SLR with AR(p) Errors

The simple linear regression model with AR(p) errors:

$$y_t = \beta_0 + \beta_1 x_t + \epsilon_t, \tag{1}$$

where

$$\epsilon_t = \phi_1 \epsilon_{t-1} + \phi_2 \epsilon_{t-2} + \cdots + \phi_p \epsilon_{t-p} + a_t, \tag{2}$$

and a_t are i.i.d $N(0, \sigma_a^2)$.

SLR with AR(p) Errors

- Durbin-Watson tests for autocorrelation at lag 1.
- A partial autocorrelation (PACF) plot of the residuals can allow us to **assess the order** of the AR structure of our errors.

ACF vs PACF

Consider a sequence of observations $x_1, x_2, \cdots$.

- Autocorrelation (ACF) at lag h of a sequence measures the correlation of any two points, x_t, x_{t+h}, in the sequence that are h apart in the sequence.
- PACF at lag h of a sequence measures the correlation of any two points, x_t, x_{t+h}, in the sequence that are h apart in the sequence, after **removing the connections with all the points between** x_t and x_{t+h}.

By definition, the ACF and PACF at lag 1 are equal.

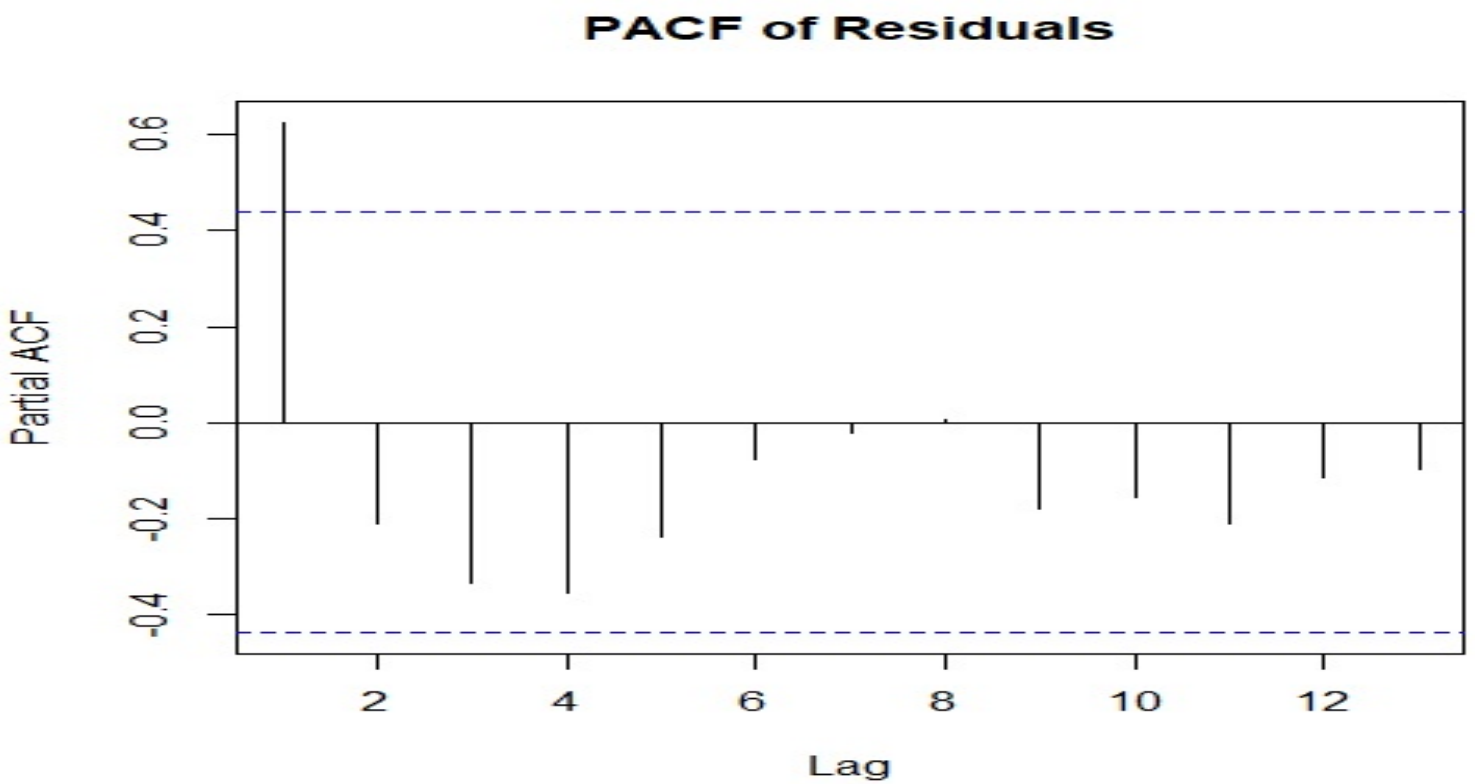
ACF vs PACF

- ACF is a measure of correlation, but cannot be used to inform us of the order of the AR structure of the observations.
- PACF can be used to inform us of the order of the AR structure of the observations.

Check for significance of PACF at various lags. The largest lag that gives a significant PACF is the order of the AR structure of the observations.

PACF Plot

From tutorial example,



The PACF plot using `pacf()` starts at lag 1, whereas the ACF plot using the `acf()` starts at lag 0.