Assignment_3 QMM

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Transportation Problem :

```
tab <- matrix(c(22,14,30,600,100,
                16,20,24,625,120,
                80,60,70,"-","-") , ncol=5 , byrow=TRUE)

colnames(tab) <- c("Warehouse1","Warehouse2","Warehouse3","Prod cost","Prod Capacity")
row.names(tab) <- c("Plant A","Plant B","Demand")
tab <- as.table(tab)
tab
```

```
##         Warehouse1 Warehouse2 Warehouse3 Prod cost Prod Capacity
## Plant A 22         14         30         600       100
## Plant B 16         20         24         625       120
## Demand  80         60         70         -         -
```

transportation problem can be formoulated as

$$\text{Min } TC = 622X_{11} + 614X_{12} + 630X_{13}X + 641X_{21} + 645X_{22} + 649X_{23}$$

/text{subject to}

#Production Capacity constraints Production plant A :

$$X_{11} + X_{12} + X_{13} \leq 100$$

Production Plant B :

$$X_{21} + X_{22} + X_{23} \leq 120$$

#Demand Constraints

Demand Warehouse 1 :

$$X_{11} + X_{21} \geq 80$$

Demand Warehouse 2 :

$$X_{12} + X_{22} \geq 60$$

Demand Warehouse 3 :

$$X_{13} + X_{23} \geq 70$$

$$X_{14} + X_{24} \geq 10$$

Non-negativity of the variables

$$X_{ij} \geq 0$$

Where

$$i = 1, 2, 3$$

And

$$j = 1, 2, 3$$

Since Demand not equal to supply so it is unbalanced one we have created the dummy row as warehouse_4

```
library(lpSolveAPI)
library(lpSolve)
#the cost matrix
Transport_cost <- matrix(c(622,614,630,0,
                           641,645,649,0) , ncol=4 , byrow=TRUE)
#defining rows and columns
colnames(Transport_cost) <- c("Warehouse_1","Warehouse_2","Warehouse_3","Dummy")
rownames(Transport_cost) <- c("Plant_A", "Plant_B")
Transport_cost
```

```
##         Warehouse_1 Warehouse_2 Warehouse_3 Dummy
## Plant_A         622         614         630     0
## Plant_B         641         645         649     0
```

```
#constraint signs and right-hand sides(Production side)
row.signs <- rep("<=",2)
row.rhs <- c(100,120)

#Demand side constraints#
col.signs <- rep(">=",4)
col.rhs <- c(80,60,70,10)

#solving the model
lptrans <- lp.transport(Transport_cost,"min",row.signs,row.rhs,col.signs,col.rhs)

lptrans$solution
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    0   60   40    0
## [2,]   80    0   30   10
```

When solved the transportation problem, the values of the variables as

$$x_{12} = 60$$

$$x_{13} = 40$$

$$x_{21} = 80$$

$$x_{23} = 30$$

$$x_{24} = 10$$

in Otherwords

80 AEDs in Plant 2 - Warehouse_1 60 AEDs in Plant 1 - Warehouse_2 40 AEDs in Plant 1 - Warehouse_3 30 AEDs in Plant 2 - Warehouse_3 This is the production in each plant and distribution to the three wholesaler warehouses to minimize the overall cost of production as well as shipping.

```
lptrans$objval
```

```
## [1] 132790
```

the minimum combined cost of production and shipping founded for the optimal solution is 132790

2. Formulation of the dual of the transportation problem

As we know the primal was to minimize the transportation cost the dual of it would be to maximize the value added(VA). K and L will be the variables for the dual respectively

```
cost_2 <- matrix(c(622,614,630,100,"k1",
641,645,649,120,"k2",
80,60,70,220,"-",
"l1","l2","l3","-","-"),ncol = 5,nrow = 4,byrow = TRUE)
colnames(cost_2) <- c("Warehouse_1", "Warehouse_2","Warehouse_3","Production Capacity","Supply(Dual)")
rownames(cost_2) <- c("Plant_A","Plant_B","Demand","Demand(Dual)")
cost_2
```

```
##              Warehouse_1 Warehouse_2 Warehouse_3 Production Capacity
## Plant_A      "622"       "614"       "630"       "100"
## Plant_B      "641"       "645"       "649"       "120"
## Demand       "80"        "60"        "70"        "220"
## Demand(Dual) "l1"        "l2"        "l3"        "-"
##              Supply(Dual)
## Plant_A      "k1"
## Plant_B      "k2"
## Demand       "-"
## Demand(Dual) "-"
```

$$\text{Max } \; Z = 100K_1 + 120K_2 + 80L_1 + 60L_2 + 70L_3$$

Subject to the following constraints

$$K_1 + L_1 \leq 622$$
$$K_1 + L_2 \leq 614$$
$$K_1 + L_3 \leq 630$$
$$K_2 + L_1 \leq 641$$

