

① Travelling Waves

- Wave Equation
- History & Snapshot Graphs
- Phase Shift
- Doppler Shift

② Superposition

- Interference
~~→ Stephen example~~
- Discontinuities on a string
- Standing Waves
→ Stephen example
- Beats
 - Desmos demo

③ Ideal Gases

- moles \rightarrow grams
- $PV = nRT$
- mention 4 processes
- WORK

④ First Law of Thermo

- heat vs. thermal energy vs. T
- WORK
~~vs. P~~

• Ideal Gas Processes

⑤ Calorimetry

- Specific heats (C_p and C_v)

• Adiabatic

⑥ Micro/Macro

- mean free path
- V_{rms}
- Temperature
- Thermal energy
- degrees of freedom

⑦ Heat Engines & Refrigerators

- Q_H and Q_C
- heat engines
- Carnot Engines
- is it impossible
- Refrigerators
- 2nd Law
- Entropy

The Wave Model

a Def: An organized disturbance that propagates thru a medium

two types of waves

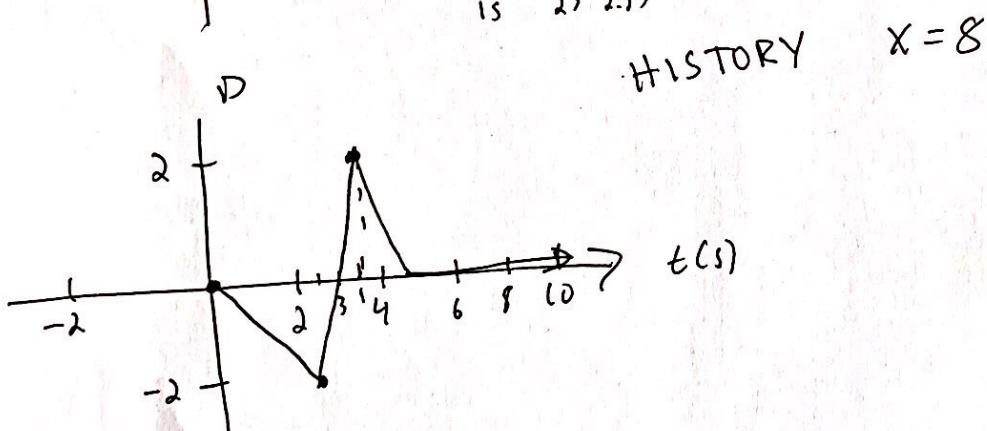
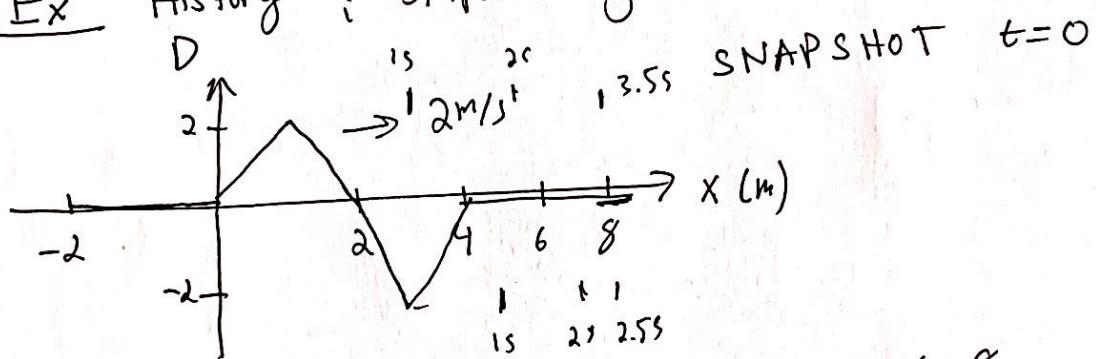
1. Longitudinal (sound)

2. Transverse (light, wave on string)

$$D(x, t) = A \sin(\kappa x - \omega t + \phi)$$

$$\kappa = \frac{2\pi}{\lambda} \quad \omega = \frac{2\pi}{T}$$

Ex History & Snapshot graphs



$$D(x,t) = A \sin(kx - \omega t + \phi)$$

velocity of particles : $\frac{dD}{dt} = -A\omega \cos(kx - \omega t + \phi)$

velocity of wave : $v = 2f$

$E_x \perp b$) $v = 50 \text{ m/s}$

$$\lambda = 10 \text{ m}$$

$$f = \frac{v}{\lambda} = \frac{50}{10} \frac{1}{s}$$

Phase of a Wave

$$D(x,t) = A \sin(kx - \omega t + \phi) \quad \sin(kx - \omega t) = \sin(kx - \omega t + 2\pi)$$

What is phase?

$$S_1 = A \sin(kx_1 - \omega t + \phi_{10})$$

$$S_2 = A \sin(kx_2 - \omega t + \phi_{20})$$

$$\phi_1 - \phi_2 = (kx_1 - \omega t + \phi_{10}) - (kx_2 - \omega t + \phi_{20}) \\ = k\Delta x + \Delta\phi$$

for constructive interference: $k\Delta x + \Delta\phi = m \cdot 2\pi$ ($m = 0, 1, 2, \dots$)

for destructive interference: $k\Delta x + \Delta\phi = (m + \frac{1}{2}) \cdot 2\pi$ ($m = 0, 1, 2, \dots$)

