Kinematics

- Kinematics studies how mechanical systems behave
- Why do we need to understand the mechanical behavior of robots?

Kinematics

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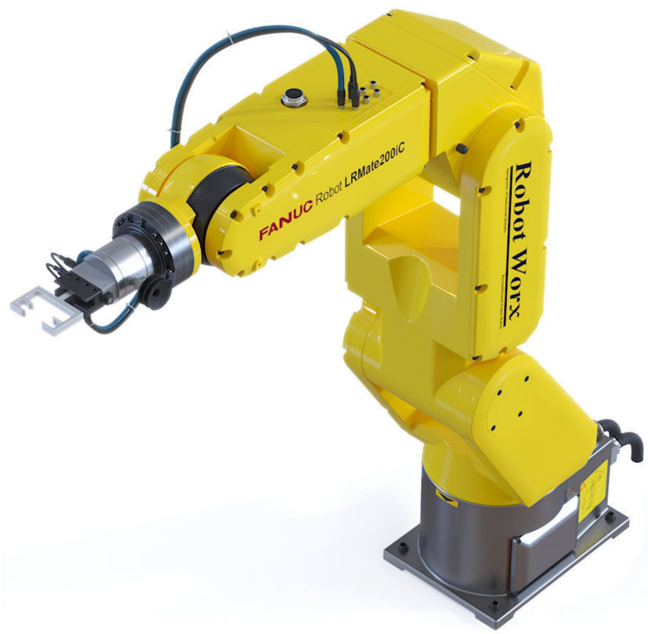
- Why do we need to understand the mechanical behavior of robots?
  - Design appropriate robots for the tasks
  - Understand how to create **control software** for robot **hardware**

- Helps define a robot's a robot's workspace

- Used for **position** and **motion** estimation

Kinematics

- What are key differences between **manipulator robots** and **mobile robots**?



Manipulator robot

Mobile robot

Kinematics

- What are key differences between **manipulator robots** and **mobile robots**?
  - There is no direct way to **measure a robot's position instantaneously**

- Must integrate the motion over time

- Motion estimation can be **inaccurate** and extremely challenging due to **slippage**

Kinematics

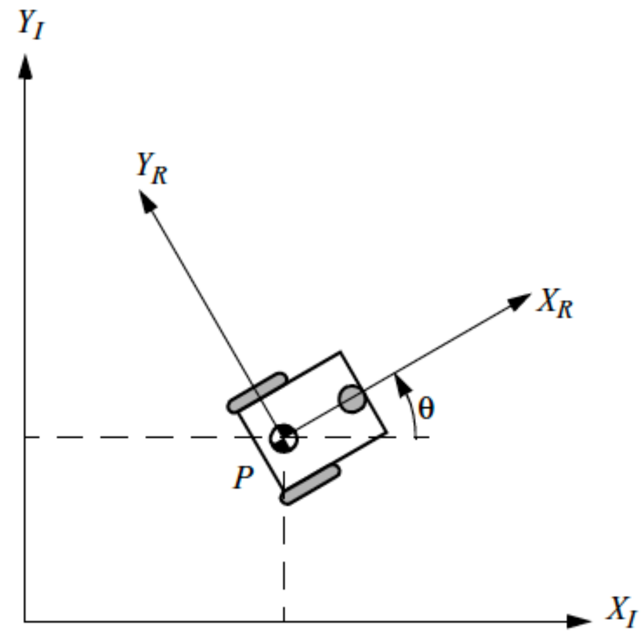
- Deriving a model for the whole robot's motion is a bottom-up process

- Each **Wheel**
  - **Contributes** to the robot motion
  - Imposes **constraints** on the robot's motion

- Forces and constraints of each wheel must be expressed with respect to a clear and consistent **reference frame.**

