***COURSERA: STATS W/ R SPECIALIZATION***

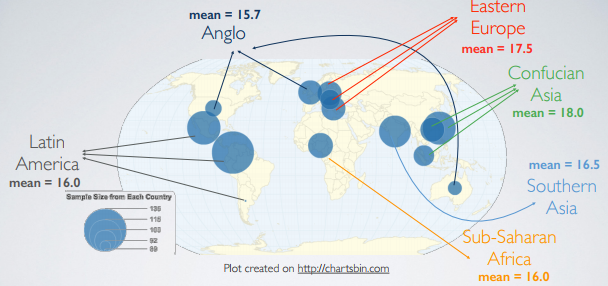
***COURSE 2 - Inference***

**WEEK 3 - Inference for Comparing Means**

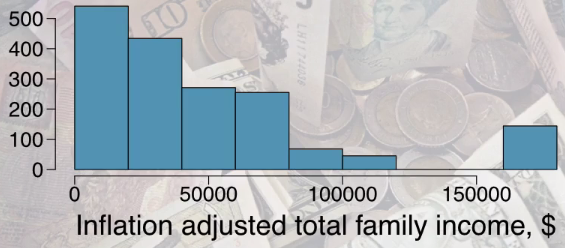
***4.3.1 t-distribution and Comparing 2 Means***

**Introduction**

* Acceptability of Workplace Bullying = study that explores relationship between culture + acceptability of workplace bullying across the globe.
* Researchers collected data using a survey from 1484 alumni + current MBA students from 14 counties on 6 continents + asked some questions on acceptability of **work related bullying**
* **Work related bullying** = giving tasks w/ unreasonable deadlines or exposing workers to an unreasonable workload, so on + so forth.



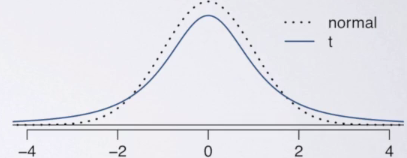
* See a geographic distribution of countries included in the study w/ sizes of circles = how large sample sizes (SS) are from each country.
* SS’s are somewhat consistent across globe + it seems like a pretty even geographic distribution
* Study further categorizes 14 countries into 6 continents + those are the 6 groups we're considering.
* We calculate mean acceptability of work related bullying score for each group (low score = bullying is unacceptable in the workplace, high score = is actually acceptable)
* Can see that the average acceptability is higher in Asia + lowest in Anglo countries.
* But just looking at sample statistics = not possible to determine if differences we're observing are **statistically significant**.
* Want to compare many means to each other
* Look at distribution of inflation-adjusted total family income in the US from a random sample of Americans collected as part of the General Social Survey in 2012



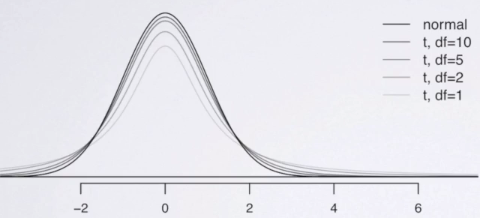
* Distribution is, as expected, pretty right-skewed.
* Suppose we‘d like to estimate typical total family income in the US.
* The CLT provided the basis for constructing a confidence interval for the mean, but what if we're not interested in the mean, ***but the median***?
* *No CLT for the median*.
* New technique for creating CI’s = **bootstrapping =** accomplishing an impossible task = a simulation-based method that doesn't have *as* rigid conditions as the CLT + therefore also works for many estimates beyond the mean

**t-distribution**

* **t-distribution** = useful for describing the distribution of a sample mean when *population SD, sigma, is unknown (almost always)*
* Remember, what purpose does a large sample serve?
* As long as observations are independent + the populations distribution is *not* extremely skewed, a large sample ensures you have a nearly normal sampling distribution of the mean + that the estimate of the **standard error** (SE = S / sqrt(n), *best estimate for unknown pop. SD*) is reliable
* So, if the sample size n is large enough, chances are SE (s) is indeed a good estimate for sigma, + therefore your overall SE estimate is reliable.
* In the age of “big data “why are we talking about small samples.
* It’s true in certain disciplines (especially w/ automatically-recorded data like webpage clicks or Twitter streams), small sample sizes might be irrelevant.
* However, there are disciplines where this is not the case (lab experiment or a study that follows a near-extinct mammal species).
* WE need methods that work well for BOTH large + small samples.
* Uncertainty of the SE estimate = addressed by using the **t-distribution** = also has a bell shape (unimodal + symmetric) + looks a lot like the normal distribution but w/ thicker tails