Ex 2: Interference

$$\frac{2\pi}{\lambda} \Delta x = m \cdot 2\pi \\ \Delta x = m \lambda$$



when $L = 20 \text{ cm} \rightarrow$ perfect constructive interference

when $L = 60 \text{ cm} \rightarrow$ destructive "

$$\frac{\lambda}{2} = 60 \text{ cm} - 20 \text{ cm} = 40 \text{ cm}$$

$$\boxed{\lambda = 80 \text{ cm}}$$

$$60 \text{ cm} + \frac{\lambda}{2} = 60 + 40 = \boxed{100 \text{ cm}}$$

Principle of Superposition

1.2A.7.1

→ When two or more waves "travel" thru each other, at a single point in space; the displacement of the medium at that point is sum of displacements due to each individual wave

$$D_{\text{net}} = \sum_i D_i$$

$$(A \sin(\omega t - \phi_1)) + (A \sin(\omega t - \phi_2)) = A \cdot (\sin \phi_1 + \sin \phi_2)$$

Phase Difference ; Interference

→ Let's assume we have 2 sinusoidal waves w/ same f, A

$$D_1 = A \sin(kx_1 - \omega t + \phi_{10}) = A \sin \phi_1$$

$$D_2 = A \sin(kx_2 - \omega t + \phi_{20}) = A \sin \phi_2$$

→ phase constant tells us what source is doing @ $t=0$

in phase: $D_1(x) = D_2(x)$ for each x

out of phase: $D_1(x) = -D_2(x)$

PHASE

uniting up to 100%

- the phase difference $\Delta\phi$ between two waves depends only on the ratio of their separation Δx to wavelength λ

$$\Delta\phi = \phi_2 - \phi_1 = (kx_2 - \omega t + \phi_0) - (kx_1 - \omega t + \phi_0)$$

$$= k(x_2 - x_1) = k\Delta x = \frac{2\pi\Delta x}{\lambda}$$

$$\Delta\phi = (\phi_0 + \omega t + kx_2) - (\phi_0 + \omega t + kx_1) = \Delta\theta$$

$$\Delta\phi = (\phi_0 + \omega t + kx_2) - (\phi_0 + \omega t + kx_1) = \Delta\theta$$

Q.E.D. $\Delta\phi$ is more than λ & $\lambda/2$ then $\phi_2 - \phi_1$

$$\times \text{less } \omega t \quad (\Delta\phi = \phi_2 - \phi_1 \text{ : angle } \omega t)$$

$$(\omega_2\theta_2 - \omega_1\theta_1) \text{ : angle } \theta_2 - \theta_1$$

Doppler Shift

Consider a moving source

for an approaching source $f_+ = \frac{f_0}{1 - \frac{v_s}{v}}$

for a receding source

$$f_- = f_0 \frac{1 + \frac{v_s}{v}}{1}$$

consider moving observer

for approaching observer $f_+ = \left(1 + \frac{v_s}{v}\right) f_0$

for receding observer $f_- = \left(1 - \frac{v_s}{v}\right) f_0$

Ex: Doppler effect

$$f_{\pm} = \frac{f_0}{1 \mp \frac{v_o}{v}}$$

$$v = \frac{C}{T} = \frac{2\pi r}{T}$$

~~$$f = 100 \text{ rpm} \cdot \frac{1 \text{ min}}{60 \text{ sec}} =$$~~

$$f = 100 \text{ rpm} \cdot \frac{60}{1} = 6000 \text{ Hz}$$

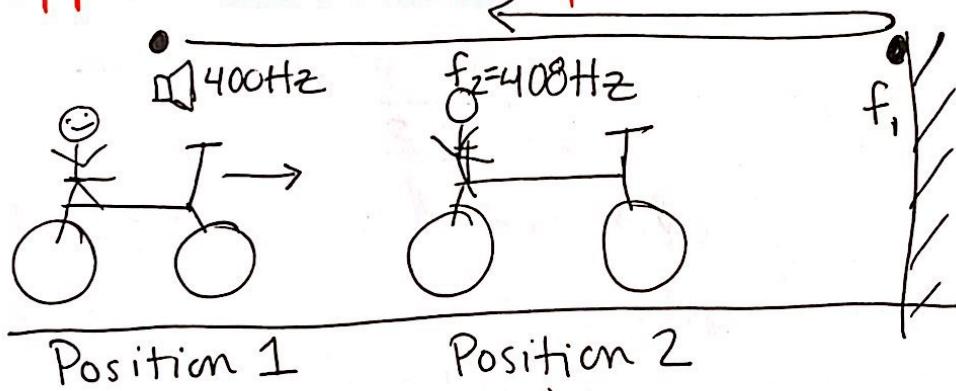
$$v = 2\pi r \cdot f$$

$$f = 100 \text{ rpm} \cdot \frac{1}{60} = \frac{100}{60} \text{ Hz}$$

$$v = \cancel{\pi} \cdot 1 \cdot \frac{10\cancel{\pi}}{\cancel{3}} = \frac{10}{3} \pi$$

