Application Note

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Lab 4: Frequency Space is the Place

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Table of Contents

1. Abstract

In this application note, we will investigate how gain can be frequency dependent by designing and building first and second order active and passive low pass, high pass, and bandpass filters. We will analyze how the filters behave in the frequency domain and time domain with mathematical analysis, simulation, and measured results. We will use these filters to create a basic audio equaliser.

2. Introduction

Audio filters are used to control frequencies which pass through a system. Many forms of audio equipment (e.g: equalisers, synthesizers) use audio filters to alter sound. There are many different types of filters available. High pass filters will allow only high frequencies to pass through. Inversely, low pass filters will allow only low frequencies to pass through. These filters and its components are frequency dependent - as in changing values of components and frequency will change the gain and cutoff frequencies of the circuit. Circuit analysis can be completed in the frequency domain to show how each circuit component affects the overall transfer function of the circuit.

3. Background

3.1 Inverting Opamp

An inverting operational amplifier configuration is where negative feedback is fed from the output of an opamp to its inverting input through a resistor called the “feedback resistor” . This results in a different signal on the inverting input terminal than the actual input voltage and therefore, the two signals must be separated using another resistor. This resistor is called the “input resistor”. The non-inverting terminal of the opamp is connected to common ground. This setup results in an amplifier with a closed-loop gain that can be controlled by adjusting the values of the feedback and input resistors. This gain can be calculated according to

$$A_V = -\frac{R_f}{R_{in}}$$

Different values for the feedback and input resistors of an inverting opamp can be used to produce the same gain, however, these values should be chosen according to a number of different factors which are discussed in this application note.

3.2 Potentiometer

A potentiometer is a three-terminal component that provides different resistances by sliding a wiper across a resistive strip. Adjusting a knob on the potentiometer causes a wiper to slide along the resistive strip which provides a “variable” resistance based on the position of the knob. Two terminals are connected across the resistor, creating a fixed maximum resistance. The third terminal is connected to the wiper which creates a break in resistance between the other two terminals. Therefore, a potentiometer can be seen as a ratio of two resistors for mathematical analysis. A linear potentiometer has a resistance between a terminal and the wiper that is proportional to the distance between them.

3.3 Speaker

A loudspeaker, or speaker, is an electroacoustic transducer that is used to produce a sound based on an electrical audio signal. The speaker uses a diaphragm attached to a coil that moves back and forth to produce sound waves. This movement of this coil occurs when an alternating current electrical audio signal is applied to it. Speakers also have a resistance.

3.4 Capacitor

A capacitor is a component created out of two metal plates and an insulating material called a dielectric. The metal plates are placed very close to each other, in parallel, but the dielectric sits between them to make sure they don’t touch. The dielectric material can be made out of various insulating materials: paper, glass, rubber, ceramic, or plastic. The plates are made of a conductive material: aluminum, tantalum, silver, or other metals. Capacitor plates with more overlapping surface area provide more capacitance, while more distance between the plates equate to less capacitance. The total capacitance of a capacitor can be calculated with the equation:

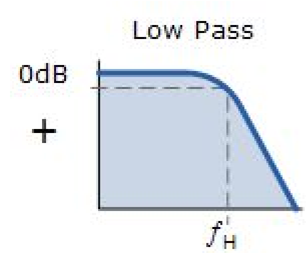
$$C = \varepsilon_{\mathrm{r}} \frac{A}{4\pi d}$$

Where ε_r is the dielectric relative permittivity, A is the amount of area the plates overlap each other, and d is the distance between the plates.

When electrical charge flows through the capacitor the positive and negative charges on the plates will attract each other and ultimately store charge. A capacitor can store electrical charge and discharge electrical charge.

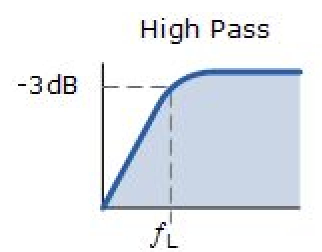
3.5 Low Pass Filter

A low pass filter only allows low frequency signals from 0Hz to its cutoff frequency point to pass while blocking those any higher. Using an operational amplifier, we can create an active low pass filter which provides amplification and gain control. The frequency response of a passive low pass filter can be seen on the right.



3.6 High Pass Filter

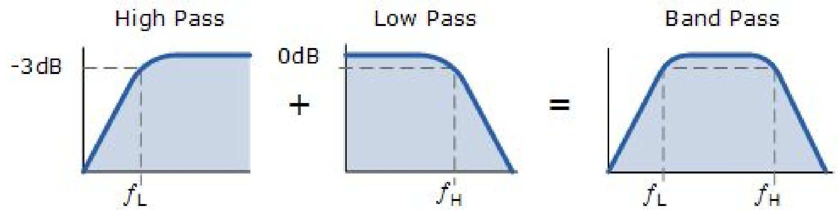
A high pass filter only allows high frequency signals from its cutoff frequency point and higher to infinity to pass through while blocking those any lower. Using an operational amplifier, we can create an



active low pass filter which provides amplification and gain control. The frequency response of a passive pass filter can be seen on the right.

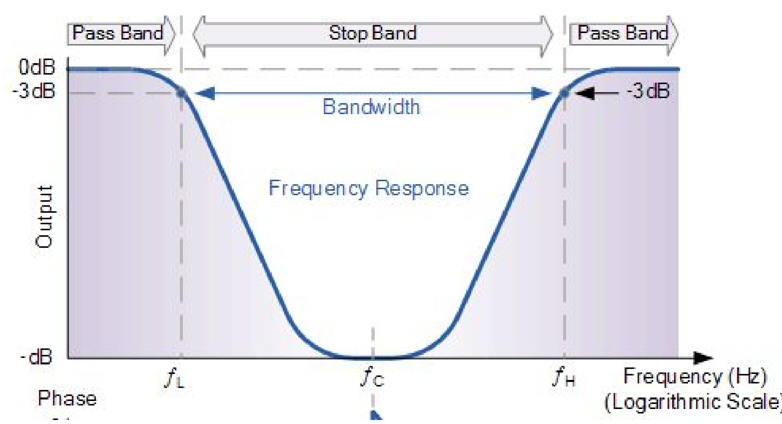
3.7 Band Pass Filter

A band pass filter allows signals falling within a certain frequency band setup between two points to pass through while blocking both the lower and higher frequencies on either side of this frequency band. The frequency response of a band pass filter is found by combining the response of a highpass filter and a low pass filter as seen below:



3.8 Notch Filter (Band Stop)

A band stop filter is the opposite of a band pass filter where it blocks frequencies falling within a certain frequency band setup between two points. The band stop filter allows only frequencies on either side of the frequency band. The frequency response of a band stop filter can be seen on the right.



3.9 The Frequency Domain

The frequency domain refers to the analysis with respect to frequency rather than time. A time-domain graph shows how a signal changes over time. A frequency domain graph shows how much of the signal lies within each given frequency band over a range of frequencies. A frequency-domain representation can also include information on phase shifts.

3.10 Bode Plots

A bode plot is a graph of the frequency response of a system which usually combines a bode magnitude plot expressing the magnitude (in decibels) of the frequency response and a bode phase plot expressing the phase shifts of a system.

4. Discussions and Results

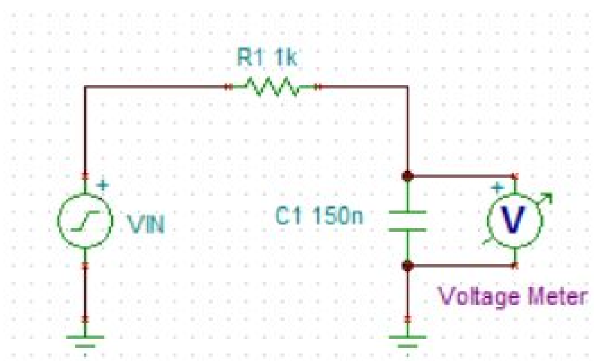
4.1 Passive low pass filter characteristics

Consider the following passive low pass filter,

$V_{IN} = 1Vp$

$R_1 = 1061\Omega$

$C_1 = 0.15\mu F = 150nF$



The cutoff frequency for this circuit can be found using the following equation:

$$f_c = \frac{1}{2\pi RC}$$

The resistor and capacitor values in this circuit were chosen to obtain a cutoff frequency (gain is -3dB down) of 1KHz.

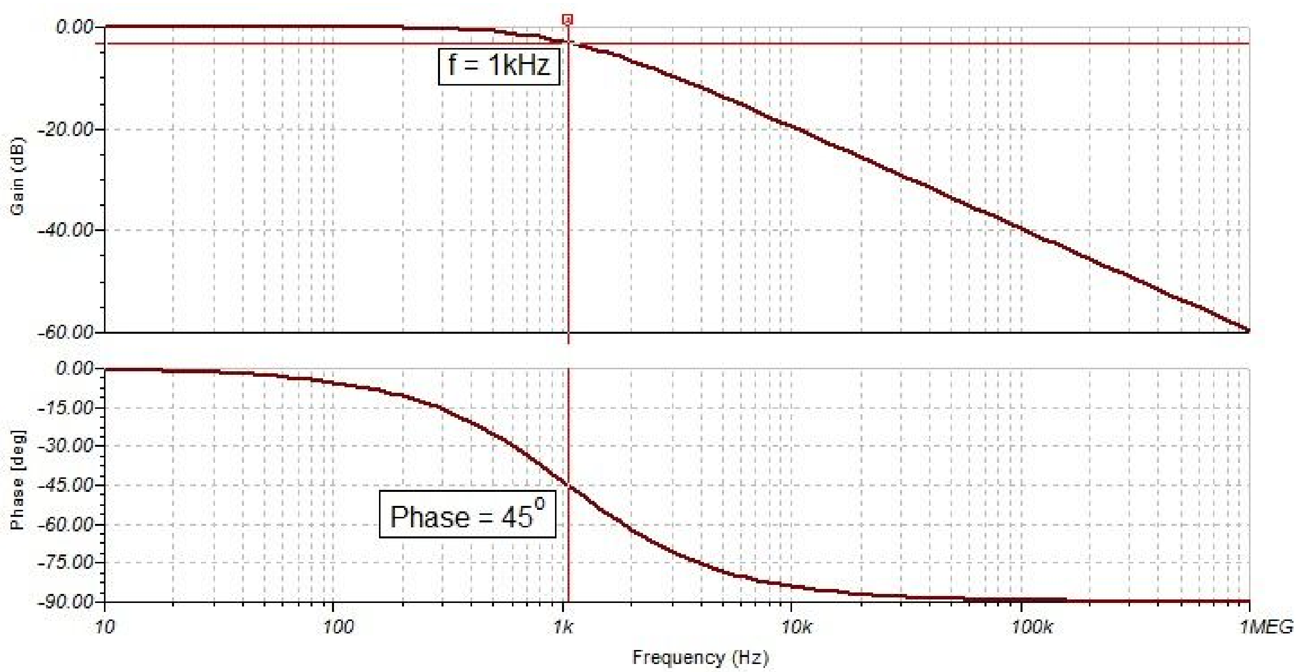
$$f_c = \frac{1}{2\pi(1061\Omega)(0.15\mu F)} = 1061Hz$$
$$f_c = 1061Hz$$

The phase at the cutoff frequency can be calculated as,

$$Phase\ Shift\ \phi = \tan^{-1}(\frac{1}{2\pi fRC})$$

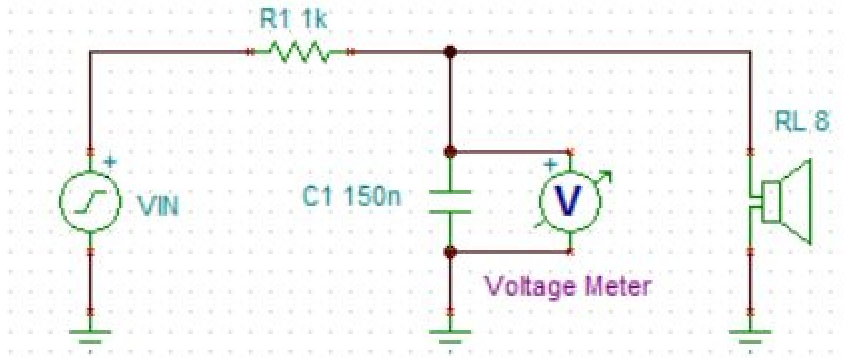
$$\phi = \tan^{-1}(\frac{1}{2\pi \cdot 1000Hz \cdot 1k\Omega \cdot 0.15\mu F}) \approx 45^{\circ}$$

The circuit is calculated to have a phase shift of 45° at the cutoff frequency.
The following is a bode plot for the low pass filter with no load (simulation),



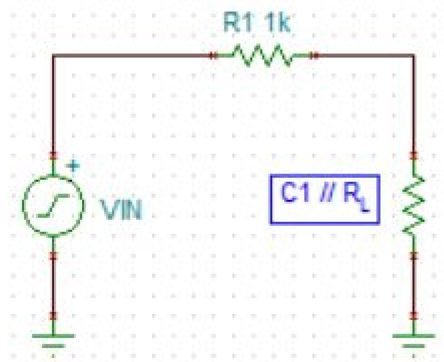
The simulation shows a cutoff frequency approximately at 1KHz (-3dB) and a phase shift of 45° at the cutoff frequency as per our calculations.

Consider the following passive low pass filter with a load attached,



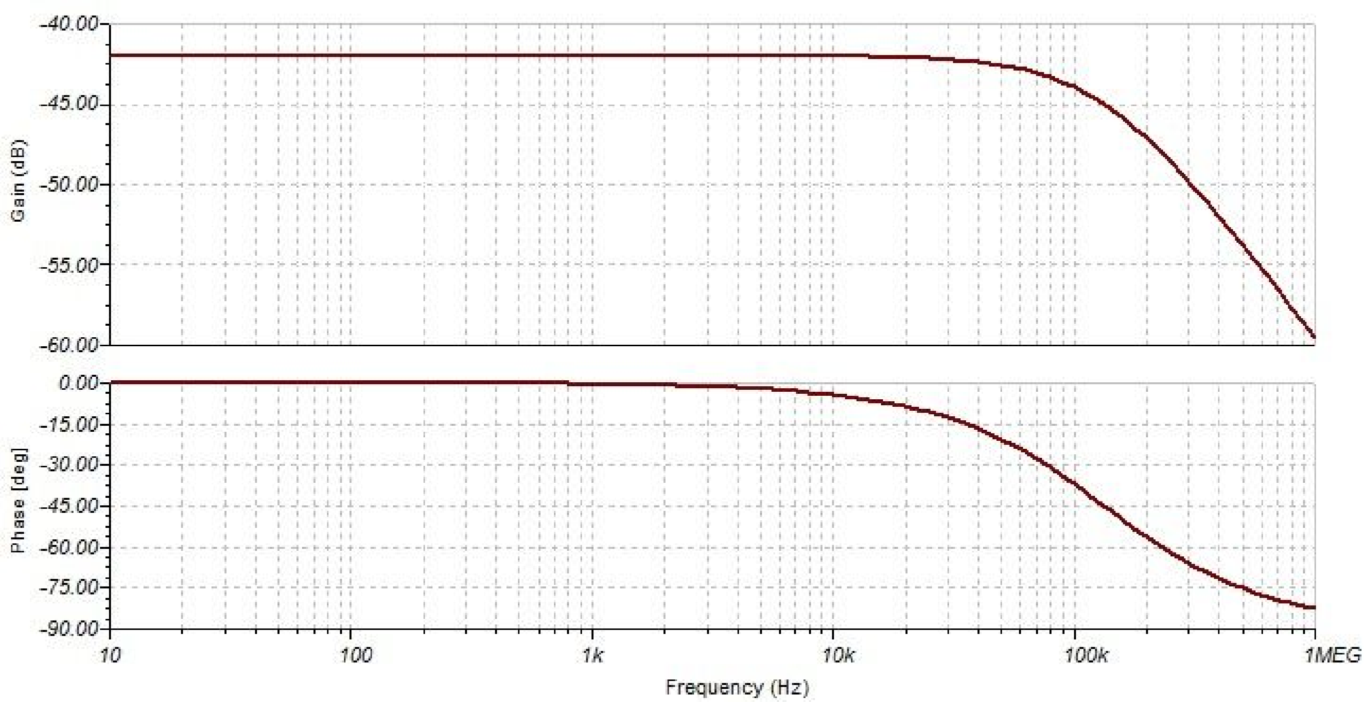
$$V_{IN} = 1Vp$$
$$R_1 = 1k\Omega$$
$$C_1 = 0.15\mu F$$
$$SP_1 = 8\Omega$$

The capacitor and load are connected in parallel. The circuit can be redrawn such as,



Now, R1 and the capacitor with load resistor in parallel can be visualized as a voltage divider.

The following is a bode plot for the low pass filter with a load attached (simulation),



The cutoff frequency for the circuit increased drastically when the load was attached to the circuit (and is no longer at -3dB).

To recalculate the cutoff frequency of the circuit, we need to find the transfer function of the circuit.

Transfer Function $H(j\omega) = \frac{v_o}{v_{in}}$

Where, $v_o = vin(\frac{output\ impedance}{total\ impedance})$

As discussed previously, R2 and the capacitor in parallel is a voltage divider with R1,

The impedance of a capacitor the s-domain can be written as, $\frac{1}{j\omega C}$

$$H(j\omega) = \frac{R_2 || \frac{1}{j\omega C}}{R_2 || \frac{1}{j\omega C} + R_1} = \frac{\frac{R_2}{j\omega C R_2 + 1}}{\frac{R_2}{j\omega C R_2 + 1} + R_1}$$

$$H(j\omega) = \frac{R_2}{R_2 + R_1 + j\omega R_1 R_2 C}$$

$$H(j\omega) = \frac{1}{(\frac{R_2 + R_1}{R_2}) + j\omega C R_1}$$

Rearranging the above equation,

$$H(j\omega) = \frac{1}{CR_1} \cdot \frac{1}{j\omega + \frac{R_1 + R_2}{CR_1 R_2}}$$

The cutoff frequency can be calculated as,

$$\omega = \frac{R_1 + R_2}{R_1 R_2 C}$$

$$2\pi f = \frac{R_1 + R_2}{R_1 R_2 C}$$

$$f = \frac{\frac{R_1 + R_2}{R_1 R_2 C}}{2\pi}$$

The values of components,

$R_1 = 1k\Omega$

$R_2(speaker) = 8\Omega$

$C_1 = 0.15\mu F$

Cutoff frequency, $f = \dfrac{1k\Omega + 8\Omega}{2\pi \cdot 1k\Omega \cdot 8\Omega \cdot (0.15 \cdot 10^{-6}F)}$

$f = 133690Hz = 133.7kHz$

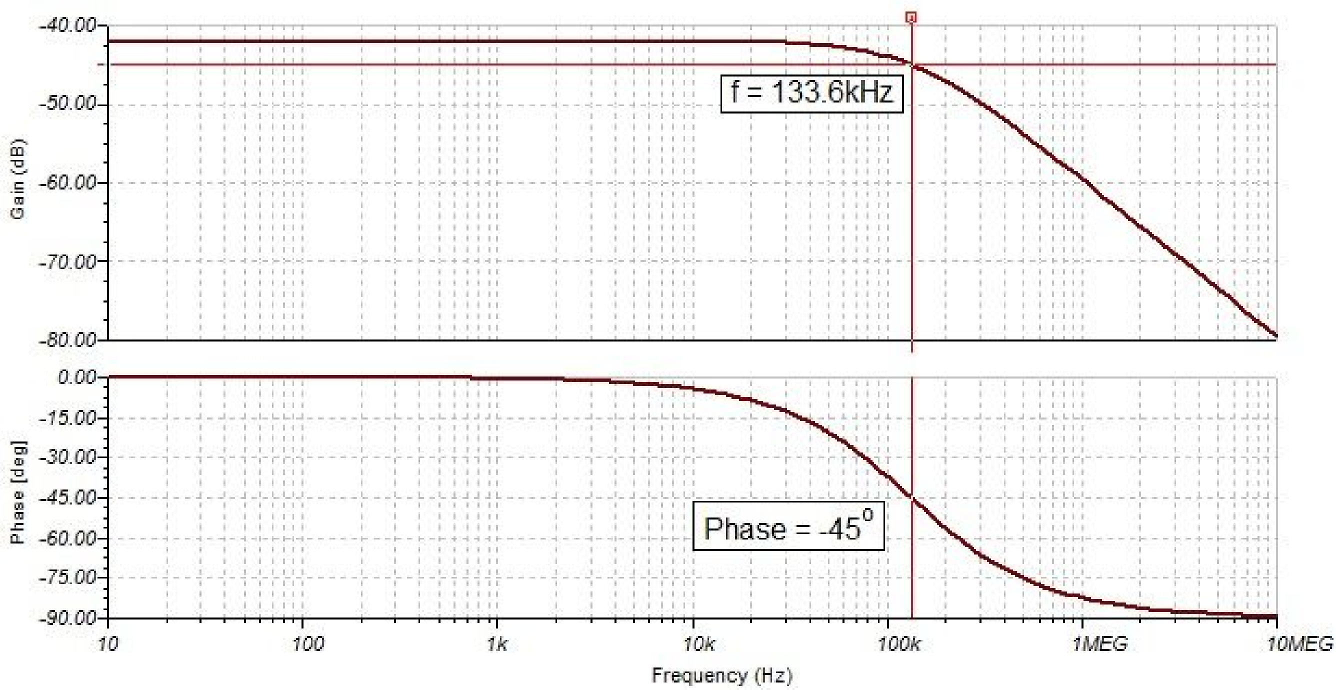
Calculating for the phase angle,

$$Phase\ Angle\ \phi = -\tan^{-1}(\frac{\omega}{\omega_0}) = -\tan^{-1}(\frac{\omega}{\frac{R_1+R_2}{R_1R_2C}}) = -\tan^{-1}(\frac{\omega C_1 R_1 R_2}{R_1 + R_2})$$

$$\phi = -\tan^{-1}(\frac{(2\pi \cdot 133.6kHz)(0.15 \cdot 10^{-6}F)(1k\Omega)(8\Omega)}{1k\Omega + 8\Omega})$$