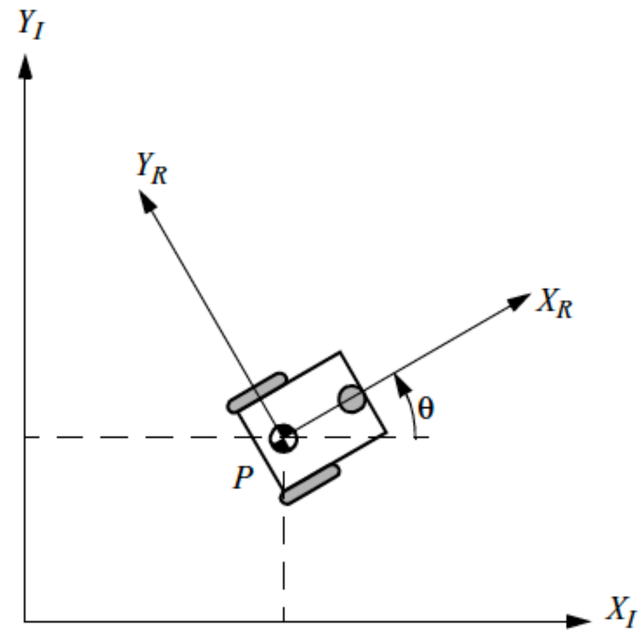
Kinematics

- We define a robot as a **rigid body on wheels,** operating on a horizontal plane.
- How can we specify the position of the robot on the plane?



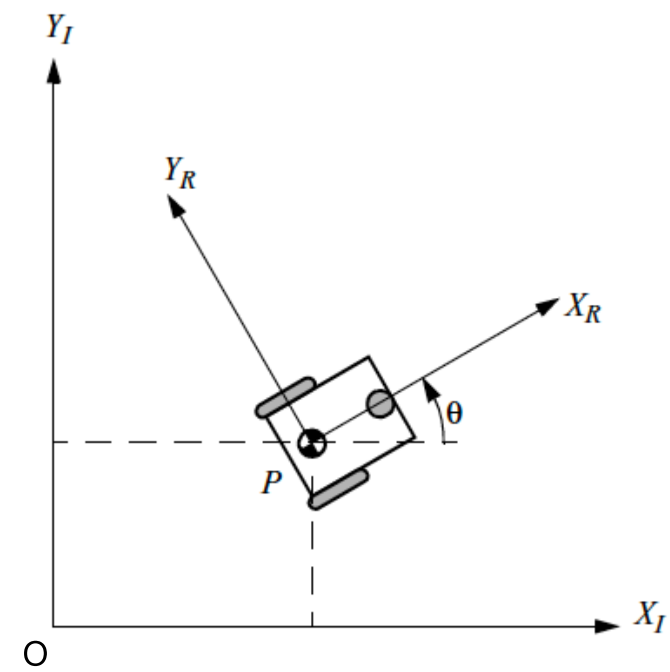
Kinematics

- We define a robot as a **rigid body on wheels,** operating on a horizontal plane.
- How can we specify the position of the robot on the plane?
- Establish relationship between **global reference frame** and **local reference frame**



Kinematics

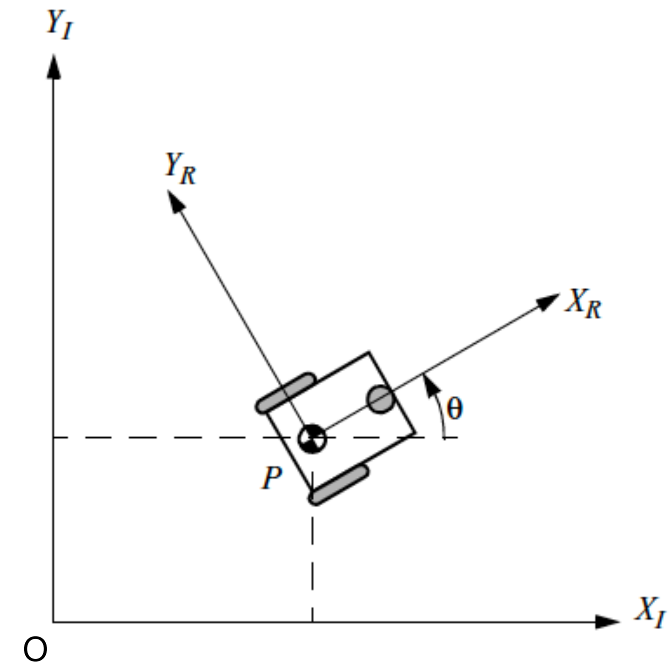
- **Local (robot) reference frame**: Three dimensional, describes position on the plane and orientation, $\{X_R, Y_R, \boldsymbol{\theta}\}$
- **Global Reference Frame:** Defined by axes X_I and Y_I with some origin O, $\{X_I, Y_I\}$
- Angular difference between global and reference frames is $\boldsymbol{\theta}$.
- Point P on the robot in the global reference frame is specified by coordinates (x, y).



