***COURSERA: STATS W/ R SPECIALIZATION***

***COURSE 1 - Introduction to Probability and Data***

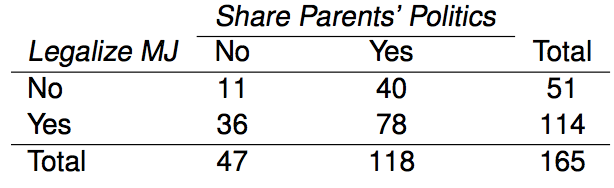
**WEEK 3 - Introduction to Probability**

***Defining Probability***

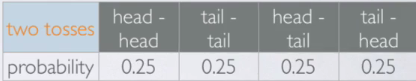
* **Random process** 🡪 know what outcomes *could* happen but not which particular one *will* happen
* Ex: coin toss, die roll, shuffle on music player, stock market
* Sometime helpful to model process as random even if it truly isn’t so
* P(A) = probability of event A
* Several interpretations of probability, but almost all agree on mathematical rule: 0 <= P(A) <= 1
* **Frequentist interpretation =** a **relative frequency =** proportion of times an outcome would occur if we ran the process an infinity number of times
* **Bayesian interpretation** = **subject degree of belief** = for same event, 2 people could have different viewpoints + as such assign different probabilities to it
* Allows for prior info to be integrated into the inferential framework
* Largely popularized by revolutionary advances in computation technology + methods in past 20 years
* **Law of Large Numbers =** as more observations are collected, proportion of occurrences of a particular outcome converges to the probability of that outcome (1/2 for coin toss, 1/6 for die roll)
* More surprising to see 3 heads in 100 coin flips compared to 10, + even more so for 1k flips
* **Independence** 🡪 coin toss P(H on toss 10) = P(H on toss 11) 🡪 coin is not **due** a heads
* Common misunderstanding of law of large numbers = **Law of Averages (Gambler’s fallacy:** random processes are supposed to compensate for what happened in the past (I’m due a good roll/hand/spin, etc.)
* But say you get 100’s of heads in a row on a coin flip, coin is most likely not fair

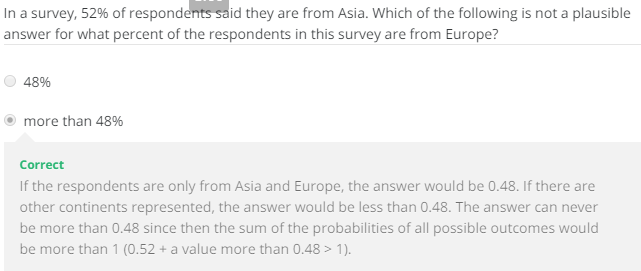
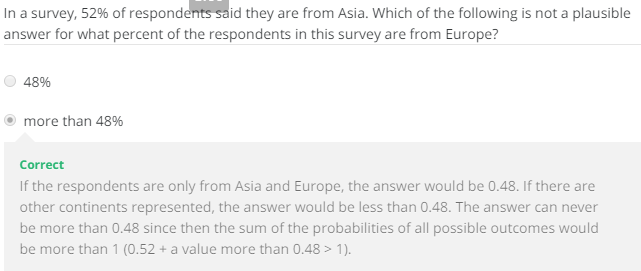
**Disjoint Events + General Addition Rule**

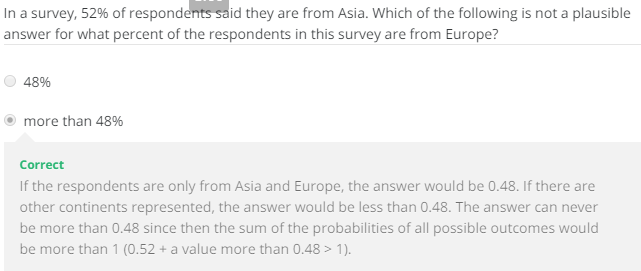
* **Disjoint events =** mutually exclusive = both events cannot happen (coin toss cannot be H AND T)
* P(A and B) = 0 🡺 P(A U B) = 0
* **Non-disjoint events** CAN happen at the same time (get an A in stats + in economics
* P(A and B) != 0 🡺 P(A U B) != 0 🡪 some # between 0 and 1
* **Union** of DISJOINT events 🡪 Probability of one event happening or the other happening?
* Drawing a Jack or a 3 from a deck? 🡪 P(J or 3) = P(J) + P(3) = 4/52 + 4/52 = 2/13
* So for disjoint events, **P(A OR B) = P(A) + P(B)**
* **Union** of NON-disjoint events:
* Probability of drawing Jack or red (could red jack) 🡪 **P(J or Red) = P(J) + P(Red) – P(J U Red)**  = 4/52 + 26/52 – 2/52 = 28/52 = 14/26 = 7/13
* So for NON-disjoint events, **P(A OR B) = P(A) + P(B) – P(A U B)**
* *Note that they can be the same formula b/c for disjoint events P(A U B) = 0*
* 