$\phi \approx 45^\circ$

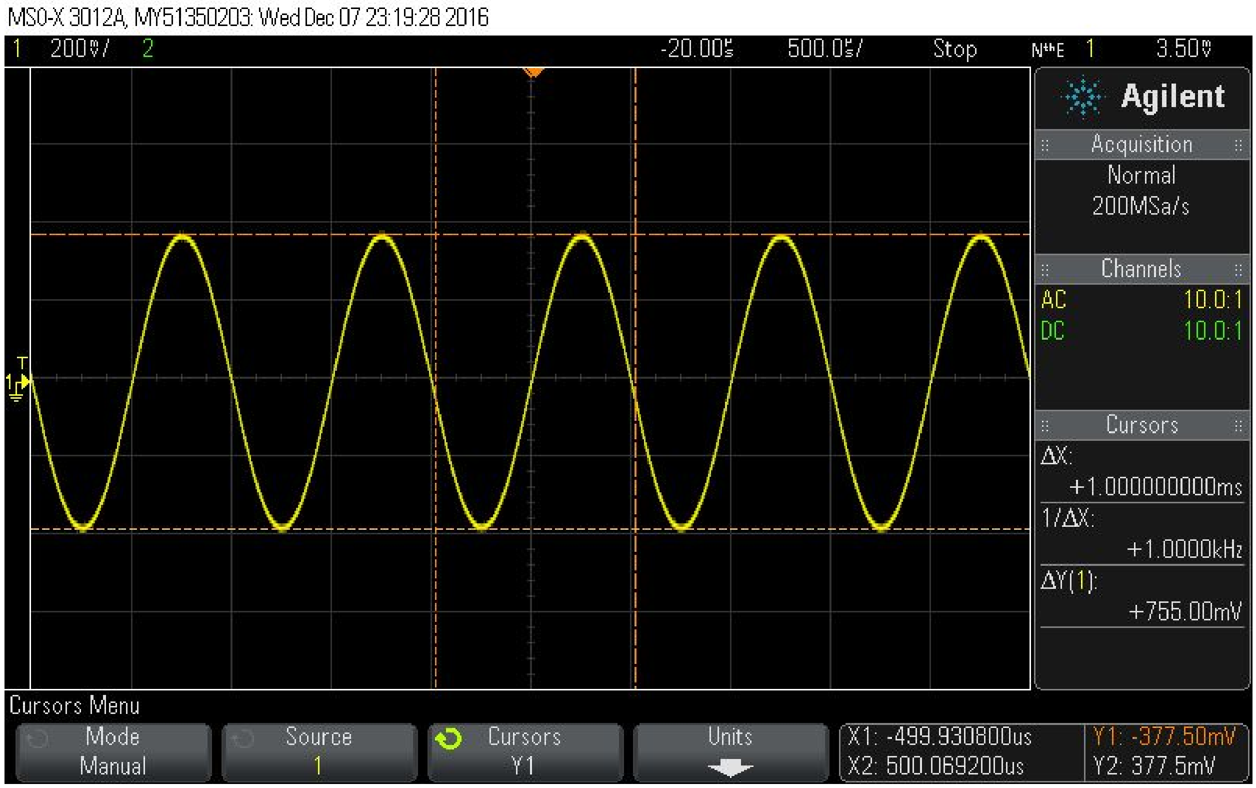
Refer to the following bode plot of the loaded low pass filter,



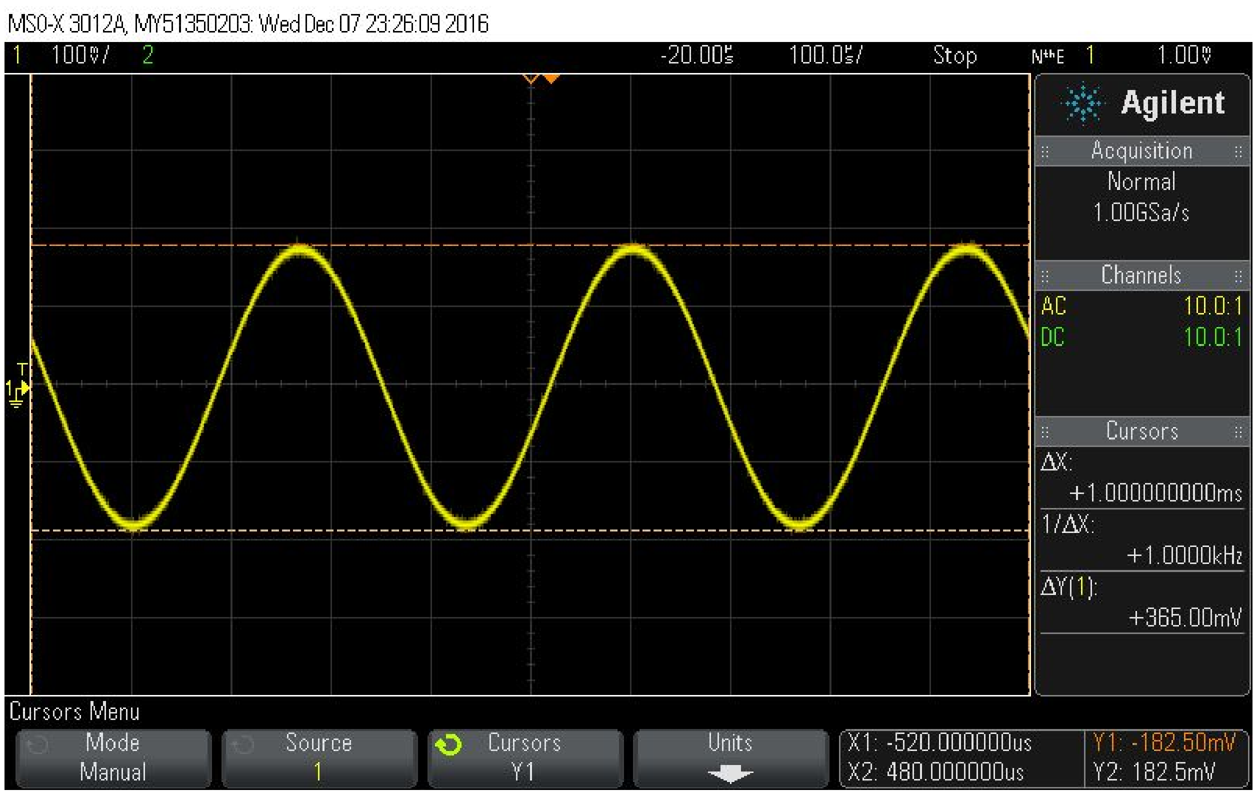
The mathematical calculation for the cutoff frequency and phase angle at the cutoff frequency corresponds with the simulation results exactly.

The output voltage decreases when the load is connected.

The following is the measured output of the unloaded passive low pass filter with an input of a 1kHz sine wave (time domain) (output 755mV),

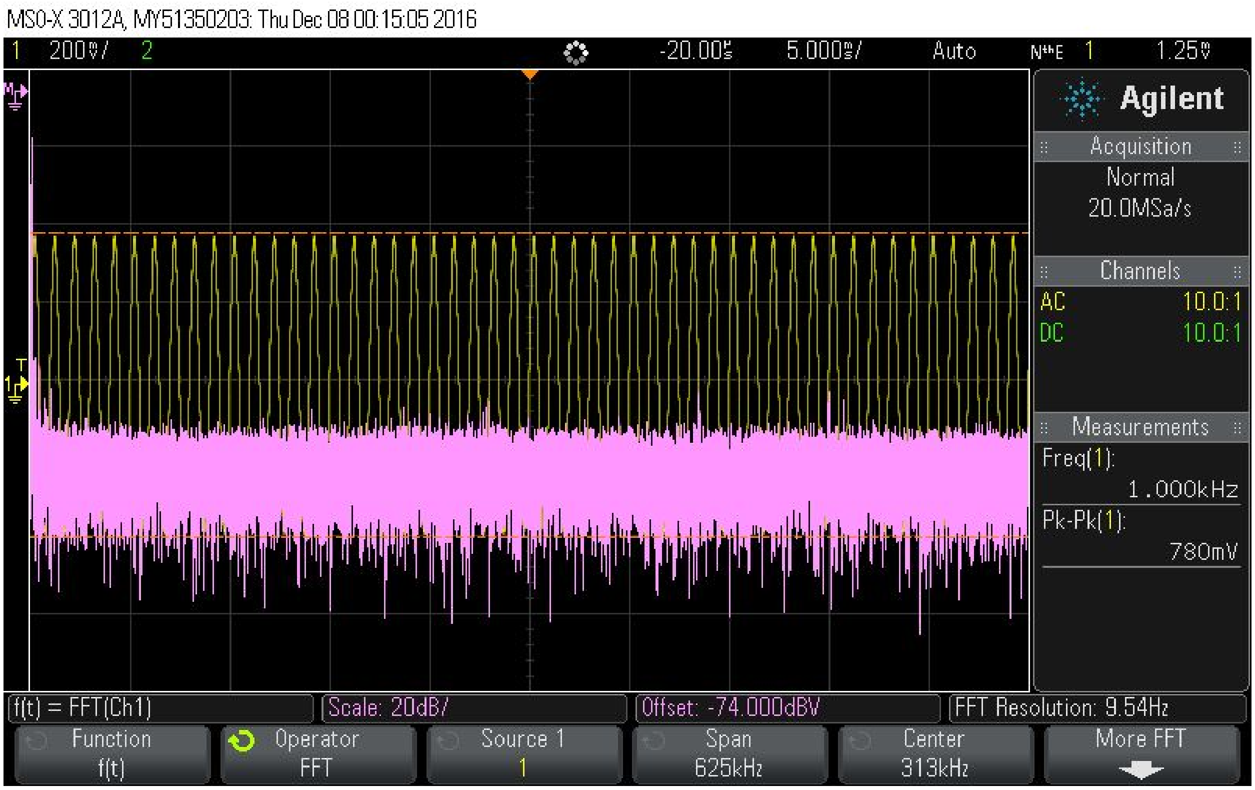


The following is the measured output of the unloaded passive low pass filter with an input of a 3kHz sine wave (time domain) (output 365mV),



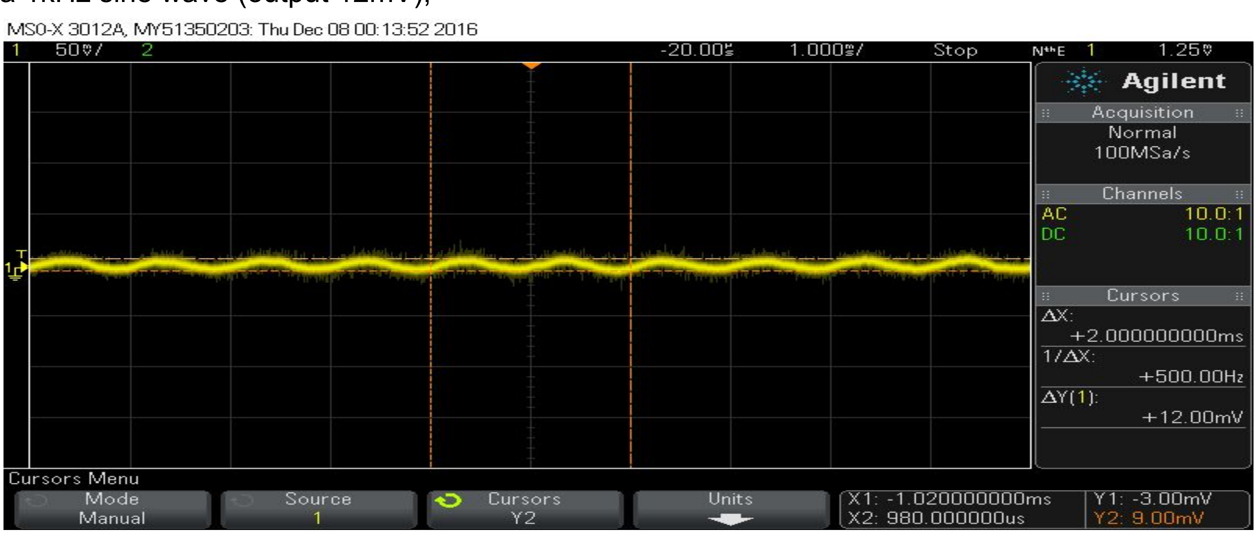
The passive low pass filter can be seen to be functioning because the amplitude of the output voltage decreases as the frequency is increased (755mV to 365mV).

The following is the frequency domain plot of the unloaded passive low pass filter using the FFT function on the oscilloscope and a 1kHz sine wave input,



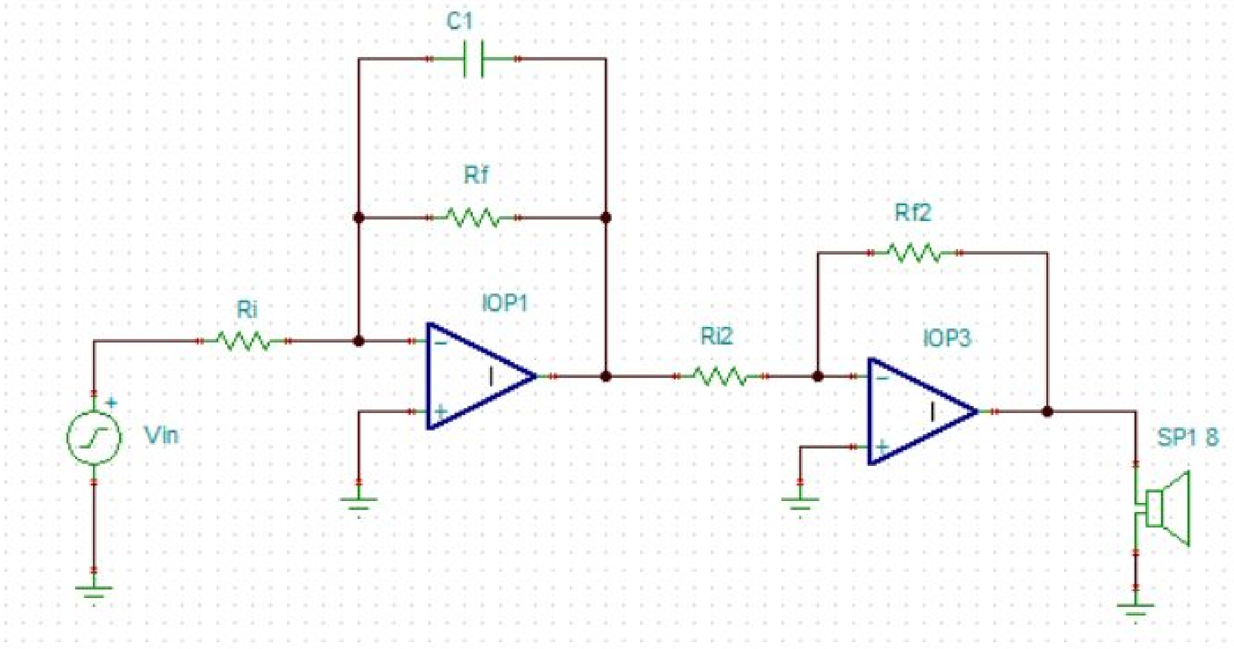
The gain can be seen to be decreasing as frequency increases.

When the speaker (load) is connected (connected in parallel to the capacitor) to the passive low pass filter the output voltage decreases drastically (in correlation to calculations) with an input of a 1kHz sine wave (output 12mV),



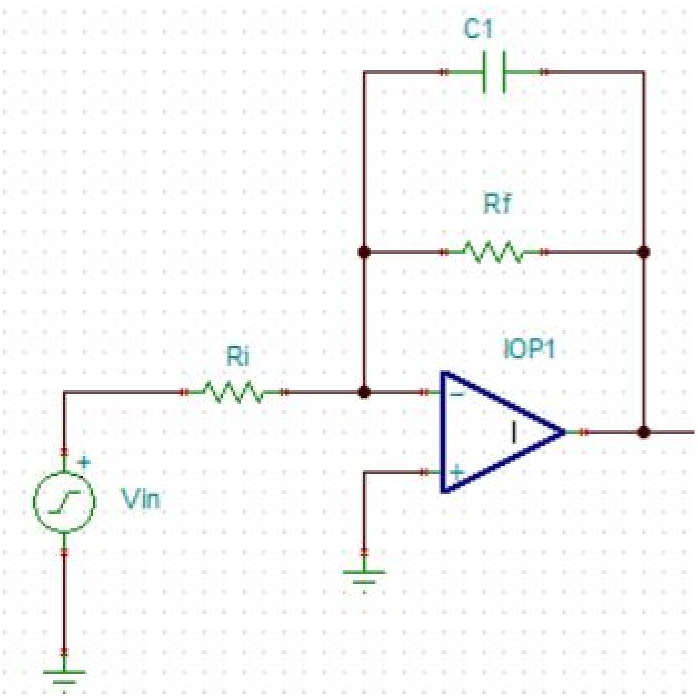
4.2 Active first order low-pass filter

Consider the following active first order low-pass filter circuit,



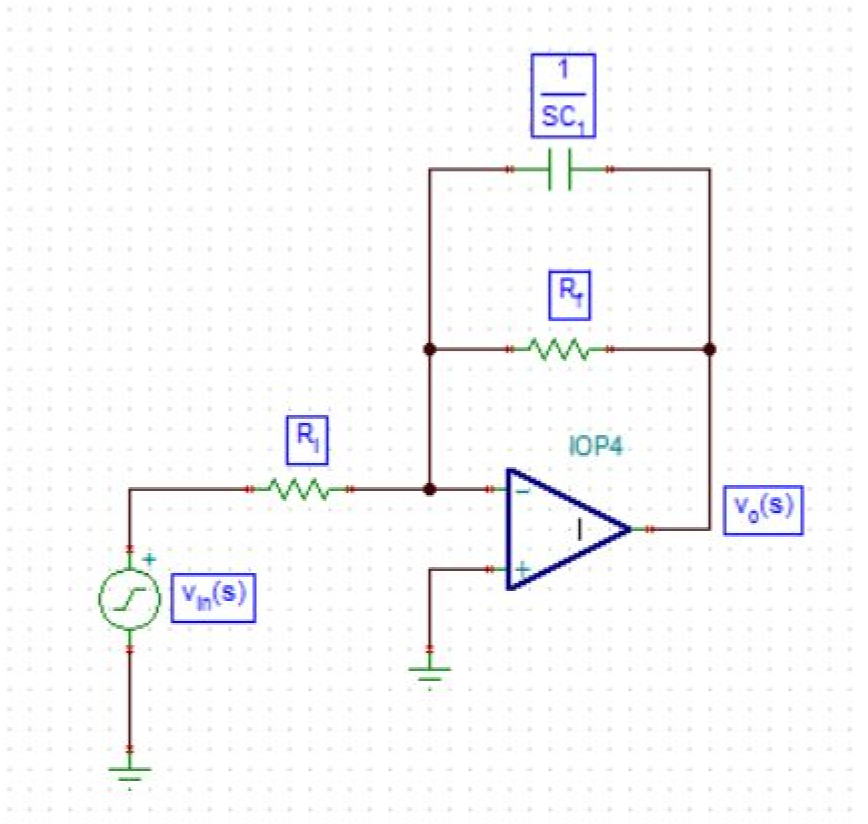
The second operational amplifier is added as buffer to separate the speaker (load) from the filter.

Focusing on the first part of the circuit,



The active low-pass filter should provide a DC gain of +14dB and a cutoff frequency of 2kHz.

We need to find the transfer function for this portion of the circuit. The circuit in the s-domain,

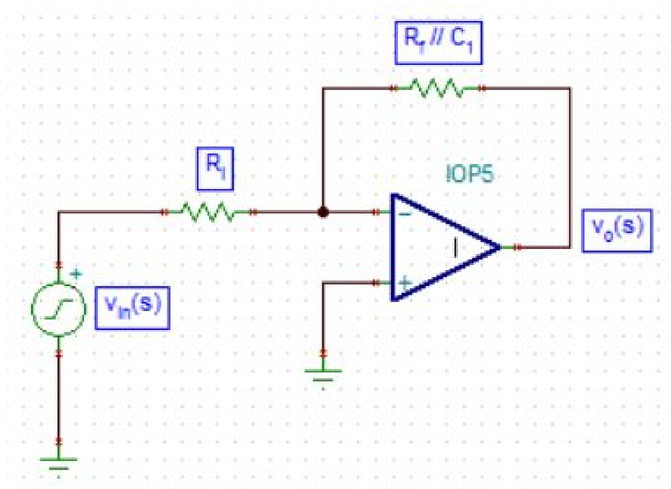


The impedance of the capacitor and the feedback resistor can be combined in parallel,

$$Z_f = \frac{1}{j\omega C_1}||R_f$$

$$Z_f = \frac{\frac{1}{j\omega C_1} \cdot R_f}{R_f + \frac{1}{j\omega C_1}}$$

$$Z_f = \frac{R_f}{j\omega C_1 R_f + 1}$$



For an inverting operational amplifier circuit in the s-domain,

$$H(j\omega) = \frac{v_{out}}{v_{in}}$$

$$H(j\omega) = -\frac{Z_f}{Z_{in}}$$

$$H(j\omega) = -\frac{\frac{R_f}{j\omega C_1 R_f + 1}}{R_i}$$

Therefore, the transfer function for this circuit,

$$H(j\omega) = -\frac{R_f}{R_i(1 + j\omega C_1 R_f)}$$

The gain for +14dB can be calculated such as,

$$+14db = 20log(\frac{R_f}{R_{in}})$$

We need to pick values for the resistors and capacitors such that the circuit provides a DC gain of +14dB and a cutoff frequency of 2000Hz.

