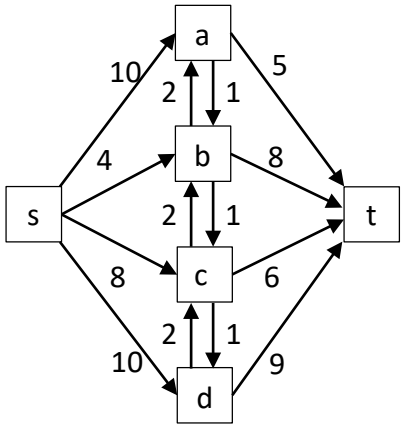
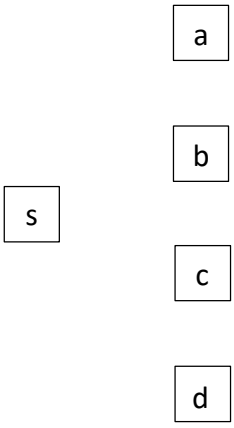
Max Flow Sample Problems

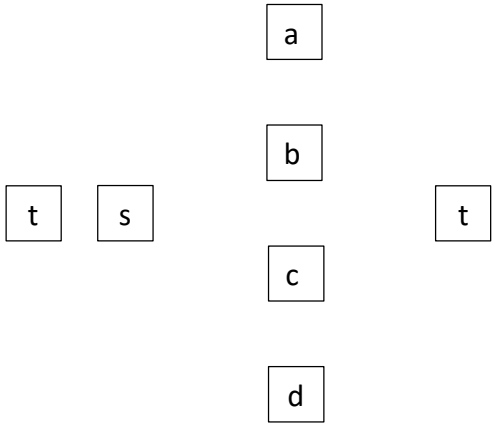
1. Find a max flow in this network from s to t, and show the final flow value along each edge. Also indicate the vertex partition that forms a min cut, and show the edges that cross the min cut.



Show max flow:



Show min cut:



2. What three things does the max-flow min-cut theorem state are equivalent:

i. _____

ii. _____

iii. _____

3. Let G be a flow network with source s, sink t, and integer capacities. Suppose that we are given a max-flow in G together with residual flow graph G'. Suppose that we now decrease the capacity of a single edge, u = (x,y), by 1. Give an efficient algorithm to update the maximum flow and state the running time of your algorithm.

   How would your algorithm change if the capacity of the edge u was reduced by k instead of just 1?

4. ***26.3-1***
   Run the Ford-Fulkerson algorithm on the flow network in Figure 26.8(c) and show the residual network after each flow augmentation. Number the vertices in L top to bottom from 1 to 5 and in R top to bottom from 6 to 9. For each iteration, pick the augmenting path that is lexicographically smallest. What is the size of the corresponding maximum matching?