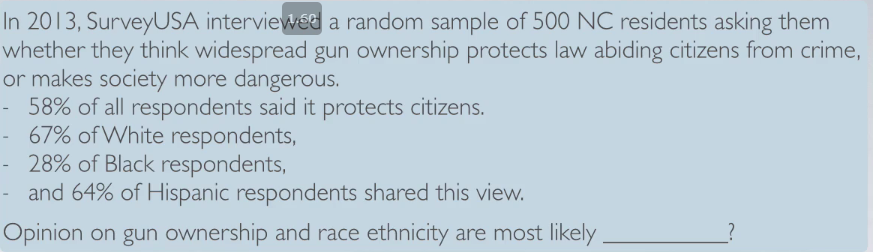
* NON-disjoint 🡪 P(M) + P(P) – P(M U P)
* 114/165 + 118/165 – 78/165 = **(114 + 118 – 78) / 156**
* The above encapsulates the **General Addition Rule**
* **Sample space (SS) =** all possible outcomes of a trial/experiment
* Couple has 2 kids 🡪 SS of genders = **S = {MM, FF, MF, FM}**
* **Probability distribution** = lists out all possible outcomes in a SS + probabilities w/ which they occur

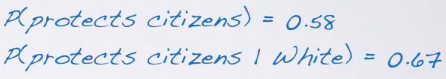
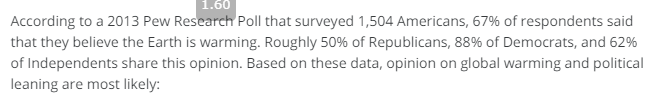
* Can create similar probability distribution for any *discrete* event of interest
* 3 Broad Rules of Probability Distributions
* Events listed must be disjoint
* Each P is between 0-1
* Sum of all probabilities w/in the distribution sum up to 1
* **Complementary events** = 2 disjoint events whose P’s sum to 1 🡪 P(A) + P(A(c)) = 1
* Coin 🡪 P(H(c)) = P(T) b/c tail is the complement of head
* For 2 coins, P(HH(c)) = sum of P(HT) + P(TH) + P(TT) 🡪 sum of all other events to 1st event so that the sum of all events equals 1
* Dividing the SS into 2 such that the sum of the 2 probabilities equals 1 (even though there’s 4 total)
* *Disjoint + Complementary are NOT the same* 🡪 not all disjoint events sum up to 1 (more than 2 events in a SS), while all complementary events sum up to 1
* So, complementary events are always disjoint, while disjoint events are not always complementary
* 

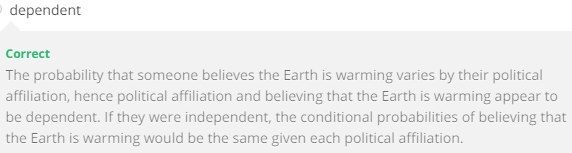


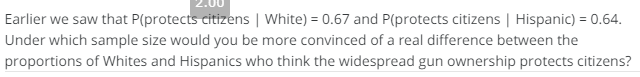
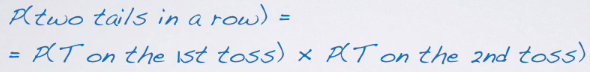
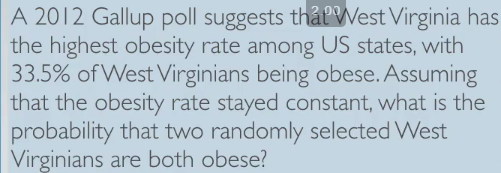
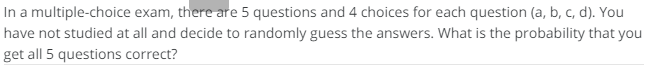
**Independence**

* 2 processes are **independent** if knowing the outcome of 1 provides no useful info about the outcome of the other
* Knowing a coin lands on H does nothing to help us know what they 2nd coin toss will land on
* But knowing 1st card drawn from a deck helps us update P(any other card), if w/out replacement
* Checking for independence rule:
* If probability of Event A happening given Event B happened is the same as the original probability of event A, they are independent 🡪 **P(A | B) = P(A), then** A + B are independent
* Knowing B tells us nothing about A
* 



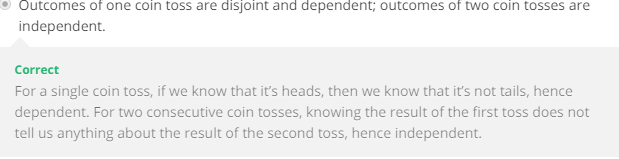
* This is b/c knowing P(protects) = 0.58, but knowing their white gives P(protects | w) = 0.67
* 
* The same goes for Blacks and Hispanics, so the opinions on gun ownership vary greatly by race, so they can be thought to most likely be **dependent**
* Knowing someone’s race may give us helpful info about their opinion on gun ownership
* 



