F10/ From F.9 : $\dot{\underline{x}}^q = A^q \underline{x}^q$, $\underline{x}^q(0)$ given

This eq. has the solution: $\underline{x}^q(t) = M^q_m \, \underline{x}^m(t)$

$$\underline{x}^q(t) = M^q_m \, e^{\Lambda^m t} \left(M^q_m\right)^{-1} \underline{x}^q(0)$$

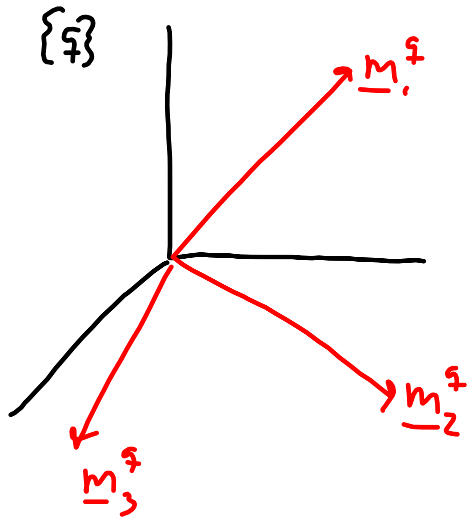
$$\underline{x}^q(t) = e^{M^q_m \Lambda^m (M^q_m)^{-1} t} \, \underline{x}^q(0)$$

If M is invertible:

$$e^{MXM^{-1}} = M e^{X} M^{-1}$$

$$\underline{x}^q(t) = e^{A^q t} \, \underline{x}^q(0)$$

where
$$e^{A^q t} = \underbrace{M^q_m \, e^{\Lambda^m t} \left(M^q_m\right)^{-1}}_{\phi(t,0)} = \underbrace{\left(I + \frac{1}{1!} A^q t + \frac{1}{2!} A^q A^q t^2 + \dots \right)}_{\text{Used in numerical calculations}}$$



$$\underline{x}^q(t) = M^q_m\, \underline{x}^m(t) = \underline{m}_1^q x_1^m(t) + \dots + \underline{m}_n^q x_n^m(t)$$

$$= \underline{m}_1^q e^{\lambda_1 t} x_1^m(0) + \dots + \underline{m}_n^q e^{\lambda_n t} x_n^m(0)$$

NB! We have assumed that we have distinctive eigenvalues $\Rightarrow$ linearly independent eigenvectors. But we can also have complex conjugated eigenvalues (which gives us complex eigenvectors). In the figure above we have assumed that the eigenvalues also are real.

Part B DYNAMICS

Dynamics includes:

1) Kinematics
   - Describing the motion by mathematics (Part A)

2) Kinetic
   - Relation between the motion and the forces that makes the motion (math + physics), f. eks Newton's laws.

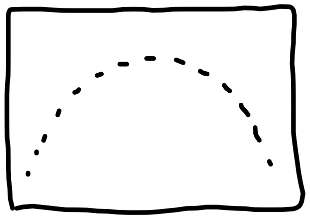
Terms

- Reference space - inertial frame
- Conected to a physical system
- Coordinate systems (frame, units, function)
- Particles (modeled by points and mass)
- Pos., vel., acc. (modeled by vectors)
- Rigid bodies (modeled by a frame and mass)
- Attitude (modeled by frames)

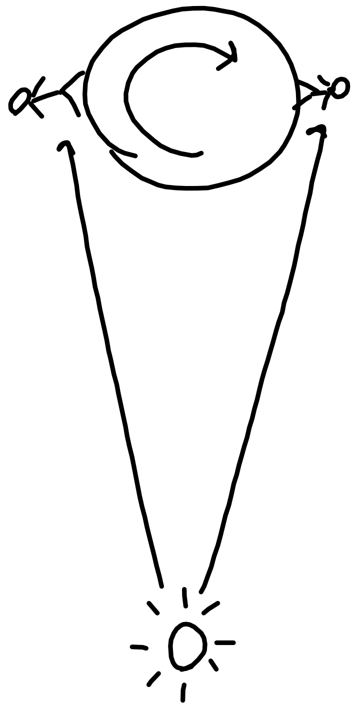
- Affine space
- Mathematical model of a reference space
- $\underrightarrow{F}_A^a : \{O_a, \vec{a}_1, \vec{a}_2, \ldots, \vec{a}_n\}$
- P : points
- $\vec{v}$: vectors

You can take a look at: Grunnleggende prinsipper i klassisk mekanikk.

Aristotles.



