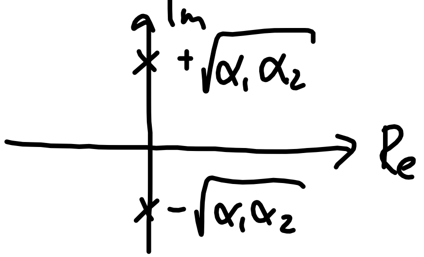
F.14 From F.13, page 11.



We know the solution has the form:

$$\boxed{\omega_y(t) = A\sin(\omega t+\phi)\,,\quad \omega = 2\pi f}$$

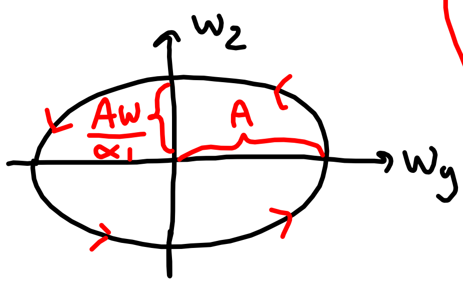
From d.e. $\dot{\omega}_y = -\alpha_1\omega_z$

$$\boxed{\omega_z(t) = -\frac{1}{\alpha_1}A\,\omega\cos(\omega t+\phi)}$$

$$\dot{\omega}_z(t) = \frac{1}{\alpha_1}A\omega^2\sin(\omega t+\phi) = \alpha_2\omega_y$$

$$\Rightarrow \omega^2 = \alpha_1\alpha_2$$

Given $\underline{\omega}_b^{ib}(t_0)$



We can show:

$$\omega = \omega_{x0}\sqrt{\frac{(J_{xx}-J_{zz})(J_{xx}-J_{yy})}{J_{yy}J_{zz}}}$$

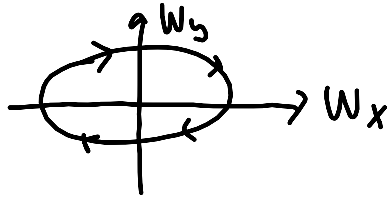
$$A = \frac{\omega_y(0)}{\sin\phi}$$

$$\phi = \arctan\left(\frac{-\omega\,\omega_y(0)}{\alpha_1\omega_z(0)}\right)$$

2) Rotation around b_3-axis.

Assume $|\omega_z| \gg |\omega_x| \approx |\omega_y|$, $\omega_x\omega_y \approx 0$

$\Rightarrow$ Complex conjugated eigenvalues and $\omega_z(t) = \omega_{z0}$



3) Rotation around b_2-axis (y-axis)

Assume $|w_y| \gg |w_x| \approx |w_z| \Rightarrow w_x w_z \approx 0$

$$\dot{w}_x = \frac{J_{yy}-J_{zz}}{J_{xx}} w_{y0} w_z = \beta_1 w_z \,, \quad \beta_1 > 0$$

$$\dot{w}_y = \frac{J_{zz}-J_{xx}}{J_{yy}} w_x w_z = 0 \Rightarrow w_y(t) = w_{y0}$$

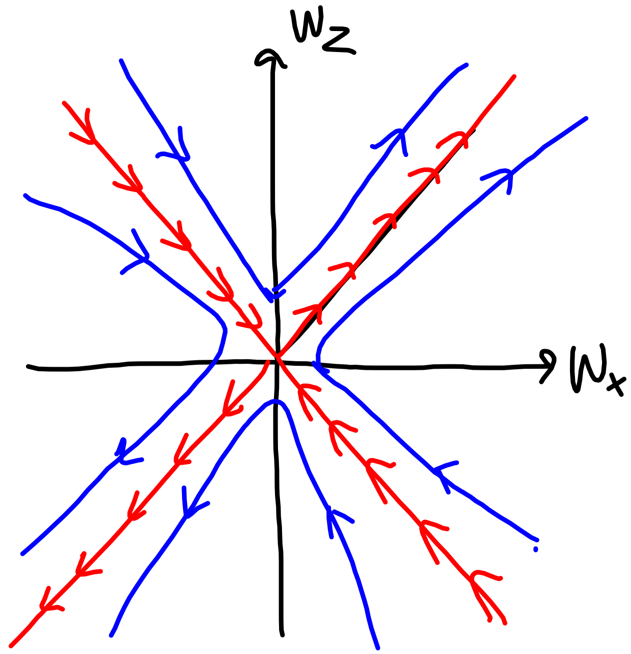
$$\dot{w}_z = \frac{J_{xx}-J_{yy}}{J_{zz}} w_{y0} w_x = \beta_2 w_x \,, \quad \beta_2 > 0$$

$$\begin{bmatrix} \dot{w}_x \\ \dot{w}_z \end{bmatrix} = \begin{bmatrix} 0 & \beta_1 \\ \beta_2 & 0 \end{bmatrix} \begin{bmatrix} w_x \\ w_z \end{bmatrix}$$