Let $R_f = 10k\Omega$

$$+14dB = 20log(\frac{10k\Omega}{R_{in}})$$
$$R_{in} = 1995\Omega$$

After rounding the value of R_{in}, the values for our resistors are,

$$R_f = 10k\Omega$$
$$R_{in} = 2k\Omega$$

The gain of our circuit can be recalculated such as,

$$A_v = 20log(\frac{10k\Omega}{2k\Omega}) = +13.9794$$

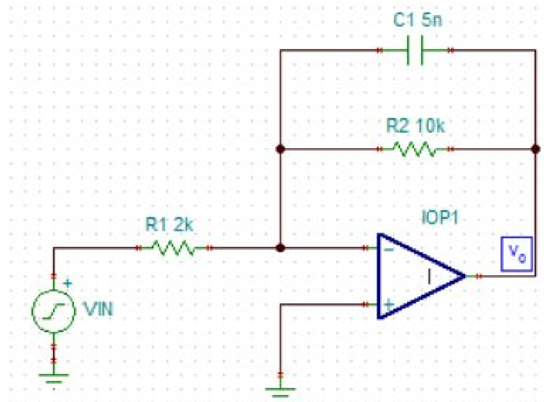
To obtain the value of the capacitor in the circuit we will set the transfer function equal to the output gain and solve for capacitance,

Note, $\omega = 2\pi f$

$f = 2000Hz$

$$+13.9794 = -\frac{R_f}{R_i(1 + j\omega C_1 R_f)} = -\frac{10k\Omega}{2k\Omega(1 + (2\pi 2000Hz) \cdot 10k\Omega \cdot C_1)}$$

$C_1 = 5.11 \cdot 10^{-9} F \approx 0.005 \mu F$

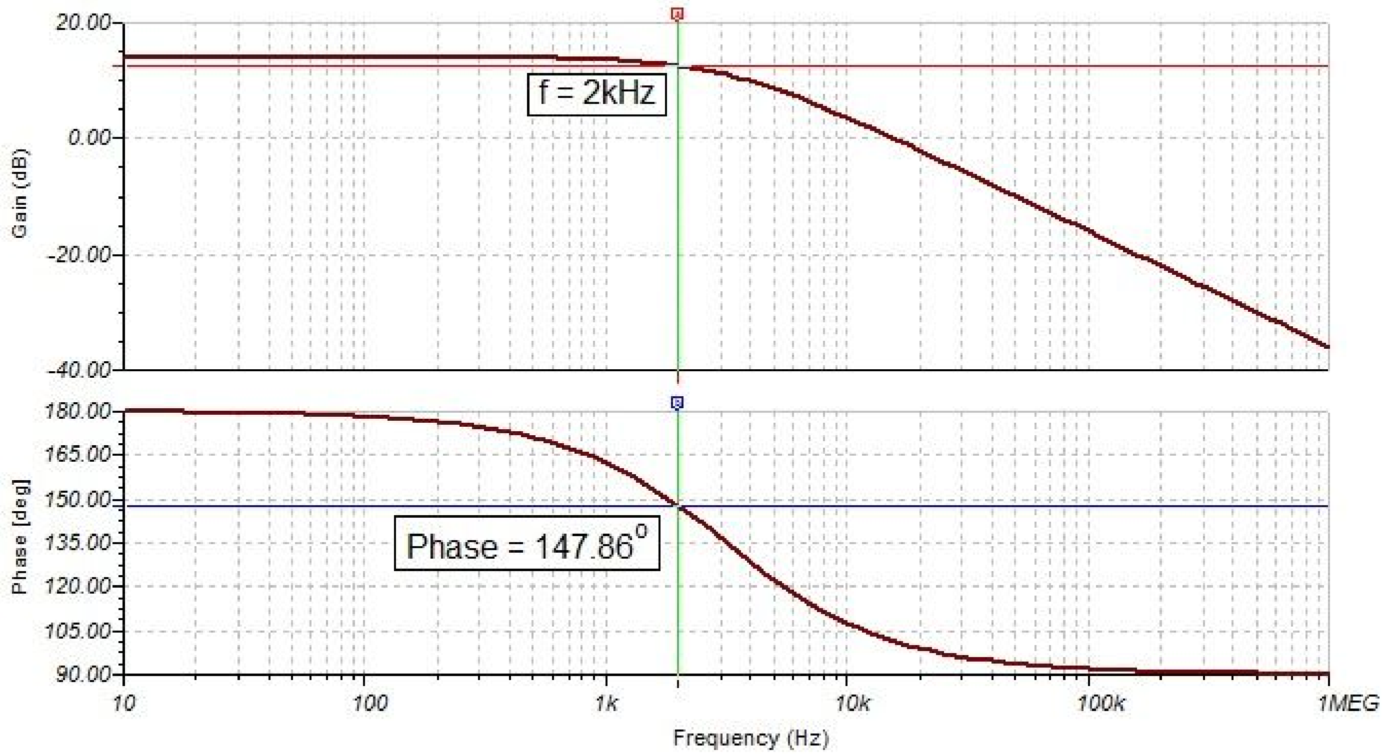


The component values of this circuit are,
$R_f = 10k\Omega$
$R_i = 2k\Omega$
$C_1 = 0.005\mu F$

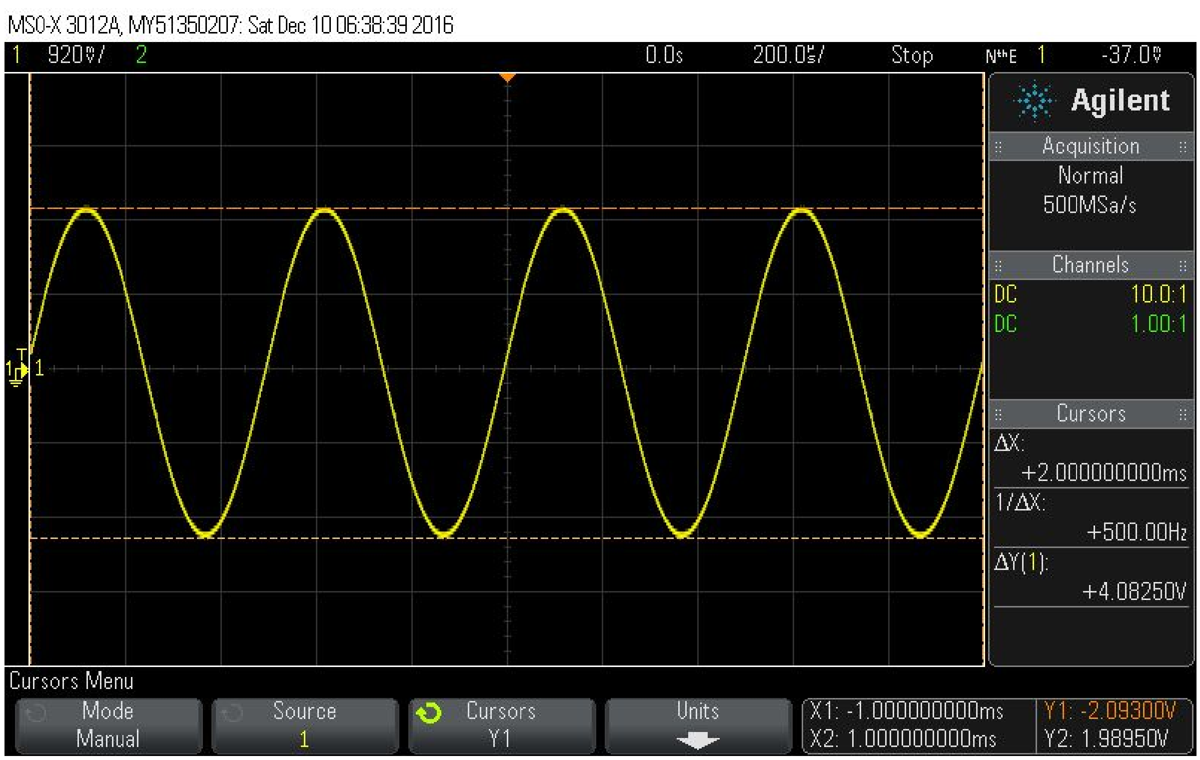
Calculating for phase angle at the cutoff frequency,

$$Phase\ Angle\ \phi = 180° - \tan^{-1}(2\pi \cdot f C_1 R_f)$$
$$\phi = 180° - \tan^{-1}(2\pi \cdot 2000Hz \cdot 0.005 \cdot 10^{-6} F \cdot 10k\Omega)$$
$$\phi = 180° - 32.14°$$
$$\phi = 147.86°$$

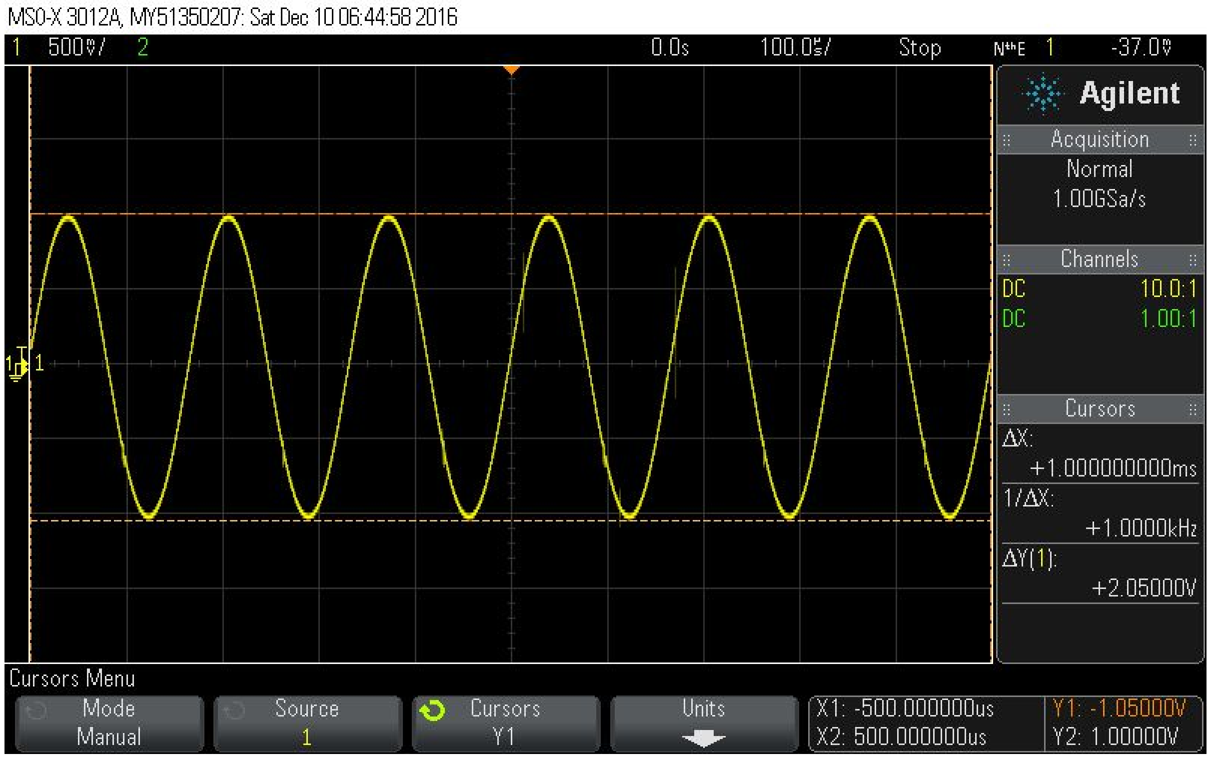
Analyzing the transfer function, the gain will decrease as the frequency is increased passed 2kHz.
Simulation (bode plot) of the calculated circuit with a cutoff frequency of 2kHz, +14dB gain, and a phase angle of 147.86° at the cutoff frequency as calculated,



The following is the measured output of the active low pass filter in the time domain using an input sine wave at a frequency of 2kHz (4.08V output),

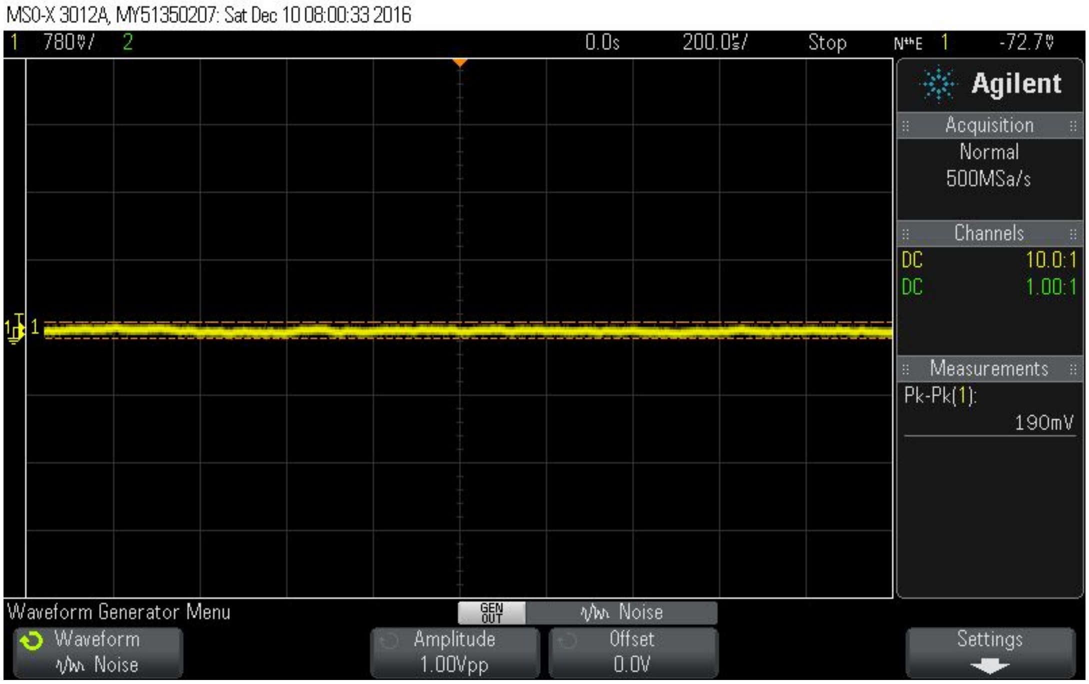


The following is the measured output of the active low pass filter in the time domain using an input sine wave at a frequency of 6kHz (2.05V),

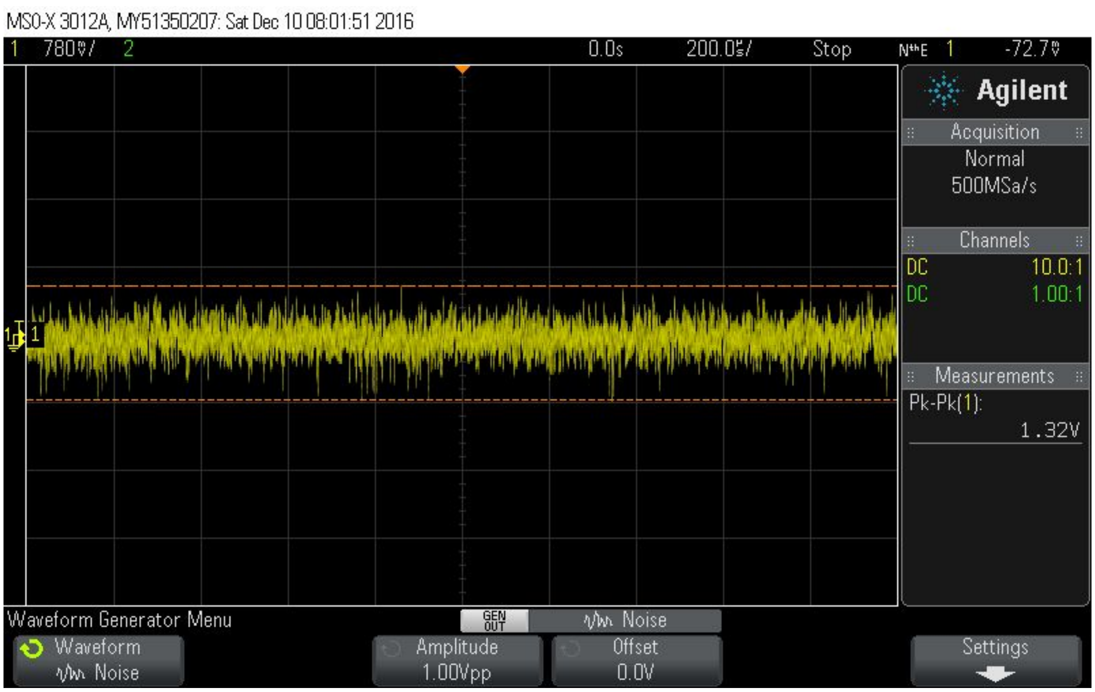


Comparing the two time domain plots we can determine the active low pass filter is functioning because the output voltage is decreasing as the frequency is increased past the cutoff (4.08V to 2.05V).

The following is the time domain plot of the active low pass filter when white noise is the input (output 190mV),

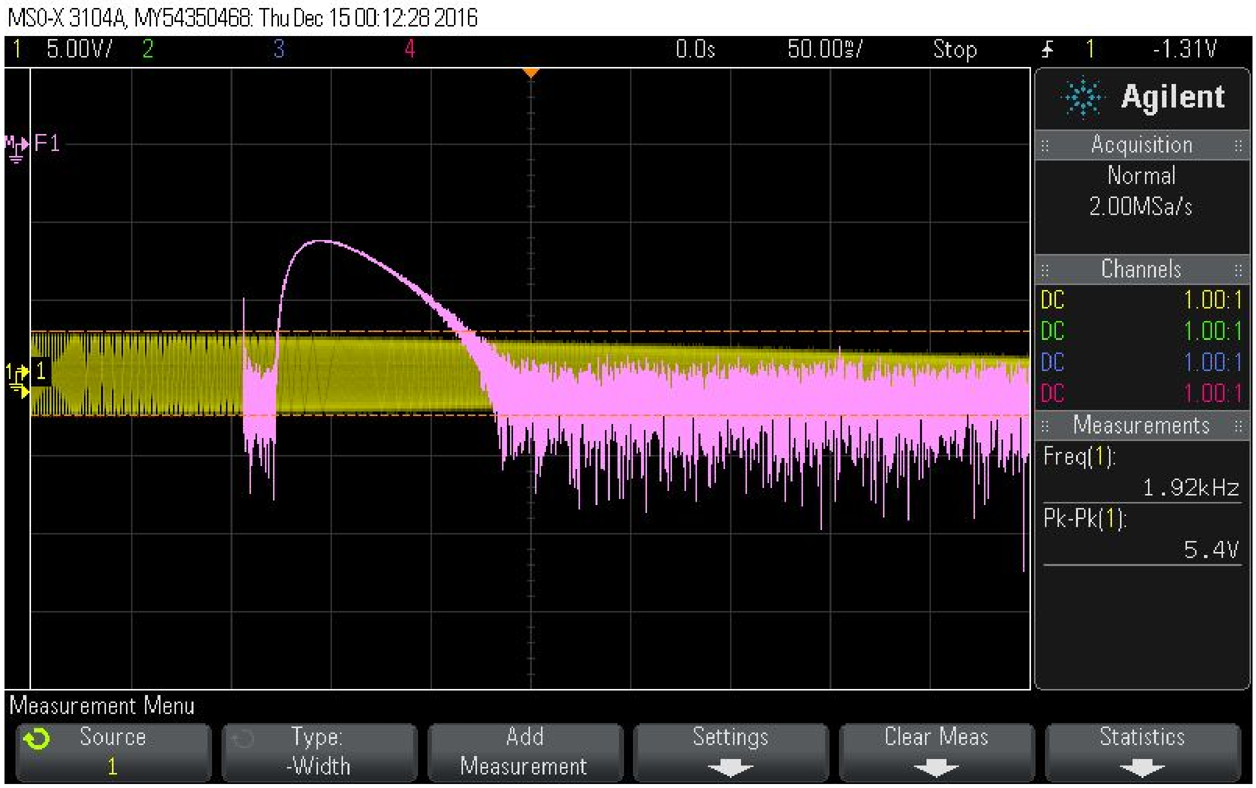


The following is the domain plot of the active low pass filter when the capacitor (C1) is removed from the circuit and white noise is used as the input (output 1.32V),



The output voltage increases when the capacitor is removed from the filter. This is happening because the circuit is no longer filtering certain frequencies and allows for a more broader spectrum of frequencies to pass through and therefore cause an increased output voltage.

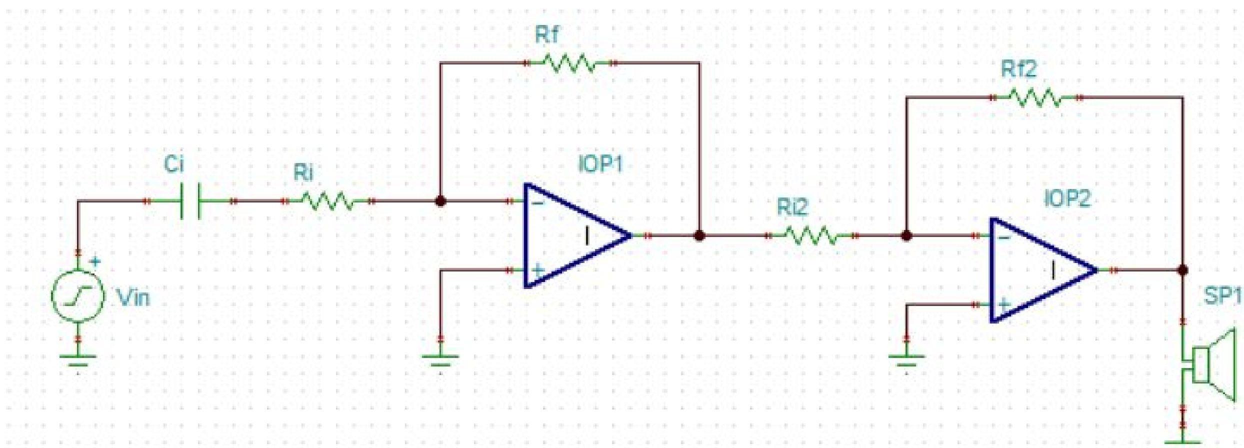
The following is the measured frequency domain plot of the active low pass filter using the FFT function on the oscilloscope. The sweep function on the function generator was used to sweep a sine wave from a frequency of 100Hz to 6kHz on a logarithmic scale (1s interval). The sweep function was used to get a more accurate frequency domain approximation on the oscilloscope,



The frequency domain approximation on the oscilloscope (FFT) corresponds with the simulations and mathematical calculations.