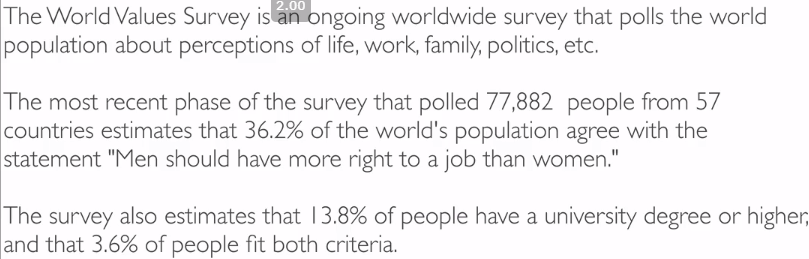
* Using phrase “most likely” b/c this is all *sample data* + we’re not using inference tools yet
* If we observe a difference between **conditional probability** based on the sample, we can say “the data *suggest* dependence”
* Then the next step would be to conduct a hypothesis test to see if this observed difference was due to change/natural random sampling, or if there’s some real effect/difference in the population
* Can do some speculation based on the size of the observed difference as well as based on the sample size
* If difference is large/varies greatly = stronger evidence difference is real
* If sample size if large = even SMALL difference in conditional probabilities provides strong evidence difference is real
* 
* 
* Product Rule for Independence
* **If A + B are independent, P(A U B) = P(A) \* P(B)**
*  **= 0.5 \* 0.5 = 0.25**
* Can be expanded to as many independent events as we have
* 
* 
* Given P(obese) = 0.335
* Given 2 individuals are randomly selected 🡺 they are therefore *independent*
* So, P(obese1 U Pobese2) = P(obese1) \* P(obese2) = P(o) + P(o) = 0.335^2 = 0.11 = 11% chance 2 randomly selected West Virginians are obese
* 

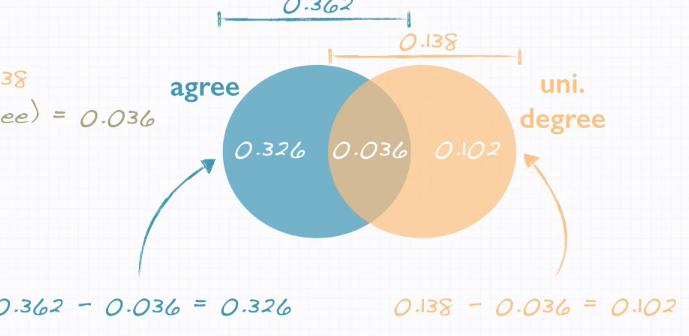
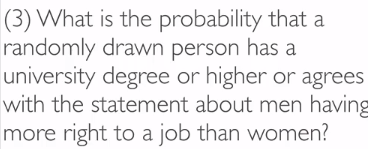
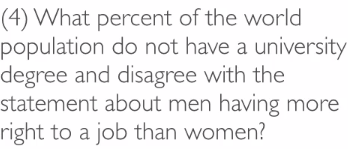


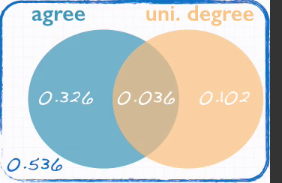
**Spotlight: Disjoint vs. Independent**

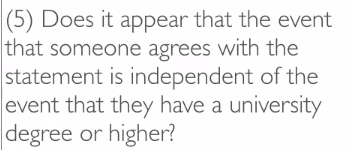
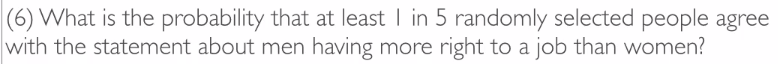
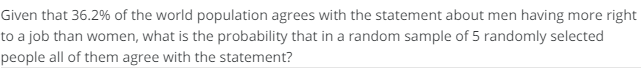
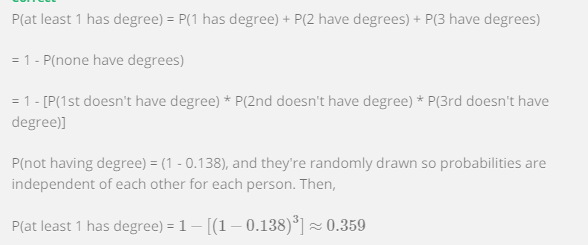
* **Disjoint** = mutually exclusive = both cannot happen at same time = P(A U B) = 0
* **Independent** = knowing outcome of 1 event gives no knowledge of outcome of another = P(A | B) = P(A) if independent
* Ex: Babies have 3 possible eye colors, Bl, Gr, Br 🡪 disjoint (cannot happen at same time)
* 2 babies 🡪 now 1st has Bl eyes 🡪 also disjoint
* if not related, gives no knowledge of outcome for 2nd baby = independent 🡪
* if related, may give some knowledge = dependent (may be more likely to have bl eyes
* Also note that outcomes for eye color for 1 baby are dependent 🡪 knowing a baby’s eye color to be blue means we have knowledge they aren’t brown or green
* Can generalize this to say Disjoint events w/ non-zero probability are always dependent on each other (if we know 1 happened we know the other cannot)
* 