Absolute points

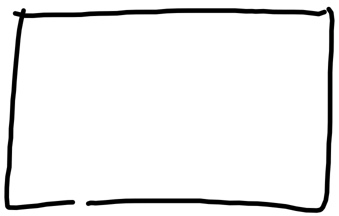


Newton

$$\vec{f} = \frac{d}{dt}(m \cdot \vec{v})$$

$$\vec{v}_3 = \vec{v}_1 + \vec{v}_2$$

Einstein



$\|v\| \leqslant c$ (speed of light)

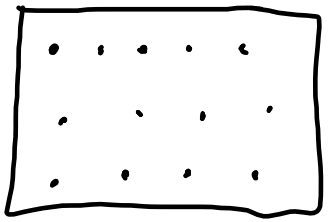
$$\vec{v}_3 \neq \vec{v}_1 + \vec{v}_2$$

Part B Dynamics

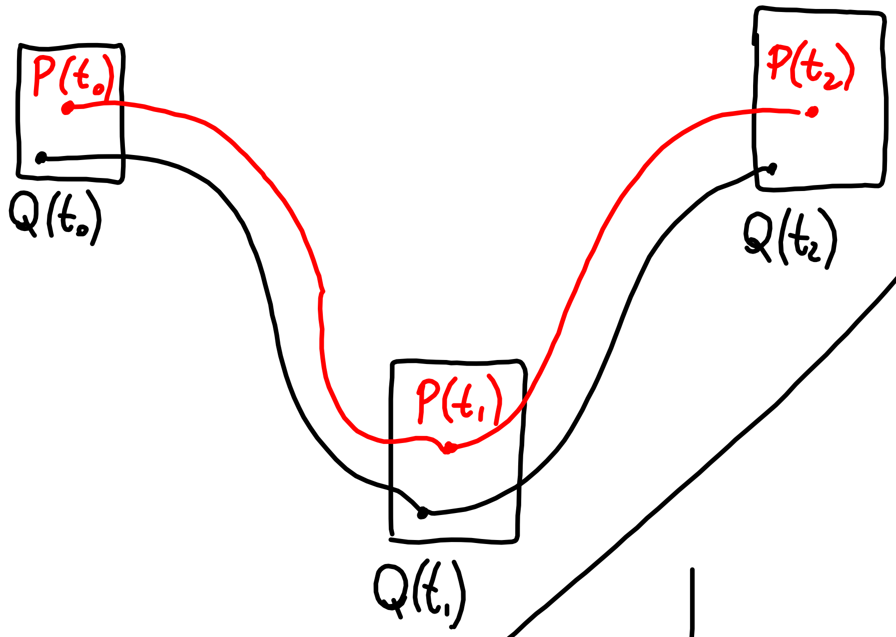
B.1 Kinematics

B.1.1 Kinematic description of particles

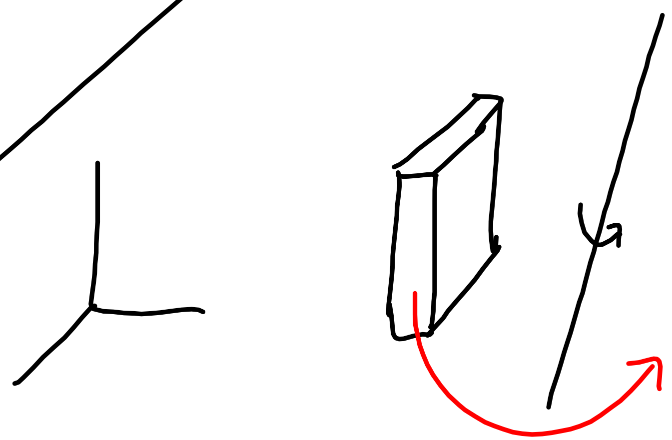
Def. B.1 Rigid body



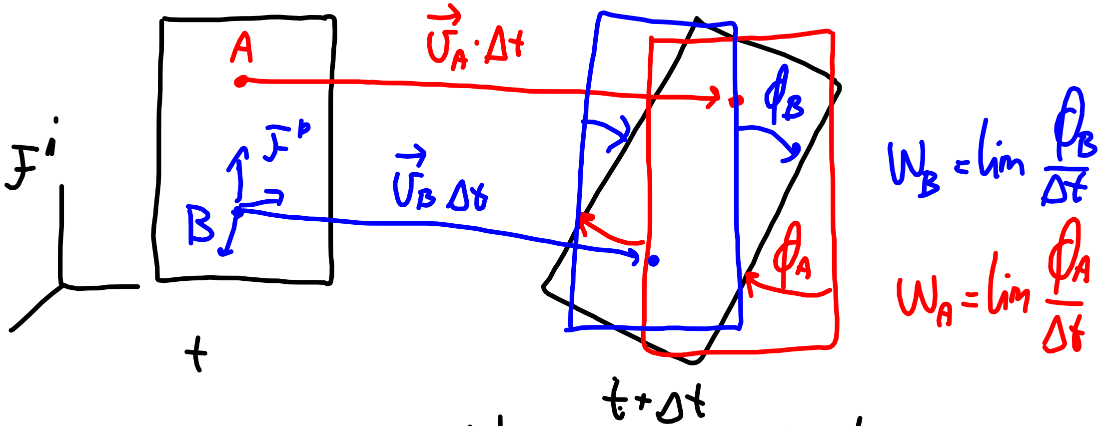
Def. B.2 Pure translation of rigid bodies



Def B.3 Rotation of a rigid body



Teorem B.1 (Chasley's teorem) ***Dekomponering i translasjon og rotasjon***
Bevegelsen av et stivt legeme relativt et k.s. kan settes sammen av translasjon og rotasjon. Dette kan gjøres på følgende måte :
1) Velg et punkt A (B) i legemet. Anta at alle punktene i legemet har samme hastighet, $\vec{v}_A\,(\vec{v}_B)$, hvor $\vec{v}_A\,(\vec{v}_B)$ er hastigheten relativt vårt k.s.
2) Superponer en ren rotasjon om punktet A med vinkelhastighet $\vec{\omega}$ relativt vårt k.s. (NB : $\vec{\omega} = \vec{\omega}_A = \vec{\omega}_B$, mens generelt er $\vec{v}_A \neq \vec{v}_B$ ($\vec{v}_A = \lim_{\Delta t \to 0}(\Delta \vec{r}_A/\Delta t)$).)