Kinematics

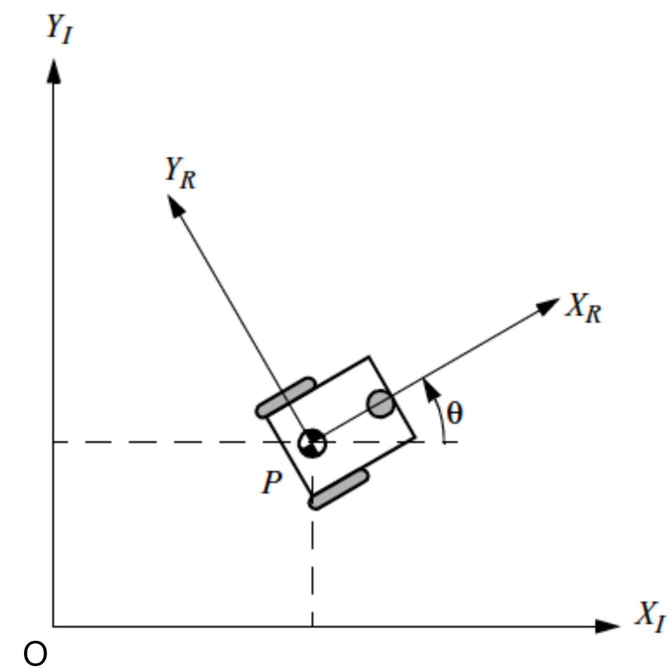
- **We can describe the pose (global) as a vector:**

$$\xi_I = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$



Kinematics

- How can we describe the robot **motion** in terms of component motions?
- We must first map motion along the axes of the **global reference frame** along the axes of the robot's **local reference frame**.
- This mapping is a function of the current pose of the robot.



Kinematics

- The **orthogonal rotation matrix (ORM) R(*θ*)** is used to handle mappings between the **global** $\{X_I, Y_I\}$ and local **reference frames** $\{X_R, Y_R\}$.
  - Computation of this operation depends on ***θ***.

$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Given the ORM definition, what is R(π / 2)?

$$\dot{\xi}_R = R(\theta)\dot{\xi}_I$$

Kinematics

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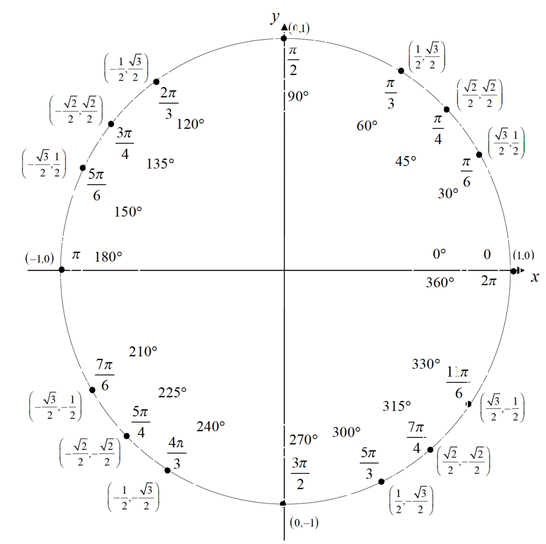
$$R(\frac{\pi}{2}) = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



For any ordered pair on the unit circle (x, y) : $\cos\theta = x$ and $\sin\theta = y$