$$\dot{\underline{x}} = A \underline{x}$$

$$\dot{w}_x = \beta_1 w_z$$

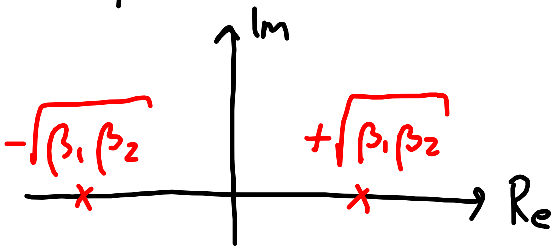
$$\dot{w}_z = \beta_2 w_x$$



Eigenvalues :

$$|\lambda I - A| = \begin{vmatrix} \lambda & -\beta_1 \\ -\beta_2 & \lambda \end{vmatrix} = \lambda^2 - \beta_1\beta_2 = 0$$

$$\lambda_{1,2} = \pm\sqrt{\beta_1\beta_2}$$

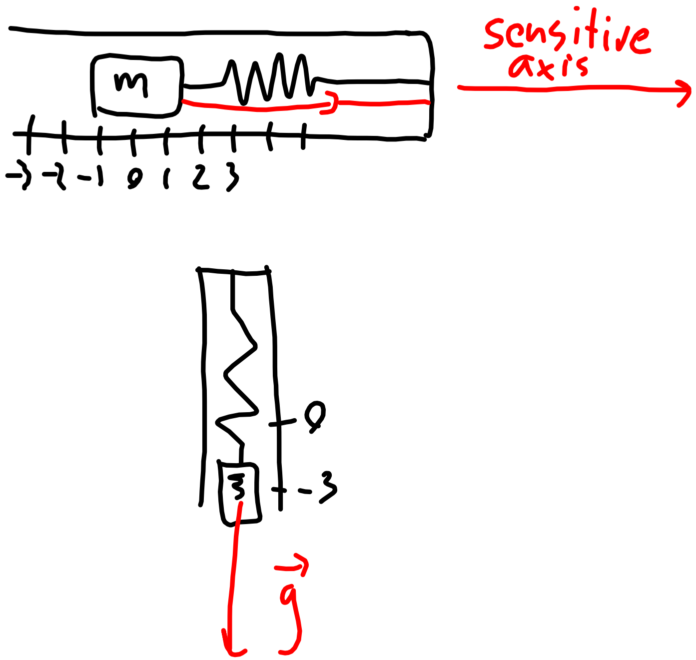


$$\begin{bmatrix} w_x(t) \\ w_y(t) \end{bmatrix} = M \begin{bmatrix} e^{\lambda_1 t} & 0 \\ 0 & e^{-\lambda_1 t} \end{bmatrix} M^{-1} \begin{bmatrix} w_x(0) \\ w_y(0) \end{bmatrix}$$

<u>Unstable !</u>

Part E: Inertial navigation system (INS)