**Probability Examples**



* What do we know?
* P(agree) = 0.362 P(degree) = 0.138 P(A U D) = .036
* 
* To be disjoint P(A U D) should = 0, and it does not 🡪 NOT disjoint
* 
* 32.6% agree w/ no degree, 10% don’t agree w/ a degree
* 
* P(D) = .138, P(A) = .362 🡪 P(D) OR P(A) for NON-disjoint events **= P(A) + P(B) – P(A U B)** from **general addition rule/additivity axiom**
* 0.138 + 0.362 - 0.036 = 0.464 🡪 **46.4%** of randomly drawing a person w/ a degree or a person who agrees w/ the statement
*  🡺 P(D(c)) = .862, P(A(c)) = .638 🡪 P(no degree nor agree) 🡪 complement of P(A or D) 🡪 1 - .464 = .536 🡪 area in Venn diagram outside both circles

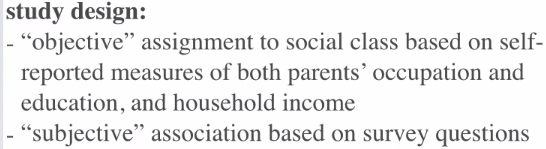


*  🡺 If A + B are independent, P(A U B) = P(A) \* P(B)
* This is NOT true so they are NOT likely to be independent
* 
* P(A) = .362, SS = {0,1,2,3,4,5} where it’s possible none agree, 1 agrees, 2 agrees, up to all agree
* Only care if at least 1 does, so divide SS into {0, >= 1}
* Subtract complement of at least 1 agrees = none agree 🡪 P(>1 agree) = 1 – P(none) 🡪 P(Dis, Dis, Dis, Dis, Dis)
* P(Dis) = .638 (the complement of agreeing(, and these are independent, so .638^5 = 0.1057
* So we have 1 - 0.1057 = **.8943 = 89.43% at least 1 in 5 randomly selected people agree**
* 
* P(A) = .362, SS = {0,1,2,3,4,5} 🡪 split SS = {all agree, >= 1 disagree} 🡪 P(all) = P(A)^5 b/c they’re independent = .362^5 = **.006 = 0.6% chance all of 5 randomly sampled persons agree**
* 
* P(Deg) = .138, SS = {0,1,2,3} 🡪 split into SS = {none, at least 1}
* P(>= 1) = 1 - P(none) 🡪 1 – P(No,No,No) 🡪 P(No) = 1 – P(Deg) = 1 - .138 = .862
* P(no)^3 = .862^3 = .641 🡪 1 - .641 = **.359**
* 

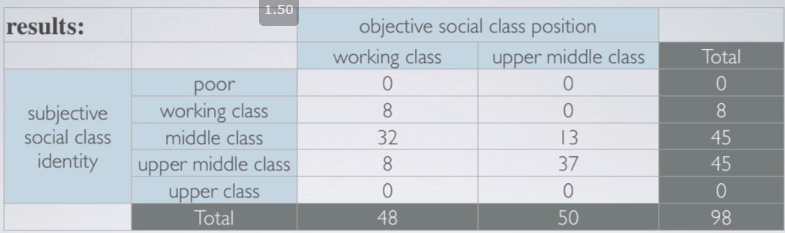
**Conditional Probabilty**

* 

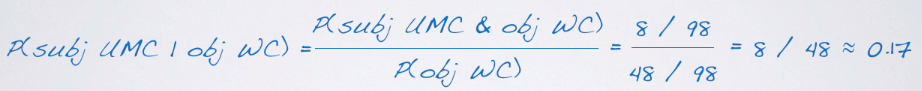


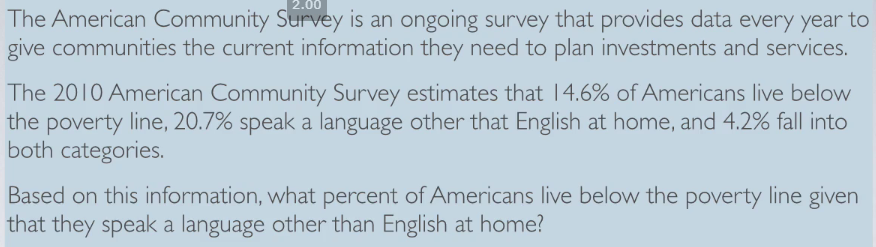


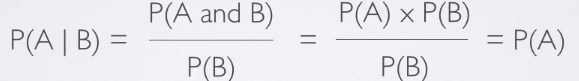
* The social class here is what we’re taking to be the “truth”
* Given 2 categorical variables, we can create a **contingency table** of objective + subjective social class

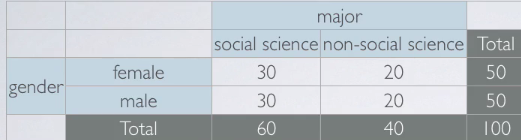


* **Marginal Probability** 🡪 counts used to calculate P() come from the **margins** of the contingency table 🡪 bottom row, last col (the TOTAL row + TOTAL col)
* P(objective = upper-middle class) 🡺 P(obj UMC)
* Look at objective upper-middle class column 🡪 see 50 students in this category out of 98 total = 50/98 = ~1/2 = ~51%
* P(sub UMC) = see 45 students in this row out of 98 total = 45/98 = .46
* **Join Probability** 🡪 P() at intersection of 2 events of interest (intersection of Venn diagram)
* P(sub UMC &obj UMC) 🡪 look for cell intersection of specified row + col 🡪 37/98 = .38
* P(sub WC & obj WC) = 8/98 = .08
* **Conditional Probability**
* P() a student objectively in WC associates w/ UMC 🡪 P() they believe they’re UMC given that they’re WC 🡪 **P(sub UMC | obj WC) 🡪 8 WC who think UMC /48 WC = .17**
* P(sub MC | obj UMC) = 13/50 = .26
* Calculate conditional probabilities w/ Bayes’ Theorem 🡪 **P(A | B) = P(A & B) / P(B)**