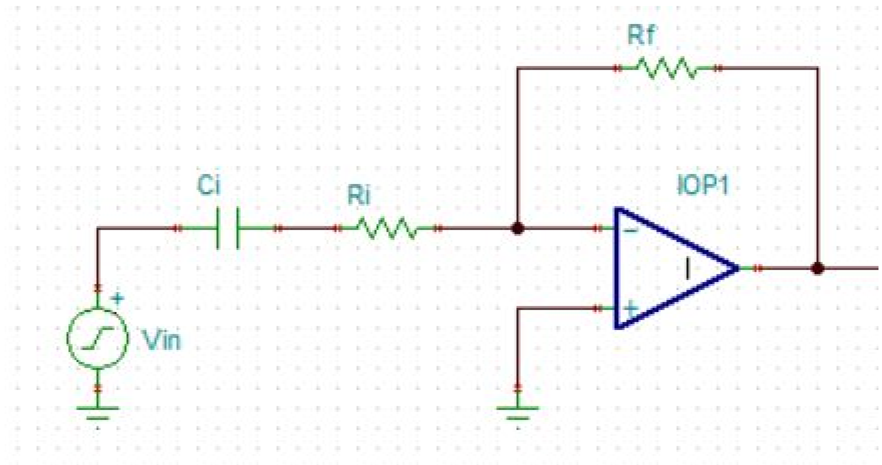
4.3.1 Active first-order high pass filter

Consider the following active high-pass filter circuit,

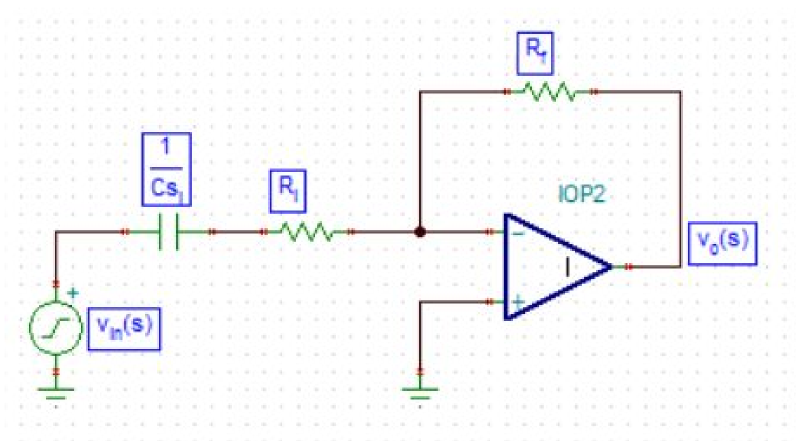


The second operational amplifier is added as buffer to separate the speaker (load) from the filter.

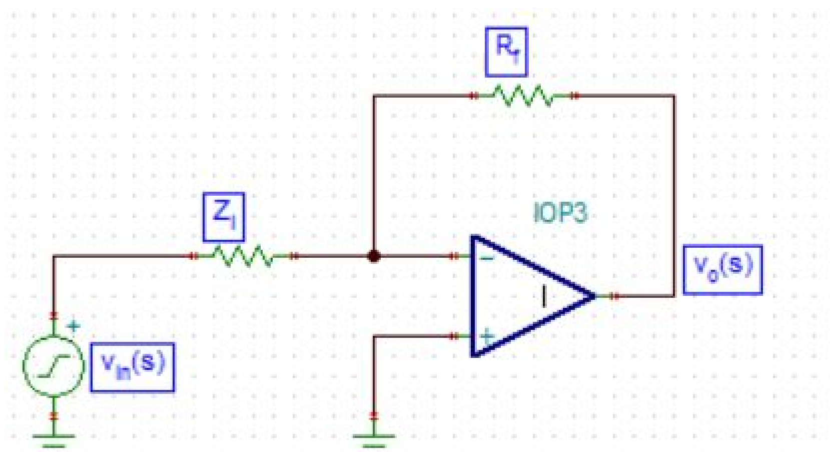
Focusing on the first portion of the circuit (high pass filter),



The active high-pass filter should provide a DC gain of +14dB and a cutoff frequency of 500Hz. We need to find the transfer function for this portion of the circuit. The circuit in the s-domain,



The input capacitor and input resistor impedance can be combined in series. The circuit can be rewritten such as,



Where Z_i equals the impedance of the input resistor and capacitor in series.

The impedance of the input resistor and capacitor in series,

$$Z_i = R_i + \frac{1}{j\omega C_i}$$

The impedance of the feedback portion is simply the feedback resistor,

$$Z_f = R_f$$

For an inverting opamp circuit in the s-domain,

$$H(j\omega) = -\frac{Z_f}{Z_i}$$

Therefore, the transfer function for the circuit is,

$$H(j\omega) = \frac{-R_f}{R_i + \frac{1}{j\omega C_i}} = \frac{-j\omega C_i R_f}{1 + j\omega RiC_i}$$

The gain for +14dB can be calculated such as,

$$+14dB = 20log(\frac{R_f}{R_i})$$

We need to pick values for the resistors and capacitor such that the circuit provides a DC gain of +14dB and a cutoff frequency of 500Hz.

Let $R_f = 100k\Omega$

$$+14dB = 20log(\frac{100k\Omega}{R_{in}})$$

$$R_{in} = 19952\Omega$$

After the rounding the value of R_{in}, the values for our resistors are,

$$R_{in} = 20k\Omega$$
$$R_f = 100k\Omega$$

The gain of our circuit can be recalculated such as,

$$A_v = 20log(\frac{100k\Omega}{20k\Omega}) = +13.9794$$

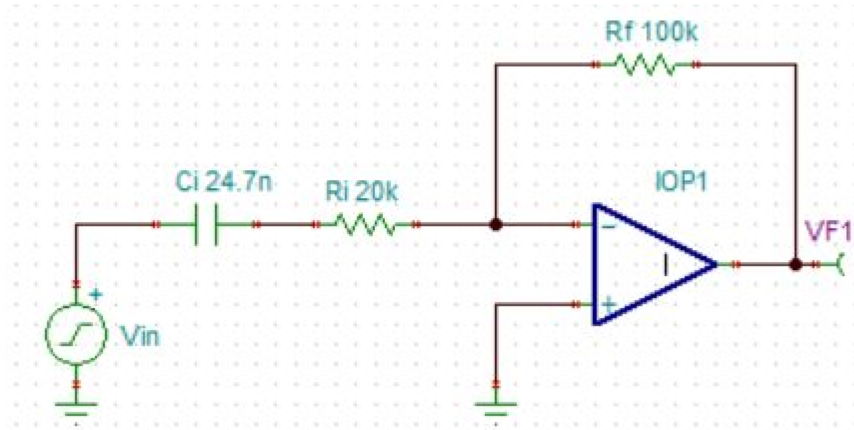
To obtain the value of the capacitor in the circuit we will set the transfer function equal to the output gain and solve for capacitance.

Note, $\omega = 2\pi f$

$$f = 500Hz$$

$$+13.9794 = \frac{-\omega C_i R_f}{1+\omega RiC_i} = \frac{-(2\pi \cdot 500Hz)(100k\Omega)(C_i)}{1+(2\pi \cdot 500Hz)(20k\Omega)(C_i)}$$

$C_i = 2.47 \cdot 10^{-8} F = 0.0247\mu F$



The component values of this circuit are,

$R_f = 100k\Omega$
$R_i = 20k\Omega$
$C_i = 0.025\mu F$

Calculating for phase angle at the cutoff frequency,

$$Phase\ Angle\ \phi = 180° - \tan^{-1}(\frac{1}{2\pi \cdot fR_iC_i})$$

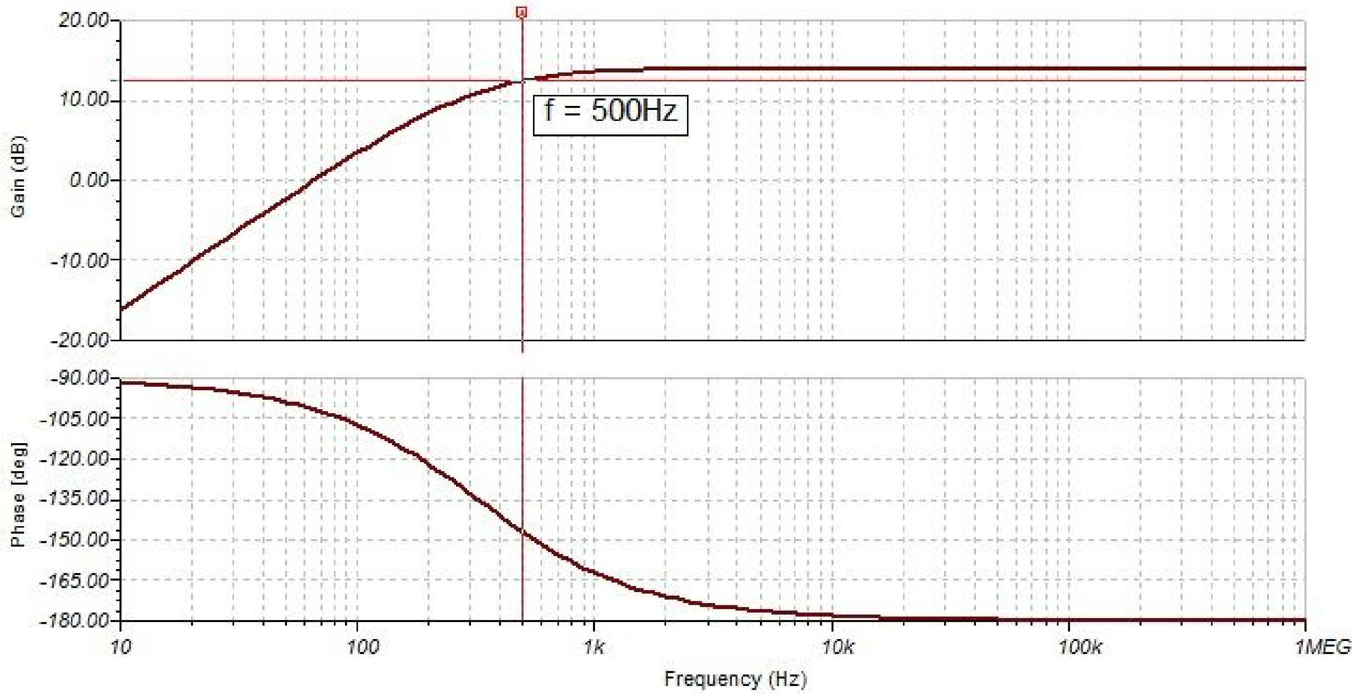
$$\phi = 180° - \tan^{-1}(2\pi \cdot 500Hz \cdot 0.0247\mu F \cdot 20k\Omega)$$

$$\phi = 180° - 32.7958°$$

$$\phi = 147.204°$$

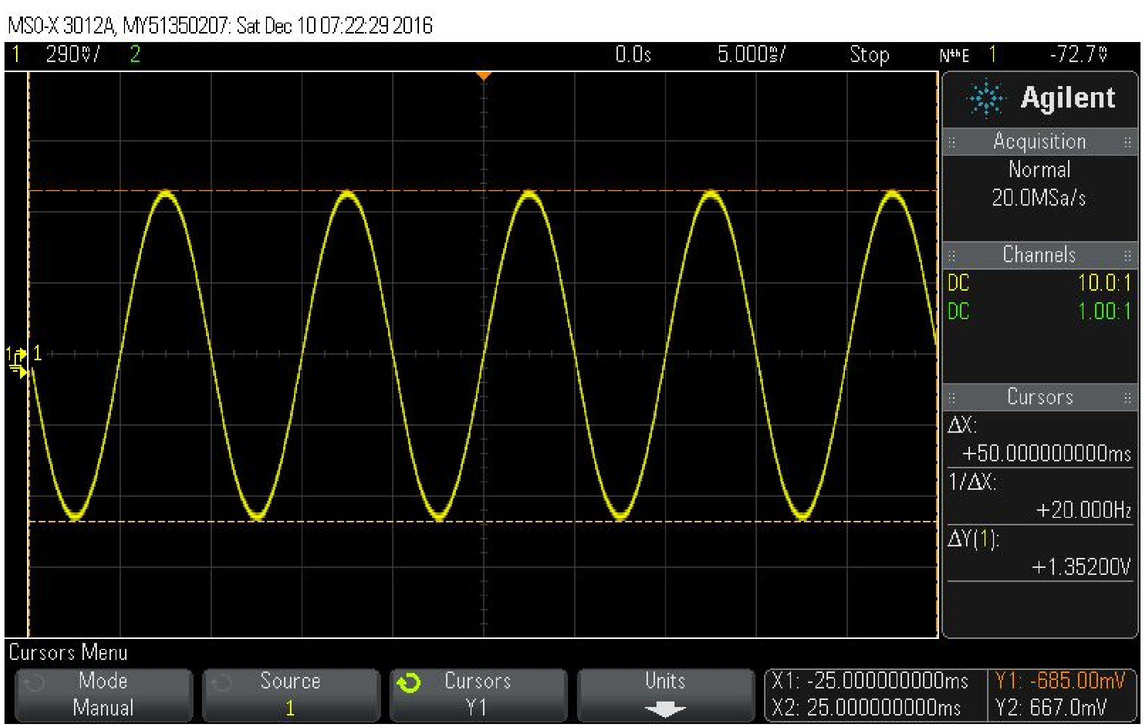
Analyzing the transfer function, the gain of the circuit will increase as frequency increases towards 500Hz.

Simulation (bode plot) of the calculated circuit with a cutoff frequency of 500Hz, +14dB gain, and a phase angle of 147.204° as calculated,

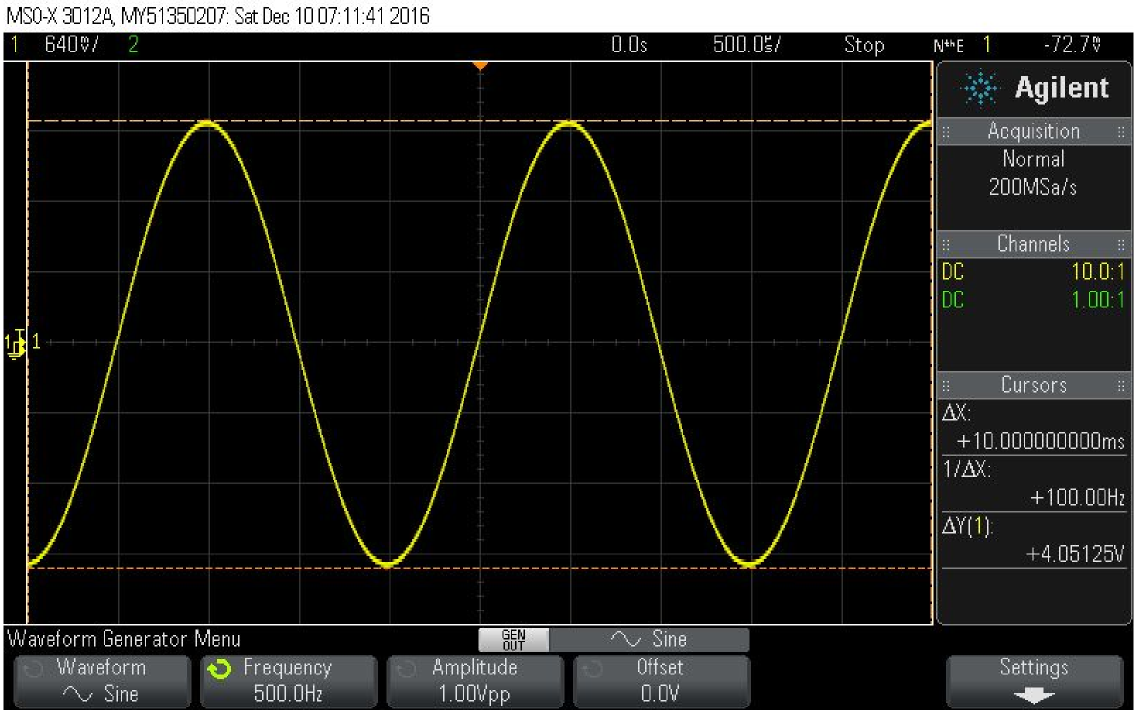


Mathematical calculations correspond with the simulations.

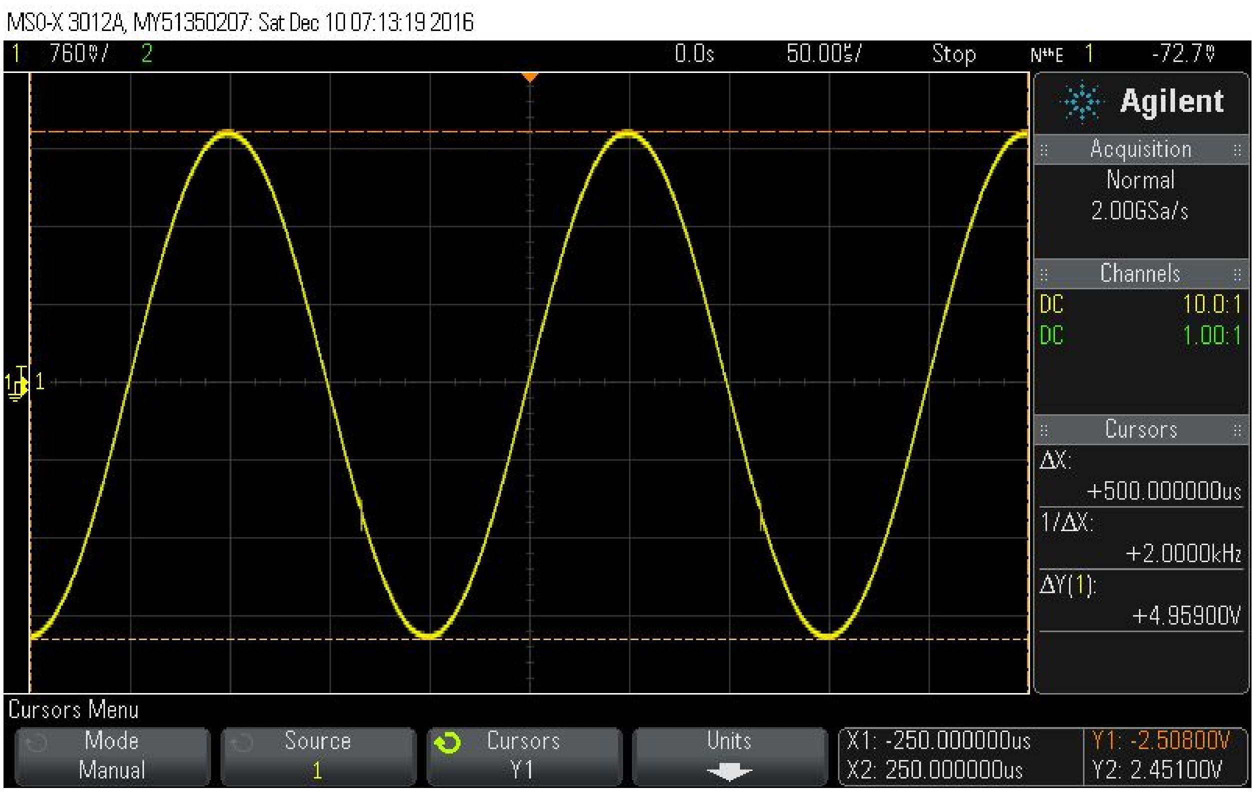
The following is the measured output of the active high pass filter in the time domain using an input sine wave at a frequency of 100Hz (output 1.35V),



The following is the measured output of the active high pass filter in the time domain using an input sine wave at a frequency of 500Hz (output 4.05V),

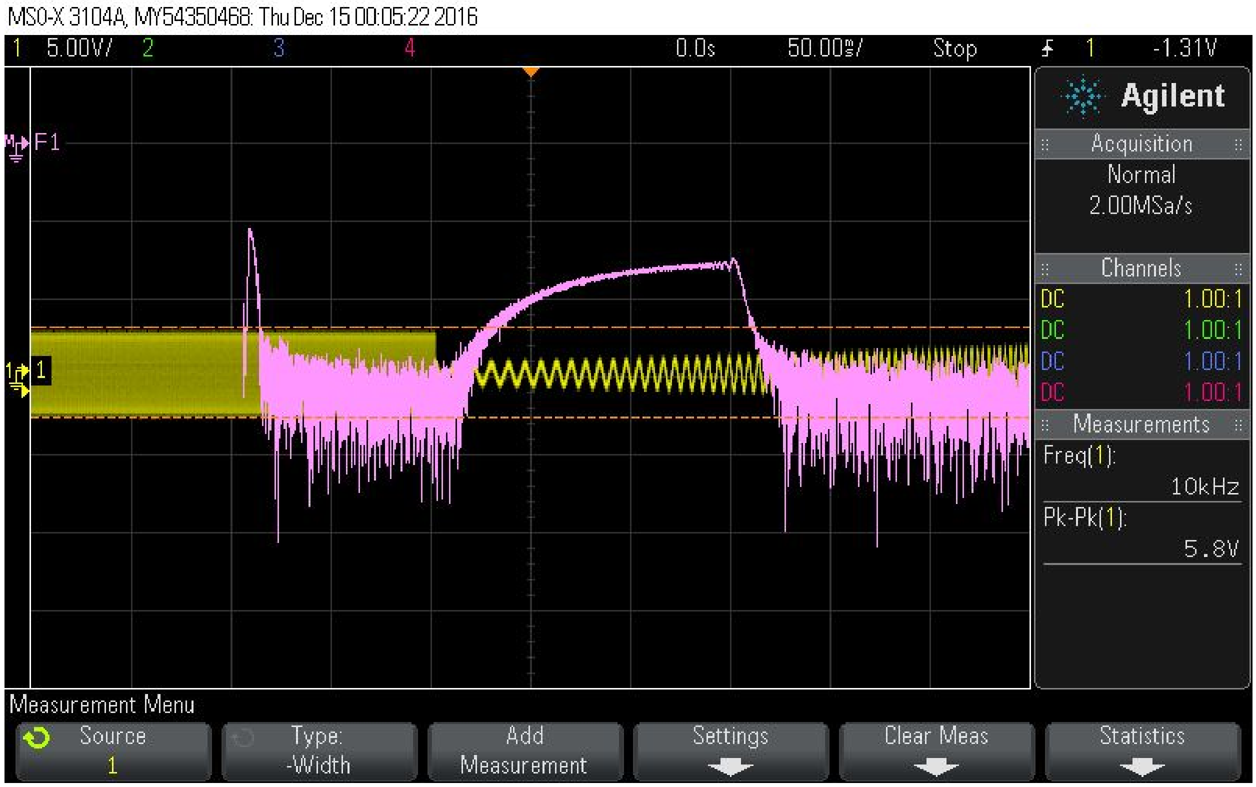


The following is the measured output of the active high pass filter in the time domain using an input sine wave at a frequency of 5kHz (output 4.95V),



The active high pass filter is functioning as seen by the output voltage increasing as the frequency increases towards the cutoff frequency.

