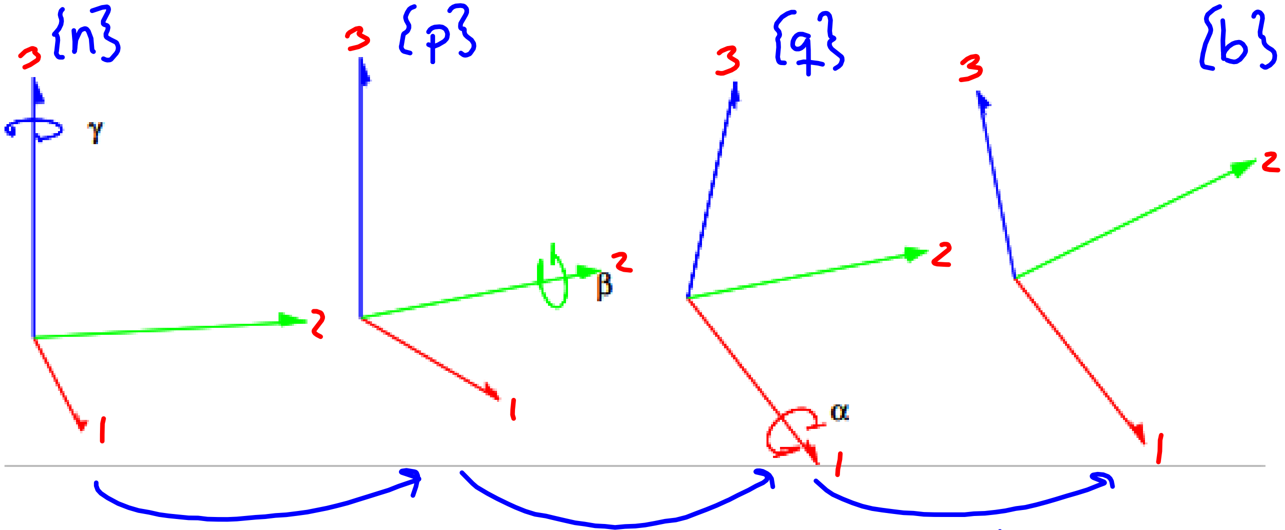
F.6/



$R_3(\gamma)$ $R_2(\beta)$ $R_1(\alpha)$

$R_{np} = R_p^n$ $R_{pq} = R_q^p$ $R_{qb} = R_b^q$

$$R_{nb} = R_b^n = R_p^n R_q^p R_b^q = R_3(\gamma) R_2(\beta) R_1(\alpha)$$

Eulers symmetrical parameter representation of the DCM

Definisjon A.12 *Eulers symmetriske parametre*
Vha vinkel-akse representasjonen kan vi definere en 4-parameter representasjon på følgende måte :

$$\underline{k}^{q} = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix}, \theta$$

$$\left.\begin{array}{ll} \varepsilon_1 = k_1 \sin(\theta/2) & \varepsilon_3 = k_3 \sin(\theta/2) \\ \varepsilon_2 = k_2 \sin(\theta/2) & \varepsilon_0 = \cos(\theta/2) \end{array}\right\} \Rightarrow \varepsilon_0^2 + \varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2 = 1 \qquad \text{(A- 45)}$$

$$\sin^2\left(\frac{\theta}{2}\right) \underbrace{\left(k_1^2 + k_2^2 + k_3^2\right)}_{1} + \cos^2\left(\frac{\theta}{2}\right) = 1$$

Quaternion representation of the DCM

A quaternion (those with unit length) can represent the DCM R_p^q if it is defined using Eulers symmetrical parameters in the following way :

$$\boxed{\mathcal{E}_p^q = \varepsilon_0 + \varepsilon_1 \mathbb{i} + \varepsilon_2 \mathbb{j} + \varepsilon_3 \mathbb{k}}$$

$$\|\mathcal{E}_p^q\| = \left(\sum_{i=0}^{3} \varepsilon_i^2\right)^{1/2} = 1$$

$$\mathbb{i}\mathbb{i} = \mathbb{j}\mathbb{j} = \mathbb{k}\mathbb{k} = -1$$

E.1. viewed as a complex number

$$\mathbb{i}\mathbb{j} = -\mathbb{j}\mathbb{i} = \mathbb{k}$$

$$\mathbb{j}\mathbb{k} = -\mathbb{k}\mathbb{j} = \mathbb{i}$$

$$\mathbb{k}\mathbb{i} = -\mathbb{i}\mathbb{k} = \mathbb{j}$$

View $\mathbb{i}$, $\mathbb{j}$ and $\mathbb{k}$ as vectors in 3D (o.n.) and the product of two vectors as the "$\times$"-product.