* Peak of t-distribution doesn't go as high as normal distribution = *t-distribution is somewhat squished in the middle + additional area is added to the tails.*
* This means, under the t-distribution
* observations = *more likely to fall 2 SDs away from the mean than under the normal distribution*
* CI’s constructed using a t-distribution = wider/more conservative than those constructed w/ the normal distribution
* Thick tails = helpful for mitigating the effect of a less-reliable estimate for the SE of the sampling distribution caused by using the sample SD instead of the population SD in its calculation.
* t-distribution (like the standard normal) = always centered at 0 + has 1 parameter = **Degrees of freedom** = determines thickness of the tails.
* In contrast, the normal distribution has 2 parameters 🡪 mean + SD.
* As dF increases, the shape of the t distribution increases + approaches the normal distribution



* We **use the t distribution for inference on a single mean** or **for comparing 2 means when population SDs are unknown (basically always)**
* Calculate t statistic T just like a Z statistic + find the p-value = probability of observed or more extreme outcome values given the null is true (same definition as before)



* Calculate
* probability the absolute value of Z is greater than 2, which is .0455 B
* probability the absolute value of t w/ 50 dF freedom > 2
* Remember t = thicker tails + higher % of observations falling further than 2 SDs from mean
* We're starting to see the effect
* probability the absolute value of t w/ 10 dF freedom > 2

> (pnorm(2,0,lower.tail = F)\*2) # only 1-sided hypothesis

[1] 0.04550026

> pt(2, 50, lower.tail = F)\*2

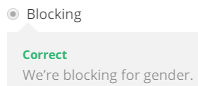
[1] 0.05094707

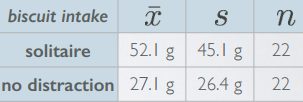
> pt(2, 10, lower.tail = F)\*2

[1] 0.07338803

* So, **as we go from the normal to a t distribution w/ a somewhat high dF to a t distribution w/ low dF, the probability of the test statistic being more than 2 SDs away from the mean increases.**
* Suppose you have a 2-sided hypothesis test + your test statistic = 2.
* Under which of the above scenarios would you be able to reject the null at the 5% significance level?
* 1st scenario = p = 4.55% which is < 5% = reject the null (barely)
* 2nd = p > .05 = fail to reject the null (barely)
* Last scenario = definitely fail to reject the null.
* As we get more conservative w/ a t distribution (lower dF = wider CI’s), we also become less likely to be able to reject the null (more likely to have it in the CI)
* Generally, dF is tied to sample size 🡪 if n is low, it is not as easy to reject the null + stronger evidence is needed in order to be able to do so.
* *This* t-distribution = **student's t distribution 🡪** William Gosset = head experimental brewer at Guinness in early 1900's w/ main role = to experimentally brew + gradually improve a consistent + economical barrel of the Guinness stout.
* This required sometimes working w/ small samples b/c maybe he’d just have few batches to try
* So, much development of the t-distribution comes from trying to make Guinness taste better
* Since Guinness was worried about trade secrets getting out, Gosset was asked to publish any work he was doing under a pseudonym and “Student” was the name that he chose for.
* While others, like Fisher, continued to work on the t-distribution, even Gosset's own foundational work, the distribution is still named after his pseudonym

**Inference for a Single Mean**

* Study = Playing A CPU Game During Lunch Effects Fullness, Memory For Lunch + Later Snack Intake.
* In this study, researchers evaluated the relationship between being distracted + recall of food consumed + snacking, w/ the idea that if you're distracted while you're eating, you may not remember what you eat.
* They also hypothesized failure to recall food consumed might lead to increased snacking later on.
* Sample = study consisted of 44 volunteer patients, half men, half women, who were randomized into 2 groups, 1 asked to play solitaire on the CPU while eating + to win as many games as possible, + the other group was asked to eat lunch w/out any distractions, focusing on what they're eating + thinking about the taste of the food + that they're eating.
* 
* 
* Both groups were provided the same amount of lunch + afterwards, while they were waiting around, they were offered biscuits to snack on.
* Researchers measured how many biscuits subjects consumed



