SLR with AR(p) Errors

The simple linear regression model with AR(p) errors:

$$y_t = \beta_0 + \beta_1 x_t + \epsilon_t, \tag{1}$$

where

$$\epsilon_t = \phi_1 \epsilon_{t-1} + \phi_2 \epsilon_{t-2} + \cdots + \phi_p \epsilon_{t-p} + a_t, \tag{2}$$

and a_t are i.i.d $N(0, \sigma_a^2)$.

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Cochrane-Orcutt method transforms the variables based on the AR structure of the errors in (2):

- $y'_t = y_t - \phi_1 y_{t-1} - \phi_2 y_{t-2} - \cdots - \phi_p y_{t-p}$
- $x'_t = x_t - \phi_1 x_{t-1} - \phi_2 x_{t-2} - \cdots - \phi_p x_{t-p}$

Subbing (1) into these transformations, the model becomes

$$y'_t = \beta'_0 + \beta'_1 x'_t + a_t \tag{3}$$

so the errors from (3) are now i.i.d. $N(0, \sigma_a^2)$.

Building the Regression Model with AR Errors

The procedure in building the regression model with AR errors (Cochrane-Orcutt method):

1. Use ordinary least squares (OLS) to estimate the coefficients in (1).
2. Examine the AR structure of the **sample residuals** from step 1 using a PACF Plot.
3. Estimate the coefficients $\phi_1, \cdots, \phi_p$ using ARIMA estimation.
4. Use the estimated $\hat{\phi}_1, \cdots, \hat{\phi}_p$ to compute y_t' and x_t'.
5. Use OLS to estimate (3) using y_t' and x_t'.
6. If the steps above are done properly, the residuals at the end of step 5 should be uncorrelated, with mean 0 and constant variance.