Kinematics

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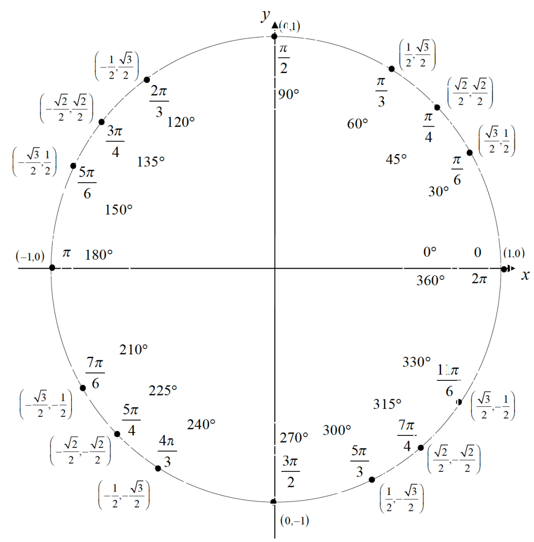
- Given the ORM definition, what is R(2π)?

$$R(2\pi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

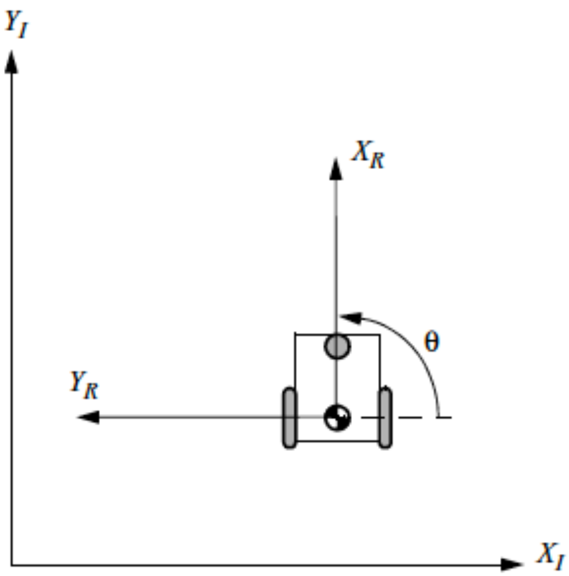


For any ordered pair on the unit circle (x, y) : $\cos\theta = x$ and $\sin\theta = y$

Kinematics

- Assume:

$$\dot{\xi}_I = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} \qquad R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



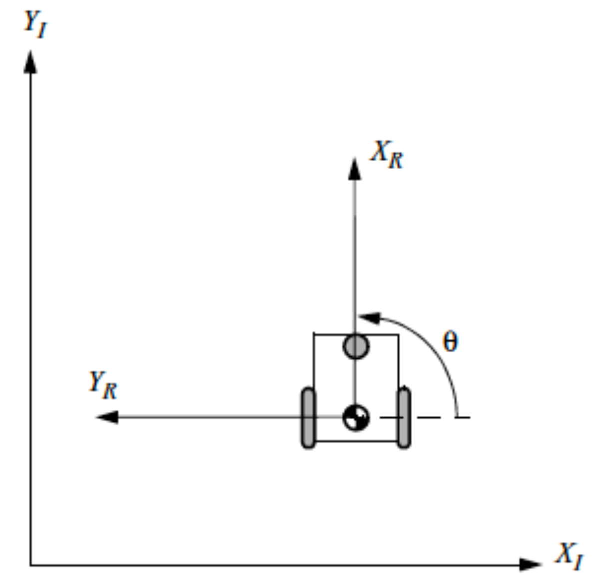
- What is the robot motion along its **local** axes X_R and Y_R if $\boldsymbol{\theta} = \pi / 2$?

$$\dot{\xi}_R =$$

Kinematics

- Assume:

$$\dot{\xi}_I = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} \qquad R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



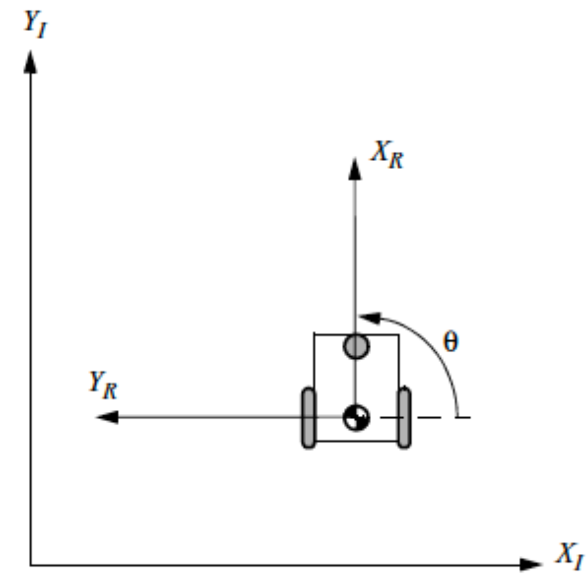
- What is the robot motion along its **local** axes X_R and Y_R if $\boldsymbol{\theta} = \pi / 2$?

