The phase space diagram gives us all possible values of position and momentum at a <u>fixed energy</u>.

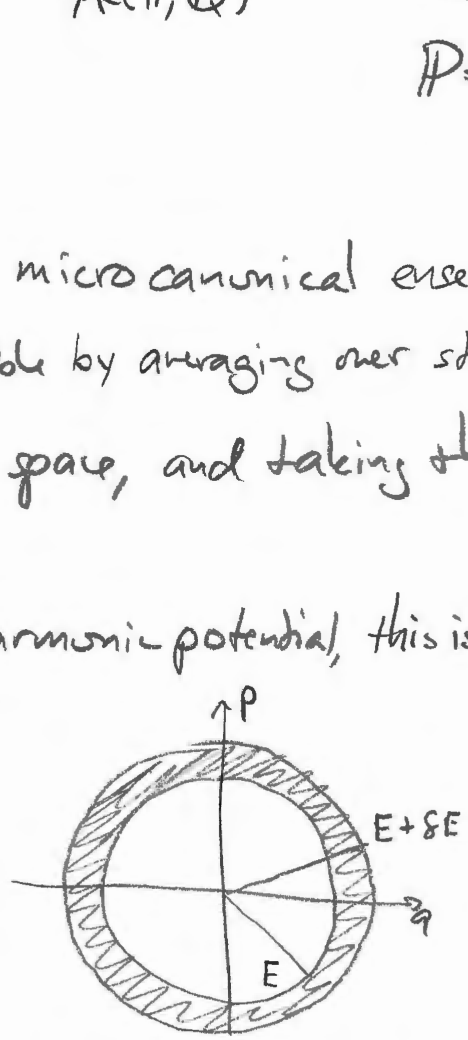
For harmonic oscillators, the Hamiltonian is simple (momentum and position are parabolic).

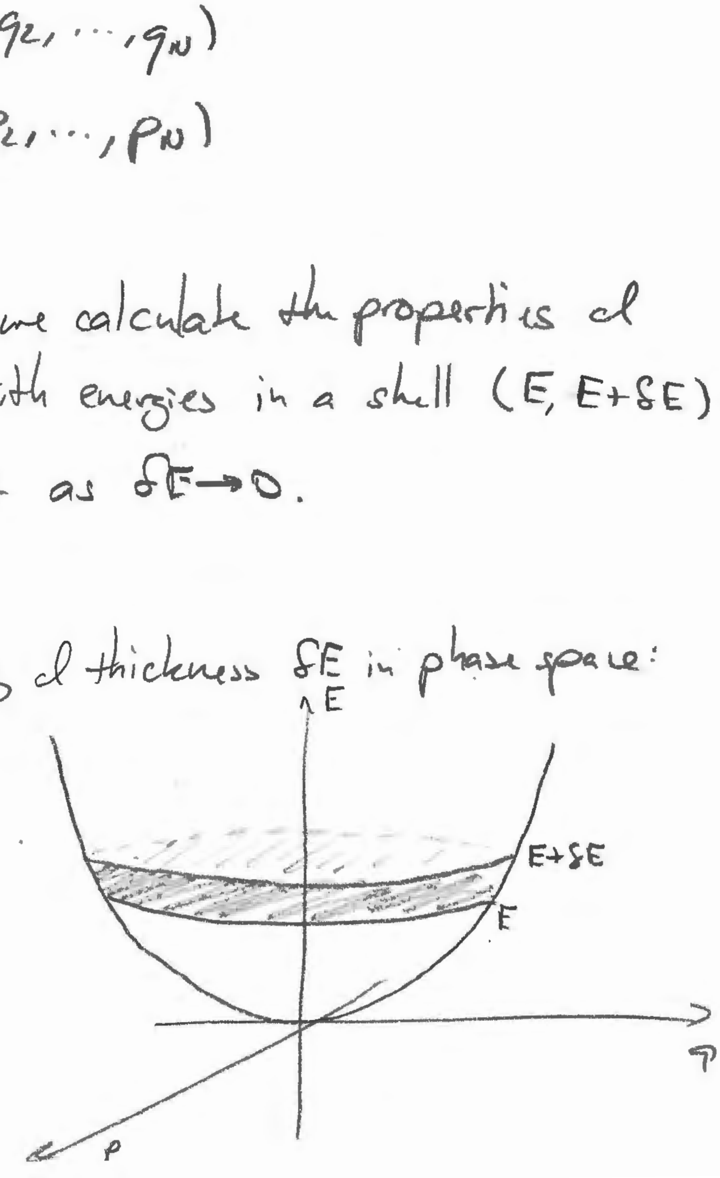
For large N, we must define our Hamiltonian in general, and in terms of the generalized $\mathbb{P}, \mathbb{Q}$:

$$\mathcal{H}(\mathbb{P}, \mathbb{Q}) \qquad \mathbb{Q} = (q_1, q_2, \cdots, q_N)$$
$$\mathbb{P} = (p_1, p_2, \cdots, p_N)$$

In the microcanonical ensemble, we calculate the properties of our ensemble by averaging over states with energies in a shell $(E, E+\delta E)$ in phase space, and taking the limit as $\delta E \to 0$.