$$f_{\pm} = \frac{600}{1 \mp \frac{10/3 \pi}{343}}$$

Doppler Shift Example #2



$$f_o = f \left(\frac{v \pm v_o}{v \mp v_s} \right)$$

First - find frequency at the wall (f_1)
(wall is observer)

$$v_o = 0$$

$$v_s = v_b \text{ (moving toward wall)}$$

$$f_1 = f \left(\frac{v + 0}{v - v_b} \right) \quad v = 340 \text{ m/s}$$

$$v_b = ?$$

Second - treat the situation as if the wall emitted frequency f_1

$$f_2 = f_1 \left(\frac{v + v_b}{v - 0} \right) \quad v_s = 0$$

$$f_2 = f_1 \left(\frac{v + v_b}{v} \right) \rightarrow f_2 = f \left(\frac{\cancel{v}}{v - v_b} \right) \left(\frac{v + v_b}{\cancel{v}} \right)$$

↓

$$408 \text{ Hz}$$

↓

$$f_1 = f \left(\frac{v}{v - v_b} \right)$$

$$f_2 = f \frac{(v + v_b)}{(v - v_b)}$$

$$408 \text{ Hz} = 400 \text{ Hz} \frac{(340 \text{ m/s} + v_b)}{(340 \text{ m/s} - v_b)}$$

$$\frac{408}{400} = \frac{340 + v_b}{340 - v_b}$$

$$1.02(340 - v_b) = 340 + v_b$$

$$346.8 - 1.02 v_b = 340 + v_b$$

$$6.8 = v_b (1 + 1.02)$$

$$6.8 = v_b (2.02)$$

$$\frac{6.8}{2.02} = v_b = \boxed{3.7 \text{ m/s}}$$

Superposition

$$D_{\text{net}} = D_1 + D_2 + D_3 + \dots$$

If wave encounters discontinuity ...

some energy is transmitted & some reflected

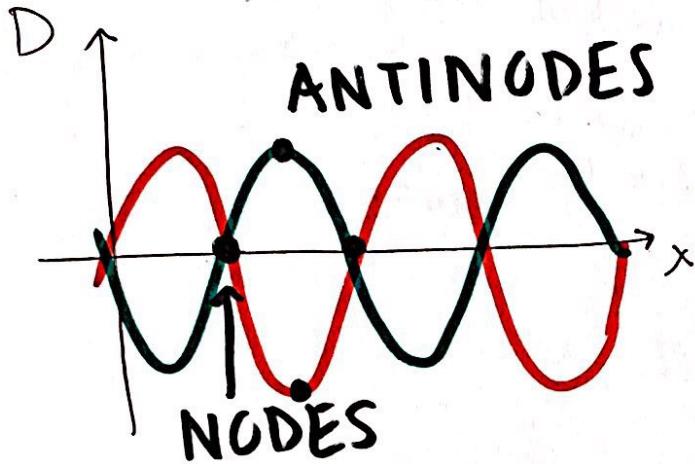
- this can cause the wave to either speed up, slow down, invert, or split

if encounters discontinuity where wave speed decreases, the reflected wave is inverted
(phase change of π)

if encounters a boundary, none of the wave can be transmitted. So it's all reflected
(same amplitude, but inverted)

Standing Waves

if two waves w/ the same amplitude, frequency & wavelength interact.