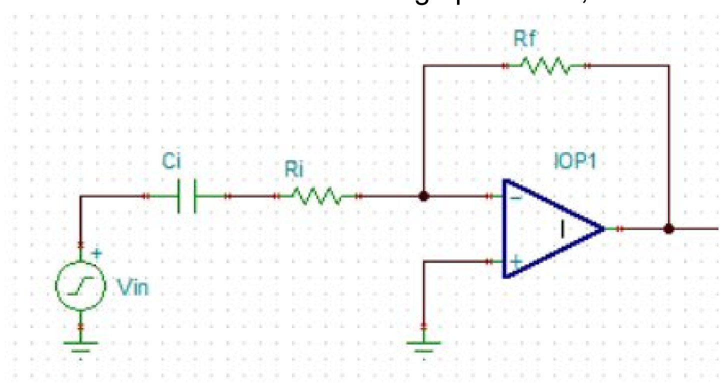
The following is the measured frequency domain plot of the active high pass filter using the FFT function on the oscilloscope. The sweep function on the function generator was used to sweep a sine wave from a frequency of 100Hz to 6kHz on a logarithmic scale (1s interval). The sweep function was used to get a more accurate frequency domain approximation on the oscilloscope,



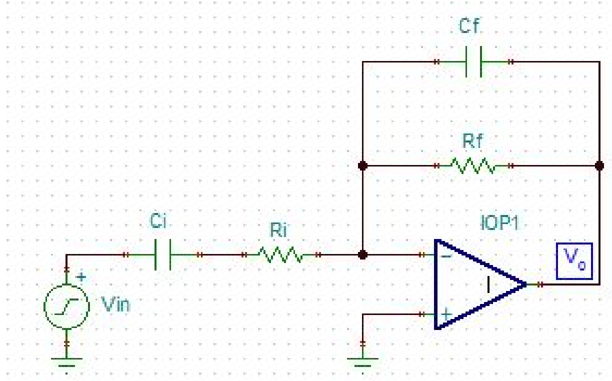
The frequency domain approximation on the oscilloscope (FFT) corresponds with the simulations and mathematical calculations.

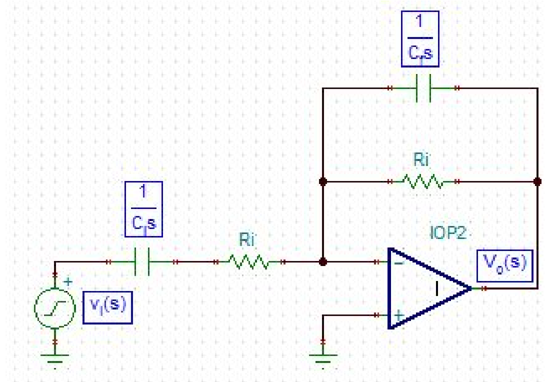
4.3.2 More: Additional capacitor in high pass filter

Recall the circuit for an active high pass filter,



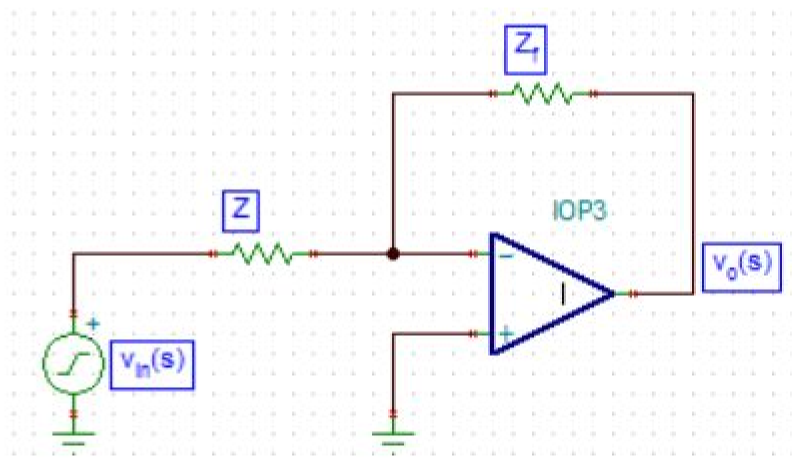
Now, an additional capacitor is added in parallel with the feedback resistor R_f (left),





We can calculate for a new transfer function for this circuit. The circuit in the s-domain can be rewritten as shown above (right).

The input capacitor and input resistor impedance can be combined in series. The feedback capacitor and feedback resistor impedance can be combined in parallel. The circuit can be rewritten such as,



Where Z_i equals the impedance of the input capacitor and input resistor; Z_f equals the impedance of the feedback capacitor and the feedback resistor.

The impedance of the feedback capacitor and feedback resistor in parallel,

$$Z_f = \frac{R_f}{1 + j\omega R_f C_f}$$

The impedance of the input capacitor and input resistor in series,

$$Z_i = R_i(1 + \frac{1}{j\omega R_i C_i})$$

For an inverting opamp circuit in the s-domain,

$$H(j\omega) = -\frac{Z_f}{Z_i}$$

$$H(j\omega) = \frac{-R_f}{(1 + j\omega R_f C_f)(1 + \frac{1}{j\omega R_i C_i})R_i}$$

$$H(j\omega) = \frac{\frac{-R_f}{R_i}}{(1 + j\omega R_f C_f)(1 + \frac{1}{j\omega R_i C_i})}$$

Therefore, the transfer function for this circuit is calculated such as,

$$H(j\omega) = \frac{-j\omega C_i R_f}{(j\omega C_i R_i + 1)(j\omega C_f R_f + 1)}$$

By analyzing the denominator of the transfer function we can see the circuit has two cutoff frequencies,

$$f_{c1} = \frac{1}{\omega R_i C_i} \; f_{c2} = \frac{1}{\omega R_f C_f}$$

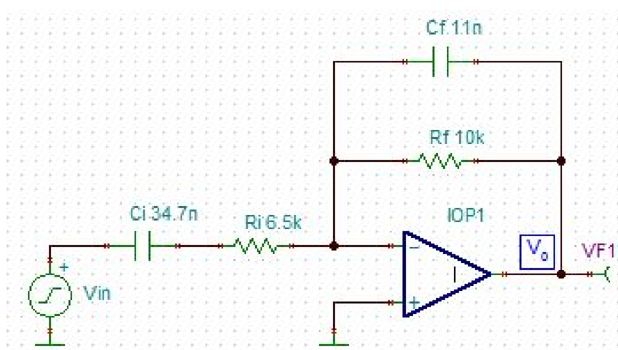
Where $\omega = 2\pi f$

The circuit has a DC gain of,

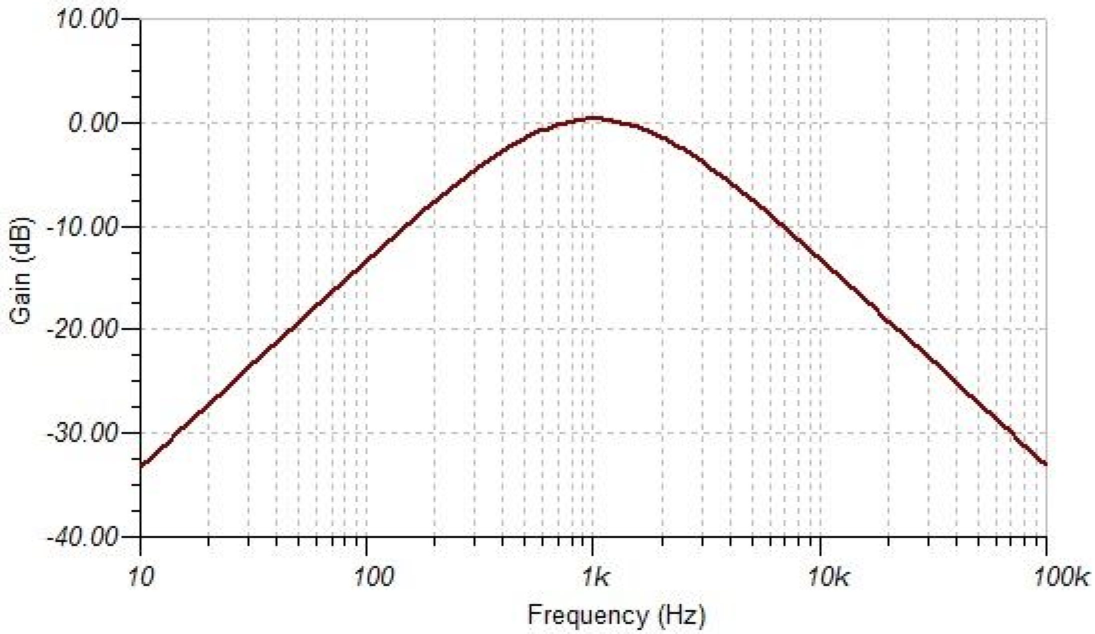
$$A_v = -\frac{R_f}{R_i}$$

Therefore we can see that adding a capacitor in parallel with the feedback resistor effectively transforms the high pass filter into a bandpass filter with two cutoff frequencies.

To prove this we will simulate a bode plot of the following circuit,



The simulated bode plot of the test circuit,



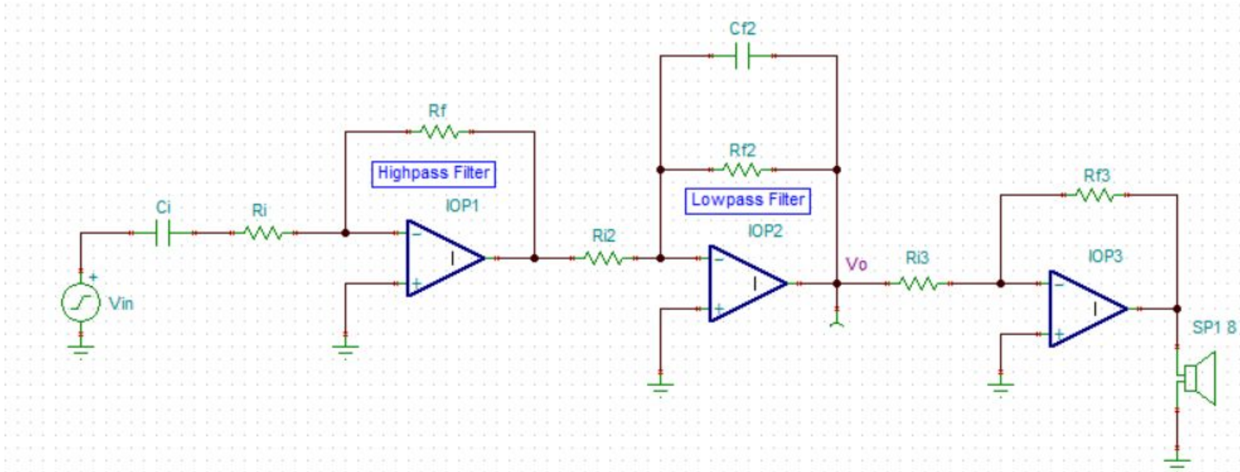
The simulation proves that adding the capacitor effectively creates a bandpass filter. A more in-depth analysis and discussion of bandpass will continue into the next section (4.4).

Besides using the additional capacitor to make a bandpass filter, the capacitor is especially important as it can be used to **limit DC gain** and extraneous noise at high frequencies by setting the cutoff frequency of the high pass portion to a high frequency value. A high pass filter will continue to allow frequencies above the cut-off frequency to pass towards infinity. Therefore, the DC gain of the circuit will also increase towards infinity. Having the additional capacitor in the high pass is important in limiting DC gain as the frequency is increased toward positive infinity.

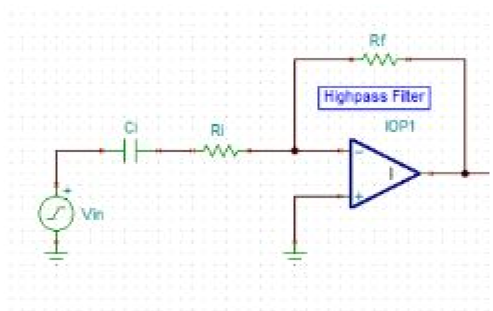
4.4 Bandpass filter (BPF)

As discussed in the previous section (4.3.2) we can make a simpler bandpass filter by adding a capacitor in parallel with the feedback resistor. However, in order to demonstrate simplistic fundamental understanding we will brute forcing a bandpass filter by doubling up a low pass filter and a high pass filter (the most basic and easy-to-understand method [subjective]).

Combining the active low-pass filter and active high-pass filter from section 4.2 and section 4.3 will result in a bandpass filter,



The third operational amplifier is added as buffer to separate the speaker (load) from the filter. Each stage (highpass stage, lowpass stage) has a gain of +14dB. We want to distribute the gain for each stage so the final output voltage isn't too large (+14dB was too loud for our tastes during testing).



High Pass Filter Stage:

Recall the transfer function for the highpass filter,

$$H(j\omega) = \frac{-R_f}{R_i + \frac{1}{j\omega C_i}} = \frac{-j\omega C_i R_f}{1 + \omega RiC_i}$$

We will set the gain for the first stage to be +3.75dB. The gain for for +3.75dB can be calculated such as,

$$+3.75dB = 20log(\frac{R_f}{R_i})$$

Let $R_f = 10k\Omega$

$$+3.75dB = 20log(\frac{10k\Omega}{R_i})$$

$R_i = 6493\Omega \approx 6500\Omega$

After rounding the value of R_i, the values for our resistors are,

$R_i = 6500\Omega$

$R_f = 10k\Omega$

The gain of our circuit can be recalculated such as,

$$A_v = 20log(\frac{10k\Omega}{6.5k\Omega}) = +3.74$$

To obtain the value of the capacitor in the circuit we will set the transfer function equal to the output gain and solve for capacitance when the **cutoff frequency for this stage is 500Hz**.

Note, $\omega = 2\pi f$

$f = 500Hz$

$$+3.74 = \frac{-j\omega C_i R_f}{1 + j\omega RiC_i} = \frac{-(2\pi \cdot 500Hz)(10k\Omega)(C_i)}{1 + (2\pi \cdot 500Hz)(6.5k\Omega)(C_i)}$$

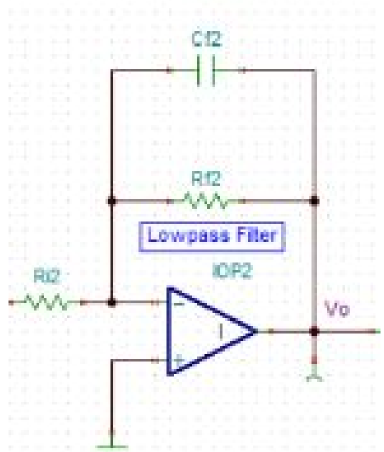
$C_i = 3.47 \cdot 10^{-8}F = 0.0347\mu F$

The component values of this circuit stage (highpass) are,

$R_f = 10k\Omega$

$R_i = 6.5k\Omega$

$C_i = 0.0347\mu F$



Low Pass Filter Stage:

Recall the transfer function for the lowpass filter,

$$H(j\omega) = -\frac{R_{f2}}{R_{i2}(1 + j\omega C_{f2} R_{f2})}$$

We will set the gain for the first stage to be +3.75dB. The gain for +3.75dB can be calculated such as,

$$+3.75dB = 20log(\frac{R_{f2}}{R_{i2}})$$

Let $R_f = 10k\Omega$

$$+3.75dB = 20log(\frac{10k\Omega}{R_{i2}})$$
$$R_{i2} = 6493\Omega \approx 6500\Omega$$

After rounding the value of R_{i2}, the values for our resistors are,

$$R_{i2} = 6500\Omega$$
$$R_{f2} = 10k\Omega$$

The gain of our circuit can be recalculated such as,

$$A_v = 20log(\frac{10k\Omega}{6.5k\Omega}) = +3.74$$

To obtain the value of the capacitor in the circuit we will set the transfer function equal to the output gain and solve for capacitance **when the cutoff frequency for this stage is 2000Hz**.

Note, $\omega = 2\pi f$

$f = 2000Hz$

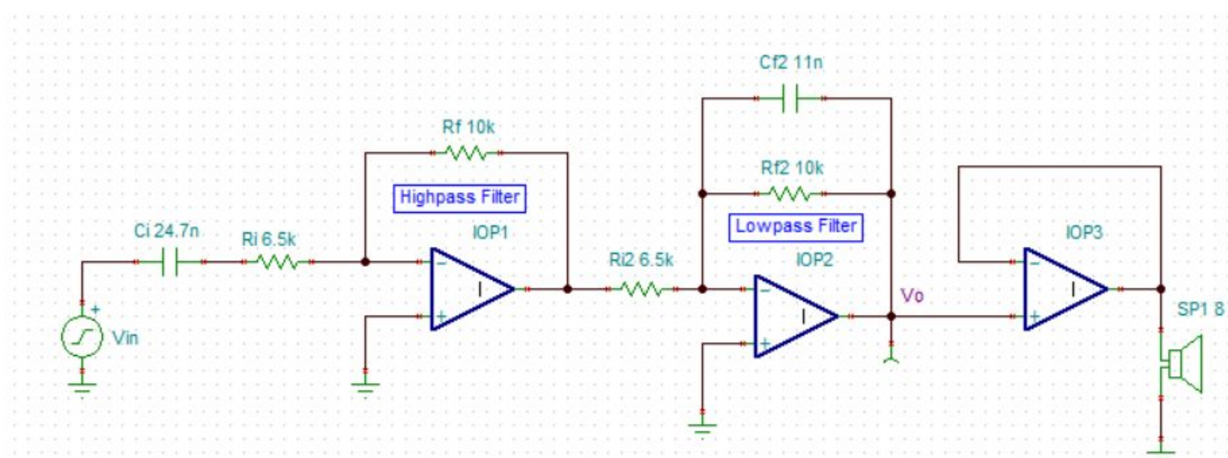
$$+3.74 = -\frac{R_{f2}}{R_{i2}(1 + j\omega C_{f2}R_{f2})} = -\frac{10k\Omega}{6.5k\Omega(1 + (2\pi 2000Hz) \cdot 10k\Omega \cdot C_{f2})}$$

$$C_{f2} = 1.123 \cdot 10^{-8}F = 0.0112\mu F$$

The component values of this circuit stage (lowpass) are,

$$R_{f2} = 10k\Omega$$
$$R_{i2} = 6.5k\Omega$$
$$C_{f2} = 0.011\mu F$$

The final bandpass filter circuit with the stages combined,



The final components of this complete circuit are,
(highpass)
$R_f = 10k\Omega$
$R_i = 6.5k\Omega$
$C_i = 0.0347\mu F$
(lowpass)
$R_{f2} = 10k\Omega$
$R_{i2} = 6.5k\Omega$
$C_{f2} = 0.011\mu F$

The phase angle of this bandpass filter circuit can be calculated by,

$$Phase\ Angle\ \phi_{final} = \phi_{lowpass} - \phi_{highpass}$$

Calculating for the phase angle of the low pass filter stage,

$$Phase\ Angle\ \phi_{lowpass} = 180^\circ - \tan^{-1}(2\pi \cdot fC_{f2}R_{f2})$$
$$\phi_{lowpass} = 180^\circ - \tan^{-1}(2\pi \cdot 2000Hz \cdot 0.011 \cdot 10^{-6}F \cdot 10k\Omega)$$
$$\phi_{lowpass} = 180^\circ - 54.117^\circ$$
$$\phi_{lowpass} = 125.883^\circ$$