Accellerometer

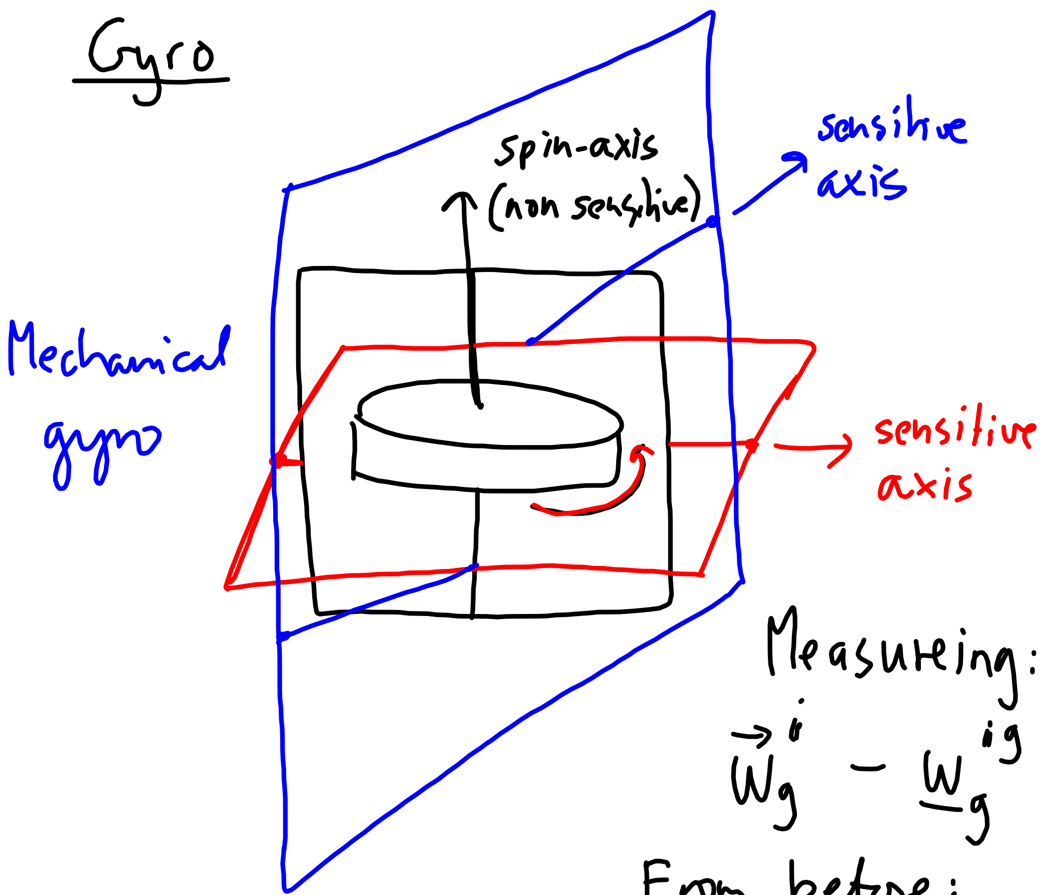


Measuring specific force:

$$\underline{f}^a = \underline{a}^{iia} - \underline{g}^a$$

$$\underline{a}^{iia} = \underline{f}^a + \underline{g}^a$$

Gyro



Measureing:

$$\vec{\omega}_g^{\,i} - \underline{\omega}_g^{ig}$$

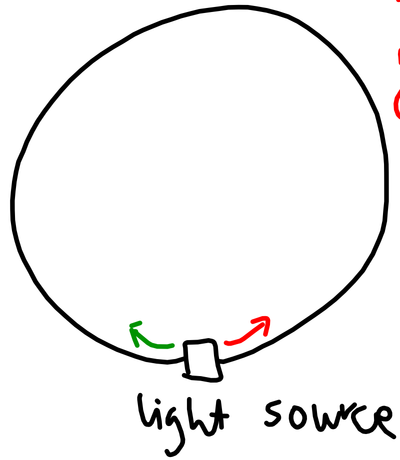
From before:

$$\dot{R}_g^i = R_g^i S(\underline{\omega}_g^{is})$$

Types of gyros:

- Mehanical gyros
- Ring laser gyro (RLG)
- Fiber optics gyo (FOG)
- MEMS

USING LIGHT



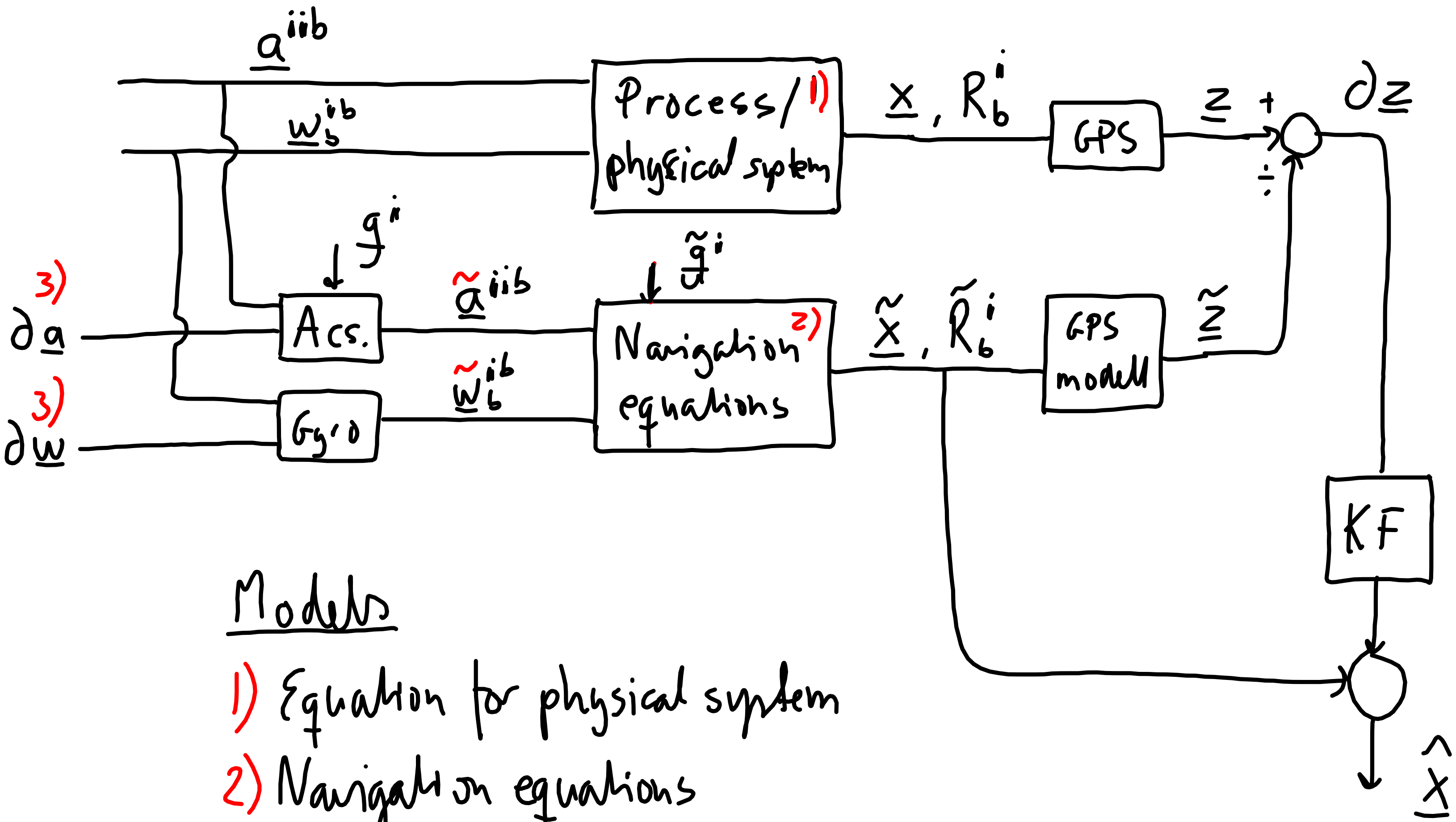
· Navigation equations

$\mathcal{F}^a = \mathcal{F}^g = \mathcal{F}^b$ - body frame

$$\boxed{\begin{aligned}
\dot{\tilde{\underline{p}}}^i &= \tilde{\underline{v}}^i \\
\dot{\tilde{\underline{v}}}^{ii} &= \tilde{R}^i_b\, \tilde{\underline{f}}^b + \tilde{\underline{g}}^i \\
\dot{\tilde{R}}^i_b &= \tilde{R}^i_b\, S(\tilde{\underline{\omega}}^{ib}_b)
\end{aligned}}$$

$\tilde{(\cdot)}$: measured component

$\tilde{(\cdot)}$: calculated component



Models

1) Equation for physical system
2) Navigation equations
3) Error models

Part C: Mathematical modelling of airplane

This part builds on:

Chin-Fang Lin

Advanced Control Systems Design

chapter 13.4

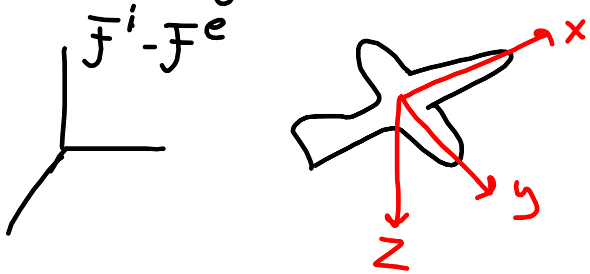
We will write Newtons 2. law and the law of angular momentum in the form (notation) that is normal to give the d.e. for airplanes.

We will also give the relationship between a state vector as in our equations (given in part B) and the one used as standard for airplanes.

We will not deal with airodynamics, i.e. the relation between the planes orientation, velocity and "pådrag" and those forces and torques this results in due to the air flow.

application

Law of angular momentum in $\mathcal{F}^b$



From part B:

$$\underline{n}_c^b = J_c^b \, \dot{\underline{w}}_b^{ibb} + S(\underline{w}_b^{ib}) J_c^b \, \underline{w}_b^{ib}$$

$\mathcal{F}^i$: fixed to the earth

$\mathcal{F}^b$: fixed to the airplane

Assume that $\mathcal{F}^b$ coinsides with the main axis of the plane => Eulerequations

$$\text{(1)} \quad \begin{aligned} n_x &= J^b_{xx}\dot{\omega}_x + \omega_y\omega_z(J^b_{zz} - J^b_{yy}) \\ n_y &= J^b_{yy}\dot{\omega}_y + \omega_z\omega_x(J^b_{xx} - J^b_{zz}) \\ n_z &= J^b_{zz}\dot{\omega}_z + \omega_x\omega_y(J^b_{yy} - J^b_{xx}) \end{aligned}$$

Compare (1) with (13-72)

$$\underline{n}_c^b = [n_x, n_y, n_z]^T = [L, M, N]^T$$

$$\underline{w}_b^{ib} = [w_x, w_y, w_z]^T = [P, Q, R]$$

$$J_c^b = \mathrm{diag}(J_{xx}, J_{yy}, J_{zz}) = \mathrm{diag}(I_{xx}, I_{yy}, I_{zz})$$