Nodes are spaced $\frac{\lambda}{2}$ apart
they never move

$$\left\{ \lambda = \frac{2L}{m} \quad m=1, 2, 3, 4. \right\}$$

Standing wave can exist if has one of these wavelengths

$$f_m = \frac{v}{\lambda_m}$$

$$f = \frac{v}{(2L/m)} = \frac{mv}{2L} \quad m=1, 2, 3, 4. \dots$$

fundamental frequency $m=1$

$$f_1 = \frac{v}{2L} \quad (\text{lowest allowed})$$

$$f_m = mf_1 \quad (\text{Harmonics})$$

Open-open or Closed-closed

$$\lambda_m = \frac{2L}{m} \quad f_m = \frac{mv}{2L} = mf_1$$

$$m = 1, 2, 3, 4. \dots$$

Open-closed

$$\lambda_m = \frac{4L}{m} \quad f_m = \frac{mv}{4L} = mf_1$$

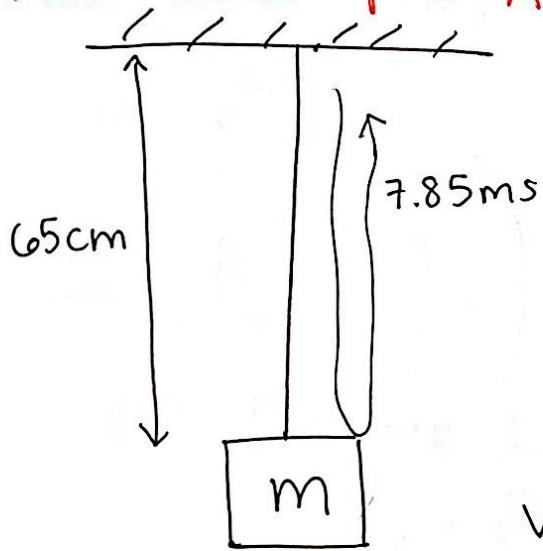
$$m = 1, 3, 5, 7, \dots$$

BEATS

$$f_{\text{beat}} = |f_1 - f_2|$$

when 2 waves of slightly varying frequency interact

Wave Example: A heavy stone



$$m_w = 8.25 \text{ g}$$

$$L = 65 \text{ cm}$$

$$t = 7.85 \text{ ms}$$

$$\mu = \frac{m_w}{L}$$

$$v_{\text{string}} = \sqrt{\frac{T_{\text{string}}}{\mu}}$$



$$T = mg$$

$$mg$$

$$v_{\text{string}} = \sqrt{\frac{mg}{\mu}} = \sqrt{\frac{mg}{\frac{m_w}{L}}} = \sqrt{\frac{mgL}{m_w}}$$

$$d = rt$$

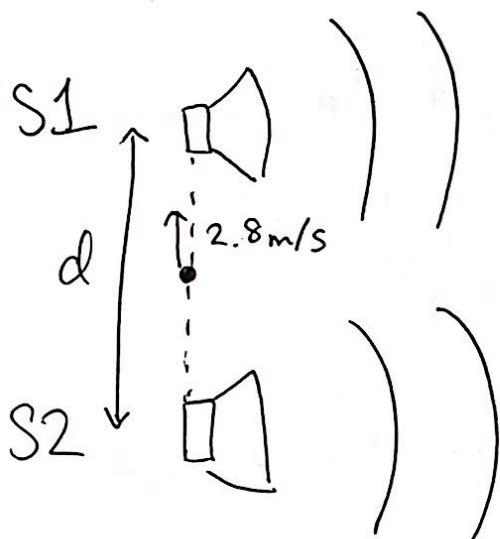
$$v_{\text{string}} = \frac{d}{t} = \frac{2(65 \text{ cm})}{7.85 \text{ ms}}$$

$$v_{\text{string}} = \frac{d}{t} = \frac{2(65 \text{ m})}{(7.85 \times 10^{-3}) \text{ s}} = 165.81 \text{ m/s}$$

$$m = \frac{v_s^2 m_w}{Lg} = \frac{(165.81 \text{ m/s})^2 (8.25 \text{ g} \times 10^{-3} \text{ kg})}{(0.65 \text{ m})(9.8 \text{ m/s}^2)}$$

$$m = 35.6 \text{ kg}$$

(Q4) Beats



$f = 1536 \text{ Hz}$ (both speakers)
 You will experience a Doppler shift & observe different frequencies for each speaker

(i.) Speaker 1 - you're moving toward

$$f_0 = f \left(\frac{v \pm v_o}{v \mp v_s} \right) \quad v_s = 0$$

$$f_0 = f \left(\frac{v + v_o}{v} \right) \quad v = 340 \text{ m/s}$$

$$f_0 = (1536 \text{ Hz}) \left(\frac{340 + 2.8 \text{ m/s}}{340} \right) = \cancel{1548.65 \text{ Hz}}$$

$$f_0 = 1548.65 \text{ Hz}$$

(ii.) Speaker 2 - you're moving away

$$f_0 = (1536 \text{ Hz}) \left(\frac{340 - 2.8}{340} \right) \quad v_s = 0$$

$$v = 340 \text{ m/s}$$

$$v_o = 2.8 \text{ m/s}$$

$$f_0 = 1523.35 \text{ Hz}$$

(iii.) Beat frequency

$$f_b = |f_1 - f_2| = |1548.65 - 1523.35|$$

$$f_b = 25.3 \text{ Hz}$$

Thermodynamics

number of mols in a substance

$$n = \frac{N}{N_A} \quad N_A \sim 6 \times 10^{23} \text{ particles}$$

How many molar in 100 g of oxygen gas?

$$m_{\text{molecule}} = 32 \text{ u} \cdot \frac{1.66 \times 10^{-27} \text{ kg}}{1 \text{ u}} = 5.31 \times 10^{-26} \text{ kg}$$

$$N = \frac{M_{\text{gas}}}{m_{\text{molecule}}} = \frac{0.1 \text{ kg}}{5.31 \times 10^{-26} \text{ kg}}$$

$$n = \frac{N}{N_A} = \frac{M_{\text{gas}}}{m_{\text{molecule}}} \frac{1}{N_A} = \frac{0.1 \text{ kg}}{5.31 \times 10^{-26} \text{ kg}} \frac{1}{6 \times 10^{23}}$$

