

Stat 6021: Review of Matrices

Read this before starting the Guided Notes.

1 Matrix Notation

Consider two $m \times n$ (where m denotes the number of rows and n denotes number of columns) matrices \mathbf{A} and \mathbf{B} . Sometimes a matrix could be written as $\mathbf{A}_{m \times n}$ with the dimensions in the subscript. The entries in matrix are denoted by a_{ij} for the (i, j) th entry in matrix \mathbf{A} .

Example: Consider the following matrix

$$\mathbf{A} = \begin{pmatrix} 4 & 2 & 1 \\ 6 & 7 & 3 \end{pmatrix}.$$

\mathbf{A} is a 2×3 matrix. The value of the a_{21} th entry is 6.

2 Matrix Addition

Two matrices can only be added together if and only if they have the same dimension. The (i, j) th entries are added together.

Example: Consider the following matrices

$$\mathbf{A} = \begin{pmatrix} 4 & 2 & 1 \\ 6 & 7 & 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 5 & 11 & 8 \\ 9 & 4 & 2 \end{pmatrix}.$$

The matrices \mathbf{A} and \mathbf{B} can be added since they have the same dimension, and their addition becomes

$$\mathbf{A} + \mathbf{B} = \begin{pmatrix} 4+5 & 2+11 & 1+8 \\ 6+9 & 7+4 & 3+2 \end{pmatrix} = \begin{pmatrix} 9 & 13 & 9 \\ 15 & 11 & 5 \end{pmatrix}.$$

Some properties with matrix addition:

- Matrix addition is commutative, i.e., $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$.
- Matrix addition is associative, i.e., $\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C}$.

3 Matrix Multiplication

Consider the matrices $\mathbf{A}_{m \times n}$ and $\mathbf{E}_{n \times q}$. The product of two matrices only exist if the number of columns of the first matrix is equal to the number of rows of the second matrix. The product is a $m \times q$ matrix whose (i, j) th entry is the inner product between the i th row of \mathbf{A} and the j th column of \mathbf{E} .

Example: Consider the following matrices

$$\mathbf{A} = \begin{pmatrix} 4 & 2 & 1 \\ 6 & 7 & 3 \end{pmatrix}, \mathbf{E} = \begin{pmatrix} 5 \\ 0 \\ 1 \end{pmatrix}.$$

The product \mathbf{AE} exists since \mathbf{A} has 3 columns and \mathbf{E} has 3 rows, and \mathbf{AE} will be a 2×1 matrix. The product \mathbf{EA} does NOT exist since \mathbf{E} has 1 column and \mathbf{A} has 2 rows. The product \mathbf{AE} is written as

$$\mathbf{AE} = \begin{pmatrix} 4 & 2 & 1 \\ 6 & 7 & 3 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 21 \\ 33 \end{pmatrix}$$

- The $(1, 1)$ th entry in \mathbf{AE} is found by taking the inner product between row 1 of \mathbf{A} and column 1 of \mathbf{E} , i.e. $4 \times 5 + 2 \times 0 + 1 \times 1 = 21$.
- The $(2, 1)$ th entry in \mathbf{AE} is found by taking the inner product between row 2 of \mathbf{A} and column 1 of \mathbf{E} , i.e. $6 \times 5 + 7 \times 0 + 3 \times 1 = 33$.

Some properties with matrix multiplication:

- Matrix multiplication is associative, i.e., $\mathbf{F}(\mathbf{GH}) = (\mathbf{FG})\mathbf{H}$.
- Matrix multiplication is distributive, i.e., $(\mathbf{F} + \mathbf{G})\mathbf{H} = \mathbf{FH} + \mathbf{GH}$.
- Matrix multiplication is NOT commutative, i.e., $\mathbf{FG} \neq \mathbf{GH}$. The order matters.

4 Transpose

The transpose of a matrix $\mathbf{A}_{m \times n}$ is a matrix whose rows are the columns of \mathbf{A} . The transpose is written as \mathbf{A}' . The (j, i) th entry in \mathbf{A}' is the (i, j) th entry in \mathbf{A} . Using the same matrix \mathbf{A} defined earlier,

$$\mathbf{A}' = \begin{pmatrix} 4 & 6 \\ 2 & 7 \\ 1 & 3 \end{pmatrix}$$

A few properties with the transpose:

- $(\mathbf{A}')' = \mathbf{A}$.

- $(A + B)' = A' + B'$.
- $(AE)' = E' A'$.
- $(FGH)' = H' G' F'$.

5 Inverse

Some additional terminology:

- A matrix \mathbf{K} is square if the dimensions of its column and row are the same.
- The identity matrix \mathbf{I} is a square matrix where the diagonal entries are equal to 1, and the other entries are all equal to 0.

The inverse of \mathbf{K} , if it exists, is the matrix \mathbf{K}^{-1} such that $\mathbf{K}\mathbf{K}^{-1} = \mathbf{I}$. We say that \mathbf{K} is invertible if its inverse exists. A few properties with the inverse:

- $(AB)^{-1} = B^{-1}A^{-1}$.
- $(A^{-1})^{-1} = A$.
- $(A')^{-1} = (A^{-1})'$.