Determinant of R_p^q: $|R_p^q| = 1 \iff \|\mathcal{E}_p^q\| = 1$

Renormalization of $\mathcal{E}_p^q$ is much simpler than renormalization of R_p^q $\left(|R_p^q| = 1,\ \underline{p}_1^q \perp \underline{p}_2^q \perp \underline{p}_3^q\right)$

Calculation rules for quaternions.

$$\boxed{\begin{aligned} R_c^a = R_b^a R_c^b \quad &\Longleftrightarrow \quad \mathcal{E}_c^a = \mathcal{E}_c^b \, \mathcal{E}_b^a \\ R_b^a \underline{r}^b \quad &\Longleftrightarrow \quad (\mathcal{E}_b^a)^* \, \mathbb{r}^b \, \mathcal{E}_b^a \end{aligned}}$$

where:

$$(\mathcal{E}_b^a)^* = \varepsilon_0 - \varepsilon_1 \mathbb{i} - \varepsilon_2 \mathbb{j} - \varepsilon_3 \mathbb{k}$$

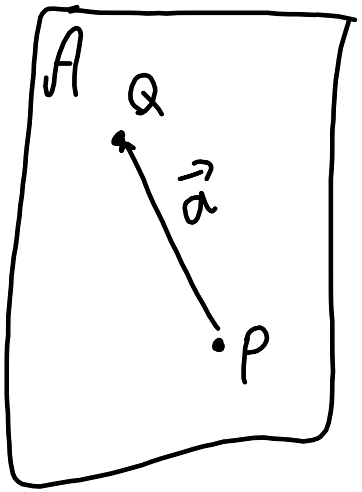
$$\mathbb{r}^b = r_1^b \mathbb{i} + r_2^b \mathbb{j} + r_3^b \mathbb{k}, \qquad \underline{r}^b = \begin{bmatrix} r_1^b \\ r_2^b \\ r_3^b \end{bmatrix}$$

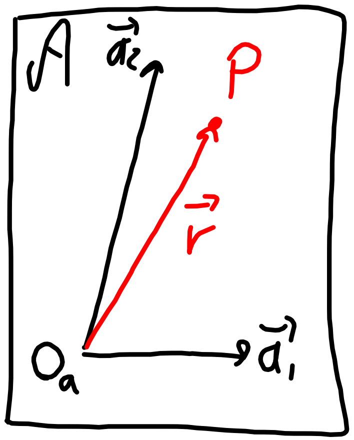
Parametrisering	Notasjon	Fordel	Ulempe	Vanilige anvendelser
RKM	C_p^q (R_p^q) o.n.	Ingen singulariteter, ingen trigonometriske funksjoner, enkel produktregel for suksesive rotasjoner	Seks redundante parametre	I analysen, for å transformere vektorer fra et k.s. til et annet
Eulervinkler	φ, θ, ψ	Ingen redundante parametre, klar fysisk tolkning	Trigonometriske funksjoner, singulariteter for visse vinkler, ingen enkel produktregel for suksessive rotasjoner	Analytiske studier, 3-akset stillingskontrol av legemer
Vinkel-akse	$\underline{k}, \theta$	Klar fysisk tolkning	En redundant parameter, aksen er udefinert når $\sin\theta = 0$, trigonometriske funksjoner	Reorienteringsmanøvre (slew)
Kvaternioner	ε	Ingen singulariteter, ingen trigonometriske funksjoner, enkel produktregel for suksessive rotasjoner	En redundant parameter, ingen klar fysisk tolkning	Treghetsnavigasjonsberegninger

$\|\vec{k}\| = \theta$

A.3 Affine spce

Definisjon A.14 ***Affint rom***
La $\mathcal{A}$ være en ikke-tom mengde av punkter, og la $\mathcal{V}$ være et vektorrom over skalarkroppen $\mathbb{K}$. Anta at for vilkårlige punkt $P \in \mathcal{A}$ og $\vec{a} \in \mathcal{V}$ er det definert en addisjon $P + \vec{a} \in \mathcal{A}$ som tilfredstiller følgende betingelser :
1. $P + \vec{0} = P$ ($\vec{0}$ er nullvektoren i $\mathcal{V}$)
2. $(P + \vec{a}) + \vec{b} = P + (\vec{a} + \vec{b})$ for $\forall \vec{a}, \vec{b} \in \mathcal{V}$
3. For enhver $Q \in \mathcal{A}$ eksisterer en entydig vektor $\vec{a} \in \mathcal{V}$ slik at $Q = P + \vec{a}$
Da er $\mathcal{A}$ et affint rom.