$$K_2 + L_2 \leq 645$$
$$K_2 + L_3 \leq 649$$

Where K1 = Warehouse_1

K2 = Warehouse_2

L1 = Warehouse_3

L2 = Plant_1

L3 = Plant_2

```
#The Objective function
f.obj <- c(100,120,80,60,70)


#The tranposed from the constraints matrix in the primal is
f.con <- matrix(c(1,0,1,0,0,
1,0,0,1,0,
1,0,0,0,1,
0,1,1,0,0,
0,1,0,1,0,
0,1,0,0,1), nrow = 6, byrow = TRUE)
f.dir <- c("<=",
"<=",
"<=",
"<=",
"<=",
"<=")
f.rhs <- c(622,614,630,641,645,649)
lp("max",f.obj,f.con,f.dir,f.rhs)
```

```
## Success: the objective function is 139120
```

```
lp("max",f.obj,f.con,f.dir,f.rhs)$solution
```

```
## [1] 614 633   8   0  16
```

So Z=139,120 dollars and variables are:

$$U_1 = 614$$

which represents Plant A

$$M_2 = 633$$

represents Plant B

$$N_1 = 8$$

represents Warehouse 1

$$N_3 = 16$$

represents Warehouse 3

3)The Economic Interpretation of the dual

The maximum combined shipping and production costs will be 139,120 dollars based on the given information and constraints. There is a minimum Z=132790 (Primal) and maximum Z=139120 (Dual).We observed that we should not ship from Plant(A/B) to all three warehouses at the same time.Instead-

$$60x_{12}$$

which is 60 Units from Plant A to Warehouse 2.

$$40x_{13}$$

which is 40 Units from Plant A to Warehouse 3.

$$80x_{21}$$

which is 80 Units from Plant B to Warehouse 1.

$$30x_{23}$$

which is 30 Units from Plant B to Warehouse 3.

We will Maximize the profit from each distribution to the respective capacity.

We have the following:

$$K_1^0 - L_1^0 \leq 622$$

then we subtract

$$K_1^0$$

to the other side to get

$$K_1^0 \leq 622 - L_1^0$$

To compute it would be $614 <= (-8+622) which is correct. continuing to evaluate these equations:

$$K_1 \leq 622 - L_1 => 614 \leq 622 - 8 = 614 => correct$$

$$K_1 \leq 614 - L_2 => 614 \leq 614 - 0 = 614 => correct$$

$$K_1 \leq 630 - L_3 => 614 \leq 630 - 16 = 614 => correct$$

$$K_2 \leq 641 - L_1 => 633 \leq 614 - 8 = 633 => correct$$

$$K_2 \leq 645 - L_2 => 633 \leq 645 - 0 = 645 => Incorrect$$

$$K_2 \leq 649 - L_3 => 633 \leq 649 - 16 = 633 => correct$$

Learning from the duality and sensitivity, we test the shadow price by updating each of the columns.In our linear programming transportation problem, we change 100 to 101 and 120 to 121.Here we can see it R.

```
row.rhs1 <- c(101,120)
row.signs1 <- rep("<=",2)
col.rhs1 <- c(80,60,70,10)
col.signs1 <- rep(">=",4)
row.rhs2 <- c(100,121)
row.signs2 <- rep("<=",2)
col.rhs2 <- c(80,60,70,10)
col.signs2 <- rep(">=",4)

lp.transport(Transport_cost,"min",row.signs,row.rhs,col.signs,col.rhs)
```

```
## Success: the objective function is 132790
```

```
lp.transport(Transport_cost,"min",row.signs1,row.rhs1,col.signs1,col.rhs1)
```

```
## Success: the objective function is 132771
```

```r
lp.transport(Transport_cost,"min",row.signs2,row.rhs2,col.signs2,col.rhs2)
```

```
## Success: the objective function is 132790
```

As the minimum of this function the number goes down by 19 indicates the shadow price is 19 so adding 1 to every plant is required. Plant B does have a shadow price

From the dual variable

$$L_2$$

where Marginal Revenue <= Marginal Cost. The equation was

$$K_2 \leq 645 - L_2 => 633 \leq 645 - 0 = 645 => Incorrect$$

and this was found by using

$$K_1^0 - L_1^0 \leq 622$$

then we subtract

$$L_1^0$$

to the other side to get

$$K_1^0 \leq 622 - L_1^0$$

```r
lp("max", f.obj,f.con, f.dir,f.rhs)$solution
```

```
## [1] 614 633   8   0  16
```

$$n_2 = 0$$

.

The interpretation from above: from the primal:

$$60x_{12}$$

is 60 Units from Plant A to Warehouse 2.

$$40x_{13}$$

is 40 Units from Plant A to Warehouse 3.

$$80x_{21}$$

is 80 Units from Plant B to Warehouse 1.

$$30x_{23}$$

is 60 Units from Plant B to Warehouse 3.

from the dual

Our goal is to get MR = MC.In five of the six cases, MR <= MC. Only Plant B to Warehouse 2 fails to satisfy this requirement.From the primal, we can see that no AED devices will be shipped there.