_m^2 C_l$$
$$M = k_M \rho V_m^2 C_m$$
$$N = k_M \rho V_m^2 C_n$$

Measured in wind tunnels

$\underline{x} = [V_m, \alpha, \beta, \phi, \theta, \psi, P, Q, R]^T$

$\underline{u}: [\delta_r, \delta_e, \delta_a, T_x]$

ρ: atmospherical density

k_F, k_M: constans depending on the vehical geometry

$$C_x = C_{x0}(\alpha, \beta, M_m) + C_{x\delta_e}(\alpha, \delta_e, M_m) + C_{x\delta_a}(\alpha, \delta_a, M_m) + C_{x\delta_r}(\alpha, \delta_r, M_m)$$
$$C_y = C_{y0}(\alpha, \beta, M_m) + C_{y\delta_e}(\alpha, \delta_e, M_m) + C_{y\delta_a}(\alpha, \delta_a, M_m) + C_{y\delta_r}(\alpha, \delta_r, M_m)$$
$$C_z = C_{z0}(\alpha, \beta, M_m) + C_{z\delta_e}(\alpha, \delta_e, M_m) + C_{z\delta_a}(\alpha, \delta_a, M_m) + C_{z\delta_r}(\alpha, \delta_r, M_m)$$
$$C_l = C_{l0}(\alpha, \beta, M_m) + C_{l\delta_e}(\alpha, \delta_e, M_m) + C_{l\delta_a}(\alpha, \delta_a, M_m) + C_{l\delta_r}(\alpha, \delta_r, M_m)$$
$$C_m = C_{m0}(\alpha, \beta, M_m) + C_{m\delta_e}(\alpha, \delta_e, M_m) + C_{m\delta_a}(\alpha, \delta_a, M_m) + C_{m\delta_r}(\alpha, \delta_r, M_m)$$
$$C_n = C_{n0}(\alpha, \beta, M_m) + C_{n\delta_e}(\alpha, \delta_e, M_m) + C_{n\delta_a}(\alpha, \delta_a, M_m) + C_{n\delta_r}(\alpha, \delta_r, M_m)$$

δ_r : rudder (yaw)

δ_e : elevator (pitch)

δ_a : aileron (roll)

The equations can be written as:

$$\dot{\underline{x}} = \underline{f}(\underline{x}, \underline{u})$$

I.e. a non-linear and determanistic equations.

If we want to create an autopilot that force the vehicle to follow a given trajectory we expand the state vector with the vehicle position in $\mathcal{F}^i$. i.e.

$$\underline{x} := [\underline{x}, \underline{p}^i]$$

If we want to follow a trajectory with a spesific velocity.

Given $\tilde{\underline{p}}(t)$ and $\tilde{\underline{v}}(t)$

$$\partial \underline{p} = \underline{p}(t) - \tilde{\underline{p}}(t) \quad , \quad \partial \underline{v} = \underline{v}(t) - \tilde{\underline{v}}(t)$$

We desire an autopilot that forces:

$$\partial \underline{p} \to 0 \,, \; \partial \underline{v} \to 0$$

Create a nominal solution:

$$\dot{\tilde{\underline{x}}} = \underline{f}(\tilde{\underline{x}}, \tilde{\underline{u}})$$

Linearize around $\tilde{\underline{x}}$ and $\tilde{\underline{u}}$:

$$\partial \underline{x} = \underline{x} - \tilde{\underline{x}} \quad , \quad \partial \underline{u} = \underline{u} - \tilde{\underline{u}}$$

$$\dot{\underline{x}} = \underline{f}(\underline{x}, \underline{u})$$

$$(\dot{\tilde{\underline{x}}} + \partial \dot{\underline{x}}) = \underline{f}(\tilde{\underline{x}} + \partial \underline{x}, \tilde{\underline{u}} + \partial \underline{u})$$

$$\dot{\tilde{\underline{x}}} + \partial \dot{\underline{x}} = f(\tilde{\underline{x}}, \tilde{\underline{u}}) + \underbrace{\left.\frac{\partial \underline{f}}{\partial \underline{x}^T}\right|_{\tilde{x},\tilde{u}}}_{F(\tilde{x},\tilde{u})} \cdot \partial \underline{x} + \underbrace{\left.\frac{\partial \underline{f}}{\partial \underline{u}^T}\right|_{\tilde{x},\hat{u}}}_{L(\tilde{x},\tilde{u})} \partial \underline{u} + h.o.t$$

I order approximation:

$$\partial \dot{\underline{x}} = F(\tilde{\underline{x}}, \tilde{\underline{u}}) \partial \underline{x} + L(\tilde{\underline{x}}, \tilde{\underline{u}}) \partial \underline{u}$$

Regulation problem:

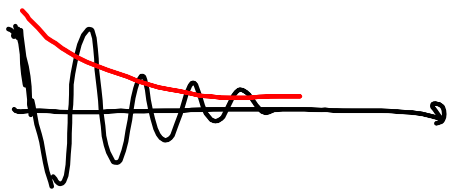
Designe a feedback that forces $\partial \underline{x} \to 0$

$$\partial \underline{u} = G(t) \partial \underline{x}$$

$$\underline{u} = \tilde{\underline{u}} + G(t) \partial \underline{x}$$

$\underline{x}(t)$: calculated by KF (INS) $\hat{\underline{x}}(t)$

$$\partial \dot{\underline{x}} = (F + LG) \partial \underline{x}$$



Part D: Mathematical modelling of robots

We go throng the document: "Matematisk modellering av roboter" witten by Oddvar Hallingstad.

- Based on Craig.

- Craig only use algebraic vectors. $\underline{r}^b$ (not geometric $\vec{r}$)

① $\underline{r}^a = R_b^a \underline{r}^b$

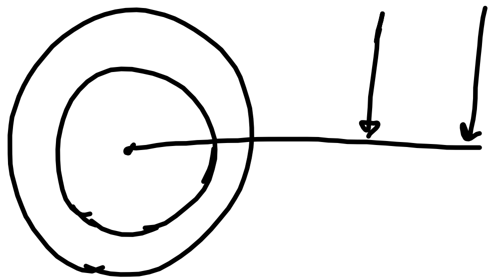
② $R_b^a = \begin{bmatrix} \underline{x}_b^a & \underline{y}_b^a & \underline{z}_b^a \end{bmatrix}$

③ $\underline{y}^b = R_b^a \underline{x}^b$

$$\tilde{R}_p^q = \begin{bmatrix} R_p^q & \underline{r}_{pq}^q \\ 0,0,0 & 1 \end{bmatrix}, \quad \tilde{\underline{r}}^p = \begin{bmatrix} \underline{r}^p \\ 1 \end{bmatrix}, \quad \tilde{\underline{r}}^q = \begin{bmatrix} \underline{r}^q \\ 1 \end{bmatrix}$$

$$\left(R_b^a\right)^{-1} = R_a^b = \left(R_b^a\right)^T$$

<u>o.n basis</u>



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