Kinematics

- Assume the robot has a velocity of (2 cm/s, 3cm/s, 5 rad/s) with respect to the robot's local reference frame, what is the robot's velocity with respect to the global reference frame?

$$\dot{\xi}_I = R\left(\frac{\pi}{3}\right)^{-1} \dot{\xi}_R =$$

$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R(\theta)^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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Kinematics

- Are all mobile robots the same?

Kinematics

- Are all mobile robots the same?

- What could cause mobile robots to differ?

Kinematics

- **Forward kinematics** provide an estimate of the robots position given its geometry and speed of its wheels

- Requires accurate measurement of the wheel velocities over time

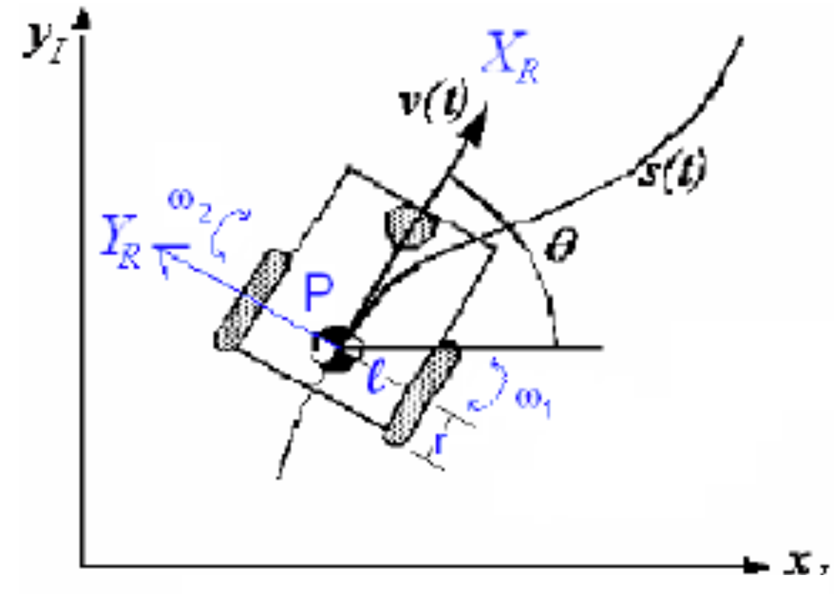
- However, position error (accumulation error) grows over time

- A differential drive robot with wheels that have speed $\dot{\phi}_1$ and $\dot{\phi}_2$ has the following forward kinematic model.

$$\dot{\xi}_I = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = f(l, r, \theta, \dot{\phi}_1, \dot{\phi}_2)$$

Kinematics

- Assume: $\dot{\xi}_I = R(\theta)^{-1}\dot{\xi}_R$



- Linear velocity in direction of X_R

$$\frac{r\dot{\varphi}_1}{2} + \frac{r\dot{\varphi}_2}{2}$$

- Y_R motion = 0, Why?

- Right wheel (1) CCW rotation, Left wheel CW rotation about point P (radius = 2l). $\omega_1 = \frac{r\dot{\varphi}_1}{2l}$ $\omega_2 = \frac{-r\dot{\varphi}_2}{2l}$

$$\dot{\xi}_I = R(\theta)^{-1}\begin{bmatrix} \frac{r\dot{\varphi}_1}{2} + \frac{r\dot{\varphi}_2}{2} \\ 0 \\ \frac{r\dot{\varphi}_1}{2l} + \frac{-r\dot{\varphi}_2}{2l} \end{bmatrix}$$

Kinematics

- Compute the Contribution of each wheel

$$\frac{r\dot{\varphi}_1}{2} + \frac{r\dot{\varphi}_2}{2}$$

- $\mathbf{Y_R = 0}$ (Always)

- Compute the rotational velocity.

$$\left[\frac{r\dot{\varphi}_1}{2l} + \frac{-r\dot{\varphi}_2}{2l}\right]$$

Kinematics

$$\theta = \frac{\pi}{2} \qquad r = 1 \qquad l = 1 \qquad \dot{\phi}_1 = 4 \qquad \dot{\phi}_2 = 2$$

$$\dot{\xi}_I = R(\theta)^{-1} \begin{bmatrix} \frac{r\dot{\varphi}_1}{2} + \frac{r\dot{\varphi}_2}{2} \\ 0 \\ \frac{r\dot{\varphi}_1}{2l} + \frac{-r\dot{\varphi}_2}{2l} \end{bmatrix}$$

$$R(\theta)^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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$\theta = \frac{\pi}{2}$ $\qquad r = 1$ $\qquad l = 1$ $\qquad \dot{\phi}_1 = 4$ $\qquad \dot{\phi}_2 = 2$

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$$= \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}$$

Kinematics

- A robot is positioned at a 60 degree angle with respect to the global reference frame and has wheels with a radius of 1cm. These wheels are 2 cm from the center of the chassis. If the speed of wheel 1 and 2 are 4 cm/s and 2 cm/s respectively. What is the robot velocity with respect to the global reference frame?