$$n = 3.13 \text{ mols}$$

Ideal Gases

Ideal Gas Model

- 1) Gases are sparse (low density)
- 2) Gases are non interacting

Ideal Gas Law

$$PV = nRT$$

Example: Ideal Gas Processes

Initially $P_1 V_1 T_1$

isothermal expansion $\rightarrow \Delta T = 0$

$$P_1 V_1 = P_2 \cdot V_2$$

$$V_2 = 2V_1$$

$$P_1 V_1 = P_2 \cdot 2V_1$$

$$P_2 = \frac{1}{2} P_1$$

Ideal Gas Processes

1. Isothermal process

$$\Delta T = 0$$

2. Isobaric Process

$$\Delta P = 0$$

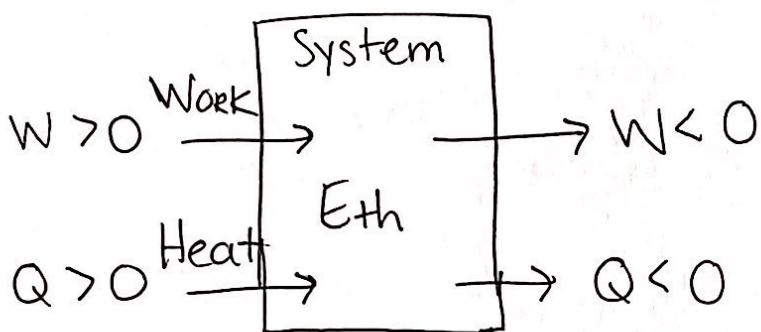
3 Isochoric Process

$$\Delta V = 0$$

First Law of Thermodynamics

$$\Delta E_{\text{th}} = W + Q$$

Environment



Work is the result of mechanical interactions
(i.e. forces)

Heat is the result of thermal interactions

Work

$$W = - \int_{V_i}^{V_f} p dV$$

$W > 0 \rightarrow$ gas is compressed
• energy transferred in

$W < 0 \rightarrow$ gas is expanded
• energy leaves system

Heat: energy transferred between a system & environment as a consequence of the temperature difference between them

- requires no macroscopic motion
- faster molecules collide w/ slower molecules until all molecules have the same avg speed
- thermal equilibrium - no ~~temperature~~ temperature difference

Thermal Energy

$$E_{\text{th}} = \frac{1}{2} k_B T$$

Work in ideal gas processes

isothermal process

$$\Delta T = 0 \quad (\Delta E_{th} = 0) \quad W = Q = N k_B T \log\left(\frac{V_f}{V_i}\right)$$

isochoric

$$\Delta V = 0$$

$$W = 0 \rightarrow \Delta E_{th} = Q$$

isobaric

$$\Delta P = 0$$

$$W_{ext} = -P \Delta V$$

adiabatic

$$Q = 0$$

$$\Delta E_{th} = W$$

$$P V^\gamma = \text{const.}$$

What happens if you change the thermal energy (ΔE_{th}) of system?

(1.) Temperature will change

(2.) The system can undergo phase change

Calorimetry

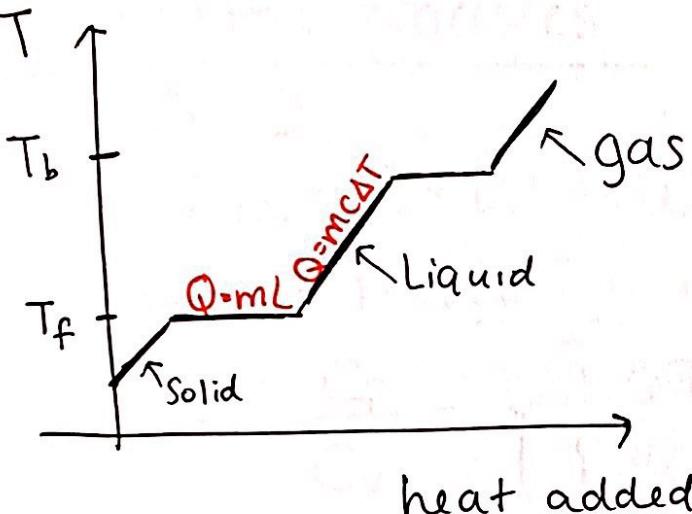
$$Q = M c \Delta T \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{temperature change}$$

$$= n C \Delta T \quad \left. \begin{array}{l} \\ \end{array} \right\}$$

$$Q = \pm M L_f \text{ (melt/freeze)} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{phase change}$$

$$\pm M L_v \text{ (boil/condensing)} \quad \left. \begin{array}{l} \\ \end{array} \right\} (+) \text{ if heat entering}$$

$$(-) \text{ if heat leaving}$$



Heat from hotter system enters colder system
Isolated System

$$Q_{\text{net}} = Q_1 + Q_2 + Q_3 + \dots = 0$$

$$\Delta T = T_f - T_i$$

$Q = +ML$ if heat is added (solid \rightarrow liquid, liquid \rightarrow gas)

$Q = -ML$ if heat is leaving (gas \rightarrow liquid, liquid \rightarrow solid)

Specific Heat of Gases

Specific heat (c) - how much energy is needed to change temperature of a material

Solids or liquids - depends on material

Gases - depends on type of gas (diatomic or monatomic) & process by which it changes (constant pressure or constant volume)

C_p and C_v

$$\star C_p = C_v + R \quad R = 8.314 \frac{\text{J}}{\text{mol} \cdot \text{K}}$$

