***Learning Statistics with R - University of Adelaide***

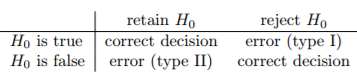
***Part IV – Statistical Theory***

**Chapter 11 – Hypothesis Testing**

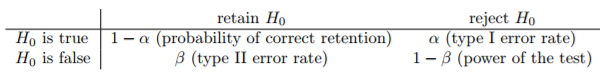
* “The process of induction is the process of assuming the simplest law that can be made to harmonize w/ our experience. This process, however, has no logical foundation but only a psychological one. It is clear there are no grounds for believing that the simplest course of events will really happen. It is a hypothesis that the sun will rise tomorrow: and this means that we do not know whether it will rise.” – Ludwig Wittgenstein
* **Estimation** was 1 of the 2 big ideas in **inferential statistics** + the other big idea is **hypothesis testing**.
* In its most abstract form, hypothesis testing really a very simple idea: researcher has some theory about the world + wants to determine whether or not the data actually support that theory.
* However, details are messy + most people find the theory of hypothesis testing to be the most frustrating part of statistics.
* Simple example study: Seek to test whether clairvoyance exists 🡺 Each participant sits at a table + is shown a card by an experimenter, which is black on 1 side + white on the other.
* Experimenter takes card away + places it on a table in an adjacent room black or white side up completely at random, w/ the randomization occurring only after experimenter has left the room w/ the participant.
* 2nd experimenter comes in + asks the participant which side of the card is facing upwards.
* Purely a 1-shot experiment: Each person sees only 1 card + gives only 1 answer + *at no stage is the participant actually in contact w/ someone who knows the right answer.*
* Dataset is very simple = asked the question of N = 100 people + some number X = 62 got the answer right, a surprisingly large number, sure, but is it large enough to claim evidence for ESP?
* This is the situation where hypothesis testing comes in useful.
* 1st distinction you need to keep clear is between **research hypotheses** and **statistical hypotheses**.
* In ESP study, overall scientific goal = to demonstrate clairvoyance exists
* Clear research goal: hoping to discover *evidence* for ESP
* In other situations, might actually be more neutral than that, so might say research goal = to determine whether or not clairvoyance *exists*.
* Basic point: a **research hypothesis** involves making a substantive, testable scientific claim
* If you’re a psychologist, your research hypotheses are fundamentally about psychological constructs
* Any of the following would count as research hypotheses:
* Listening to music reduces ability to pay attention to other things = a claim about causal relationship between 2 psychologically meaningful concepts (listening to music + paying attention to things), so it’s a perfectly reasonable research hypothesis.
* Intelligence is related to personality = a relational claim about 2 psychological constructs (intelligence + personality), but claim is weaker: *correlational*, NOT causal.
* Intelligence is speed of information processing: This hypothesis has a quite different character:
* not actually a relational claim at all but an **ontological** claim about the *fundamental character of intelligence*
* Actually worth expanding on this one
* Usually easier to think about how to construct experiments to test research hypotheses of the form “*does X affect Y?”* than to address claims like “*what is X?*”
* In practice, usually you find ways of *testing relational claims that follow from ontological ones.*
* Ex: If I believe intelligence is speed of information processing in the brain, my experiments will often involve looking for *relationships* between *measures of intelligence* + *measures of* *speed*.
* As a consequence, most everyday research questions tend to be **relational** in nature, but are almost always motivated by *deeper ontological questions* about the *state of nature*.
* Notice in practice, research hypotheses could overlap a lot.
* Ultimate goal in ESP experiment might be to test an ontological claim “ESP exists”, but I might operationally restrict myself to a *narrower* hypothesis like “Some people can ‘see’ objects in a clairvoyant fashion”.
* That said, there are some things that really don’t count as proper research hypotheses in any meaningful sense:
* Love is a battlefield. too vague to be testable.
* While it’s okay for a research hypothesis to have a degree of vagueness to it, it has to be possible to **operationalize** theoretical ideas
* Difficult to see how this can be converted into any concrete research design.
* If that’s true, this isn’t a scientific research hypothesis, it’s a pop song.
* Doesn’t mean it’s not interesting: a lot of deep questions humans have fall into this category
* Maybe 1 day science will be able to construct testable theories of love, or to test to see if God exists, + so on; but right now we can’t
* The first rule of tautology club is the first rule of tautology club: Not a substantive claim of any kind