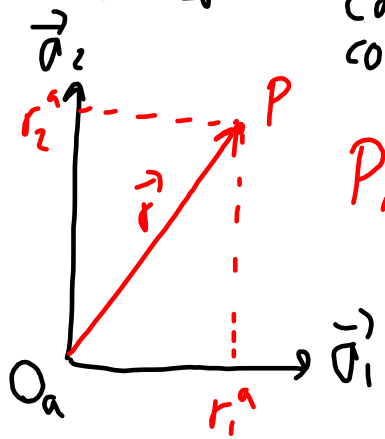


Given the frame $\mathcal{F}_A^a = \{O_a, \vec{a}_1, \vec{a}_2, \vec{a}_3\}$
There is a clear relation
between P, $\vec{r}$, $\underline{r}^a$

A3.2 Coordinate systems and frames

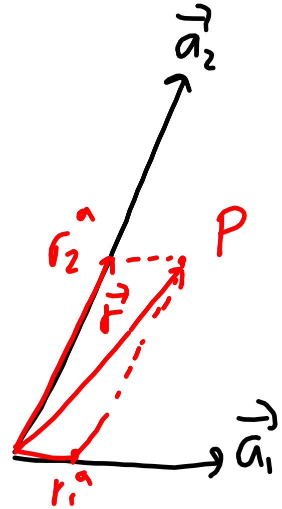
Definisjon A.15 ***Affine koordinater***
La $\mathcal{A}$ være et $n-$dimensjonalt affint rom og la $\mathcal{F}^e = (O_e; \vec{e}_1, \vec{e}_2, \ldots, \vec{e}_n)$ være en **ramme**, *hvor O_e, kallt origo, er et punkt i $\mathcal{A}$, og vektorene $\{\vec{e}_i\}$ er et sett basisvektorer for for det tilhørende vektorrom $\mathcal{V}$. Da er de inhomogene* **koordinatene** *til et vilkårlig punkt $P \in \mathcal{A}$ med hensyn til ramma $\mathcal{F}^e$ gitt av n-tuppelet $\{p_1^e, p_2^e, \ldots, p_n^e\}$ (vi vil sette disse sammen til en algebraisk vektor, $\underline{p}^e$), hvor $P = O_e + \sum_{i=1}^n p_i^e \vec{e}_i$. Dersom $\mathcal{A}$ også har strukturen til et Euclidsk rom (se nedefor) og vi lar ramma $\mathcal{F}^e = (O_e; \vec{e}_1, \vec{e}_2, \ldots, \vec{e}_n)$ bestå av ortogonale basisvektorer sier vi at vi har et* **rektangulært koordinatsystem** *(eller ortogonalt k.s.), dersom basisvektorer har lengde 1, $\langle \vec{e}_i, \vec{e}_j \rangle = \delta_{ij}$,kalles k.s. for* **kartesisk**. *Generelt vil $\langle \vec{e}_i, \vec{e}_j \rangle = c_{ij}$ og vi sier vi har et* **oblikt koordinatsystem** *(eller ikke-ortogonalt k.s.). Dersom vi har en oblik ramme hvor lengden på basisvektorene er $\langle \vec{e}_i, \vec{e}_i \rangle = 1$ er vinkelen mellom basisvektoren gitt av $c_{ij} = \cos \angle \vec{e}_i \vec{e}_j$*

Example



Cartesian coordinate system

$P, \vec{r} \rightarrow \underline{r}^a$



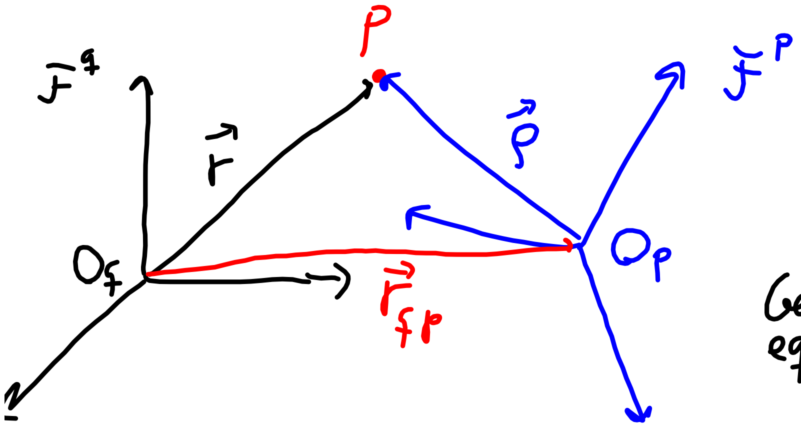
Obligt c.s.

