**Unit 1: Probability models and axioms**

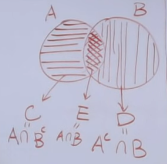
**Lecture 3: Solved problems**

**The Probability Of The Difference Of Two Events**

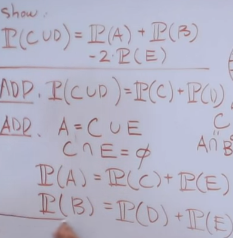
* Give a mathematical derivation of the formula

for the probability that exactly one of the events A and B will occur. Your derivation should be a sequence of steps, with each step justified by appealing to one of the probability axioms.

* So we start with 2 events, A + B, + there might be some intersection between the 2 events.
* The set of points/samples in A + not in B = a set **C = A intersection B(c)**
* Similarly, for all points in B + not in A, we'll call it **D = A(c) intersection B.**
* Finally, for points that are in the intersection of A + B, call it E = **A intersection B.**



* Rewrite our objective as:
* Show that the P(C U D) = P(A) + P(B) – 2P(E)
* Review what the axioms are,
* 1) Non-negativity = Take any event A, then P(A) must be *at least 0*.
* 2) Normalization = probability of the entire space Ω must be equal to 1.
* 3) Additivity axiom (ADD) 🡪 if there are 2 events, A + B, *that are* ***disjoint*** (have nothing in common), their intersection = the empty set.
* Then the probability of their union **P(A U B) = P(A) + P(B)**
* 1st invoke the additivity axioms to argue the P(C U D) = P(C) + P(D)
* b/c set C + set D are completely disjoint from each other.
* In a similar way, also notice A = union of sets C + E, and C + E are disjoint w/ each other, b/c C + E, by definition, don't share any points.
* Therefore, P(A) = P(C) + P(E)
* In a similar way, P(B) = P(D) + P(E) b/c event B is the union of D + E and D + E are disjoint
* This should be enough to prove our final claim.



* We have the P(C U D) = **P(C) + P(D)**
* Insert 2 terms to make the connection with a 2nd part of the equation more obvious.
* Write P(C) + P(D) **= P(C) + P(E) + P(D) + P(E)**
* We added 2 terms, P(E), so to make the equality valid we'll subtract out 2P(E)
* We know P(A) = P(C) + P(E) and P(B) = P(D) + P(E), so we can sub them back in to get
* P(C U D) = P(A) + P(B) – 2P(E), the final equation.
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