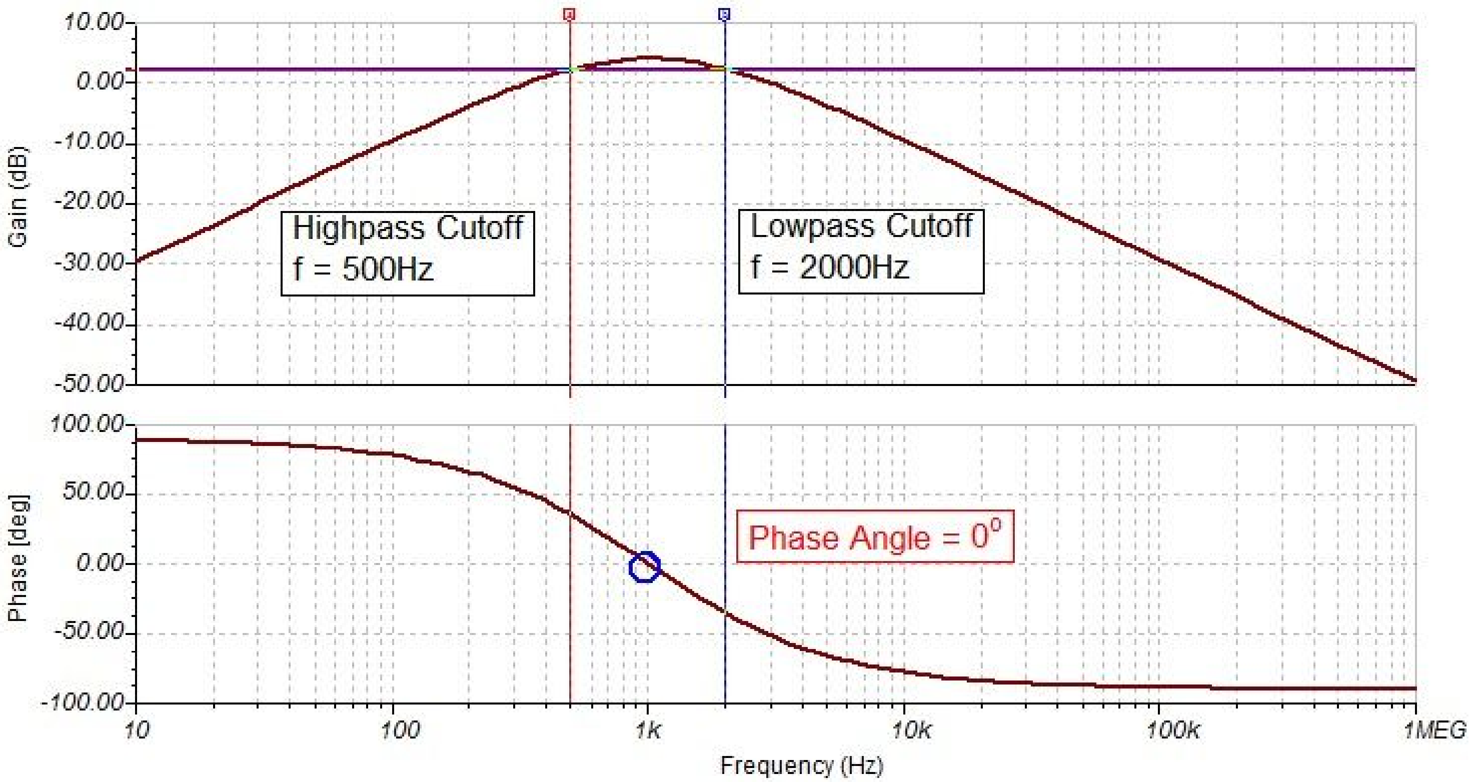
Calculating for the phase angle of the high pass filter stage,

$$Phase\ Angle\ \phi_{highpass} = 180^\circ - \tan^{-1}(\frac{1}{2\pi \cdot fR_iC_i})$$
$$\phi_{highpass} = 180^\circ - \tan^{-1}(2\pi \cdot 500Hz \cdot 0.0347\mu F \cdot 6.5k\Omega)$$
$$\phi_{highpass} = 180^\circ - 54.679^\circ$$
$$\phi_{highpass} = 125.321^\circ$$

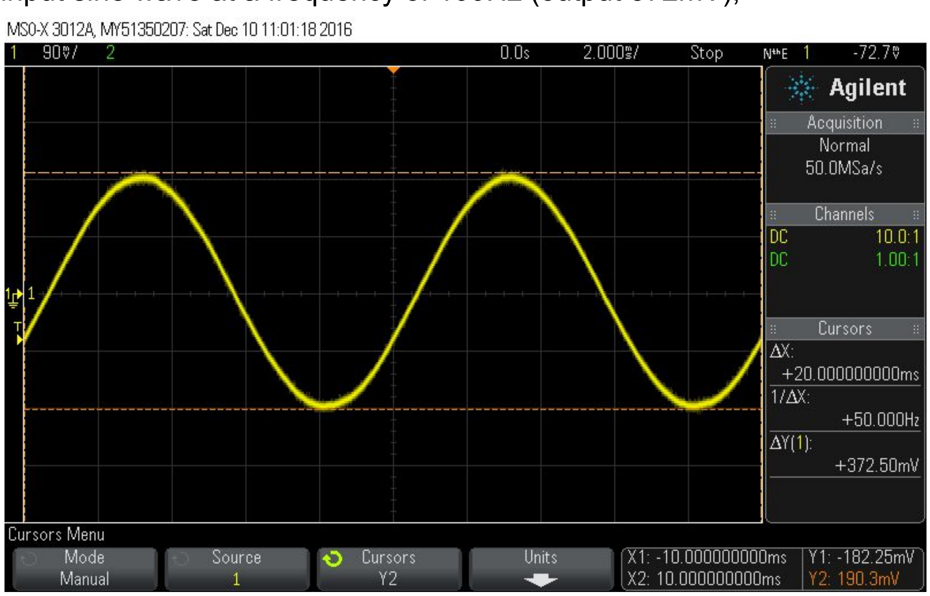
Calculating for the total phase angle of the circuit,

$$Phase\ Angle\ \phi_{final} = \phi_{lowpass} - \phi_{highpass}$$
$$\phi_{final} = (125.883^\circ) - (125.321^\circ)$$
$$\phi_{final} = 0.562^\circ \approx 0^\circ$$

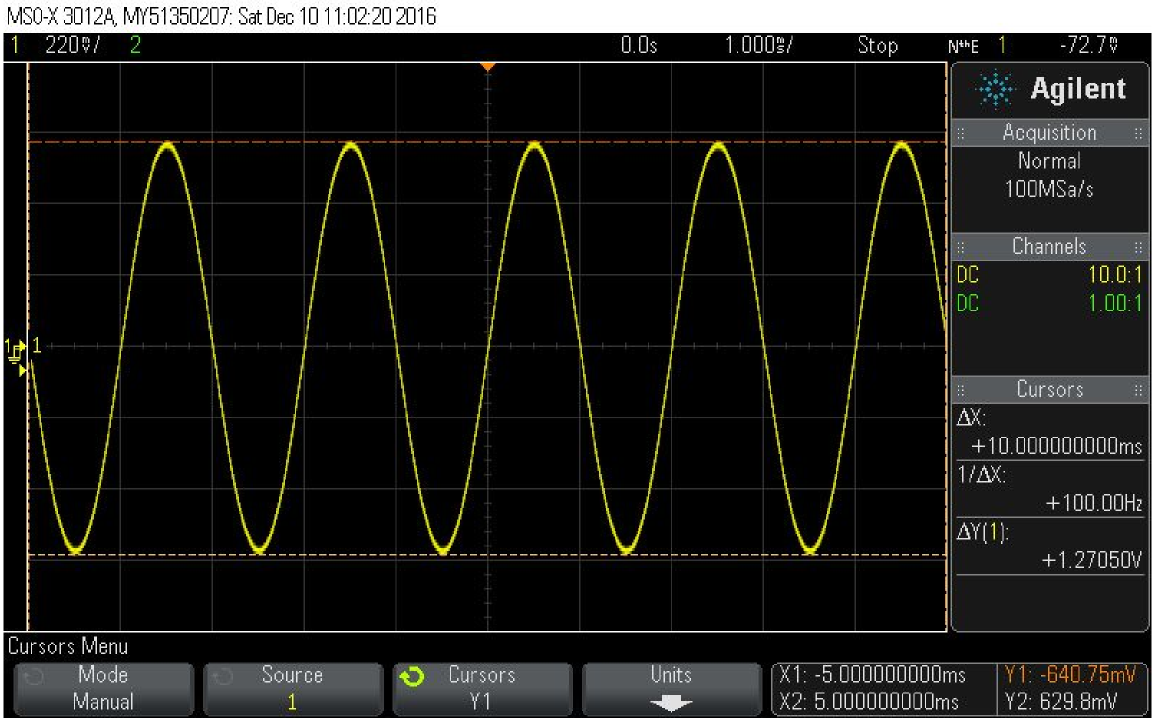
Simulation (bode plot) of the calculated bandpass filter circuit with a high pass cutoff frequency of 500Hz and a low pass cutoff frequency of 2000Hz at +3.74dB gain. The phase angle of the completed circuit is at 0° as calculated.



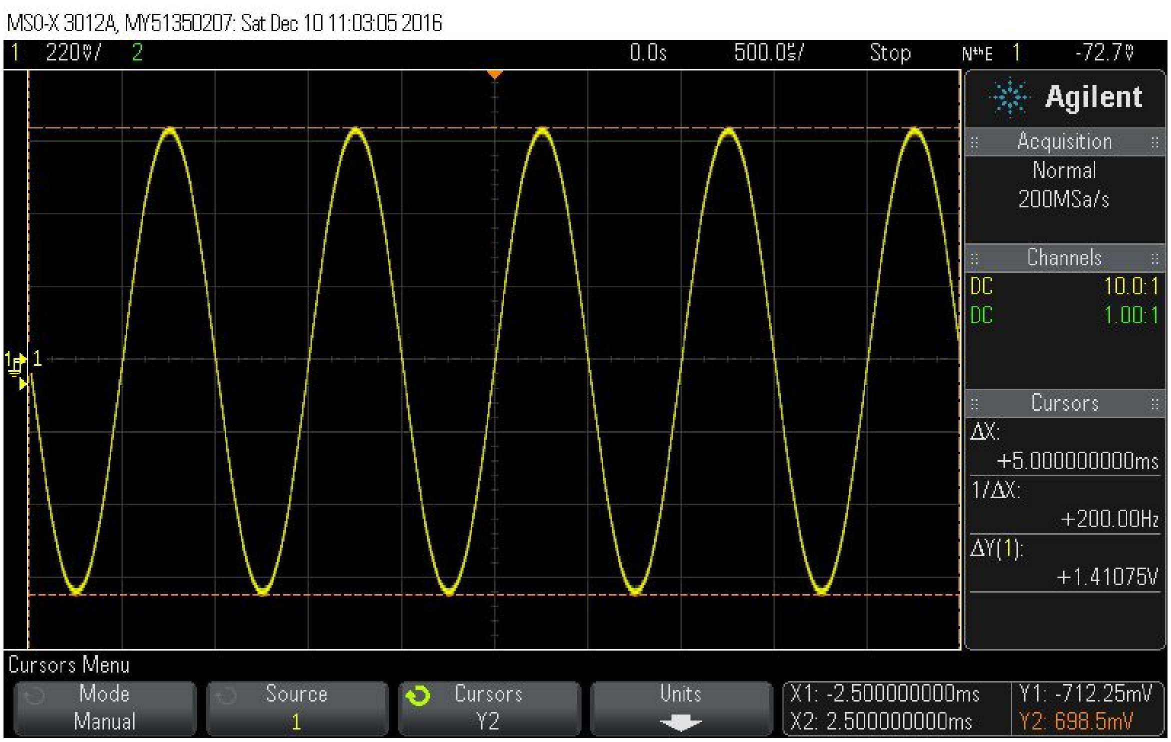
Mathematical calculations correspond with the simulations.
The following is the measured output of the active band pass filter in the time domain using an input sine wave at a frequency of 100Hz (output 372mV),



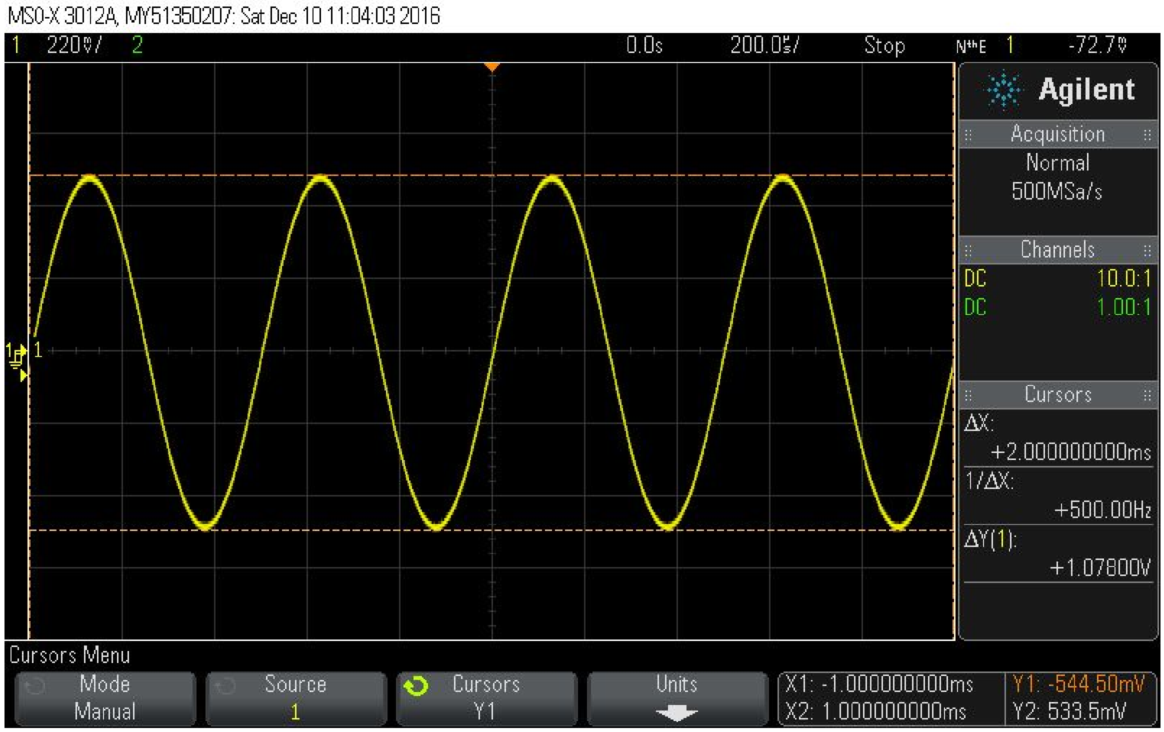
The following is the measured output of the active band pass filter in the time domain using an input sine wave at a frequency of 500Hz (output 1.27V),



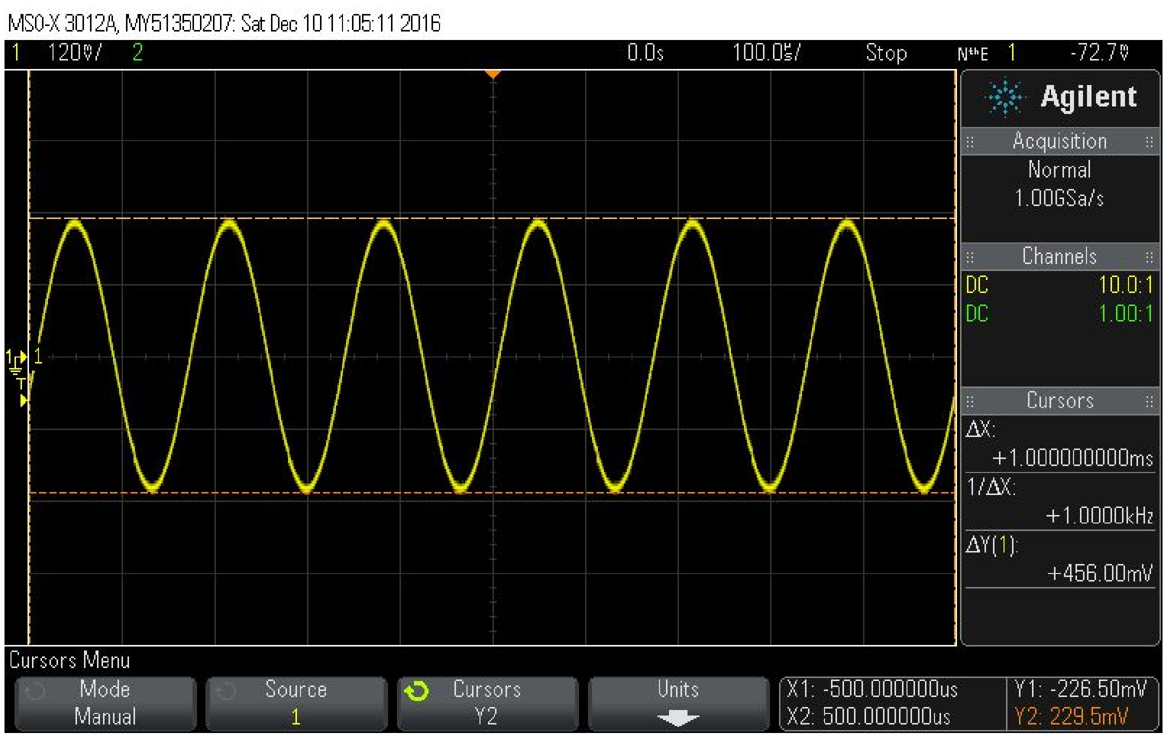
The following is the measured output of the active band pass filter in the domain using an input sine wave at a frequency of 1kHz (output 1.41V),



The following is the measured output of the active band pass filter in the time domain using an input sine wave at a frequency of 2kHz (output 1.07V),



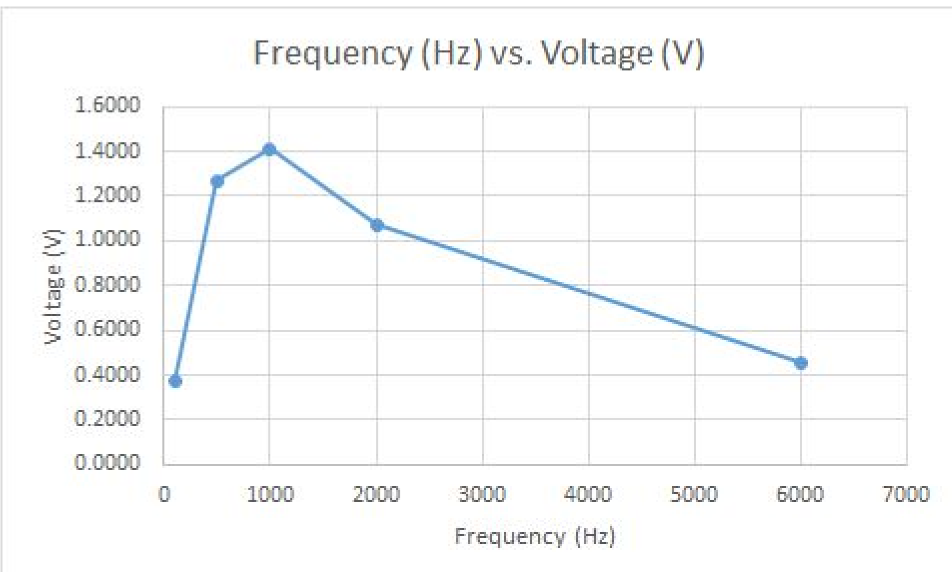
The following is the measured output of the active band pass filter in the time domain using an input sine wave at a frequency of 6kHz (output 456mV),



Compiled data from the time domain plots above,

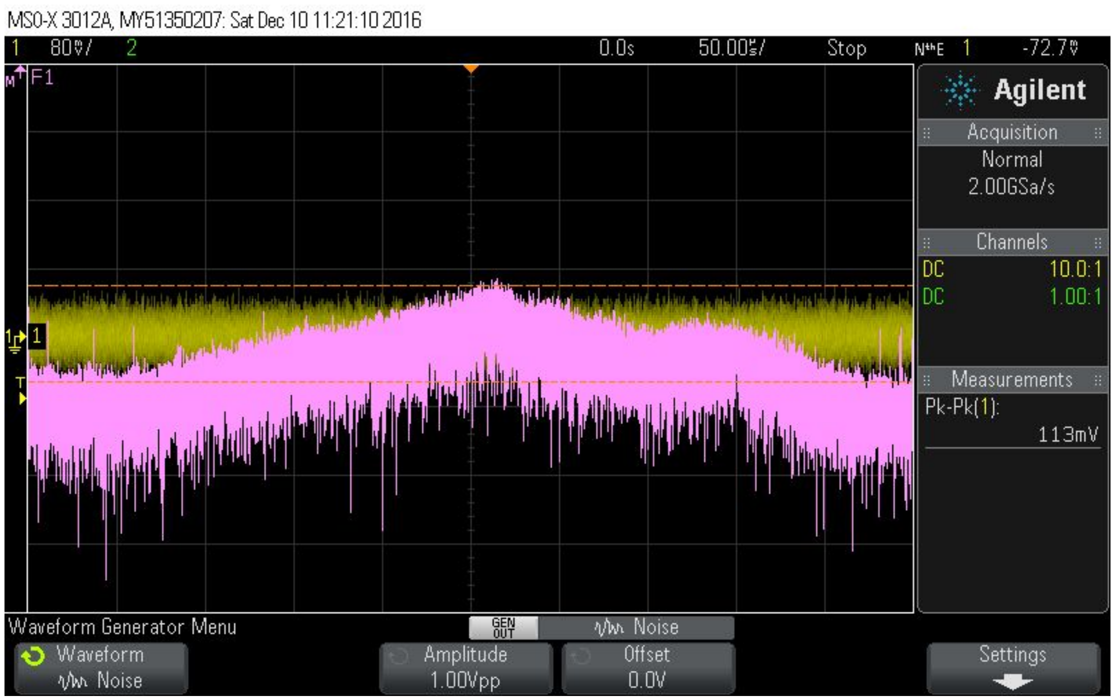
Frequency (Hz)	Voltage (V)
100	0.3725
500	1.2700
1000	1.4100
2000	1.0700
6000	0.4560

Scatter plot of the compiled data,

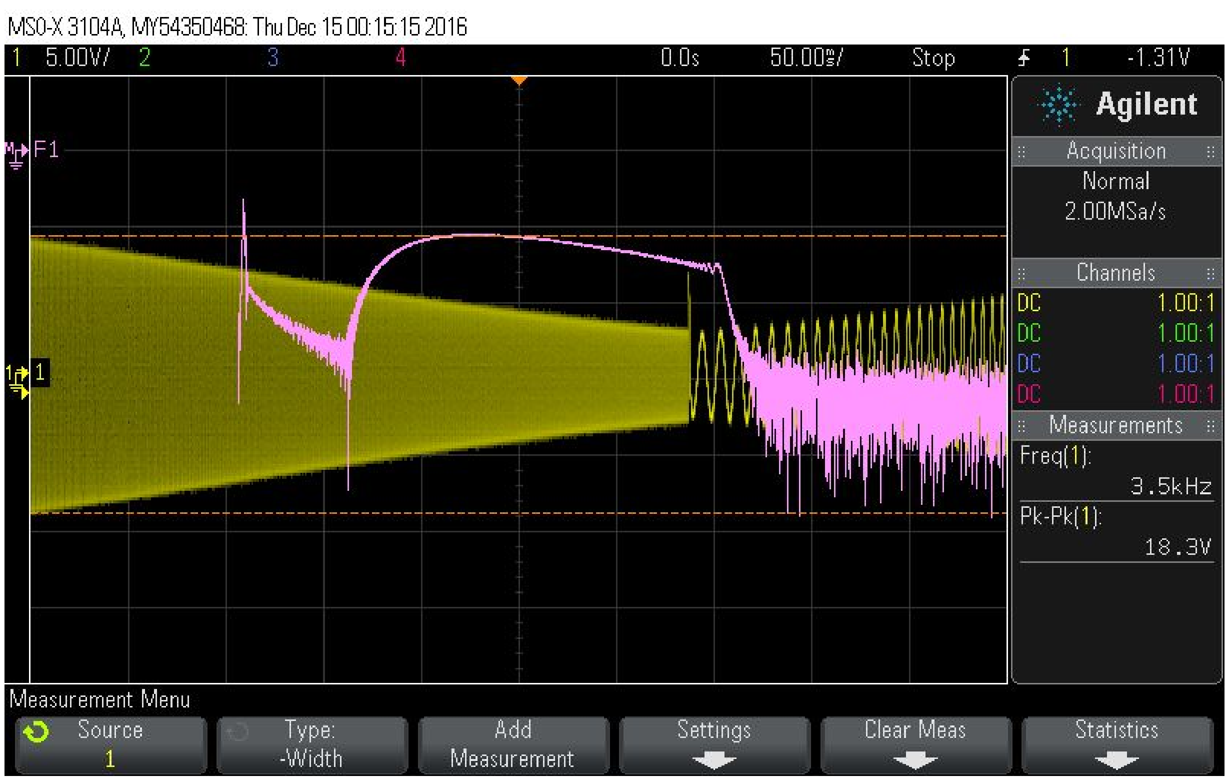


The scatter plot of compiled raw data above resembles the bode plot simulation of a bandpass filter previously shown.