$P, \vec{r} \rightarrow \underline{r}^a$

A.3.3 Matrix representation of points and vectors

Inhomogenuos representation.

Def. A.17 Position vector



$$P = O_q + \vec{r} = O_p + \vec{\rho}$$

$$\vec{r} - \vec{\rho} = O_p - O_q = \vec{r}_{qp}$$

Def. $\vec{r}_{qp} := \vec{r}_{O_q O_p}$

Geometrical equation:	$\vec{r} = \vec{r}_{qp} + \vec{\rho}$
Algebraic equation:	$\underline{r}^q = \underline{r}^q_{qp} + \underline{\rho}^q = \underline{r}^q_{qp} + \underline{R}^q_p \underline{\rho}^p$

We see that when representing points in two different frames the coordinates transform as:

$$\boxed{\underline{r}^q = \underline{r}^q_{qp} + R^q_p \underline{\rho}^p}$$ Inhomogenous form

But vectors in vector space transform as:

$$\boxed{\underline{v}^q = R^q_p \underline{v}^p}$$ Homogenous form

Homogenous representation of coordinates to a point.

$$(1a)\quad \boxed{\underline{r}^q_p = \underline{r}^q_{qp} + R^q_p \underline{\rho}^p_p}$$

$$\tilde{\underline{r}}^q_p = \begin{bmatrix} r_1^q \\ r_2^q \\ r_3^q \\ 1 \end{bmatrix}$$

Define:

$$\tilde{\underline{r}}^q_p = [\underline{r}^q_p\ ;\ 1]$$

$$\tilde{\underline{\rho}}^p_p = [\underline{\rho}^p_p\ ;\ 1]$$

; 4 dim. matrix rep. of the point P in $\mathcal{F}^p$ and $\mathcal{F}^q$

We get:

$$(1b)\quad \boxed{\tilde{\underline{r}}^q_p = T^q_p \tilde{\underline{\rho}}^p_p ,}\qquad T^q_p = \left[\begin{array}{c|c} R^q_p & \underline{r}^q_{qp} \\ \hline [0,0,0] & 1 \end{array}\right]$$

T^q_p : transformation matrix

For normal vectors (not matrix rep. of points) we have:

$$\boxed{\underline{v}^q = R^q_p \underline{v}^p} \quad (2a)$$

Define:

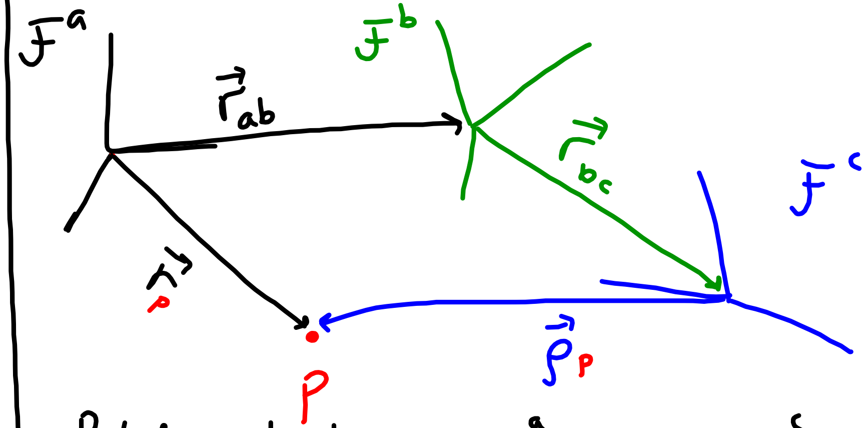
$$\underline{\tilde{v}}^q = [\underline{v}^q ; 0]$$

$$\underline{\tilde{v}}^p = [\underline{v}^p ; 0]$$

These vectors transform as:

$$\boxed{\underline{\tilde{v}}^q = T^q_p \underline{\tilde{v}}^p} \quad (2b)$$

Example: 3 frames.



Relation between $\underline{r}^a_p$ and $\underline{\rho}^c_p$

$$\vec{r}_p = \vec{r}_{ab} + \vec{r}_{bc} + \vec{\rho}_p$$

$$\boxed{\underline{r}^a_p = \underline{r}^a_{ab} + R^a_b \underline{r}^b_{bc} + R^a_b R^b_c \underline{\rho}^c_p} \quad (3a)$$

$$\boxed{\underline{\tilde{r}}^a_p = T^a_b T^b_c \underline{\tilde{\rho}}^c_p} \quad (3b)$$