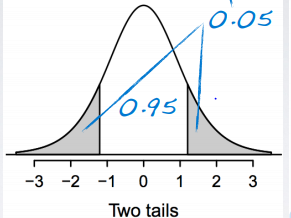
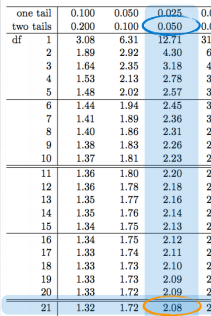
* This summary statistics suggest distracted eating groups snack more after lunch (x.bar.s = 52.1 g of biscuits compared to 27.1 g (x.bar.n) for the other group.
* We're also given the SDs for both groups, as well as the sample sizes, n, both = 22
* Goal = estimate average snacking level for distracted eaters.
* Estimating a population parameter entails a **CI = point estimate +/- a margin of error**.
* **Margin of error** = a critical value \* the standard error
* Since we're doing **inference on the mean** = use **t statistic**
* This SE of x.bar = s / sqrt(n)



* To figure out t 🡪 need to determine the dF associated w/ the t-distribution needed for this data
* When working w/ data *from only 1 sample* + *estimating a single mean*, the dF = n-1.
* We lose 1 dF b/c we're *estimating the SE* of the sample mean *using the sample SD*.
* Putting all of this together, the CI for a *single population mean* can be estimated using x.bar +/- t\* w/ n-1 dF \* s /Sqrt(n)



* There are variety of ways of finding the critical t score 🡪 t-table w/ dF = 22 – 1 = 21 for the row + corresponding tail area for desired confidence level.

* If we a 5% confidence level, we have 95% of our data in the center of the distribution (want the middle 95% in our CI), so we have 5% left for the 2 tails.

> ## find critical value of t for sample size of 22

> n <- 22

> dF <- n - 1

> abs(qt(p = .025, df = dF)) # find percentile

[1] 2.079614

* Note we *always use a positive critical value* + the confidence level = always the middle symmetric area in the center of the curve
* Once you mark that, you can easily determine the tail areas + use that value to find critical t-values We finally have all of our building blocks
* Now construct the CI for the average snacking level of distracted eaters.

> x.bar.s <- 52.1

> s <- 45.1

> t.crit <- abs(qt(p = .025, df = dF))

> SE <- s / sqrt(n)

> mOe <- t.crit \* SE

> (lower <- x.bar.s - mOe)

[1] 32.10378

> (upper <- x.bar.s + mOe)

[1] 72.09622

* **We are 95% confident distracted eaters consume between 32.1 to 72.1 grams of snacks post meal.**
* Next, suppose suggested serving of these biscuits = 30 g.
* *Do these data provide convincing evidence the amount of snacks consumed by distracted eaters post lunch is different than the suggested serving size?*
* Givens = sample mean, sample SD, sample size, SE calculated earlier = 9.62.
* Null: the population mean mu = 30 grams
* Alternative: mu != 30 (interested in any difference from mu, i.e. in either direction)
* The test statistic, t, can be calculated as sample mean - the null value divided by SE

> null.mu <- 30

> (t <- (x.bar.s - null.mu) / SE)

[1] 2.298408

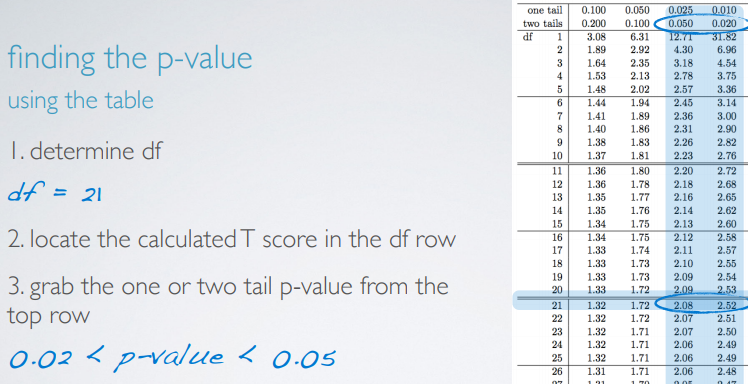
* Our observed test statistic = 2.3 *and* -2.3 (2-sided alternative hypothesis = shade both tails)