$\vec{v}_A \neq \vec{v}_B$, $w_A = w_B$

i.e. we can model the motion of a rigid body as the motion of the frame F^b relative F^i

The motion of a rigid body is put together by $\vec{v}_{Ob}^{\,i}(t)$ and $\vec{w}_b^{\,i}(t)$, i is inertial frame

B.2 Kinetic

Newton's laws for a particle

Teorem B.2 (Newtons 1.lov) *Dersom en partikkel er langt borte fra innflytelsen fra alle andre partiler i universet, vil den bevege seg med konstant hastighet mht et treghetssystem,* **i** *(kan egentlig utledes fra Newtons 2.lov).* (N.1 is a special case of N.2)

Teorem B.3 (Newtons 2.lov) *Dersom det lineære moment,* $\vec{p}^{\mathbf{i}}$*, for en partikkel i et treghetssystem* **i** *endres med tiden, sies partikkelen å være påvirket av en kraft,* $\vec{f}$*, gitt ved :*

$$\vec{f} = \dot{\vec{p}}^{\mathbf{i}} \quad hvor \quad \vec{p}^{\mathbf{i}} = m\vec{v}^{\mathbf{i}} \tag{B- 139}$$

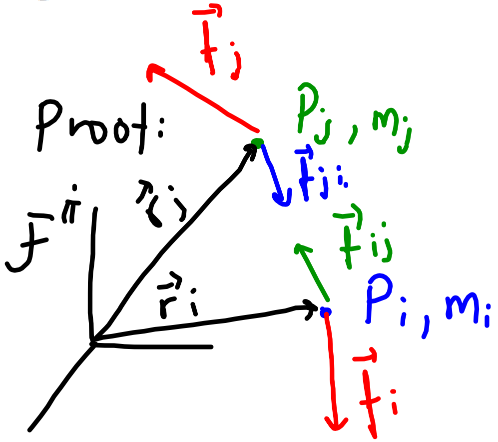
Teorem B.4 (Newtons 3.lov) *Dersom to isolerte partikler interakterer med hverandre vil den krafta partikkel nr 1 utsetter partikkel nr 2 for være lik i størrelse, men motsatt rettet den krafta partikkel nr 2 utsetter partikkel 1 for. Dvs : aksjon = reaksjon eller kraft = motkraft.*