$$\dot{\xi}_R = R\left(\frac{\pi}{2}\right)\dot{\xi}_I$$

Kinematics

- Assume:

$$\dot{\xi}_I = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} \qquad R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



- What is the robot motion along its **local** axes X_R and Y_R if $\boldsymbol{\theta} = \pi / 2$?

$$\dot{\xi}_R = R(\frac{\pi}{2})\dot{\xi}_I = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}$$

Matrix Multiplication Refresher

$$\begin{array}{cccc}
 & & \begin{array}{cc} \vec{b_1} & \vec{b_2} \\ \downarrow & \downarrow \end{array} & \\
\begin{array}{c} \vec{a_1}\rightarrow \\ \vec{a_2}\rightarrow \end{array} & \begin{bmatrix} 1 & 7 \\ 2 & 4 \end{bmatrix} \cdot & \begin{bmatrix} 3 & 3 \\ 5 & 2 \end{bmatrix} & = \begin{bmatrix} \vec{a_1} \cdot \vec{b_1} & \vec{a_1} \cdot \vec{b_2} \\ \vec{a_2} \cdot \vec{b_1} & \vec{a_2} \cdot \vec{b_2} \end{bmatrix} \\
 & A & B & C
\end{array}$$

Matrix Multiplication Refresher

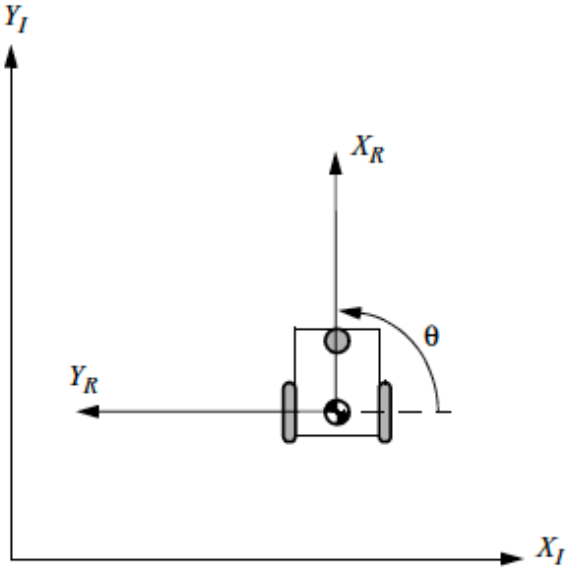
$$\begin{array}{c} \vec{a_1}\rightarrow \\ \vec{a_2}\rightarrow \end{array} \underset{A}{\begin{bmatrix} \boxed{1 \quad 7} \\ 2 \quad\ 4 \end{bmatrix}} \cdot \underset{B}{\begin{array}{cc} \vec{b_1} & \vec{b_2} \\ \downarrow & \downarrow \\ \multicolumn{2}{c}{\begin{bmatrix} \boxed{\begin{matrix}3\\5\end{matrix}} & \begin{matrix}3\\2\end{matrix} \end{bmatrix}} \end{array}} = \underset{C}{\begin{bmatrix} \vec{a_1}\cdot\vec{b_1} & \vec{a_1}\cdot\vec{b_2} \\ \vec{a_2}\cdot\vec{b_1} & \vec{a_2}\cdot\vec{b_2} \end{bmatrix}}$$

$$a_1 \cdot b_1 = \begin{bmatrix}1\\7\end{bmatrix} \cdot \begin{bmatrix}3\\5\end{bmatrix} = (1\cdot 3) + (7 \cdot 5) = 38$$

Kinematics

- Assume:

$$\dot{\xi}_I = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} \qquad R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



- What is the robot motion along its **local** axes X_R and Y_R if $\boldsymbol{\theta} = \pi / 2$?