> # probability of obtaining this mean x.bar.s t w/ 21 dF if null = 30 is true

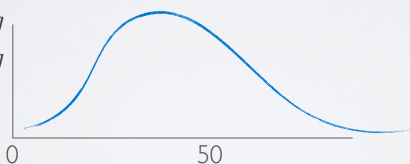
> pt(t, dF, lower.tail = F)\*2

[1] 0.03190849

* Or w/ table



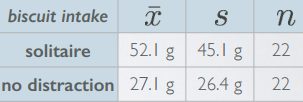
* Focusing on row of the table for our dF, locate the calculated t-score = 2.3 (work w/ the absolute value of the calculated t-score) + we grab the 1 or 2-tailed p-value from top of the table (depending on our alternative).
* In this case, we had a 2-sided alternative so our p is going to be somewhere between 0.02-0.05.
* This answer is less precise than the exact value R gives, but we still have sufficient info on the p-value to compare it to the significance levels of the test + make a decision.
* To recap, we focused on 1 group from the study (distracted eaters) + were provided some sample statistics on this group
* We calculated a 95% CI ranging from 32.1 to 72.1 g + did the hypothesis test where we compared how much these people ate to the suggested serving size.
* We found a p-value = ~3.18%, which < standard significance level of 5% = rejected the null + concluded these data DO provide convincing evidence distracted eaters consume an amount different than the suggested serving size.
* Since both *the estimation + the testing* were done using the *same underlying inferential framework* + the *same distribution*, the results should agree w/ each other.
* The null sets mu = 30 + we rejected this null.
* Similarly, the CI does NOT contain the null value of 30.
* Therefore, these two methods agree.
* 1 important task we skipped over = initially checking the conditions.
* We DO have a random assignment + 22 < 10% of all distracted eaters (we can assume).
* Therefore, we assume that 1 distracted eater in the sample is independent of another
* We're not given a visualization of the distribution of biscuit consumption to check the sample size skew condition.
* However given the sample statistics, we can kind of sketch it out.



* Sample mean = 52 + there's a natural boundary at 0 (one cannot < 0 g of biscuit)
* The 68, 95, 99.7 rule is not going to apply here (> 1 SD below the mean = hits natural boundary of 0 g)
* Therefore, the data are likely right-skewed
* The t distribution is pretty robust of skewness, but ideally we’d like to see a visualization of this distribution + asses this sample size condition accordingly, especially given the low sample size.

**Inference for Comparing 2 Independent Means**

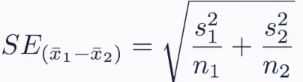
* Stick w/ the distracted eaters study.



* Want to estimate how much more/less distracted eaters snack compared to non-distracted ones
* Need a **CI** of this = Point estimate +/- Margin of error = (different between 2 sample averages) +/- critical value \* **standard error of difference between the 2 sample means**



* This is a new SE = square root of sum of squared variances divided by sample size of each group



* Note we *add* 2 variances even though we're looking for the SE of the *difference* of the 2 means
* Conceptually, think about it as bringing together 2 measures w/ an inherent variability around them = the 2 sample means.
* **When you bring 2 unknowns together, the result should always be *more variable*, regardless of whether you're adding them or subtracting them.**
* Also have a new dF calculation for 2 *INDEPENDENT* means



* This is actually not the *exact* dF, which is quite tedious to compute by hand.
* This is a conservative estimate, since it relies on the lower of the 2 sample sizes.
* Conditions to meet (like all inferential methods).
* 1) Independence, both within + between groups.
* Verified w/in-group independence *w/ respect to the outcome variable* via random sampling/assignment + the **10% condition** (if sampling w/out replacement)
* both n1 + n2 should be < 10% of their respective populations. I
* Failure to meet the between-groups independence condition is not inherently a problem
* just means that we’d need to use methods suited for *dependent* (**paired**) groups.
* We will introduce these
* 2) sample size and skew.
* More skewed the population distributions are = the larger the samples we need from them
* Estimate difference between the average post-meal snack consumption between those who eat with + w/out distractions.
* **CI = point estimate** (difference between the 2 sample means) **+/- a margin of error** (critical T-score \* SE of the difference between the 2 sample means.

> # find t-critical value for comparing the 2 means

> n1 <- 22

> n2 <- 22

> dF <- min(n1 - 1, n2 - 1)

> (t.crit <- abs(qt(p = .025, df = dF)))

**[1] 2.079614**

> # get point estimate

> x.bar.dist <- 52.1

> x.bar.nondistract <- 27.1

> (pe <- x.bar.dist - x.bar.nondistract)

[1] 25

>

> # get margin of error

> var.dist <- 45.1

> var.nondist <- 26.4

> (se.diff.2.means <- sqrt(((var.dist^2)/n.dist) + ((var.nondist^2)/n.nondistract)))

[1] 11.14159

> (m0e <- t.crit\*se.diff.2.means)

[1] 23.1702

>

> # get CI

> (lower <- pe - m0e)

[1] 1.829798

> (upper <- pe + m0e)

[1] 48.1702

* The **CI = {1.83, 48.17} grams.**
* Next, we need a hypothesis test for evaluating whether these data provide **convincing evidence** of a difference between average post-meal snack consumption between those who eat w/ + w/out distractions
* When doing a hypothesis test, 1st step always = **set your hypotheses.**
* H0 says there's absolutely nothing going on here + the difference between the average snack consumption for those who eat / + /out distraction = 0.