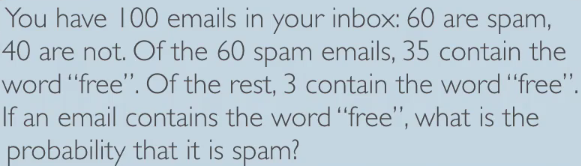
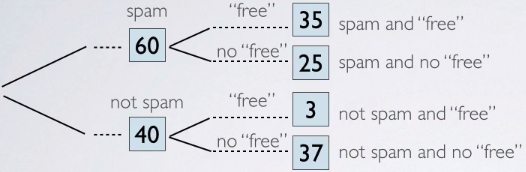
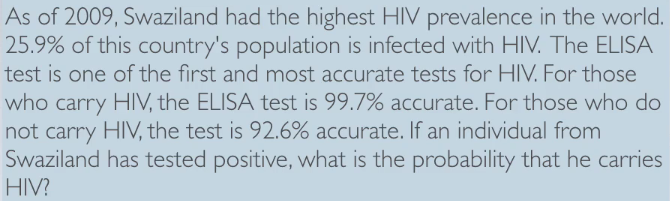
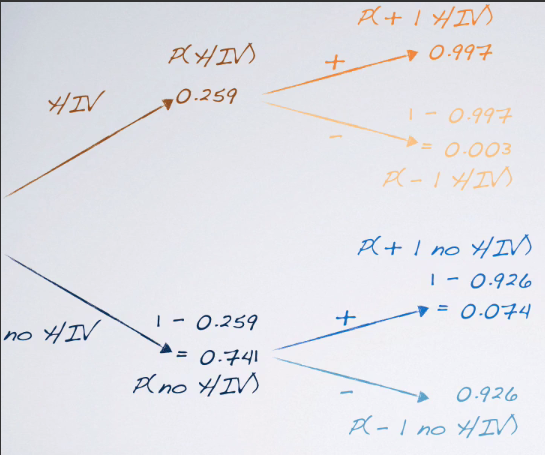
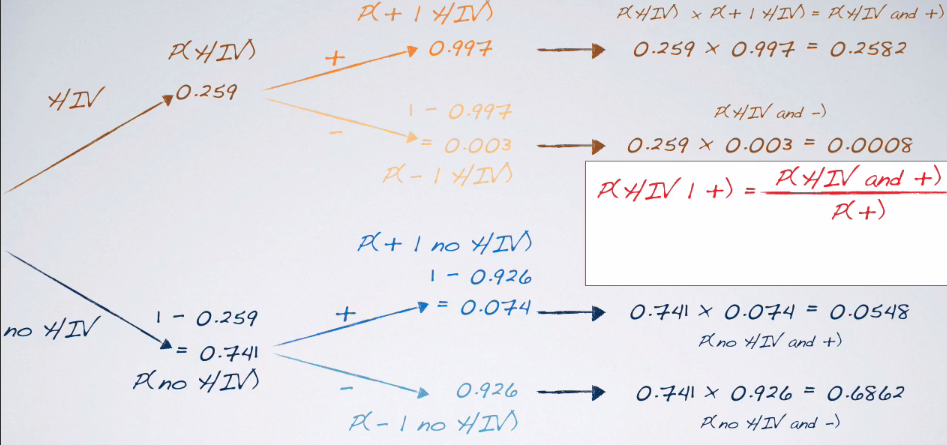


* 
* P(Below | Non-English) = P(Below & Non-English) / P(Non-English) **= 4.2 / 20.7 = ~.2**
* Can use this info to compare to the general public.
* Since we know 14.6% live below the poverty line, it seems that living below the poverty line is more prevalent for those who speak a language other than English at home
* This suggest language spoken at home and living below poverty level MAY be dependent
* P(non-English | below) = P(below & non-English) / P(below) = .042 / .146 = .29
* Remember Product Rule for Independent Events
* 
* If we believe the events are NOT independent, or we cannot check if they are, the joint probability must be calculated differently
* Bayes’s DOESN’T have a independence condition 🡪 rearrange it 🡪 P(A & B) = P(A | B) \* P(B)
* This is **general product rule**
* Generally, if P(A|B) = P(A), events A + B are said to be independent 🡪 knowing event B occurred did nothing to change the probability of A occurring
* Mathematically 🡪 if they’re independent, P(A & B) = P(A) \* P(B)