$$\dot{\xi}_R = R(\frac{\pi}{2})\dot{\xi}_I = \begin{bmatrix} \boxed{0 \quad 1 \quad 0} \\ -1 \quad 0 \quad 0 \\ 0 \quad 0 \quad 1 \end{bmatrix} \boxed{\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}} = \begin{bmatrix} ? \\ \\ \end{bmatrix}$$

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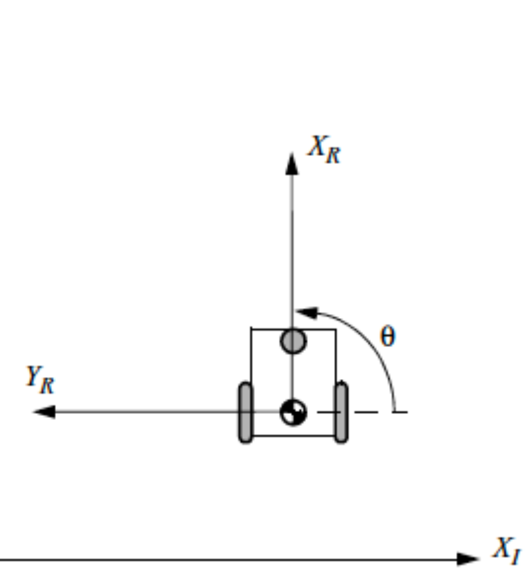

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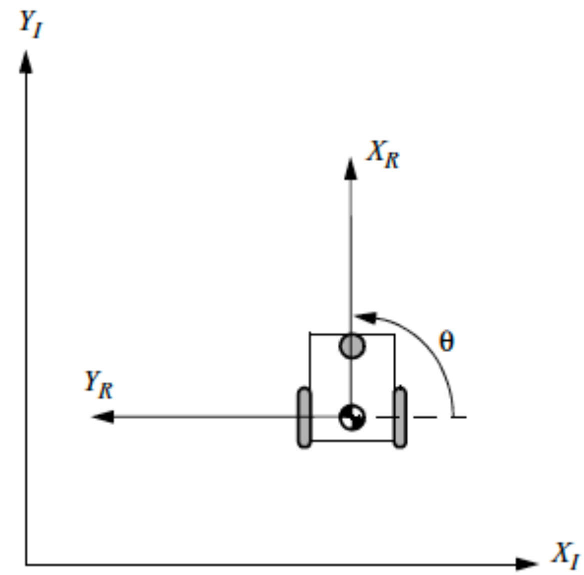

- What is the robot motion along its **local** axes X_R and Y_R if $\boldsymbol{\theta} = 2\pi$?

$$\dot{\xi}_R =$$

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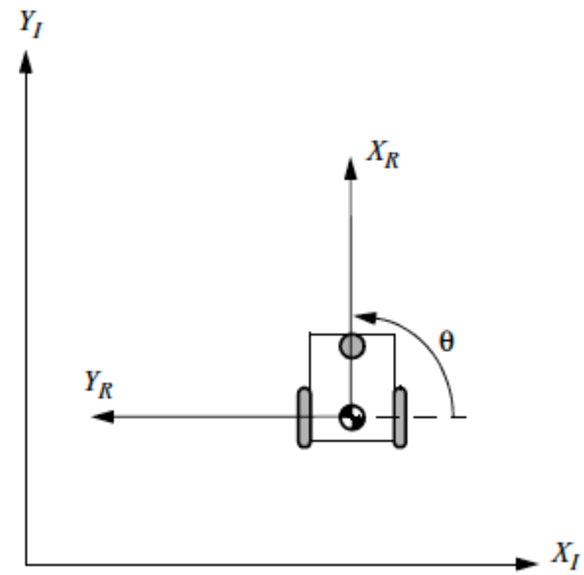