* Note we use μ + not x.bar b/c **hypotheses are always about populations + never about samples**
* We already *know the sample statistics*, so we *don't need to hypothesize about them*.
* Want to *use sample statistics* to *say something about the unknown population parameters.*
* The alternative = there *is* a difference between the 2 population means/is not 0.

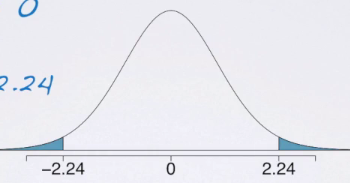


* Our T-score w/ 21 dF = the observed difference - null value of 0 / the SE

> (t.21 <- (pe - 0)/se.diff.2.means)

[1] 2.243845

* The last step before making a decision on these hypotheses = find the p-value



> ## probability of obtaining this OBSERVED DIFFERENCE (point estimate) w/ 21 dF if null = 0 is true

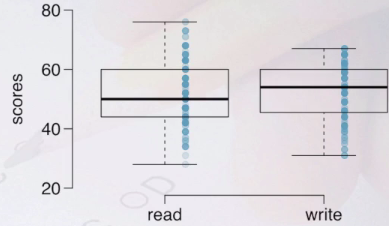
> pt(t.21, dF, lower.tail = F)\*2

[1] 0.03575082

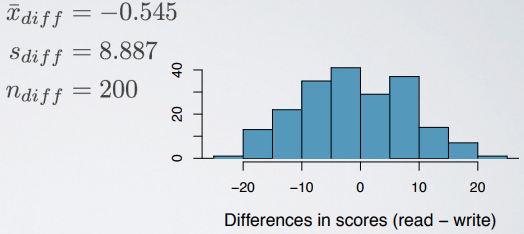
* To recap, we had / a study where researchers **randomly assigned** respondents into distracted + non-distracted eating groups + compared snack intake post-meal.
* The sample statistic suggested distracted eaters consumed more snacks on average.
* However, just b/c we observe a difference in sample means doesn't necessarily mean there is something going on that is **statistically significant** in the **actual populations**.
* So, we use **statistical inference tools** to evaluate if this apparent relationship between distracted eating + snacking more provide evidence of a **real difference** at **the population level**.
* Note, we have a **randomized control trial** here, so *if we do indeed find a significant result*, we could then talk about a **causal relationship** between these 2 variables.
* The **CI for the average difference** = {1.83, 48.17} + the hypothesis test evaluating a different between the 2 means yielded a p-value of roughly 4%.
* This means we’d reject the null (which is not in our CI) + conclude these data do indeed provide convincing evidence there is a difference between average snack intake of distracted + non-distracted eaters
* **The results of the CI + hypothesis test agree, as we used similar methods + we rejected the null (which set the difference between the 2 means to 0) + this null value was not included in our CI)**

**Inference for Comparing 2 Paired Means**

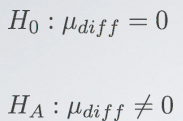
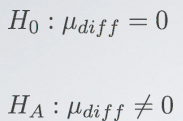
* Our methodologies change if the means we're comparing are **paired**/dependent (not much new)
* 200 observations were randomly sampled from the High School and Beyond survey.
* The same students took a reading + a writing test.



* At first glance, it appears median writing score is slightly higher than median reading score.
* Both distributions seem fairly symmetric, but reading score = slightly more right-skewed (evidenced by the fact that the median is closer to the 25th percentile than the 75th percentile)
* Reading scores are slightly more variable than the writing scores (wider box)
* *That all being said, at a first glance, it is really difficult to tell if there's a difference in the scores*
* So can the reading and writing scores for a given student be assumed to
* A student's reading score is likely NOT independent of their writing score.
* Generally high achieving students are likely to score highly on both tests.
* When 2 sets of observations have a special correspondence (are NOT independent) they are **paired**.
* To analyze paired data, it is often useful to look at the *difference in outcomes of each pair of observations.*
* Ex: For each student, subtract writing score from reading score + create a new variable = **diff** = the difference between the 2 scores 🡪 calculate this difference for each student in our data set.
* Good idea to start by defining the **parameter of interest** + the **point estimate**.
* In this case, we're interested in finding **average difference between reading + writing scores of all high school students = μ.diff**
* Since we don't have access to the whole population, we **estimate this unknown population mean w/ our sample statistic** = the average difference between reading + writing scores of these 200 sampled high school students, **x.bar.diff**
* If there was no difference between reading writing scores, we’d expect diff to be 0
* Look at the distribution of these differences.