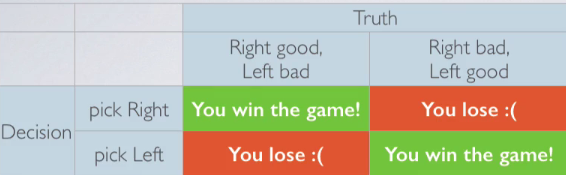
* 
* P(SS) = 60/100 = .6
* Find P(SS) if we know the randomly sampled student is a female = P(SS | G) = P(SS & F) / P(F) = (30/100) / (50/100) = 30 / 50 = .6
* Also note that **P(SS) = P(SS | M) = P(SS | F) = .6** 🡪 *all are the same P() 🡪 if we know P(A | B) = P(A), then the events are independent*
* Here, P(SS) = P(SS | either gender) 🡪 can determine gender and major are independent

**Probability Trees**

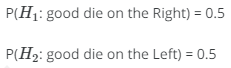
* These are helpful for solving conditional probabilities, especially when the probability we’re asked for is the opposite of what we’re given: P(A|B) 🡪 P(B|A)
* 
* P(S) = .6, P(NS) = .4, P(F|S) = 35/60, P(F|NS) = 3/40
* 
* P(S|F) = P(S & F) / P(F) 🡪 35 / (35 + 3) = 35/38 = .92
* i.e. Bayes’ 🡪 numerator = joint, denominator = marginal of what to condition on
* Not to do the same w/ probabilities and not actual counts (don’t know sample size or populations)
* 
* P(P|H) = .997, P(N|NH) = .926, P(H) = .259
* P(P|H) = P(P & H) / .259
* P(H|P) = P(H & P) / P(P) 🡪 reverse of what we’ve given
* 1st branch = always made up of marginal probabilities b/c we’re splitting up the population w/out conditioning based on any other attributes 🡪 have HIV or not
* 
* 
*  🡺 
* These events are dependent, so to get the joint probabilities (i.e. P(H & P)) we multiply them together in a new set of branches
* 
* P(H|P) = P(H & P) / P(P)
* P(P) is made up of both sets of (+) joint probabilities 🡪 P(H+ OR NH+) 🡪 disjoint 🡪 add them
* .2582 / (.2582 + .0548) = .82

**Bayesian Inference**

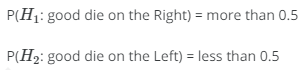
* 2 die 🡪 6 and 12 sided. Goal = guess which hand holds which die
* Will collect die by asking player to roll die and tell us if results is >= 4
* P(>= 4\_6) = 1/2 🡺 S = {1,2,3,4,5,6} P(>= 4\_12) = 3/4 🡺 S = {1,2,3,4,5,6,7,8,9,10,11,12}
* Which die would you prefer to try to roll >= 4 🡪 12 sided b/c we have 75% chance of doing so compared to 50%
* 12-sided now = “good die” = the die we want to find in player’s hand
* Rules.
* 1 die in  left hand + other in right
* Pick a hand, left or right 🡪 they roll it + tell you if outcome >= 4 or not, but NOT what the outcome actually is (can give away which die is in which hand)
* Based on that piece of info, you make a decision as to which hand holds the good die.
* Could also choose to try again (collect more data) but each round costs money 🡪 don't want to keep trying too many times.
* If you think about data collection, it's always costly + while we love large sample sizes, it takes a huge amount of resources to obtain such samples.
* These rules reflect some reality about conducting scientific studies.
* Evaluate choices we might make:
* 2 possibilities of truth 🡪 good die is in right hand or left hand



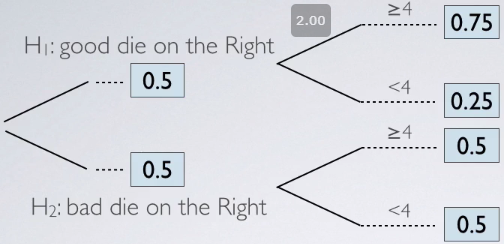
* To avoid losing, might want to collect as much data as possible, but remember, it’s costly.
* So at some point before you're entirely sure, have to just go ahead + make a guess.
* If no consequences to losing 🡪 might not care much whether you win or lose.
* But w/ money running on it, you might be conservative about calling the game *too early*.
* This is balancing the cost associated w/ making the wrong decision + losing the game against the certainty that comes w/ additional data collection.
* Before we collect any data, probabilities associated w/ the following hypotheses:



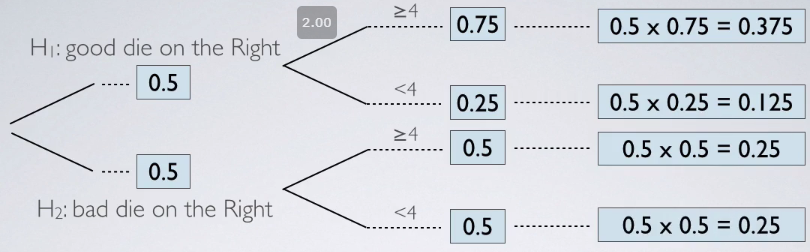
* These are the **prior probabilities** of the 2 competing claims/hypotheses = what you believe before seeing any data.
* Could have made these up, but instead chose to make an educated guess.
* May know they tend to favor the left hand + might put a higher probability of holding the good die w/ the left
* But w/ no additional info, 50-50 = going best bet.
* Round 1 – right hand 🡪 IS >= 4
* Now revaluate stance 🡪 how, if at all, do hypotheses probabilities change?