* True *by definition*: No conceivable state of nature could possibly be inconsistent w/ this claim
* As such, say this = an **unfalsifiable hypothesis** + as such it is outside the domain of science
* *Whatever else you do in science, claims must have the possibility of being wrong.*
* More people in my experiment will say “yes” than “no”: Fails as a research hypothesis b/c it’s a claim *about the data set*, not *about the psychology* (unless your actual research question is whether people have some kind of “yes” bias).
* This hypothesis is starting to sound more like a **statistical hypothesis** than research hypothesis
* **Research Hypotheses** can be somewhat messy at times + ultimately they *are* scientific claims.
* **Statistical hypotheses** are *neither* of these 2 things 🡺 MUST be mathematically precise + MUST correspond to specific claims about the *characteristics of the data generating mechanism* (i.e., the “population”).
* Even so, the intent is that statistical hypotheses bear a *clear relationship* to the substantive research hypotheses you care about
* Ex: ESP study 🡪 research hypothesis = some people are able to see through walls/whatever.
* What I want to do is to *map* this onto a statement about *how* data were generated
* Quantity I’m interested in w/in the experiment is P(correct), the true-*but-unknown* probability w/ which participants in my experiment answer the question correctly.
* Let’s use the Greek letter θ (theta) to refer to this probability.
* Here are 4 different statistical hypotheses:
* If ESP doesn’t exist + if my experiment is well designed, my participants are just guessing:
* should expect them to get it right 1/2 of the time
* so my statistical hypothesis is the true probability of choosing correctly is θ = 0.5.
* Suppose ESP does exist + participants can see the card.
* If true, people will perform better *than chance*.
* Statistical hypothesis is θ > 0.5.
* ESP does exist, but the colors are all reversed + people don’t realize it
* If that’s how it works you’d expect people’s performance to be below *chance*.
* Correspond to a statistical hypothesis that θ < 0.5.
* Suppose ESP exists, but I have no idea whether people are seeing the right or wrong color.
* Only claim I could make about the data would be the probability of making the correct answer is not equal to 50%
* Corresponds to the statistical hypothesis that θ != 0.5.
* All of these are legitimate examples of a statistical hypothesis b/c they are statements about a population parameter + are meaningfully related to my experiment.
* What this discussion hopefully makes clear is that *when attempting to construct a statistical hypothesis, test that the researcher actually has 2 quite distinct hypotheses to consider.*
* 1st: They have a **research hypothesis** (claim about psychology)
* 2nd: It corresponds to a **statistical hypothesis** (claim about the data-generating population).
* ESP example: these might be:
* Research hypothesis: “ESP exists”
* Statistical Hypothesis: θ != 0.5
* The key thing to recognize is a statistical hypothesis test is a test of the statistical hypothesis, NOT the research hypothesis.
* If a study is badly designed, the link between the research + statistical hypothesis is broken.
* Suppose the ESP study was conducted in a situation where participants can actually see the card reflected in a window
* if that happens, I’d be able to find very strong evidence that θ != 0.5, but this would tell us nothing about whether “ESP exists”.
* So, I have a **research hypothesis** that *corresponds to what I want to believe about the world* + can map it onto a **statistical hypothesis** that *corresponds to what I want to believe about how the data were generated*.
* **Null** **hypothesis, H0**, corresponds to *the exact opposite of what I want to believe*
* Now turn to focus exclusively on that, almost to the neglect of the thing I’m actually interested in, **alternative hypothesis, H1**
* ESP example 🡺 null = θ = 05, since that’s what we’d expect if ESP didn’t exist.
* Hope is that ESP is real + the alternative to this null is θ != 0.5.
* Dividing up the possible values of θ into 2 groups: those values I hope aren’t true (null) + those I’d be happy w/ if they turn out to be right (alternative). Having
* Important thing = *Recognize that the goal of a hypothesis test is NOT to show the alternative hypothesis is (probably) true but to show that the null hypothesis is (probably) false.*
* Ex: Hypothesis test = a criminal trial of the null hypothesis (defendant), researcher = prosecutor, + statistical test itself = judge.
* There is a *presumption of innocence*: null hypothesis is deemed to be true unless the researcher can prove *beyond a reasonable doubt* it is false.
* Free to design the experiment in any way (w/in reason) w/ goal to maximize the chance the data will yield a conviction for the crime of being false.
* The catch = Statistical test sets rules of a trial + they are *designed to protect null* 🡺 specifically to ensure if the null is actually true, the chances of a false conviction *are guaranteed to be low*