For the harmonic potential, this is a ring of thickness δE in phase space:





• In general, we define the phase space volume divided by δE:

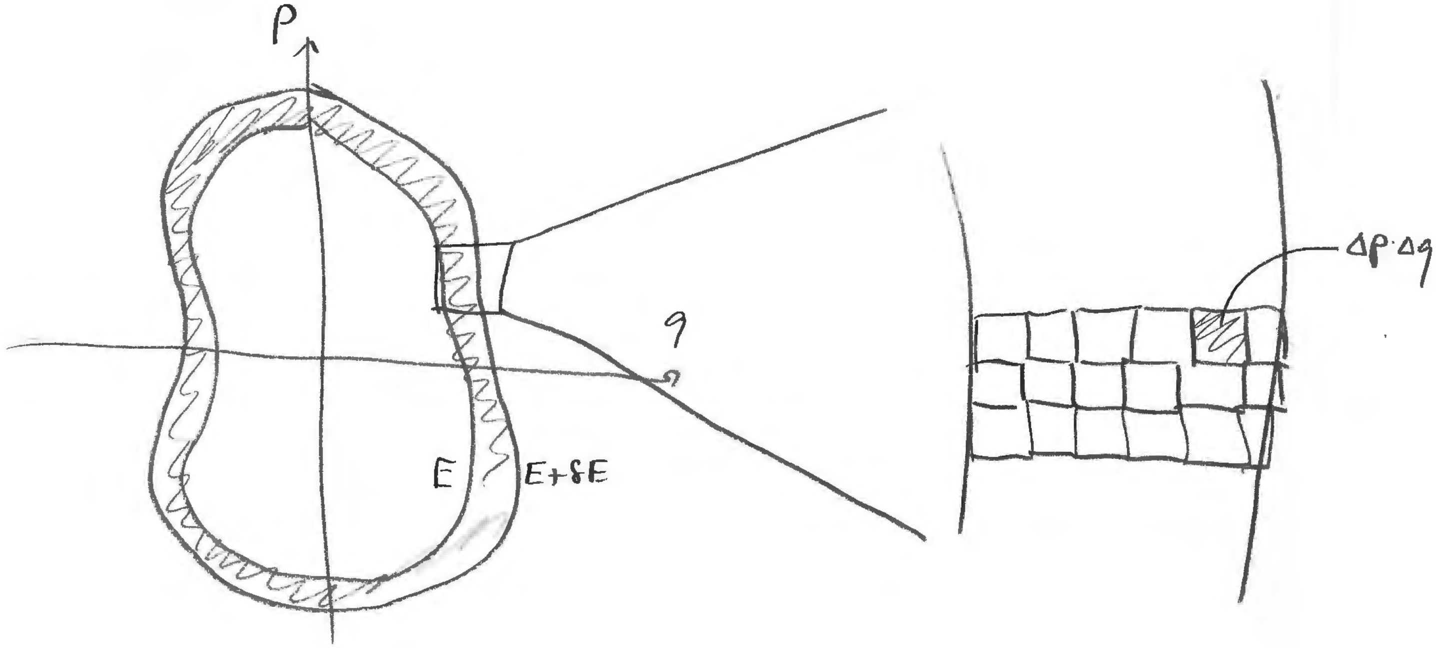
$$\Omega(E) = \frac{1}{\delta E} \int_{E < \mathcal{H}(\mathbb{P},\mathbb{Q}) < E+\delta E} d\mathbb{P}\, d\mathbb{Q}$$

This is comparable to the "Density of states" $dN/d\epsilon$ where the total number of states is filled by infinitesimal $\Delta p \Delta q$

Using this formalism, we can find the average $\langle O \rangle$ of a property of the microcanonical ensemble by

$$\langle O \rangle_E = \frac{1}{\Omega(E)\, \delta E} \int_{E < \mathcal{H}(\mathbb{P},\mathbb{Q}) < E+\delta E} O(\mathbb{P},\mathbb{Q})\, d\mathbb{P}\, d\mathbb{Q}$$

What does this look like for an arbitrary Hamiltonian?



In the microcanonical ensemble, the long-time equilibrium behavior of the system is precisely the typical behavior of all system with the same value of the conserved quantity (E in this case)

This is made explicit in our expression for $\langle O \rangle$. All points in phase space are equally likely, so the average treats them with equal weight.