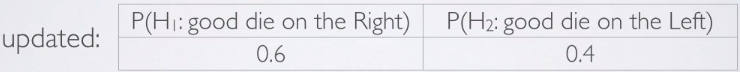
* Just rolled the die + got a high valued outcome = more likely w/ 12-sided die 🡪 probability the right is holding 12-sided die should be a little higher than initially assigned.
* Actually calculate that probability:
* Started w/ equal chances of 50/50 chance of H1 being true before starting data collection = our **priors**.
* Then, think about the **data collection** stage.
* If it is true the good die is on the right 🡪 probability of rolling a number >= 4 is going to be 75%. + the complement of that, rolling a number < 4, is going to be 25%.
* If, on the other hand, BAD die is on the right + you picking the right hand, probability of rolling a number >= 4 is only 50% + the compliment, rolling a number < 4, is also 50%.



* In probability trees, next step = calculate the **joint probabilities**.

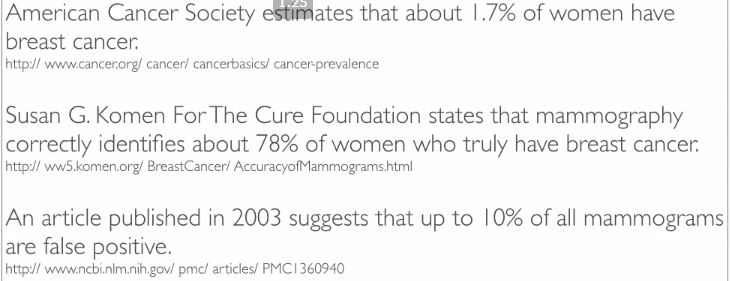


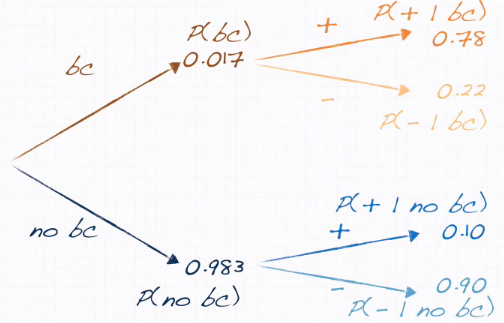
* We did indeed roll >= 4 so there are 2 outcomes we're most interested in = very top + 3rd branch
* How does the probability change for the hypothesis, if the 1st hypothesis being true?
* i.e. probability the good die is on the right, given you rolled >= 4 w/ the die on the right.
* To find this **conditional probability**, use **Bayes’ theorem** 🡪 if looking for P(A | B), find the **joint probability** of A and B divided by the **marginal probability** of B.
* P(A|B) = P(&B) / P(B) 🡺 P(good right & >=4 right ) / P(>= 4 right)
* **0.375 / (0.375 + .025) = 0.375 / 0.625 = 0.6 = 60%**
* Earlier we guessed the probability of the hypothesis being true should increase from 50% + now w/ the 1 DP observed, we can indeed see an increase up to 60%. The
* This is the **posterior probability =** probability the good die is on the right, given you rolled >= 4 w/ the die on the right.
* **Posterior probability** = probability of the hypothesis given the data.
* Or in other words, the probability of a hypothesis we set forth, given the data we just observed.
* Depends on both prior probability we set + the observed data.
* 
* Different than what we calculated at the end of the randomization tests on gender discrimination (probability of observed or more extreme data, given the null being true) which was the probability of data given the hypothesis 🡺 a **p-value**.
* This time, we're making our decision based on the posterior probability as opposed to a p-value
* In a **Bayesian approach** 🡪 evaluate claims iteratively as we collect more data.
* Next iteration = next roll 🡪 get to take advantage of what we learned from the next round’s data
* In other words: *Update our prior w/ our posterior probability from the previous iteration.*
* In the next iteration, our updated prior for the 1st hypothesis being true is going to be the 60%, (which was the posterior from the previous iteration)
* The compliment of that, 40% = the probability of the competing hypothesis.