**11.2 Two types of errors**

* Want to construct test so we never make any errors 🡺 never possible.
* Always have to accept there’s a chance we did the wrong thing + as a consequence, the goal behind statistical hypothesis testing is NOT to eliminate errors, but to *minimize them*.
* It is either the case the null is true or it is false + our test will either **reject the null** or **retain it**
* After we run the test + make our choice, 1 of 4 things might have happened:

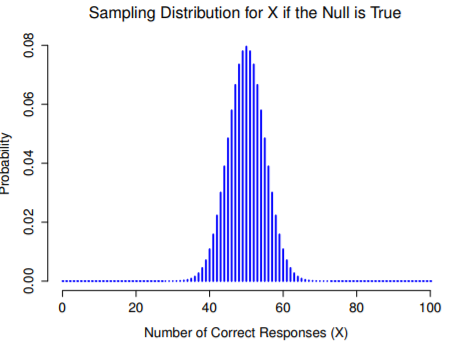


* **Type I Error** = reject a null that is actually true
* **Type II Error** = retain a null when it is in fact false
* A criminal trial requires you establish “beyond a reasonable doubt” that the defendant did it.
* All evidentiary rules are (in theory, at least) designed to ensure there’s (almost) no chance of wrongfully convicting an innocent defendant (trial is designed to protect the rights of a defendant)
* In other words, a criminal trial doesn’t treat the 2 types of error in the same way
* Punishing the innocent is deemed to be *much worse* than letting the guilty go free.
* Statistical tests are pretty much the same
* The Single Most Important Design Principle Of The Test = to *control the probability of a type I error + keep it below some fixed probability*.
* This probability, α, = the **significance level** of the test (sometimes the **size** of the test).
* A hypothesis test is said to have significance level α if the type I error rate is no larger than α.
* Would also like to keep type II error rate under control too, denoted w/ β.
* Much more common to refer to the **power** of a test = probability w/ which we reject a null when it *really is false* (good) 🡺 **1 – β**



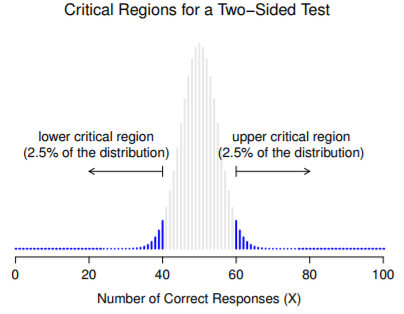
* A powerful hypothesis test = one that has a *small value of β* while *still keeping α fixed at some (small) desired level*.
* By convention, scientists make use of 3 different α levels: .05, .01, .001.
* Tests are designed to ensure the α level is kept small, but *there’s no corresponding guarantee regarding β*.
* Certainly would like type II error rate to be small + we try to design tests that keep it small, but this is very much secondary to the overwhelming need to control the type I error rate.
* It is better to retain 10 false nulls than to reject a single true one”.
* 1 thing to avoid = the word “prove”
* a statistical test really doesn’t *prove* a hypothesis is true or false.
* Proof implies certainty + statistics = never having to say you’re certain.
* Some argue you’re only allowed to make statements like “rejected the null”, “failed to reject the null”, or possibly “retained the null”

**11.3 Test statistics and sampling distributions**

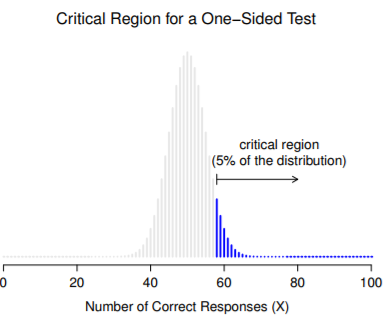
* ESP 🡺 ignore actual data obtained for the moment + think about the structure of the experiment
* The form of the data is that “X out of N people correctly IDed the color of the hidden card”
* Moreover, suppose the null really is true: ESP doesn’t exist, + the true probability anyone picks the correct color is exactly θ = 0.5.
* Would then expect the proportion of people w/ a correct response to be pretty close to 50%
* We’d say “X/N is approximately 0.5” (don’t expect this fraction to be exactly 0.5)
* If X = 99 participants got the question right, we’d feel pretty confident the null is wrong.
* if only X = 3 got the answer right, similarly confident the null was wrong.
* We have a quantity X we can calculate by looking at data + after looking at X, we make a decision about whether to believe the null is correct, or to reject it in favor of the alternative.
* **Test statistic** = what we calculate to guide our choices.
* Having chosen a test statistic, state precisely *which values* of the test statistic cause us to reject the null + which values cause us to keep it.
* In order to do so, *need to determine what the sampling distribution of the test statistic would be if the null were actually true*
* This distribution tells us *exactly what values of X our null would lead us to expect*, + therefore, we can use this it as a tool for assessing how closely the null agrees w/ our data.
* To determine the sampling distribution of a test statistic for a lot of hypothesis tests = complicated
* Sometimes it’s very easy + fortunately, ESP example provides 1 of the easiest cases.
* Our population parameter θ = just the overall probability people respond correctly when asked the question, + our test statistic X = count of people who did so out of a sample size of N.
* That’s exactly what the binomial distribution describes
* We’d say “the null predicts X is binomially distributed” = X ~ Binomial(θ, N)
* Since the null states θ = 0.5 + our experiment has N = 100, we have the sampling distribution needed
* 
* No surprises really: the null says X = 50 is the most likely outcome + says we’re almost certain to see somewhere between 40-60 correct responses.