$$\dot{\xi}_I = R(\theta)^{-1} \begin{bmatrix} \frac{r\dot{\varphi}_1}{2} + \frac{r\dot{\varphi}_2}{2} \\ 0 \\ \frac{r\dot{\varphi}_1}{2l} + \frac{-r\dot{\varphi}_2}{2l} \end{bmatrix}$$

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$\theta = \pi / 3$

$r = 1$

$l = 2$

$\dot{\varphi}_1 = 4$

$\dot{\varphi}_2 \models 2$

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$$\theta = \pi / 3$$
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- A robot is positioned at a 60 degree angle with respect to the global reference frame and has wheels with a radius of 1cm. These wheels are 2 cm from the center of the chassis. If the speed of wheel 1 and 2 are 4 rad/s and 2 rad/s respectively. What is the robot velocity with respect to the global reference frame?

$$\dot{\xi}_I = R(\theta)^{-1} \begin{bmatrix} \frac{r\dot{\varphi}_1}{2} + \frac{r\dot{\varphi}_2}{2} \\ 0 \\ \frac{r\dot{\varphi}_1}{2l} + \frac{-r\dot{\varphi}_2}{2l} \end{bmatrix}$$

$$\begin{aligned} \theta &= \pi / 3 \\ r &= 1 \\ l &= 2 \\ \dot{\varphi}_1 &= 4 \\ \dot{\varphi}_2 &\models 2 \end{aligned}$$

$$\begin{aligned} \dot{x}_{r1} &= \frac{r\dot{\varphi}_1}{2} \\ \dot{x}_{r2} &= \frac{r\dot{\varphi}_2}{2} \\ \omega_1 &= \frac{r\dot{\varphi}_1}{2l} \\ \omega_2 &= -\frac{r\dot{\varphi}_2}{2l} \end{aligned}$$

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- A robot is positioned at a 60 degree angle with respect to the global reference frame and has wheels with a radius of 1cm. These wheels are 2 cm from the center of the chassis. If the speed of wheel 1 and 2 are 4 cm/s and 2 cm/s respectively. What is the robot velocity with respect to the global reference frame?

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$\theta = \pi / 3$

$r = 1$

$l = 2$

$\dot{\varphi}_1 = 4$

$\dot{\varphi}_2 \models 2$

$\dot{x}_{r1} = \frac{r\dot{\varphi}_1}{2}$

$\dot{x}_{r2} = \frac{r\dot{\varphi}_2}{2}$

$\omega_1 = \frac{r\dot{\varphi}_1}{2l}$

$\omega_2 = -\frac{r\dot{\varphi}_2}{2l}$

$$\dot{\xi}_I = R(\theta)^{-1} \begin{bmatrix} \dot{x}_{r1} + \dot{x}_{r2} \\ 0 \\ \omega_1 + \omega_2 \end{bmatrix} = \begin{bmatrix} \cos \pi/3 & -\sin \pi/3 & 0 \\ \sin \pi/3 & \cos \pi/3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3.0 \\ 0 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 1.5 \\ 2.5981 \\ 0.5 \end{bmatrix}$$

Kinematics

- A robot is positioned at a 45 degree angle with respect to the global reference frame and has wheels with a radius of 3cm. These wheels are 2 cm from the center of the chassis. If the speed of wheel 1 and 2 are 4 cm/s and 6 cm/s respectively. What is the robot velocity with respect to the global reference frame?

Kinematics

- A robot is positioned at a 45 degree angle with respect to the global reference frame and has wheels with a radius of 3cm. These wheels are 2 cm from the center of the chassis. If the speed of wheel 1 and 2 are 4 cm/s and 6 cm/s respectively. What is the robot velocity with respect to the global reference frame?

$$\dot{\xi}_I = R(\theta)^{-1} \begin{bmatrix} \frac{r\dot{\varphi}_1}{2} + \frac{r\varphi_2}{2} \\ 0 \\ \frac{r\dot{\varphi}_1}{2l} + \frac{-r\dot{\varphi}_2}{2l} \end{bmatrix}$$

$\theta = \pi/4$
$r = 3$
[redacted]
$l = 2$
$\dot{\varphi}_1 = 4$
$\dot{\varphi}_2 = 6$

$$\Rightarrow \begin{bmatrix} \cos\pi/4 & -\sin\pi/4 & 0 \\ \sin\pi/4 & \cos\pi/4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{3\times4}{2} + \frac{3\times6}{2} \\ 0 \\ \frac{3\times4}{2\times2} + \frac{-3\times6}{2\times2} \end{bmatrix}$$

$$= \begin{bmatrix} 0.70 & -0.70 & 0 \\ 0.70 & 0.70 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 15 \\ 0 \\ -1.5 \end{bmatrix} = \begin{bmatrix} 10.66 \\ 10.66 \\ -1.5 \end{bmatrix}$$