1: $P_r = R(\theta) P_i$

$$P_i = \begin{bmatrix} 6 \text{ cm/s} \\ 4 \text{ cm/s} \\ 12 \text{ rad/s} \end{bmatrix}, \quad \theta = \pi/2$$

$$P_r = \begin{bmatrix} 6 \\ 4 \\ 12 \end{bmatrix} \begin{bmatrix} \cos\pi/2 & \sin\pi/2 & 0 \\ -\sin\pi/2 & \cos\pi/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 12 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 \text{ cm/s} \\ -6 \text{ cm/s} \\ 12 \text{ rad/s} \end{bmatrix}$$

2: $P_r = R(\theta) P_i$, or $P_i = P_r R^{-}(\theta)$

$$P_r = \begin{bmatrix} 6 \text{ cm/s} \\ 2 \text{ cm/s} \\ 18 \text{ rad/s} \end{bmatrix}, \quad \theta = 3\pi/2$$

$$P_i = \begin{bmatrix} 6 \\ 2 \\ 18 \end{bmatrix} \begin{bmatrix} \cos 3\pi/2 & -\sin 3\pi/2 & 0 \\ \sin 3\pi/2 & \cos 3\pi/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ 18 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 \text{ cm/s} \\ -6 \text{ cm/s} \\ 18 \text{ rad/s} \end{bmatrix}$$

3: Not enough information! We need to know the angle, θ, between the global and local reference frames.

4: Known variables:

$\theta = \pi/2$

$r = 6$ cm

$l = 2$ cm

$\phi_1 = 8$ rad/s

$\phi_2 = 4$ rad/s

Equation:

$$P_I = R(\theta)^{-1} \begin{bmatrix} \frac{r\phi_1}{2} + \frac{r\phi_2}{2} \\ 0 \\ \frac{r\phi_1}{2l} - \frac{r\phi_2}{2l} \end{bmatrix}$$

$$P_I = \begin{bmatrix} \cos\pi/2 & \sin\pi/2 & 0 \\ -\sin\pi/2 & \cos\pi/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{6 \cdot 8}{2} + \frac{6 \cdot 4}{2} \\ 0 \\ \frac{6 \cdot 8}{2 \cdot 2} - \frac{6 \cdot 4}{2 \cdot 2} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 36 \\ 0 \\ 6 \end{bmatrix} = \begin{bmatrix} 0 \text{ cm/s} \\ -36 \text{ cm/s} \\ 6 \text{ rad/s} \end{bmatrix}$$