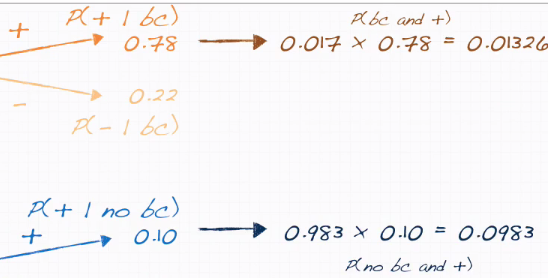
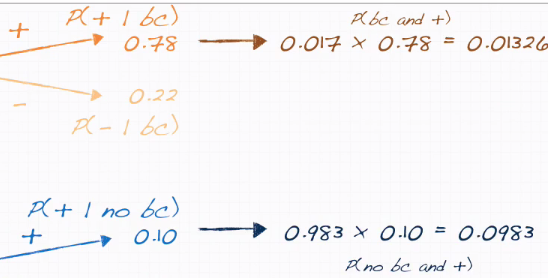
* The Bayesian approach allows us to take advantage of prior info (like a previous published study or a physical model) to naturally integrate data as collected + then update priors.
* We also get to avoid the counter-intuitive definition of a p-value (probability of observed or more extreme outcome, given the null is true) + instead can base decisions on posterior probability (probability the hypothesis is true, given the observed data)
* **A good prior helps, but a bad prior hurts.**
* Remember when we set our priors (50/50 chance for the 2 hypotheses being true), we said we were taking an *educated guess 🡪* don't want to just make up our prior probabilities.
* *But, prior matters less, the more data you have*.
* So you, even if you didn't have a great prior to begin w/, as you collect more data, you're going to be able to converge to the right probabilities.

**Examples of Bayesian Inference**

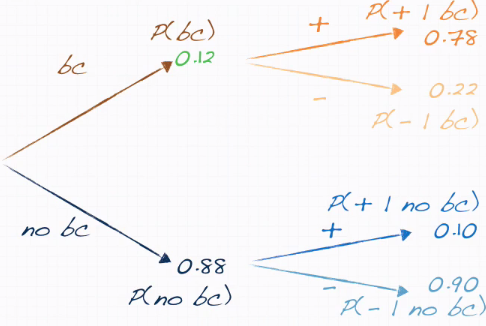
* 
* Givens: P(C) = .017 P(P | C) = .78 P(P | NC) = .1
* Prior to any testing or info exchange between patient + Dr., what probability should a Dr. assign to a female patient having breast cancer?
* 0.017 (we know nothing of their medical history) = **prior**
* When a patient goes through breast cancer screening, there are 2 competing claims: Patient has cancer or doesn't.
* If a mammogram yields a positive result what is the probability that patient has cancer?
* This is P(C | P) 🡪 reverse of given P(P | C) = .78 🡪 use probability tree

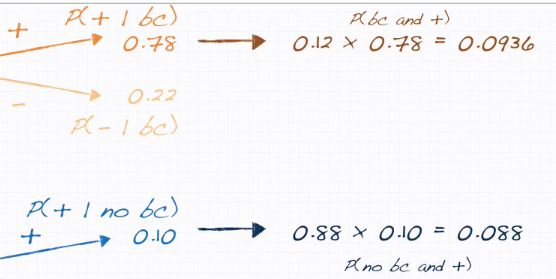
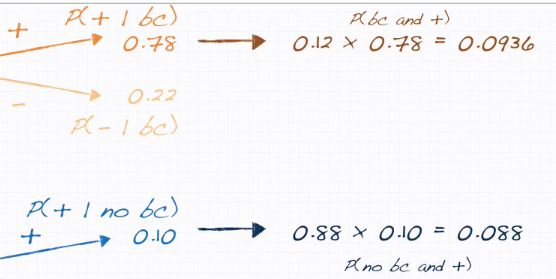


* We're given “positive” 🡪 therefore only interested in 1st + 3rd branches



* P(C | P) = P(C & P) / P(P) 🡺 (.017 \* .78) / (((.017 \* .78) + (.983\*.1)) = .01326 / (.01326 + .0983)
* = **0.12 =** 12% rate/**POSTERIOR** probability of having cancer given testing positive
* Initially, we had given a 1.7% chance to this patient having breast cancer b/c we knew nothing about them.
* Then, we tested them 🡪 positive 🡪 now have this additional info about the patient + after data collection, the probability assigned to this patient having breast cancer = slightly higher.
* Since a positive mammogram doesn't necessarily mean a patient *actually* has breast cancer, the doctor might decide to *retest* the patient.
* What is the probability of having breast cancer if the 2nd mammogram also yields a positive result?
* Once again we run through our probability tree w/ the updated prior but also *nothing about the test has changed* so the probability of testing positive given cancer is still 78% + probability of testing negative given cancer is still 22%, + similarly with the lower branch



* Once again: only interested in branches where patient tests positive
* 
* 
* P(C | P) = P(C & P) / P(P) 🡺 (.12 \* .78) / (((.12 \* .78) + (.88\*.1)) = .0936 / (.0936 + .088) = **. 5154**
* 51.45% rate/*NEW* **POSTERIOR** probability of having cancer given testing positive
* Bayesian Approach To Statistical Inference.
* Set a prior 🡪 collect data 🡪 obtain a posterior 🡪 update prior w/ the posterior
* Last semester, out of 170 students taking a particular statistics class, 71 students were “majoring” in social sciences and 53 students were majoring in pre-med. There were 6 students who were majoring in both. What is the probability a randomly chosen student is majoring in social sciences, given that s/he is majoring in pre-medical studies?
* P(SS) = 71/170 P(PM) = 53/170 P(SS & PM) = 6/170
* P(SS | PM) = P(SS & PM) / P(PM) = (6/170) / (53/170) = **6/53**