Stat 6021: Interpretation of Regression Coefficients with Log Transformation on Response Variable

Read this after Section 2 in Guided Notes.

1 Interpreting Regression Coefficients: Log Transformation on Response Variable

One of the reasons a log transformation is a popular transformation is that regression coefficients are still fairly easy to interpret. Consider a log transformation applied to the response variable. The least-squares regression equation becomes

$$\begin{aligned} \log \hat{y} &= \hat{\beta}_0 + \hat{\beta}_1 x \\ \Longrightarrow \hat{y} &= \exp(\hat{\beta}_0 + \hat{\beta}_1 x) \end{aligned} \tag{1}$$

When the predictor variable increases by one unit, (1) becomes

$$\begin{aligned} \hat{y}_{new} &= \exp(\hat{\beta}_0 + \hat{\beta}_1 (x+1)) \\ &= \exp(\hat{\beta}_0 + \hat{\beta}_1 x) \times \exp(\hat{\beta}_1) \end{aligned} \tag{2}$$

Consider the ratio of $\hat{y}_{new}$ and $\hat{y}$ using (1) and (2), i.e.,

$$\begin{aligned} \frac{\hat{y}_{new}}{\hat{y}} &= \frac{\exp(\hat{\beta}_0 + \hat{\beta}_1 x) \times \exp(\hat{\beta}_1)}{\exp(\hat{\beta}_0 + \hat{\beta}_1 x)} \\ &= \exp(\hat{\beta}_1) \\ \Longrightarrow \hat{y}_{new} &= \exp(\hat{\beta}_1) \times \hat{y} \end{aligned} \tag{3}$$

From (3), we can see how to interpret the estimated slope when a log transformation is applied to the response variable: the predicted response variable is multiplied by a factor of $\exp(\hat{\beta}_1)$ when the predictor increases by one unit.