**11.4 Making decisions**

* We’ve constructed a test statistic (X) + we chose this test statistic in such a way that we’re pretty confident if X is close to N/2, we should retain the null, + if not we should reject it.
* But exactly *which* values of the test statistic do associate w/ the null + which w/ the alternative?
* ESP: observed a value of X = 62. What to make? Believe the null or the alternative?
* The **critical region** of the test corresponds to values of X that lead to rejecting the null
* Consider what we know:
* X should be very big or very small in order to reject the null
* If the null is true, the sampling distribution of X is Binomial(0.5, N)
* If α = .05, the critical region must cover 5% of this sampling distribution
* It’s important to understand the **critical region** corresponds to values of X for which we’d reject the null + the *sampling distribution* in question *describes the probability* we’d obtain a *particular value of X* if the null were actually true.
* Suppose we chose a critical region that covers 20% of the sampling distribution + that the null is actually true 🡺 The probability of incorrectly rejecting the null = 20% 🡺 20% of getting our test statistic if the null were true
* Therefore, we’d have built a test that had α = 0.2.
* If we want α = .05, the critical region is only allowed to cover 5% of the sampling distribution of our test statistic.



* The critical regions associated w/ the hypothesis test for ESP w/ significance level α = 0.05.
* The plot shows the sampling distribution of X under the null hypothesis + the grey bars correspond to values of X for which we’d *retain the null*.
* Black bars = start of the critical regions = values of X for which we’d reject the null.
* B/c the alternative is 2-sided (allows both θ < 0.5 + θ > 0.5), the critical region covers *both tails* of the distribution.
* To ensure an α = 0.05, must ensure each of the 2 regions encompasses 2.5% of the sampling distribution
* As it turns out, those 3 things uniquely solve the problem:
* Our critical region = the most extreme values (**tails**) of the distribution.
* If we want α = 0.05, our critical regions correspond to X < 40 + X > 60
* If the # of people saying “true” is between 41-59, we should retain the null.
* If the # is between 0-40 or 60-100, we should reject the null.
* 40 + 60 are referred to as the **critical values**, since they define the edges of the critical region.
* Strictly speaking, the test just constructed has α = 0.057, which is a bit too generous.
* If I’d chosen 39 + 61 to be the boundaries for the critical region, the critical region only covers 3.5% of the distribution.
* It makes more sense to use 40 + 60 as my critical values + be willing to tolerate a 5.7% type I error rate, since that’s as close as we can get to a value of α = 0.05
* At this point, our hypothesis test is essentially complete
* (1) We choose an α level (e.g., α = 0.05
* (2) Came up w/ some test statistic (X) that does a good job (in some meaningful sense) of comparing H0 to H1
* (3) Figured out the sampling distribution of the test statistic on the assumption the null is true (binomial)
* (4) Calculated the critical region that produces an appropriate α level (0-40 + 60-100).
* All we have to do now is calculate the value of the test statistic for the real data, X = 62, + then compare it to the critical values to make our decision.
* Since 62 is greater than the critical value 60, we reject the null + say the test has produced a **significant result**
* The concept of **statistical significance** is actually a very simple one but has a very unfortunate name.
* If the data allow us to reject the null, we say “the result is statistically significant”, which is often shortened to “the result is significant”.
* **Significant** just means something like “indicated”, rather than “important”.
* **Statistically Significant** means is the data allowed us to reject a null
* Whether or not the result is *actually important* in the real world is a very different question, + depends on all sorts of other things
* If we take a moment to think about the statistical hypotheses so far, H0 : θ = 0.5 and H1 : θ != 0.5 , we notice the alternative covers both the possibility that θ > 0.5 + the possibility that θ < 0.5.
* This makes sense if we really think ESP could produce better-than-chance performance or worse-than-chance performance (there are some people who think that).
* This is an example of a **2-sided test** b/cthe alternative covers the area on both sides of the null + as a consequence, the critical region of the test covers both tails of the sampling distribution (2.5% on either side if α = 0.05)
* It might be the case I’m only willing to believe in ESP if it produces *better than chance* performance.
* If so, my alternative only covers the possibility that θ > 0.5 + as a consequence the null now becomes θ <= 0.5 and therefore H0 : θ <= 0.5 and H1 : θ > 0.5
* This is a **one-sided test** + when this happens the critical region only covers 1 tail of the sampling distribution



* The critical region for a 1-sided test when the alternative is θ = 0.05
* We only reject the null for large values of X.
* As a consequence, the critical region only covers the upper tail of the sampling distribution, specifically the upper 5% of the distribution