$\star \Delta E_{\text{th}}$ is the same for any ideal gas process that has the same ΔT

$$\Delta E_{\text{th}} = nC_v\Delta T \quad (\text{any ideal gas processes})$$

(pg 533)

Adiabatic Processes

$$Q = 0$$

$$\Delta E_{\text{th}} = Q + W = W \quad \leftarrow \quad W = nC_v \Delta T$$

$$\gamma = \frac{C_p}{C_v} = \begin{cases} 1.67 & \text{monatomic} \\ 1.40 & \text{diatomic} \end{cases}$$

$PV^\gamma = \text{constant}$ in adiabatic process
(proof pg 536)

$$P = \frac{nRT}{V} \quad \leftarrow \quad \left(\frac{nRT}{V} \right) V^\gamma = \text{constant}$$

$$\underbrace{nRT}_{\text{constant}} \frac{V^\gamma}{V} = \text{constant}$$

$$\frac{T}{V} V^\gamma = \text{constant}$$

$$T V^{\gamma-1} = \text{constant}$$

Calorimetry Example:

$$P = 793 \text{ kW} = \frac{\text{energy}}{\text{time}} \leftarrow Q (\text{heat})$$

$$\left. \begin{array}{l} T_i = 0^\circ \text{C} \\ T_f = 0^\circ \text{C} \end{array} \right\} \quad \Delta T = 0$$

$$Q = \pm M L_f \quad L_f = 3.33 \times 10^5 \text{ J/kg}$$

$$M = 2 \text{ kg}$$

$$Q = (2)(3.33 \times 10^5) = 666000 \text{ J}$$

$$P = \frac{E}{t} = \frac{Q}{t} \rightarrow t = \frac{Q}{P} = \frac{6660000J}{(793 \times 10^3) \frac{J}{s}} = 0.84s$$

Calorimetry #2

Closed System:

$$Q_{net} = 0 = Q_{coffee} + Q_{ice}$$

$$Q = mc\Delta T$$

$$T_f = 60^\circ C = 333 K$$

$$T_i = 90^\circ C = 363 K$$

$$Q_{ice} = Q_1 + Q_2 + Q_3$$

Raise T
to melting
point

$$Q = mc\Delta T$$

$$T_f = 0^\circ C = 273 K$$

$$T_i = -20^\circ C = 253 K$$

phase
change
to water

$$Q = mL_f$$

Raise T
to 60°C

$$T_f = 60^\circ C = 333 K$$

$$T_i = 0^\circ C = 273 K$$

$$Q_{net} = Q_{ice} + Q_{coffee}$$

$$Q_{net} = Q_1 + Q_2 + Q_3 + Q_{coffee} = 0$$

$$Q = m_{ice} c \Delta T$$

$$Q_1 = m_{ice} \left(2090 \frac{J}{kg \cdot K} \right) (T_f - T_i) = m_{ice} (2090)(273 - 253)$$

$$Q_1 = m_{ice} (41800 \frac{J}{kg})$$

$$Q_2 = m_{ice} L_f = m_{ice} (330,000 \frac{J}{kg})$$

$$Q_3 = m_{ice} c \Delta T = m_{ice} \left(4190 \frac{J}{kg \cdot K} \right) (333 - 273)$$

$$= m_{ice} (251400 J/kg)$$

$$Q_{\text{coffee}} = mc\Delta T$$

② ↓

$$V_{\text{coffee}} = 300 \text{ mL} = 300 \text{ cm}^3 \times \frac{1 \text{ m}}{100 \text{ cm}} \times \frac{1 \text{ m}}{100 \text{ cm}} \times \frac{1 \text{ m}}{100 \text{ cm}}$$
$$V_c = 0.0003 \text{ m}^3$$

$$\rho_{\text{coffee}} = 1000 \frac{\text{kg}}{\text{m}^3}$$

$$\rho = \frac{m}{V} \Rightarrow m = \rho V = \left(1000 \frac{\text{kg}}{\text{m}^3}\right) (0.0003 \text{ m}^3)$$

$$m_c = 0.3 \text{ kg}$$

$$Q_{\text{coffee}} = (0.3 \text{ kg}) \left(4190 \frac{\text{J}}{\text{kg} \cdot \text{K}}\right) (333 \text{ K} - 363 \text{ K})$$

$$Q_{\text{coffee}} = -37710 \text{ J}$$

$$Q = Q_1 + Q_2 + Q_3 + Q_{\text{coffee}} = 0$$

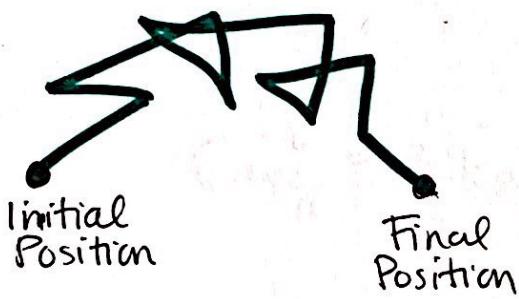
$$m_{\text{ice}} \left(41800 \frac{\text{J}}{\text{kg}}\right) + m_{\text{ice}} \left(330,000 \frac{\text{J}}{\text{kg}}\right) + m_{\text{ice}} \left(251400 \frac{\text{J}}{\text{kg}}\right)$$
$$+ (-37710 \text{ J}) = 0$$

$$m_{\text{ice}} \underbrace{(41800 + 330000 + 251400)}_{= 37710} = 37710$$

$$m_{\text{ice}} = 0.061 \text{ kg} = 61 \text{ g}$$

Micro-macro connection

The macroscopic properties of a system (i.e. temperature, & pressure) are related to AVG behavior of atoms & molecules