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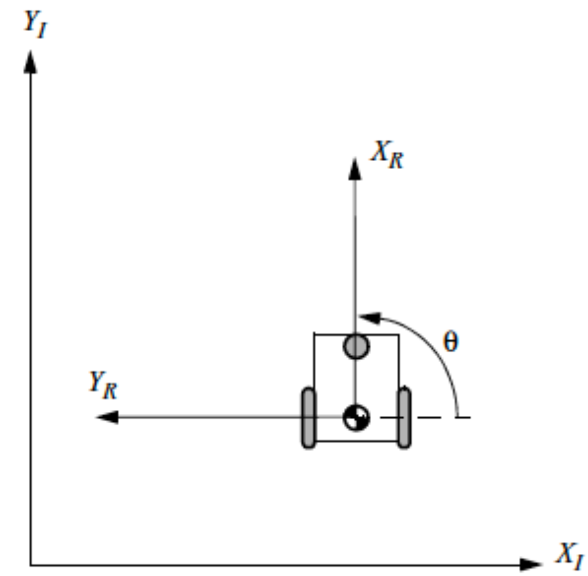

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Kinematics

- Assume the robot has a velocity of $(\dot{x}, \dot{y}, \dot{\theta})$ in the global reference frame and is positioned at P and $\boldsymbol{\theta} = \pi / 3$ with respect to the global reference frame. What is the motion along X_R and Y_R due to $\boldsymbol{\theta}$ with respect to the robot reference?

$$\dot{\xi}_R =$$

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Kinematics

$$\dot{\xi}_R = R\left(\frac{\pi}{3}\right)\dot{\xi}_I = \begin{bmatrix} \cos(\pi/3) & \sin(\pi/3) & 0 \\ -\sin(\pi/3) & \cos(\pi/3) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}$$

$$\begin{bmatrix} (\cos(\pi/3)\cdot\dot{x}) + (\sin(\pi/3)\cdot\dot{y}) + (0\cdot\dot{\theta}) \\ (-\sin(\pi/3)\cdot\dot{x}) + (\cos(\pi/3)\cdot\dot{y}) + (0\cdot\dot{\theta}) \\ (0\cdot\dot{x}) + (0\cdot\dot{y}) + (1\cdot\dot{\theta}) \end{bmatrix} = \begin{bmatrix} 0.5\dot{x} + 0.866\dot{y} \\ -0.866\dot{x} + 0.5\dot{y} \\ \dot{\theta} \end{bmatrix}$$

Kinematics

- Assume the robot has a velocity of (2 cm/s, 3cm/s, 5 rad/s) in the global reference frame and is positioned at P and $\boldsymbol{\theta} = \pi / 3$ with respect to the global reference frame. What is velocity with respect to the robot's local reference frame?

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- Assume the robot has a velocity of (2 cm/s, 3cm/s, 5 rad/s) in the global reference frame and is positioned at P and $\boldsymbol{\theta} = \pi / 3$ with respect to the global reference frame. What is velocity with respect to the robot's local reference frame?

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$$\dot{\xi}_R = R\left(\frac{\pi}{3}\right)\dot{\xi}_I = \begin{bmatrix} \cos{}^{\pi}\!/_3 & \sin{}^{\pi}\!/_3 & 0 \\ -\sin{}^{\pi}\!/_3 & \cos{}^{\pi}\!/_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix} =$$

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$$\dot{\xi}_R = R\left(\frac{\pi}{3}\right)\dot{\xi}_I = \begin{bmatrix} \cos{}^{\pi}\!/_3 & \sin{}^{\pi}\!/_3 & 0 \\ -\sin{}^{\pi}\!/_3 & \cos{}^{\pi}\!/_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 3.5981 \\ -0.23205 \\ 5 \end{bmatrix}$$

Kinematics

$$\dot{\xi}_{12} = R\left(\frac{\pi}{3}\right)\dot{\xi}_{I} = \begin{bmatrix} \cos(\pi/3) & \sin(\pi/3) & 0 \\ -\sin(\pi/3) & \cos(\pi/3) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} (\cos(\pi/3)\cdot 2) + (\sin(\pi/3)\cdot 3) + (0\cdot 5) \\ (-\sin(\pi/3)\cdot 2) + (\cos(\pi/3)\cdot 3) + (0\cdot 5) \\ (0\cdot 2) + (0\cdot 3) + (1\cdot 5) \end{bmatrix} = \begin{bmatrix} 3.59 \\ -0.23 \\ 5 \end{bmatrix}$$