# University of California Riverside

PHYS 133, Statistical Mechanics, Spring 2019

**Final Problem Set** Due June 12, 2019 @ 7:00pm

**Instructions:**

-Work alone on this problem set. While it is challenging, the point of this exam is for you to review and understand the content of the entire course. The exam is structured such that you can actively review the course while solving relevant problems.

-Email any questions. You may ask Tom any questions, but please do not discuss this exam with other students in Physics 133 or any other physics course. In order to be fair, questions should only be asked and will only be answered in email form, so that the questions and answers can be distributed among all students.

-Attempt every problem and every part. Even if you don't have time to complete a calculation, credit will be given if you explain the procedure that you would attempt to get the solution.

0. **Read and review.** Sethna Ch. 1, 2, 3, 5, and 7.
1. **Random Walks in Grade Space.** Sethna Problem 1, Chapter 2.
2. **Maxwell Relations.** Sethna Problem 11, Chapter 3.
3. **Burning Information and Maxwellian Demons.** Sethna Problem 2, Chapter 5.
4. **Entropy Increases: Diffusion.** Sethna Problem 10, Chapter 5.
5. **Phase Space Units and the Zero of Entropy.** Sethna Problem 3, Chapter 7.

6. **An Unlikely Event.** On June 12, 2019 at 6:59 pm, you are exiting the elevator, walking toward MS&E to hand in your final take home exam for Physics 133. Just a few moments before 6:59 pm, a small meteor falls from the sky and crashes into a physics textbook on the desk of a philosopher trying to understand the meaning of entropy. The meteor is burning at such a high temperature that it instantly incinerates the book and continues burning through floor after floor of the building until coming to rest near a water line adjacent to a 50 year-old water fountain. The pipe becomes super heated, compressing the water in the plumbing and causing a catastrophic failure of the building's pipes. The compression shock shakes the foundation of the building and, just as you exit the elevator, a micro-earthquake rumbles across the entire campus. Your exam slips from your hands and falls like an autumn leaf right between the crack that separates the elevator from the third floor's surface. The exam can't be recovered. Assume that the observation of this event is extremely unlikely. How much information is contained in such an event?

**a.** To answer this, let's first recall: From Shannon, we know that entropy is in fact a measure of *information*. Suppose that any event, including that described above, occurs with a probability $p$. We can associate with the observation of this event a function that quantifies the amount of information, call it $I(p)$. Find the functional form of $I(p)$ such that the following conditions are met:

1. Information is *non-negative*.
2. If two events occur independently so that their joint probability is the product of event 1 and event 2, then their information is *additive*.
3. Information is a continuous function of the probability $p$.
4. There is *no information* content to an event that is always observed.

Start by writing down the above conditions in a mathematical form. Then determine the simplest function that satisfies all 4 properties.

**b.** Find the value of $I(p)$ for the event described above (i.e., the probability of observing such an event approaches zero). How much information is contained in an event as unlikely as this one?