Mean free path (λ)

avg distance ~~to~~ between collisions

$$\lambda = \frac{L}{N_{\text{collisions}}}$$

$$N_{\text{collisions}} = \frac{N}{V} \cdot 4\pi r^2 L$$

$\frac{N}{V}$
number density

$$\frac{N}{V} = \frac{P}{k_B T}$$

Pressure in a gas is due to collisions of molecules with walls of container

Root-mean-square speed
speed of a molecule

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

$$(v^2)_{\text{avg}} = (v_x^2 + v_y^2 + v_z^2)_{\text{avg}}$$

$$= (v_x^2)_{\text{avg}} + (v_y^2)_{\text{avg}} + (v_z^2)_{\text{avg}}$$

$$v_{\text{rms}} = \sqrt{(v^2)_{\text{avg}}}$$

Gives you a sense of avg speed of molecules

$$\epsilon = \frac{1}{2}mv^2 \text{ (translational KE)}$$

$$E_{\text{avg}} = \frac{1}{2}m v_{\text{rms}}^2 \text{ (avg. translational KE)}$$

ideal gas law } see pg 507
 $P = \frac{F}{A}$ for derivation

$$E_{avg} = \frac{3}{2} k_B T \xrightarrow[\text{for temperature}]{\text{rearrange}} \quad$$

$$T = \frac{2}{3k_B} E_{avg}$$

the true meaning
of temperature

$$E_{avg} = \frac{3}{2} k_B T$$

$$E_{avg} = \frac{1}{2} m v_{rms}^2$$

$$\frac{3}{2} k_B T = \frac{1}{2} m v_{rms}^2$$

$$V_{rms} = \sqrt{\frac{3k_B T}{m}}$$

$$\text{Equilibrium } (E_{avg})_1 = (E_{avg})_2 \\ (T_1)_f = (T_2)_f$$

Example: RMS speed

$$V_{rms} = \sqrt{\frac{3k_B T}{m}}$$

$$(V_{rms})_{H_2} = (V_{rms})_{N_2}$$

$$\sqrt{\frac{3k_B T_{H_2}}{m_{H_2}}} = \sqrt{\frac{3k_B T_{N_2}}{m_{N_2}}}$$

$$\frac{T_{H_2}}{m_{H_2}} = \frac{T_{N_2}}{m_{N_2}}$$

$$m_{H_2} = 2(1 \text{ amu}) = 2 \text{ amu}$$

$$m_{N_2} = 2(14 \text{ amu}) = 28 \text{ amu}$$

$$T_{H_2} = \frac{m_{H_2}}{m_{N_2}} T_{N_2} = \frac{(2 \text{ amu}) (100^\circ C + 273)}{(28 \text{ amu})} = 26.64 \text{ K}$$

$$\boxed{T_{H_2} = -246.36^\circ \text{C}}$$

RMS Speed pt. 2

1g H₂, v_{rms} = 1800 m/s

a) $K = \frac{1}{2} m v^2$

$$v^2 = v_{rms}^2$$

$$K_{\text{thermal}} = \frac{1}{2} M_{H_2} v_{rms}^2 = \frac{1}{2} \cdot (1 \times 10^{-3} \text{ kg}) \cdot (1800^2) \text{ J}$$

b) 5 degrees of freedom

$$E_{\text{therm}} = N \frac{5}{2} K_B T$$

$$\frac{1}{2} m v_{rms}^2 = \frac{3}{2} K_B T$$

$$T = \frac{1}{3} \frac{m v_{rms}^2}{K_B}$$

$$E_{\text{therm}} = N \frac{5}{2} K_B \cdot \frac{1}{3} \frac{m v_{rms}^2}{K_B}$$

$$= N \frac{5}{6} m v_{rms}^2$$

$$N = \frac{M_{H_2}}{m_{molecule}} = \frac{1 \times 10^{-3} \text{ kg}}{4 \cdot 1.67 \times 10^{-27} \text{ kg}}$$

$$E_{\text{kin}} = \frac{M_{H_2}}{m_{molecule}} \cdot \frac{5}{6} M_{H_2} \cdot V_{rms}^2$$

$$E_{\text{kin}} = \frac{(M_{H_2})^2}{m_{molecule}} \cdot \frac{5}{6} V_{rms}^2$$

How much heat Energy?