The following is the measured frequency domain plot of the active bandpass filter using the FFT function on the oscilloscope.The input of the filter is white noise.



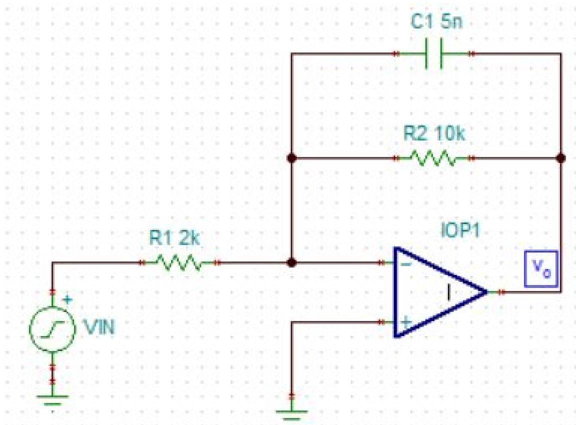
The following is the measured frequency domain plot of the active bandpass filter using the FFT function on the oscilloscope. The sweep function on the function generator was used to sweep a sine wave from a frequency of 100Hz to 6kHz on a logarithmic scale (1s interval). The sweep function was used to get a more accurate frequency domain approximation on the oscilloscope,



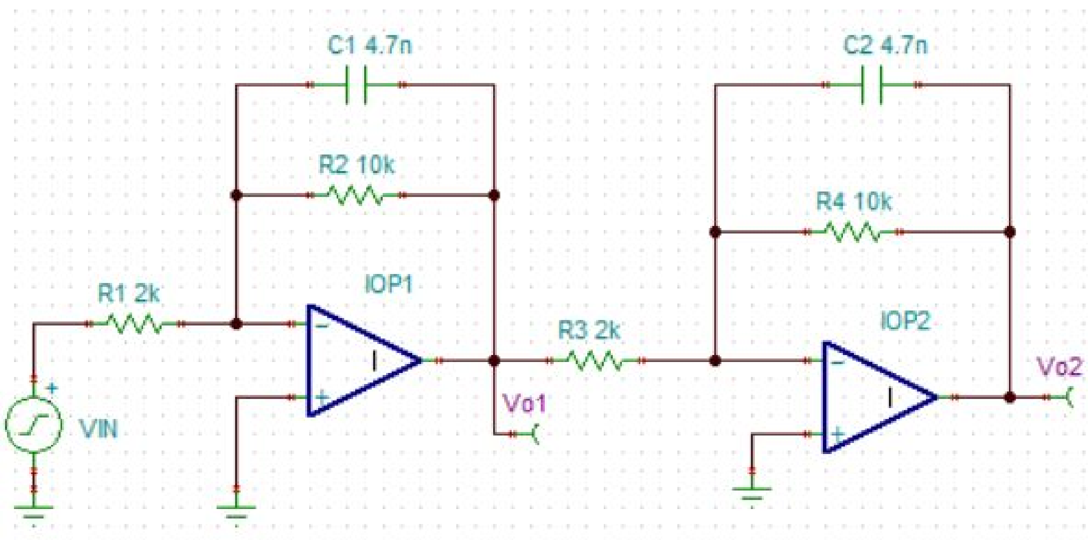
The output voltages seen by the time domain plots and the frequency domain approximations (FFT) measured in the lab validates the circuit matches with mathematical calculations and simulations performed.

4.5.1 Second-order active low pass filter

We can build a second-order active low-pass filter by cascading the active low-pass filter from section 4.2 in series.
The active first order low-pass filter from section 4.2,



The active second order low-pass filter we will now be using.



We need to pick values for the resistors and capacitors such that the circuit provides a DC gain of +14dB and a cutoff frequency of 2000Hz at each stage. We chose to keep the DC gain at +14dB in order to more clearly notice the change in slope (audibly) in the frequency domain.

Let $R_f = 10k\Omega$

$$+14dB = 20log(\frac{10k\Omega}{R_{in}})$$
$$R_{in} = 1995\Omega$$

After rounding the value of R_{in}, the values for our resistors are,

$$R_f = 10k\Omega$$
$$R_{in} = 2k\Omega$$

The gain of our circuit can be recalculated such as,

$$A_v = 20log(\frac{10k\Omega}{2k\Omega}) = +13.9794$$

To obtain the value of the capacitor in the circuit we will set the transfer function equal to the output gain and solve for capacitance,

Note, $\omega = 2\pi f$

$f = 2000Hz$

$$+13.9794 = -\frac{R_f}{R_i(1 + j\omega C_1 R_f)} = -\frac{10k\Omega}{2k\Omega(1 + (2\pi 2000Hz) \cdot 10k\Omega \cdot C_1)}$$
$$C_1 = 5.11 \cdot 10^{-9}F \approx 0.005\mu F$$

The component values of this circuit are,

$$R_f = 10k\Omega$$
$$R_i = 2k\Omega$$
$$C_1 = 0.005\mu F$$

To find the overall gain (transfer function) for the second order filter we can multiply the first and second stage of the circuit,

$$H(j\omega) = \frac{v_{o2}}{v_{o1}} \cdot \frac{v_{o1}}{v_i}$$
$$H(j\omega) = \frac{(R_{f1}/R_{i1})(R_{f2}/R_{i2})}{(1 + j\omega C_{f1}R_{f1})(1 + j\omega C_{f2}R_{f2})}$$

By analyzing the poles in the above transfer function (# of poles increase) its gain roll of decreases to -40db/decade.

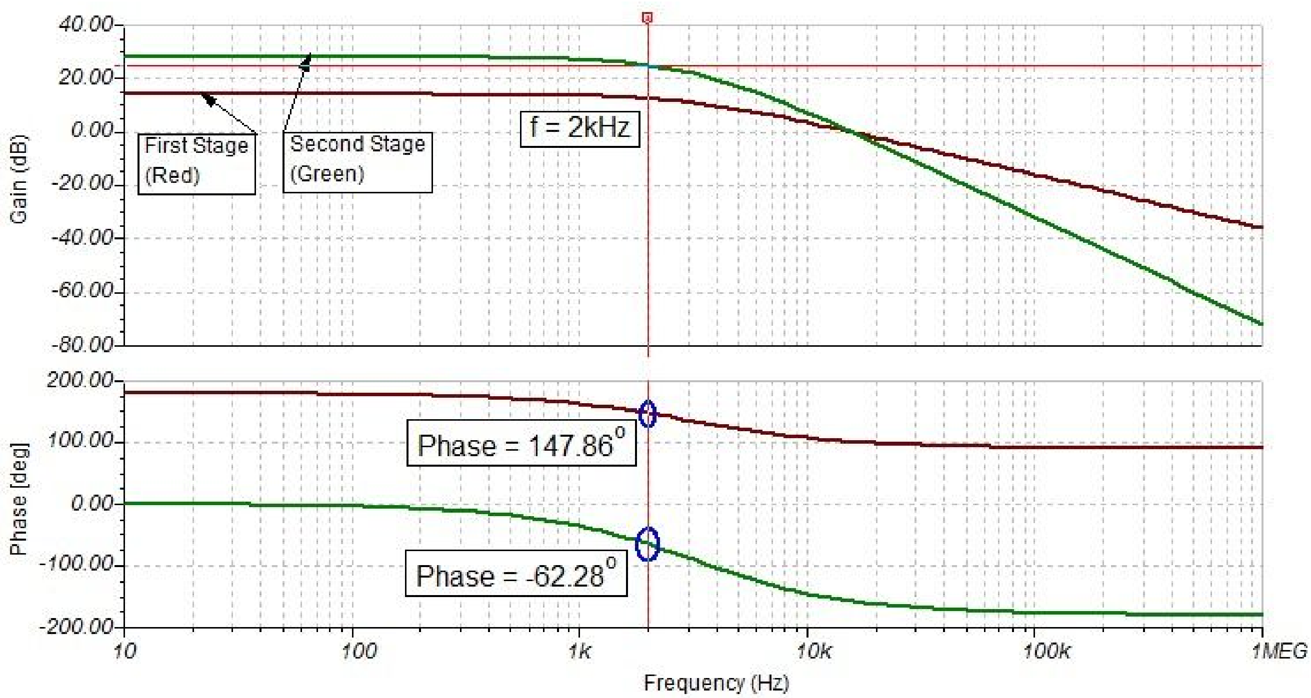
Calculating for the phase angle at the cutoff frequency for the first stage,

$$Phase\ Angle\ \phi_1 = 180^\circ - \tan^{-1}(2\pi \cdot fC_1R_f)$$
$$\phi_1 = 180^\circ - \tan^{-1}(2\pi \cdot 2000Hz \cdot 0.005 \cdot 10^{-6}F \cdot 10k\Omega)$$
$$\phi_1 = 180^\circ - 32.14^\circ$$
$$\phi_1 = 147.86^\circ$$

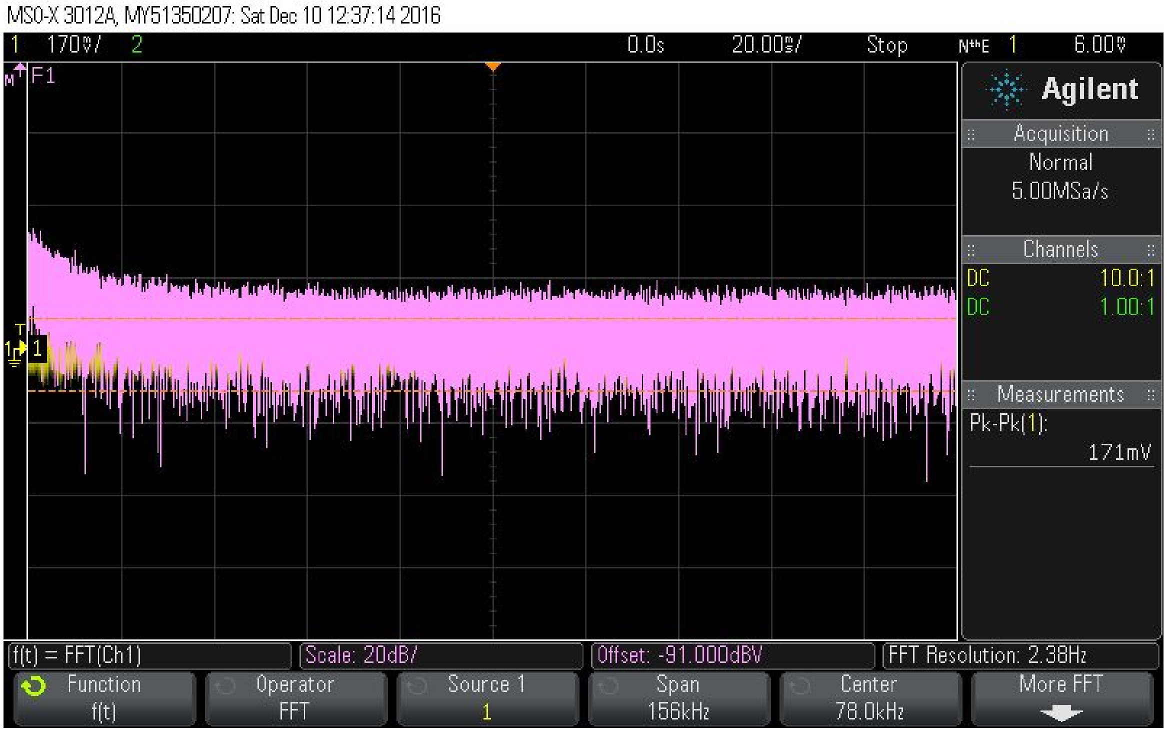
Calculating for the phase angle at the second stage,

$$Phase\ Angle\ \phi_2 = (\phi_1 - 180^\circ) - \tan^{-1}(2\pi \cdot fC_1R_f)$$
$$\phi_2 = (147.86^\circ - 180^\circ) - \tan^{-1}(2\pi \cdot 2000Hz \cdot 0.005 \cdot 10^{-6} \cdot 10k\Omega)$$
$$\phi_2 = -32.14^\circ - 32.14^\circ$$
$$\phi_2 = -62.28^\circ$$

Simulation (bode plot) of the calculated second order low pass filter with a cutoff frequency of 2kHz. The first stage has a phase angle of 147.86° and the second stage has a phase angle of -62.28° as calculated.



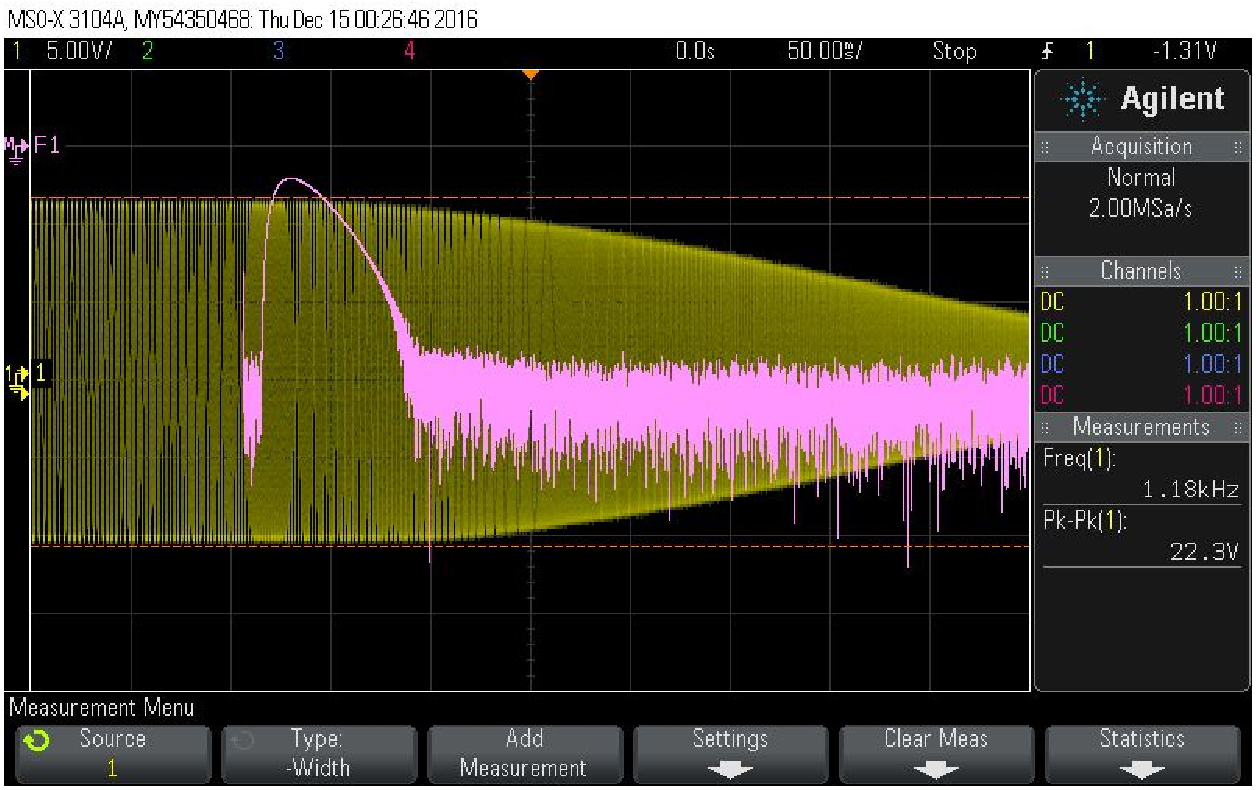
The following is the measured frequency domain plot of the **first stage** of the active second order low pass filter using the FFT function on the oscilloscope.The input of the filter is white noise.



The following is the measured frequency domain plot of the **second stage** of the active second order low pass filter using the FFT function on the oscilloscope. The input of the filter is white noise.

By comparing the measured frequency domain plots of the first and second stages individually it is clear the steepness of the slope is increasing.

The following is the measured frequency domain plot of the active second order low pass filter using the FFT function on the oscilloscope. The sweep function on the function generator was used to sweep a sine wave from a frequency of 100Hz to 6kHz on a logarithmic scale (1s interval). The sweep function was used to get a more accurate frequency domain approximation on the oscilloscope,



Comparing the above frequency domain plot (second order) to the frequency domain plot of the active first order low pass filter in section 4.2 shows a clear increase in the steepness of slope.

The measured results correspond with mathematical calculations and simulations.

4.5.2 Cascading filters increases the slope

Roll off (slope) of a filter is given by,

$$\Delta L = 20\log\left(\frac{\omega_2}{\omega_1}\right)\ \mathrm{dB/interval}_{2,1}$$

For a decade this is,

$$\Delta L = 20\log 10 = 20\ \mathrm{dB/decade}$$

Consequently, the total roll-off by a higher order circuit is given by,

$$\Delta L_T = n\Delta L = 20ndB/Decade$$

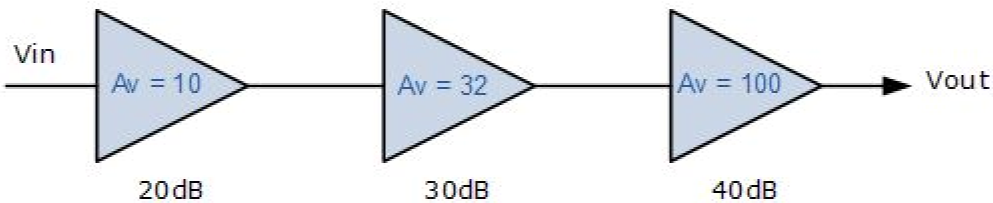
The total roll-off for a first-order filter can be calculated by,
$$\Delta L_T = (1)\Delta L = 20(1)dB/Decade = 20dB/Decade$$

The total roll-off for a second-order filter can be calculated by,
$$\Delta L_T = (2)\Delta L = 20(2)dB/Decade = 40dB/Decade$$

The increased slope for a second-order filter corresponds with the simulations and measured data found in the previous section (4.5.1).

4.5.3 Cascading filters affect on voltage gain

Consider the following cascading opamp filter configuration,



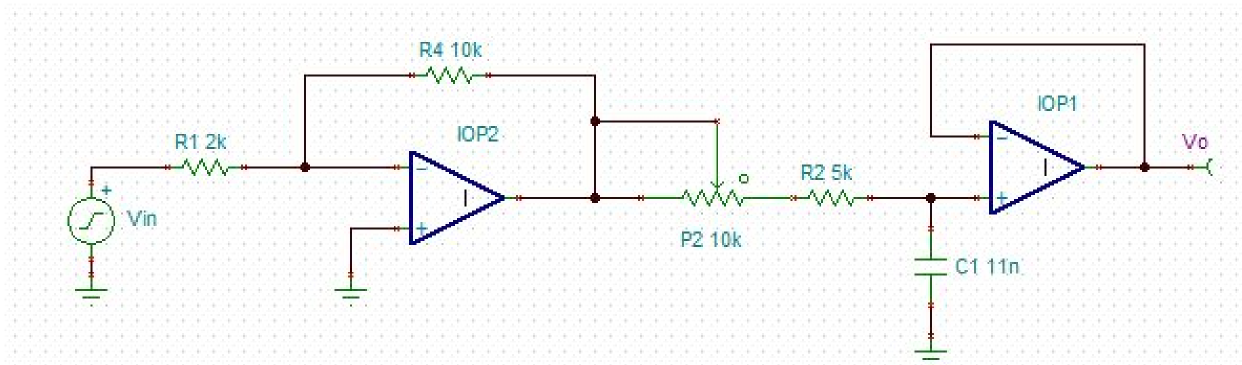
The total gain of the circuit can be calculated by multiplying each gain stage,
$$A_v = A_{v1} \cdot A_{v2} \cdot A_{v3}$$
$$A_v = 10 \cdot 32 \cdot 100 = 32000$$

The gain in decibels can be calculated by,
$$A_v(dB) = 20log_{10}(32000) = 90dB$$

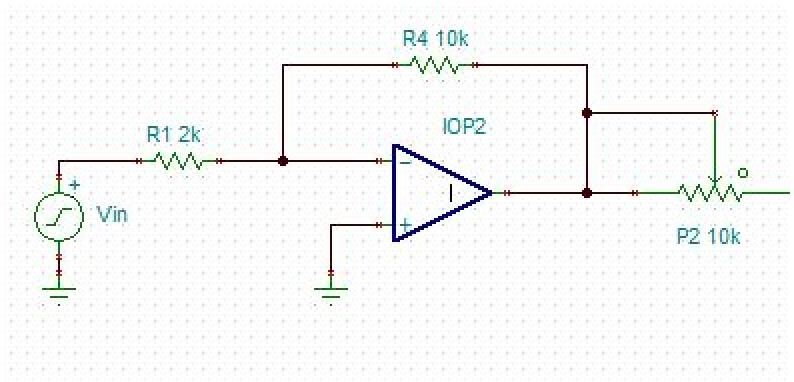
Individually calculated,
$$A_v = 20log_{10}(A_{v1}) + 20log_{10}(A_{v2}) + 20log_{10}(A_{v3})$$
$$A_v = 20log_{10}(10) + 20log_{10}(32) + 20log_{10}(100)$$
$$A_v = 20dB + 30dB + 40dB$$
$$A_v = 90dB$$

4.6 More: Adjustable cutoff frequency without affecting DC gain

In the circuits discussed previously sections the cutoff frequencies were calculated to result a specific cutoff frequency given resistor and capacitor values. To adjust the cutoff frequency of a filter we must change the resistor values (using a potentiometer) because we cannot dynamically alter capacitance values. However, adjusting resistor values in the previous circuit would also alter the DC gain which is not allowed under provided design parameters. In order to correctly solve these problems, we must isolate the two functions: DC Gain and filtering. In this section we will be designing a low pass filter with an adjustable cutoff frequency.