* We see they’re centered around 0 but the average difference is not exactly equal to 0 + we're also seeing quite a bit of variability in this distribution.
* Therefore, it's impossible to determine whether there is a statistically significant difference between average reading + writing scores simply by visually evaluating this plot.
* We need statistical inference tools once again 🡪 1st define our hypotheses.
* H0: there's nothing going on
* H1: there’s a difference between the test scores.

* We summarized our 2 columns of data (scores) into just 1 column of differences 🡪 once again we’re setting out to do inference on a *single* population mean μ.diff.
* Therefore, the structure = exactly the same as hypothesis test for doing a test on any single mean, except **we're really doing inference on a difference of paired means.**
* The mechanics, the conditions, etc. = all the same as working w/ a single population mean.

> mu.diff <- 0

> x.bar.diff <- -.545

> s.diff <- 8.887

> n.diff <- 200

> (dF <- n.diff - 1)

[1] 199

> (se.diff <- s.diff/sqrt(n.diff))

[1] 0.6284058

> (t.199 <- (x.bar.diff - mu.diff)/se.diff)

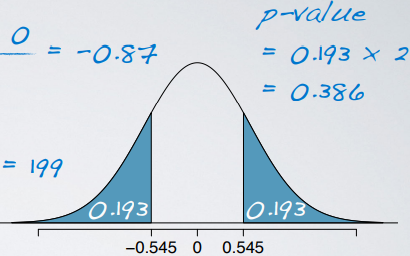
[1] -0.867274

* Then we draw our curve, mark the observed difference, + shade the tail areas corresponding to the p-value (2-sided alternative = shade both tails\_

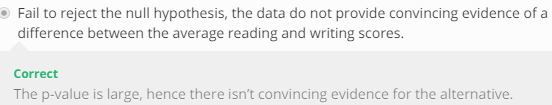
> ## probability of obtaining this OBSERVED DIFFERENCE (x.bar.diff) w/ 199 dF if null = 0 is true

> pt(abs(t.199), dF, lower.tail = F)\*2

[1] 0.3868365



* Compare the p-value to the significance level + if lower, reject the null + conclude the data DO provide convincing evidence for the alternative
* However, understanding what the p-value *actually means as a* ***conditional probability***, it usually takes a little bit more practice.
* 



* The best approach for finding the correct interpretation of the p-value as a probability are questions:
* ~~p-value = probability the average scores on the reading + writing exams are equal~~
* Basically means p-value equals the probability of the null being true = FALSE
* ~~p-value = probability the average scores on the reading + writing exams are different~~
* i.e. probability of the alternative being true = FALSE
* **p-value = probability of obtaining a random sample of 200 students where average difference between reading + writing scores is *at least* 0.545 in either direction if the true average difference between scores is in fact 0.**
* Generically reads as “probability of observed or more extreme outcome given the null is true” 🡪 indeed the definition of the p-value.
* ~~p-value = probability of incorrectly rejecting the null if in fact the null is true.~~
* This is actually the **probability of a type I error** + not the definition of p-value.
* In summary, we started off with 2 variables (reading + writing scores of the same set of students) + summarized these variables into 1 by taking the **pairwise differences**.
* In situations where we do inference for paired data, most often the null sets the average difference between the 2 paired means to = 0 (indicating no difference between them)
* Paired data can happen when we have a set of data from the same set of people, like in pre-post studies, such as a weight-loss study (post-weight will necessarily be dependent on pre-weight)
* Other studies might also take **repeated measures** on the same set of people, such as reaction time of the same set of people after they have spent the recommended amount of 7.5 hours the previous night or if they've only spent 2 hours.
* We might also use paired approaches when *we have different sets of subjects to begin with, but for some reason we believe these subjects to be not independent 🡪* **Twin studies** is an obvious example for these, or studies on partner A + partner B who are in a relationship.
* We’d design these studies as paired if we believe these individuals in the 2 groups are similar on *certain aspects* + we're evaluating their differences on other aspects.