Teorem B.5 ***Newtons 2. lov for et system av partikler***

Vi antar at Newtons 3. lov gjelder for krafta mellom partiklene, dvs $\vec{f}_{ij} = -\vec{f}_{ji}$. *Da vil den totale ytre kraft,* $\vec{F}$, *være lik total masse,* M, *ganger med massesenterets akselerasjon,* $\vec{a}_c^{\mathbf{i}}$, *sett fra treghetsramma :*

$$\boxed{\vec{F} = M \frac{{}^{\mathbf{i}}d\,{}^{\mathbf{i}}d}{dt^2}\vec{r}_c = M\, \vec{a}_c^{\mathbf{i}}} \qquad M = \sum_{i=1}^{n} m_i\,,\quad \vec{F} = \sum_{i=1}^{n} \vec{f}_i \qquad \text{(B- 140)}$$

Proof:



$$\vec{r}_c = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{r}_i \Rightarrow M\vec{r}_c = \sum_{i=1}^{n} m_i \vec{r}_i$$

$\vec{r}_c$: center of mass

$\vec{f}$: outer force

$\vec{f}_{ij}$: inner force (according to N.3 law)

m_i : mass of particle i

$\vec{r}_i$: position vector of particle i

Newtons 2. law works for all particles.

$$P_i : \; \vec{f}_i + \sum_{\substack{j=1 \\ j\neq i}}^{n} \vec{f}_{ij} = m_i \vec{a}_i^{\,i} = m_i \ddot{\vec{r}}_i^{\,ii}$$

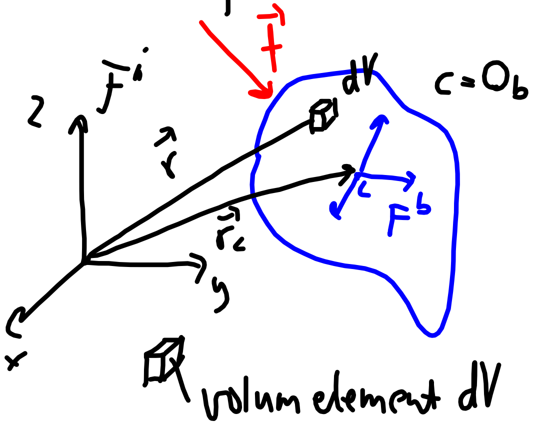
$$\begin{array}{llllllllll}
P_1 : & \vec{f}_1 & + \; 0 & + \; \vec{f}_{12} & + \; \vec{f}_{13} & + \; \cdots & + \; \vec{f}_{1n} & = & m_1 \dfrac{d^i}{dt}\left(\dfrac{d^i}{dt}\vec{r}_1\right) \\
P_2 : & \vec{f}_2 & + \; \vec{f}_{21} & + \; 0 & + \; \vec{f}_{23} & + \; \cdots & + \; \vec{f}_{2n} & = & m_2 \dfrac{d^i}{dt}\left(\dfrac{d^i}{dt}\vec{r}_2\right) \\
P_3 : & \vec{f}_3 & + \; \vec{f}_{31} & + \; \vec{f}_{32} & + \; 0 & + \; \cdots & + \; \vec{f}_{3n} & = & m_3 \dfrac{d^i}{dt}\left(\dfrac{d^i}{dt}\vec{r}_3\right) \\
\vdots & & & & & & & & \\
P_n : & \vec{f}_n & + \; \vec{f}_{n1} & + \; \vec{f}_{n2} & + \; \vec{f}_{n3} & + \; \cdots & + \; 0 & = & m_n \dfrac{d^i}{dt}\left(\dfrac{d^i}{dt}\vec{r}_n\right)
\end{array}$$

$$\sum P_i : \; \underbrace{\sum_{i=1}^{n} \vec{f}_i}_{\vec{F}} + \vec{0} \qquad \boxed{\vec{F} = M \vec{a}_c^{\,i}} \qquad = \underbrace{\frac{d^i}{dt}\left(\frac{d^i}{dt} \sum m_i \vec{r}_i\right)}_{M \ddot{\vec{r}}_c^{\,ii}}$$

Since a rigid body can be viewed as a sum of particles (molecyl) we have:

$$\boxed{\vec{f} = m\,\vec{a}_c^{\,i}}$$

where $\vec{f}$ is the total outer-force acting on the body, m is the mass and c is the center of mass.



$$m = \iiint_V k(\vec{r})\,dV$$ where $k(\vec{r})$ is the mass density.

$$\vec{r}_c = \frac{1}{m}\iiint_V \vec{r}\,k(\vec{r})\,dV = \frac{1}{m}\int_z\int_y\int_x \vec{r}\,k(\vec{r})\,dx\,dy\,dz$$