$$\Delta E_{th} = Q + W$$

$$Q + W = 0$$

$$F_{spring} = -kx$$

$$W_{gas} = + \int_P \frac{dV}{A dx} = \int \frac{F}{A} A dx$$

$$W_k = - \int_{x_i}^{x_f} k x dx = - \frac{1}{2} k x^2 \Big|_{x_i}^{x_f}$$

$$W_{spring} = - W_{gas}$$

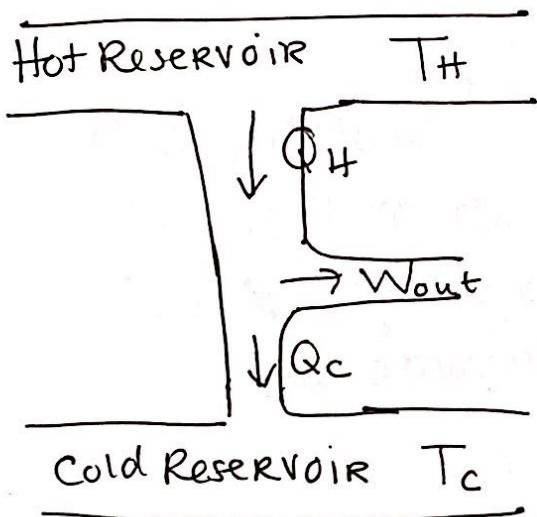
$$W_{gas} = Q$$

$$Q = + \frac{1}{2} k x^2 \Big|_{x_i}^{x_f}$$

Heat Engines & Refrigerators

Q_H = heat transferred to or from a hot reservoir

Q_C = heat transferred to or from a cold reservoir



if it were perfectly efficient
all Q_H would turn
into work

$$(\Delta E_{th})_{\text{net}} = 0 \quad (\text{any heat engine, over 1 full cycle})$$

because theoretically, temperature & thermal energy return to original value after ~~re~~
each cycle

$$(\Delta E_{th})_{\text{net}} = Q + W$$

$$(\Delta E_{th})_{\text{net}} = Q_{\text{net}} - W_s = 0$$

$$\textcircled{O} \quad \cancel{Q_{\text{net}}} = Q_H - Q_C$$

$$W_s = Q_H - Q_C$$

Efficiency (η)

$$\eta = \frac{\text{what you get}}{\text{what you had to pay}} = \frac{W_{\text{out}}}{Q_H} = 1 - \frac{Q_C}{Q_H}$$

Carnot Engine

- only isothermal & adiabatic

$$\eta_{\text{Carnot}} = 1 - \frac{T_c}{T_h}$$

} The standard for efficiency
→ no heat engine can be more efficient than this

Is it possible?

(1.) Is it more efficient than a Carnot?

(2.) Are you getting more work out than the amount of heat you put in?

Refrigerators

$$k = \frac{Q_{\text{out}}}{W_{\text{in}}}$$

$$k_{\text{Carnot}} = \frac{T_c}{T_h - T_c}$$

Carnot heat engine + ordinary fridge

$$\kappa = 2.0 = \frac{Q_{out}}{W_{in}}$$

$$T_c = 250\text{ K} \quad Q_{in_1} = 105$$

$$T_H = 350\text{ K}$$

1st consider engine

$$\eta = \frac{W_{out_1}}{Q_{in_1}} = 1 - \frac{T_c}{T_H}$$

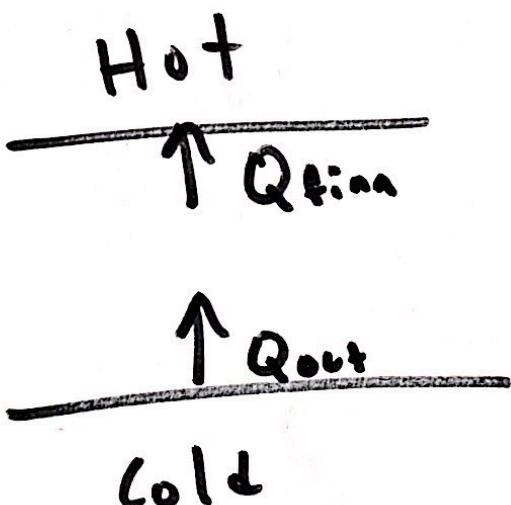
$$W_{out_1} = \left(1 - \frac{T_c}{T_H}\right) Q_{in_1} = \left(1 - \frac{250}{350}\right) 105$$

2nd consider fridge

$$\kappa = 2.0 = \frac{Q_{out}}{W_{out_1}}$$



$W_{in} \rightarrow$



$$Q_{final} = W_{in} + Q_{out}$$

$$\gamma = 2 = \frac{W_{out+1}}{Q_{out}}$$

$$Q_{out} = \frac{W_{out+1}}{2} = \frac{1}{2} \left(1 - \frac{T_c}{T_{in}}\right) Q_{in}$$

$$Q_{out} = \frac{W_{engine}}{2}$$

$$Q_{final} = Q_{out} + W_{in} = \frac{W_{engine}}{2} + W_{engine} = \frac{3}{2} W_{engine}$$

$$Q_{final} = \frac{3}{2} \left(1 - \frac{T_c}{T_H}\right) Q_{in} = \frac{3}{2} \left(1 - \frac{250}{350}\right) 10^5$$