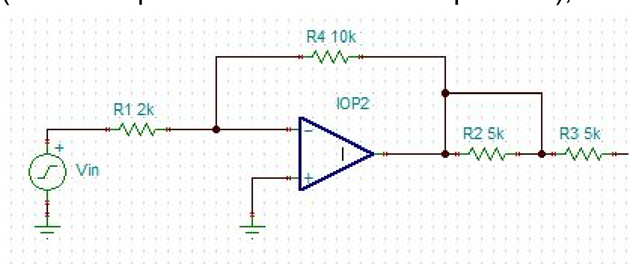
Analyzing the first portion of the circuit which controls the DC gain,



The gain of the inverting operational amplifier circuit above is,

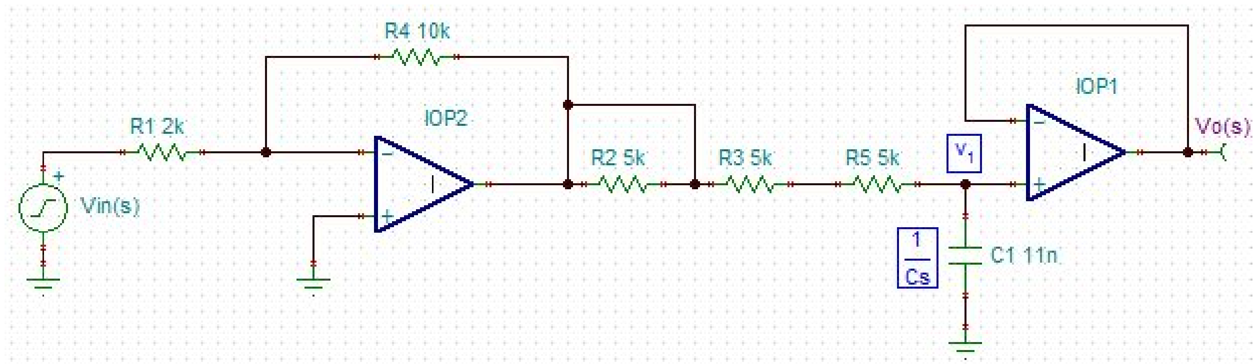
$$A_v = 20log_{10}(\frac{R_f}{Ri}) = 20log_{10}(\frac{10k\Omega}{2k\Omega}) = +13.9794dB$$

The first portion of the circuit can be rewritten to more clearly how the potentiometer is working (when the potentiometer is at 50% position),



The output of the potentiometer is connected as a feedback loop to the output of the operational amplifier. Therefore, the bottom leg of the potentiometer can be considered a variable resistor in this particular scenario.

Connecting the first portion of the circuit to the second portion (when the potentiometer is at 50% position) [converted to s-domain],



For an ideal operational amplifier the inverting terminal equals the non-inverting terminal. The second operational amplifier (IOP1) is in a buffer configuration with negative feedback. Therefore, the output of the device will equal the non-inverting terminal input.

$$H(j\omega) = \frac{v_{in}}{v_o} = \frac{v_{in}}{v_1}$$

Solving for the voltage at the non-inverting terminal (v_1) can be seen as a voltage divider with the capacitor and R3, R5 in series,

The general voltage divider formula,

$$V_{in} \cdot (\frac{R_2}{R_2 + R_1})$$

For our circuit,

$$H(j\omega) = -5V \cdot (\frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + (R_3 + R_5)}) \quad \text{or} \quad +14dB \cdot (\frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + (R_3 + R_5)})$$

$$H(j\omega) = Gain \cdot \frac{-1}{j\omega C(R_3 + R_5) + 1}$$

The gain in the transfer function is independent from the cutoff frequency. Inversely, the adjustable portion of the resistor is dependant on the value of the capacitor - which causes adjustable cutoff frequency in the circuit as the design specifications required.

Example calculation when the potentiometer is at 80% position,
$\omega = 2\pi f$

$$H(j\omega) = +14dB = \frac{-1}{2\pi f C(R_3 + R_5) + 1}$$

$$H(j\omega) = +14dB = \frac{-1}{2\pi f(11.0 \cdot 10^{-9}F)(2k\Omega + 5k\Omega) + 1}$$

Solving for the cutoff frequency,

$$f = 1919Hz$$

Example calculation when the potentiometer is at 20% position,

$\omega = 2\pi f$

$$H(j\omega) = +14dB = \frac{-1}{2\pi f C(R_3 + R_5) + 1}$$

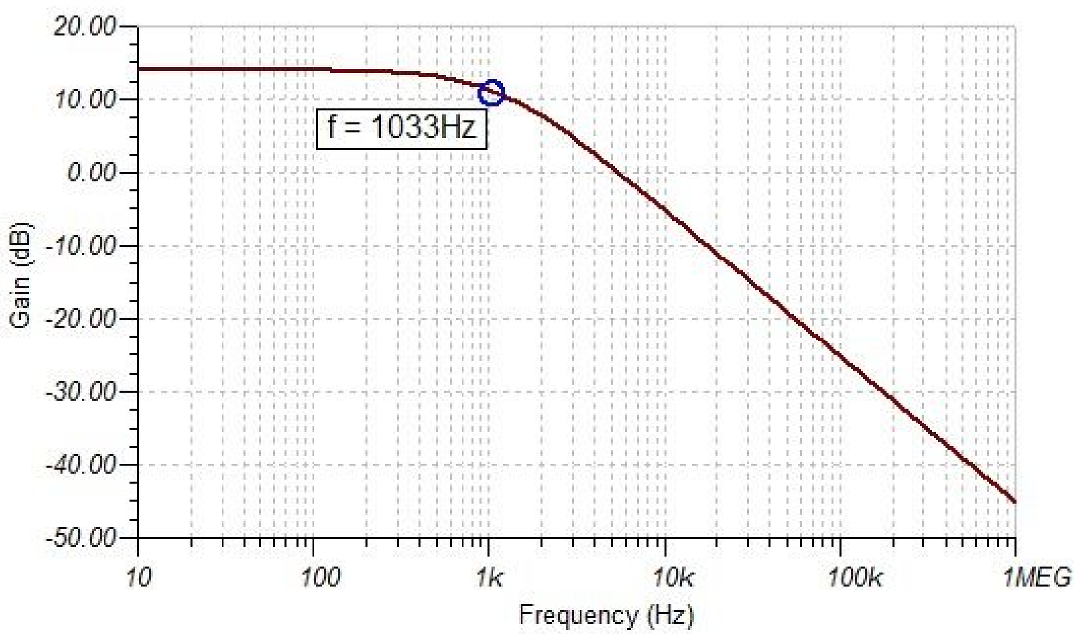
$$H(j\omega) = +14dB = \frac{-1}{2\pi f(11.0 \cdot 10^{-9}F)(8k\Omega + 5k\Omega) + 1}$$

Solving for the cutoff frequency,

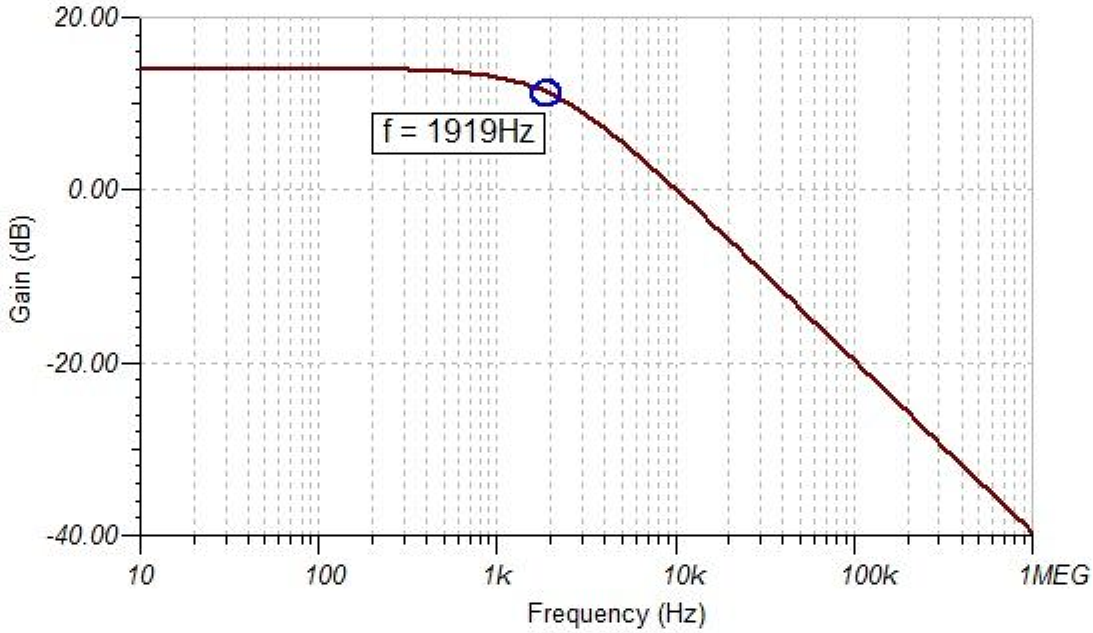
$$f = 1033Hz$$

As seen by the mathematical calculations, adjusting the potentiometer position adjusts the cutoff frequency of the circuit while keeping the DC gain of the circuit independent.

Simulated bode plot of the circuit when the potentiometer is at 80% position,



The cutoff frequency in the above simulation corresponds with mathematical analysis (1033Hz). Simulated bode plot of the circuit when the potentiometer is at 20% position,

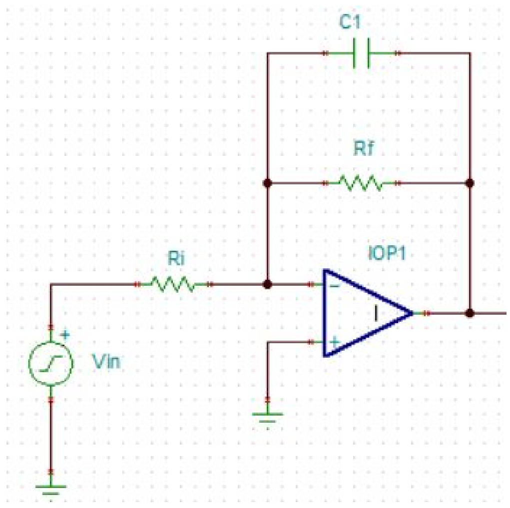


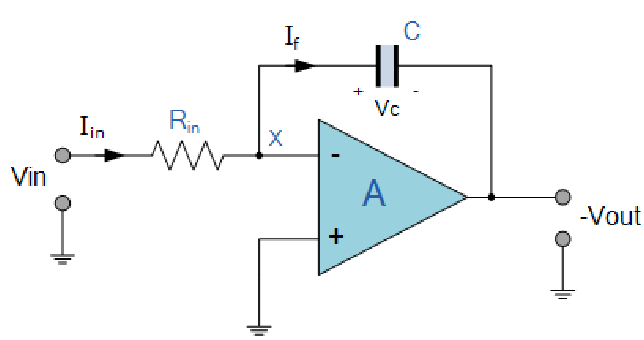
The cutoff frequency in the above simulation corresponds with mathematical analysis (1919Hz).

As noted by mathematical analysis and simulation, changing the potentiometer position alters the cutoff frequency of the circuit but overall DC gain of the circuit remains constant / independent in this configuration.

4.7.1 Fundamental math operators of low pass filters

Consider the following active low pass filter (left) which is similar to opamp integrator circuit (right),





Recall the transfer function of the active low pass filter from section 4.2,

$$H(j\omega) = \frac{v_{out}}{v_{in}} = -\frac{R_f}{R_i(1 + j\omega C_1 R_f)}$$

Which can be rewritten as,

$$v_o(j\omega)(j\omega + R_f C_1 + \frac{1}{R_i C_1}) = \frac{v_i(j\omega)}{R_i C_1}$$

Apply inverse Laplace,

$$\mathcal{L}^{-1}[v_o(j\omega)j\omega + v_o(j\omega)(R_f C_1 + \frac{1}{R_i C_1}] = \mathcal{L}^{-1}[\frac{v_i(j\omega)}{R_i C_1}]$$

$$\frac{dv_o(t)}{dt} + v_o(t)(R_f C_1 + \frac{1}{R_i C_1}) = \frac{1}{R_i C_1} v_i(t)$$

Apply integration to both sides with respect to *t* to find the time domain expression for the circuit,

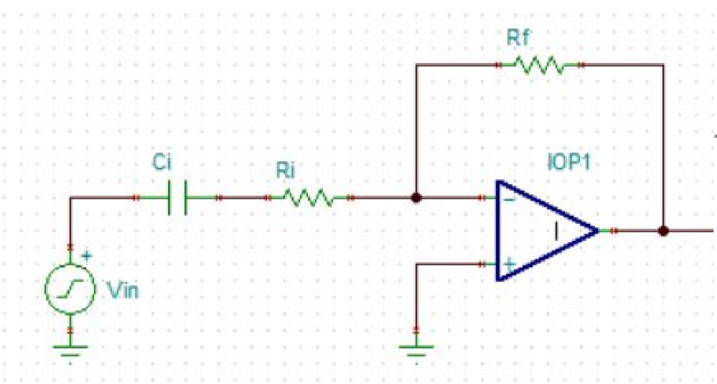
$$v_o(t) = -\int_0^t v_o(t)(R_f C_1 + \frac{1}{R_i C_1})dt + \int_0^t \frac{1}{R_i C_1} v_i(t)$$

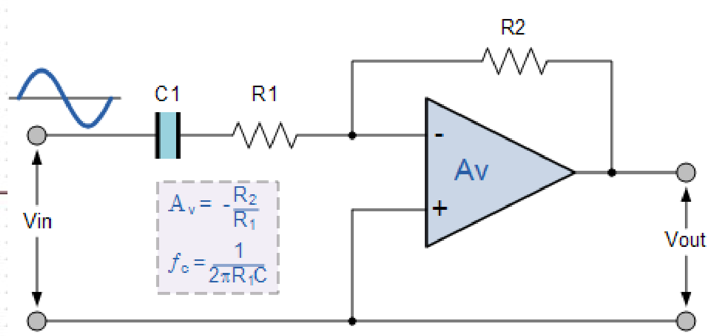
The low pass filter is an integrator,

$$v_i(t) \Longleftrightarrow \boxed{\int_0^t v_i(t)dt} \Longleftrightarrow v_o(t)$$

4.7.2 Fundamental math operators of high pass filters

Consider the following active highpass filter (left) which is similar to opamp differentiator circuit (right),





Recall the transfer function of an active high pass filter from section 4.3,

$$H(j\omega) = \frac{v_o}{v_i} = \frac{-j\omega C_i R_f}{1 + j\omega RiC_i}$$

Which can be rewritten as,

$$v_o(j\omega) = v_i(s) \cdot (-R_f C_1)(j\omega + R_i C_i)$$

$$v_o(j\omega) = j\omega v_i(j\omega)(-R_f C_1) + v_i(j\omega)(R_i C_1)(-R_f C_1)$$

Apply inverse Laplace,

$$\mathcal{L}^{-1}[v_o(j\omega)] = \mathcal{L}^{-1}[j\omega v_i(j\omega)(-R_f C_1) + v_i(j\omega)(R_i C_1)(-R_f C_1)]$$

The time domain expression for this circuit,

$$v_o(t) = (-R_f C_1)\frac{dv_i(t)}{dt} - (R_f C_1)(R_i C_1)v_i(t)$$

The high pass filter is a differentiator,

$$v_i(t) \Longleftrightarrow \boxed{\frac{dv_i(t)}{dt}} \Longleftrightarrow v_o(t)$$

4.8 TINA AC Transfer Characteristics

TINA-TI can be used to find bode frequency plots by using the AC Transfer Characteristics simulations. To simulate an accurate bode plot the program:

1. Sets the function generator to a frequency
2. Tests the circuit output
3. Plots the amplitude
4. Repeat

The frequency which is set in the function generator has no effect on the simulation because it the program iterates through a set of test frequencies regardless.

4.9.1 Practical concerns of capacitors too small

There are practical concerns with using capacitors that are too small. For example, very small capacitor values will cause individual metal plates in a breadboard to carry capacitance. In addition, small capacitors can create noise with long wires and cause instability within the circuit. There is trace inductance (whether it is a PCB or breadboard) which will limit response of the capacitor. If there are power wires nearby, a circuit may ask the wires to supply some of the current which may cause some dips. However, when noise sees the capacitor first, even with some inductance in the traces, the noise will not go into the cables and not cause any further problems, which reduces the overall noise your circuit sees by a greater factor.

4.9.2 Practical concerns of capacitors too big

There are practical concerns with using capacitors that are too big (value). Capacitors with a high capacitance value are more difficult to obtain and are generally more expensive. Additionally, a large capacitor will demand a large amount of current to charge and be capable of sourcing a large amount of current for a long time - which could damage the circuit under certain configurations. A capacitor with a larger capacitance value will also take longer to discharge which may cause a delay in your circuit (increasing the time constant).

4.10 Selection of frequencies for bandpass filter

Why did we choose frequencies of 500Hz and 2kHz for bandpass filter?

We chose the values of 500Hz and 2kHz because the spectrum of the midrange frequencies of audio are frequencies from 500Hz to 2kHz. The audio spectrum is the audible frequency range at which humans can hear. The audio spectrum range spans from 20Hz to 20kHz and can be broken down into different frequency bands, with each having a different impact on the total sound.

The seven frequency bands are:

1. Sub-bass (20Hz - 60Hz)
2. Bass (60Hz - 250Hz)
3. Low midrange (250Hz - 500Hz)
4. **Midrange (500Hz - 2kHz)**
5. Upper midrange (2kHz - 4kHz)
6. Presence (4kHz - 6kHz)
7. Brilliance (6kHz - 20kHz)

For the bandpass filter we wanted to have an increased gain on the midrange frequencies (500Hz to 2kHz). The high pass filter will cut-off at 500Hz and begin to increase gain. The low pass will cut-off at 2kHz and begin to decrease gain.

4.11 Issues & Error Notes

4.11.1 Inaccurate FFT measurements

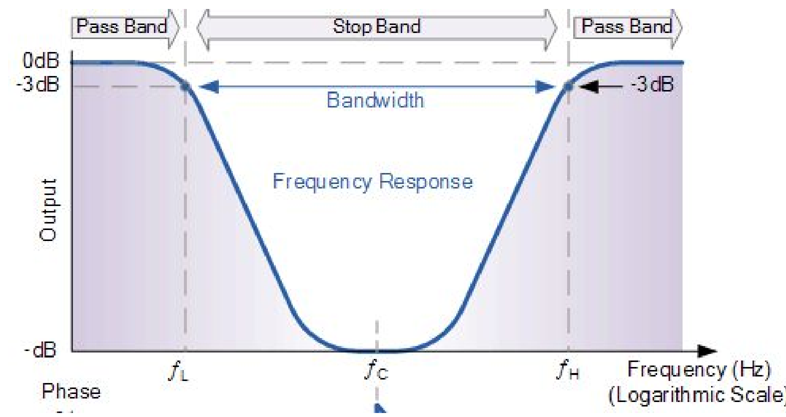
When using the oscilloscope for FFT (fast-fourier transform) measurements, we were not receiving accurate results. For example, a high pass filter would look the same as a low pass filter in the frequency domain which we knew to be incorrect.

After confirming the circuit was functioning properly we determined it was settings for the FFT on the oscilloscope. We changed the settings around for the span and center of the measurements but the results were still not receiving an expected result.

The problem was then narrowed down to the function generator. We used the sweep function on the function generator to sweep a sine wave from a frequency of 100Hz to 6kHz on a logarithmic scale (1s interval). Using the sweep function on the function generator allowed for a far more accurate frequency domain approximation using the FFT on the oscilloscope.

4.11.2 Accidental notch filter

When building the bandpass filter we accidentally switched the cutoff frequencies for the low pass and high pass filter. (low pass was given high pass cut-off and vice versa). This caused the bandpass filter to behave opposite as expected (the mid frequencies were decreasing in gain) [example shown on right].



After given advice from the professor and researching a notch filter, we discovered the basic mistake and corrected it by calculating cutoff frequencies for the respective filters.